STRONG (f) GRAVITY, DIRAC'S LARGE NUMBERS HYPOTHESIS AND THE EARLY HADRON ERA OF THE BIG-BANG UNIVERSE

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Received on June 2, 1975

ABSTRACT

The theory of f-*gravity metric* ($f_{\mu\nu}$) *and the usual Einstein metric* ($g_{\mu\nu}$) *developed by the present authors is shown to be connected with Dirac's recent suggestion of two metrics,* i.e., *an atomic metric in addition to the space-time metric. The large value of the* f-*gravity coupling constant* $G_f \sim 10^{39}\ G_N$ (G_N *being the Newtonian gravitational constant) provides the physical basis for Dirac's large numbers hypothesis. The cosmological consequences of the above theory for the early universe are discussed. The fine-structure constant and the strong interaction coupling constant appear as natural consequences of the initial conditions of the universe.*

Key words: *f*-Gravity; Large numbers hypothesis; Early hadron era.

1 INTRODUCTION

Dirac has recently revived [1, 2] interest in the large numbers hypothesis first proposed by him several years ago [3]. This hypothesis attaches a deep significance to certain remarkable numerical coincidences that arise while constructing some large dimensionless numbers ($\sim 10^{39}$) from atomic constants and cosmological parameters. For instance, the Hubble age of the universe expressed in terms of a fundamental atomic unit of time (say the interval required for a light signal to cross a characteristic elementary particle dimension ($\sim 10^{-13}$ cm) or the typical time for strong interactions, ($\sim 10^{-23}$ secs) is a large dimensionless number $\sim 10^{39}$. The ratio of electrostatic to gravitational interaction between an electron and a proton or alternately the ratio of the strong interaction between two proton to their gravitational interaction is also about 10^{39}. Furthermore, astronomical data imply that the total number of particles in the observable universe is around 10^{78}, which is the square of the number 10^{39}. Dirac's hypothesis is that such coincidences are not accidental but that all these large quantities are connected, indicating a profound relationship underlying subatomic and cosmic phenomena. Now the first of these numbers involves the age of the universe which obviously increases with time as the universe evolves, implying that this number also increases with time (t). Consequently, by the Dirac hypothesis the second number expressing the ratio of the forces between particles must also increase proportionately with time. It follows that in order to satisfy the large numbers hypothesis, G, the gravitational constant, occurring in the denominator of the second number must decrease with time. In other words, Dirac argues that G must decrease as t^{-1} in order to preserve the supposedly inbuilt equivalence between these numbers. This leads to the results that the gravitational forcesweaken with time in direct proportion to the Hubble age. It then follows that G must have had a rather large value during the early history of the universe. Now Einstein's general relativity, which is a most elegant and successful theory of gravitation, does not imply a time variation of G. Jordan was the first to develop a theory incorporating Dirac's hypothesis of $G \propto t^{-1}$ by introducing into Einstein's theory a neutral scalar field κ with zero rest mass. He substituted the Einstein variational principle, *i.e.*,

$$\delta \int (R + \kappa \propto) \sqrt{-g}\; d^4 x = 0 \tag{1}$$

by another more general expression

$$\delta \int \kappa^{\eta} \left[R + \kappa (\propto + \propto_{\kappa})\right] \sqrt{-g}\; d^4 x = 0, \tag{2}$$

where R is the contracted tensor of space-time curvature. $R = g^{\mu\nu} R_{\mu\nu}$, $\kappa = 8\pi G/c4$, $\propto$ is the lagrangian density of matter, $\propto_{\kappa}$ is the lagrangian density of the scalar field and η is a dimensionless number of order unity. The Jordan variational principle virtually coincides with the Brans-Dicke theory if one assumes the latter's scalar field ϕ to be equal to κ^{-1}. On the basis of Jordan's field equations, Brill obtained for the case of a homogeneous isotropic universe of radius R the solution:

$$\left.\begin{array}{l} \kappa = \kappa_0\, t^{-1/\eta} \\ \text{or} \\ \kappa = \kappa_0\, t^{-1}\ (\text{for } \eta = +1) \end{array}\right\} \tag{3}$$

Thus the Jordan and Brans-Dicke theories provide a mathematical substantiation of Dirac's cosmological dependence, $G \propto t^{-1}$. As we stated before this dependence implies that G must have had a rather large value during the early history of the universe. The discovery of the isotropic thermal microwave 3°K background radiation and its uusal interpretation as the primeval black body radiation predicted by big bang models constitutes compelling evidence for a hot dense early phase in the evolution of the universe. It is customary to divide the early history of the Universe into different epochs or eras, *i.e.*, the hadron era, the lepton era, etc., each epoch called after the particle species that dominates the energy balance. In what follows, we shall be primarily concerned with the earliest era, *i.e.*, the hadron era, when the universe consisted predominantly of an extremely hot compact gas of hadrons.

2. *f*-Gravity and Hadrons

The hadron era is usually assumed [4] to have started at the epoch $t \approx 10^{-23}$ secs., which shows that if G varied according to the large numbers hypothesis right down to this epoch it would have had a very large value of $G \sim 10^{39}$ times G_N (where G_N is the present day value of the Newtonian constant) at the beginning of the hadron era. We note that this large value for G at the start of the hadron era coincides with the value G_f found by us earlier [5] for the short-range strong *f*-gravity field mediated by massive spin-2^+ *f*-mesons. The existence of a nonet of spin-2 mesons in particular the f^0 meson having quantum numbers the same as the graviton, strengthens the belief that the gravitational interaction of hadrons at the quantum level proceeds *via* the interconversion of the spin-2 mesons to Einstein's gravitons analogous to the two-stage picture for hadronic electromagnetic interactions where it is well known that the ρ° meson (which forms part of the massive spin-1 nonet) plays the role of a massive photon [6, 7]. In a recent paper [8], while examining the possible role of *f*-gravity in averting gravitational singularities, we had pointed out that as matter in the early history of the universe was in a superdense state predominantly composed of hadrons it is reasonable to consider the gravitational interactions between the hadrons at short range to be due to exchange of massive *f*-mesons for which we must necessarily use the corresponding constant G_f. Now we see that the Dirac hypothesis does lead to a value of G as high as G_f during the early hadron era. Again we had shown [9] that perhaps the most natural way to include the short-range of *f*-gravity into Einstein's field equations was to reinterpret the so-called 'cosmological' constant in terms of the inverse Compton length of the *f*-meson as: (m_f being the *f*-meson mass):

$$\Lambda_f \simeq \left(\frac{m_f c}{\hbar}\right)^2. \tag{4}$$

This provides a de Sitter-type metric [7, 9] for the space (intensely curved) within a hadron. This metric is a solution of Einstein's equation with a cosmological term, for the *f*-meson field $f_{\mu\nu}$; *i.e.*, of a field equations $G_{\mu\nu} + \Lambda_f f_{\mu\nu} = 0$. These equations can be interpreted either as vacuum field equations with a cosmological constant or as field equations with an energy density term [10]:

$$\rho \sim \frac{\Lambda_f c^2}{8\pi G_f}, \tag{5}$$

with Λ_f being given as in equation (4).

Using for ρ in equation (5), the density of hadronic matter [8], *i.e.*, $\rho \sim 10^{17}$ g.cm^{-3}, gives a value of $G_f \sim 10^{39}\ G_N$. This high value for G within a hadron finds justification in other approaches also [11, 12]; for instance, the quantization condition on the gravitational charge obtained by Motz [11], *i.e.*, $Gm^2 = \hbar c$, implies a high value for G within an elementary particle of mass m. Also the present authors have demonstrated in recent work [5, 9, 13], that if we invoke this short-range strong f-gravity, it appears that general relativity may play a crucial role in determining the masses of elementary particles. In fact reasonable values were obtained for the hadron masses [5, 13]. The f-gravity model [5, 9] shows that for a typical hadron of the mass of a proton (m_p) we have a relation:

$$\frac{G_f m_p}{rc^2} \sim 1, \tag{6}$$

with r corresponding to a particle size $\sim 10^{-14}$ cm. $\approx \frac{1}{\sqrt{\Lambda_f}}$, the range of of f-gravity. The quantization condition of Motz is also seen to imply such a relation which is tantamount to the vanishing of g_{00} at the Compton length of the proton consistent with reference [9]. In fact, we had recently obtained [13, 14] relation (6) as well as the Motz quantization condition by using quantized values for the angular momentum occurring in the Kerr metric. Equation (6) indicates that we could picture hadrons as fluctuations in the f-gravity metric of order unity which occurred in the early hadron era (when $G \approx G_f$ by the Dirac hypothesis) and that these fluctuations may have been frozen in the form of quantum black holes [13, 14]. This is similar to recent conjectures that several small black holes might have been formed in the early universe as a result of density fluctuations which in turn caused fossilised fluctuations in the metric of order unity [15]. The only difference is that here the use of the large constant G_f makes these black holes the size of hadrons.

3. f-GRAVITY AND DIRAC'S ATOMIC METRIC

An important feature of Dirac's recent work is the proposal that there are really two space-time metrics, one of which is the so called atomic metric determined by atomic (or nuclear) measurements and which is not the same as the usual metric occurring in Einstein's field equations. In introducing the atomic metric Dirac was interested in reviving Weyl's generalisation of

Riemannian geometry in order to unify gravitation and electromagnetism. Now in Riemannian geometry a vector A_k when parallel transported around a contour which encloses a small two-dimensional surface $\triangle S^{lm}$ has its direction changed according to: $\triangle A_k = \frac{1}{2} R^i_{klm} A_i \triangle S^{lm}$, where R^i_{klm} is the Riemann curvature tensor. In Wey'ls geometry, not only does the direction change but also the length of the vector changes when it is parallel transported. The change in length δl will be proportional to l and to the co-ordinate change δx^μ and will be of the form: $\delta l = l\kappa_\mu \delta x^\mu$. The coefficients κ_μ together with the metric $g_{\mu\nu}$ determine the space time. It is natural to identify the four κ_μ with the electromagnetic potentials and since the $g_{\mu\nu}$ already describe the gravitational field, we have a unified geometrical framework for electromagnetism and gravitation. However, in spite of its elegance, the Weyl theory had to be abandoned soon becuase it implies changes in fundamental atomic length standards (for example the Bohr radius) at different points in space-time contrary to observations. Thus considerations of quantum phenomena with their fundamental length and time units ruled out Weyl's theory. We have seen that the gravitational constant when measured in atomic units varies with the age of the universe. To account for this variation Dirac modifies Einstein's theory by supposing that the element of distance ds in the Einstein theory given by $ds^2 = g_{\mu\nu}\, dx^\mu\, dx^\nu$ is not the same as the element of distance measured by atomic clocks. The element of distance in Einstein's theory is ds_E and the element of distance measured by atomic clocks is something different, *i.e.*, ds_A. The ratio of these two is something which depeds on the epoch, the time measured from the origin of creation. With these two ds's, we shall have the gravitational constant varying with time. The objection to Weyl's theory vanishes, since Weyl's ideas are to be applied to ds_E. Atomic clocks measure ds_A which remains invariant when we take it around a closed loop (as atomic standards are universal). With the introduction of these two metrics, Dirac is in fact led to a scalar-tensor theory which incorporates a varying G. In f-gravity while considering the mixing of f-mesons with Einstein's massless gravitons, we have two metrics [16] $f_{\mu\nu}$ for the f-meson field and $g_{\mu\nu}$ for the usual graviton field. The coupling constant for $f_{\mu\nu}$ is of the order of the strong interactions (*i.e.*, G_f) and would be measured by atomic apparatus. It would, therefore, appear very tempting to relate these two metrics to those of Dirac. In fact the atomic metric of Dirac determined by measurements of the effects of say strong interactions could be identified with the f-gravity metric $f_{\mu\nu}$. The large coupling constant was of course frozen within the particles (which as mentioned above could be pictured as fossilised fluctuations in the f-gravity metric of order unity as implied by equation (6) during the early hadron era when the coupling constants for Newtonian gravity and f-gravity were identical. We thus see why ds_A remains invariant and that the ratio of the two metrics given by (G_f/G) varies with the epoch, *i.e.*, G_f remains constant (frozen within the particles) and G varies with time when expressed in units of G_f which is an atomic unit given by $G_f \approx \hbar c/m^2$. At the beginning of the hadron era $G_f \approx G$ and the two metrics coincided. This pattern fits nicely with Dirac's two metric hypothesis. Moreover we have shown [17] that consideration of f-gravity mediated by f-mesons together with ordinary gravitation does lead to a scalar-tensor theory. Elsewhere, we have also pointed out a possible connection between strong interactions and strong f-gravity [7].

4. f-Gravity and Mach's Principle

Another attractive feature of this line of thinking is the automatic explanation for the total number of particles in the universe being of the order of 10^{78}. To show this, we first note that a number of considerations including that of Mach's principle show that for the universe as a whole with a mass M_U and Hubble radius R_H, we should have a relation [18, 19]:

$$\frac{G_N M_U}{R_H c^2} \sim 1. \tag{7}$$

Mach's principle asserts that all dynamical phenomena must be caused by matter and not by intrinsic properties of empty space. This could be interpreted to mean that the inertial mass m of any object or particle is equal to the mass $(m\psi/c^2)$ which it acquires from the gravitational potential $\psi = G_N M_U/R_H$ generated by the rest of the mass in the universe, therefore giving rise to equation (7) which indicates a geometrically closed universe. This is also consistent with two recent speculations about the overall structure of the universe. One of these, due to Pathria [20], notes that for the universe as a whole the Schwarzschild radius is of the same order as the Hubble radius, *i.e.*, 10^{28} cm., implying that the universe may not only be a closed structure (as perceived by its inhabitants at the present epoch) but may also be a black hole which would again yield equation (7). The other recent speculation due to Tryon [21] proposes that the universe is a gigantic fluctuation of the vacuum in the sense of quantum field theory with a net energy of zero, *i.e.*, its total negative gravitational energy equals the total positive energy of the matter it contains which is precisely what equation (7) states ! If N be the total number of protons in the universe, then examining the equations (6) and (7), we have:

$$N = \left(\frac{G_f}{G_N}\right) \left(\frac{R_H}{r}\right) = \left(\frac{G_f}{G_N}\right) (R_H \sqrt{\Lambda_f}). \tag{8}$$

As

$$G_f \approx 10^{39} \times G_N \text{ and } R_H \sqrt{\Lambda_f} \approx 10^{39},$$

we have:

$$N \simeq (10^{39})^2 = 10^{78}. \tag{9}$$

We come to the remarkable conclusion that the total numbr of hadrons in the universe is determined by the fact that the hadron and the universe are both built up on the same geometric pattern, *i.e.*,

$$\frac{G_f\, m_p}{rc^2} = \frac{G_N\, M_U}{R_H\, c^2} = \frac{G_N \cdot Nm_p}{R_H\, c^2}, \tag{10}$$

as implied by Equations (6) and (7). The large value of G_f makes the hadron mass and size correspondingly small. Equation (10) can be interpreted in many different ways: If $N = 1$, *i.e.*, if there were only one particle in the universe, $G_N = G_f$ and $R_H =$ ($r \sim 10^{-14}$ cm.), *i.e.*, each hadron is a miniature universe. So we see that G_f may have its large value within the particle simply because there are so many particles in the universe, whose net interaction reduces the local inertial mass because of the large binding energy of the interaction. All this is beautifully consistent with Mach's principle in the spirit of Dicke [22]. Again one can write the analogue of equation (2) for the universe, if we replace G_f by G_N and Λ_f by the overall curvature of the universe, given by $\Lambda \sim (1/R_H{}^2)$. This gives for the universe:

$$\rho_U \sim \frac{c^2}{8\pi\, G_M\, R_H{}^2}. \tag{11}$$

Comparing ρ_U with the hadron density in Equation (2), again gives the total number of hadrons in the universe. If we write Einstein's equations with a Λ-term for the universe as $G_{\mu\nu} + \Lambda g_{\mu\nu} = 0$, analogous to that for the f-meson field $G_{\mu\nu} + \Lambda_f\, f_{\mu\nu} = 0$, $\Lambda_f \sim (m_f c/\hbar)^2$ and interpret the Λ-term in both cases as corresponding to the energy density of the vacuum [10], then in one case we have a vacuum fluctuation of the scale of the universe, $\Lambda \sim 1/R_H^2$, in the spirit of Tyron of density given by Equation (11) and in the other case a vacuum fluctuation of the scale of the elementary particle with a density, given by equation (5), equal to the hadronic density. This once again places both the hadron and the universe on the same footing.

5. f-Gravity and the Early Hadron Era

We shall now briefly investigate how f-gravity and Dirac's hypothesis affect quantum gravitational considerations in the early universe. As we remarked in an earlier paper [8] by using G_f instead of G, the absurdly large density $c^5/G^2\hbar \sim 10^{94}$ g·cm.$^{-3}$ at which quantum effects of the gravitational field are expected to become important is scaled down to $\simeq 10^{17}$ g.cm^{-3}. Harrison [23] has shown that quantum fluctuations of the metric at such high densities set a limit to classical cosmology at an epoch:

$$t^* = \left(\frac{G\hbar}{c^5}\right)^{\frac{1}{2}} \approx 10^{-44}\,\text{sec.} \tag{12}$$

This indicates an upper limit for the temperature

$$T_M = 10^{10}\left(\frac{c^5}{G\hbar}\right)^{\frac{1}{4}} = \frac{1}{K_B}\left(\frac{\hbar c^5}{2G}\right)^{\frac{1}{2}} \sim 10^{32}\,{}^\circ\text{K}. \tag{13}$$

at this epoch, again an absurd value. But with f-gravity and also with Dirac's hypothesis when $G \approx G_f$ at the epoch $t^* \sim 10^{-23}$ secs. this upper limit on the temperature becomes of the order of $T_M \sim 10^{13}\,{}^\circ$K. This is a limit set purely by quantum gravitational (pertaining to f-gravity) considerations. We shall now point out how this value for the maximum temperature ties up neatly with the upper limit in the temperature predicted by the Hagedorn model [24] for hadrons with an exponential hadron mass spectrum. This model, based on experimental data on multitudes of hardon resonances, replaces the limited spectrum of elementary particles usually assumed for the hadron era by a rich hadron-level density increasing with mass exponentially, *i.e.*,

$$\rho(m) \sim Cm^a\, e^{bm} \tag{14}$$

Level densities of this form are suggested among others by the statistical bootstrap model and the Veneziano model for elementary particles [25]. It is also in accord with the presently known mass spectrum. Hagedorn observes that in very high-energy laboratory collisions the transverse momenta of particles emerging from a collision obey nearly a Boltzmann distribution in the centre of mass frame corresponding to a temperature T_0 which is almost independent of the primary energies. He therefore suggests that in the interaction region during the collision time particles are created with thermal energies corresponding to temperatures $\leq T_0$. An increase in primary energy results in an increase in the number and masses of newly created particles but not in their kinetic energies. Secondly, he observes that excited states of hadrons appear in no way different from new particles unless they have very large angular momenta. He therefore assumes that asymptotically for large energies the (rest) mass spectrum of elementary particles approaches the energy spectrum upto a polynomical factor. With this, he arrives at an exponential density $\rho(m) \sim m^a \exp.(-m/T_0)$, where

T_0 is a maximum limiting temperature. Several phenomenological and theoretical arguments further suggest that the limiting temperature is of order $T_M \sim 10^{13}$ °K. Experimental data would suggest that the early hadron era must have consisted of a rich exponential mass spectrum as suggested by Hagedorn and consequently this would have placed an upper limiting temperature $\sim 10^{13}$ °K. during that era. It is remarkable how this temperature is of the same order as that deduced above. Thus incorporation of f-gravity (or alternately the extension of Dirac's hypothesis to the early hadron era) puts the same upper limits on density and temperature on a compact gas of hadrons as the various bootstrap models, in particular, the Hagedorn model. Now it has been conjectured [26] that in the free particle (quark) model with a limited spectrum as distinct from Hagedorn's exponential spectrum, the limiting temperature is 10^{32} °K rather than 10^{13} °K and hence at energies corresponding to this temperature the gravitational interactions are strong enough to maintain thermodynamic equilibrium between hardons *via* exchange of gravitons (for instance the cross section for annihilation into gravitons goes as G^2m^2 and for photoproduction of gravitons as G^2 me) and once the universe expands, these gravitons decouple and form a graviton background of temperature 1 °K. Thus this would lead us to expect a thermal graviton background in free quark models but not in the case of the Hagedorn models [24]. However, incorporation of f-gravity or Dirac's hypothesis leads to large values of G in the early hadron era, and this would imply that gravitational interactions would be as strong as strong interactions at the epoch $t \sim 10^{-23}$ secs., and one would get a graviton background (in the present era) even in the Hagedorn model with its limiting temperature, since the characteristic 'quantum gravitational temperature' for f gravity, $1/K_B \; (\hbar C^5/2G_f)^{\frac{1}{2}}$, at which gravitational interactions are strong turns out to be the same as the Hagedorn limiting temperature. Thus observation of a thermal graviton background need not necessarily distinguish between elementary particle models as suggested by Weinberg [26]. Again one notes a remarkable relationship for the typical mass of a hadron, *i.e.*, $m \sim (\hbar^2 H/Gc)^{1/3}$, H being the Hubble constant. This relation can be cast in the form: $Gm^2/\hbar c = \hbar H/mc^2$, which shows that at the epoch $1/H \sim \hbar/mc^2$ $\sim 10^{-23}$ secs. the gravitational fine structure constant in $Gm^2/\hbar c \sim 1$ the same as the strength of the strong interactions, justifying what we said above.

6. The Fine-structure Constant and the Early Universe

We shall now consider a possible connection between the fine-structure constant and the enetropy density of the primeval black-body radiation. For a Robertson-Walker line element, with

$$T^{\mu\nu} = (\rho + p/c^2)\, U^{\mu}\, U^{\nu} - pg_{\mu\nu},$$

the Einstein field equations are:

$$\frac{2\ddot{R}}{R} - \frac{\dot{R}^2}{R^2} - \frac{8\pi Gp}{c^2} = \frac{Kc^2}{R^2} \tag{15}$$

$$\frac{\dot{R}^2}{R^2} - \frac{8\pi G\rho}{3} = \frac{Kc^2}{R^2} \tag{16}$$

For a Einstein-de Sitter universe, $K = 0$,

and

$$\frac{\ddot{R}^2}{R^2} = \frac{8\pi G\rho}{3}.$$

For black-body radiation, $\rho = aT^4/c^2$, a is the Stefan-Boltzmann constant

Hence,

$$\frac{\dot{R}}{R} = \sqrt{\frac{8\pi GaT^4}{3c^2}}. \tag{17}$$

For adiabatic expansion of radiation we have by Wien's displacement law:

$$\frac{\dot{R}}{R} \propto -\frac{\dot{T}}{T}, \quad \dot{T} = \frac{dT}{dt}$$

giving

$$-\frac{\dot{T}}{T} = \sqrt{\frac{8\pi Ga}{3c^2}}\, T^2$$

or

$$-\frac{dT}{T^3} = \left(\frac{8\pi Ga}{3c^2}\right)^{1/2} dt. \tag{18}$$

Thus

$$T = \left(\frac{3c^2}{8\pi Ga}\right)^{1/4} \frac{1}{t^{1/2}}. \tag{19}$$

Now at the apoch $t \approx 10^{-23}$ secs,

$$\sim H^{-1}\left(\frac{Gm_p^2}{\hbar c}\right)^{1/2},$$

$$T_0 \propto \frac{1}{(Gm_p^2/\hbar c)^{1/4}}, \quad \frac{1}{H^{-1/2}\left(\dfrac{Gm_p^2}{\hbar c}\right)^{1/2}} \propto \frac{1}{(Gm_p^2/\hbar c)^{3/4}} \tag{20}$$

The entropy density of the radiation is given by:

$$\frac{S_0}{K_B} = \frac{4a\,T_0^3}{3K_B} \propto \frac{1}{(Gm^2{}_p/\hbar c)^{9/4}}\,. \tag{21}$$

This quantity is directly proportional to the number of photons which is therefore proportional to $(Gm_p^2/\hbar c)^{-9/4}$. Now the total number of baryons in the universe is proportional to $(Gm^2{}_p/\hbar c)^{-2} \sim 10^{78}$.

Therefore, number of photons/No. of baryons

$$= \frac{\eta_\gamma}{\eta_B} \approx \left(\frac{\hbar c}{Gm_p^2}\right)^{\frac{1}{4}} \tag{22}$$

i.e., $\eta_\gamma/\eta_B \approx 10^{10}$, in agreement with the value usually quoted for the 3 °K radiation. This number is supposed to be a universal constant for the Einstein-De Sitter universe [26]. Suppose we now impose a cutoff for the photon wavelength in the early universe at the Schwarzschild radius of the electron, giving the relation [27, 28, 5]:

$$(\hbar c/G)^{\frac{1}{2}}\, e^{-0\cdot4/\alpha} = m_e \tag{23}$$

where α is the fine structure constant whose value is to be determined.

Using (23) in (22) gives:

$$e^{0\cdot8/\alpha} = \left(\frac{\eta_\gamma}{\eta_B}\right)^4 \left(\frac{m_p}{m_e}\right)^2,$$

or

$$\frac{0\cdot8}{\alpha} \approx ln\left[\left(\frac{\eta_\gamma}{\eta_B}\right)^4\left(\frac{m_p}{m_e}\right)^2\right] \approx 4\,ln\left(\frac{\eta_\gamma}{\eta_B}\right) + 2\,ln\left(\frac{m_p}{m_e}\right) \tag{24}$$

This gives

$$\frac{0\cdot8}{\alpha} \approx \frac{1}{110}\,,\ \text{or}\ \alpha \approx \frac{1}{137\cdot5} \tag{25}$$

The quantities (η_γ/η_B) and (m_p/m_e) occurring on the right side of equation (24) are both universal constants probably explaining why α is also a universal constant. Also these quantities occur as logarithmic expressions indicating that even if the initial conditions were very different, *i.e.*, say the universe started again with a different photon to baryon ratio, α would not have a value very different from 1/137.

If we use G_f instead of G, then equation (19) yields,

$$T = \left(\frac{3c^2}{32\pi\,G_f\,a}\right)^{\frac{1}{4}} t^{-\frac{1}{2}} \tag{26}$$

giving $T \simeq 10^{13}$ °K at $t = 10^{-23}$ secs, the beginning of the hadron era. This is indeed the Hagedorn limiting temperature. We can now argue that the strong interaction coupling constant should depend on the ratio of virtual vector mesons to baryons, *i.e.*, η_ρ/η_B, (ρ-meson replacing the photon). Equation (22) then gives:

$$\frac{\eta_\rho}{\eta_B} \sim \left(\frac{\hbar c}{G_f\, m}\right)^{\frac{1}{4}} \sim 1\cdot 1, \text{ giving } \frac{1}{g} \approx 0\cdot 7, \text{ or } g \sim 1$$

for the strong interaction constant.

The idea used above, namely, that the fine structure constant and the strong interaction coupling constant depend on the ratio of the number of virtual photons (or mesons) to the number of baryons and fixed by these initial conditions has an analogue in the polaron problem where the interaction strength between electrons due to exchange of phonons is related to the density of the virtual phonon cloud surrounding the electron.

7. CONCLUDING REMARKS

We see that at the very early stages of the universe our understanding of cosmological evolution converges on to some of the deepest and most fundamental problems regarding the structure of elementary particles and their interactions at very high energies. From what we have said above we have a remarkably consistent formalism exhibiting connection between several diverse aspects of particle physics, gravitation and cosmology and in particular, between *f*-gravity, particle models, Dirac's hypothesis and maximum temperature, hydrodynamical bootstrap particle models.

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Note:—A short preliminary report of this work appears in *Physics Letters B*, to be printed.