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Application of network analyzer in measuring the performance functions of power supply

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Abstract

Frequency response analysis is critical in understanding the steady- and transient state behaviour of any electrical network. To study the system behaviour analytical model of the system is required. If the system is well understood it may be easy to develop an analytical model, and hence frequency response can be determined. But practical systems have non-idealities (or complex) which are difficult to simulate. Hence, measuring the frequency response of any black box (electrical network) makes the job practically simpler. Network analyzer or frequency response analyzer is used to determine the frequency response of an electrical network. This paper deals with the application of network analyzer in measuring the performance functions of switched-mode power supplies, which is to validate the analytical model of power supply.

Keywords: Network analyzer, electrical network, push–pull converter.

1. Introduction

Network analyzer or frequency response analyzer is used to determine the frequency response of any electrical network. This paper examines the measurement techniques for measuring the performance functions of power supplies. To determine the performance functions of power supply it is necessary to perturb the various inputs of the power supply [1, 2]. Standard procedures to introduce perturbation in voltage, current and duty cycle are disucssed. Loop-gain measurement technique is covered in detail. Measured and analytical results are presented. The authors have used AP Instruments network analyzer for determining performance functions. The procedures are more generic and can be applied to any frequency-response analyzer.

2. Operation of network analyzer

Figure 1 shows the block diagram of the network analyzer. The source (or oscillator) generates a test frequency, which is injected into the test circuit. At the same frequency the two receivers measure the voltages of excitation signal (V_A) and response signal (V_B) of the circuit. The mathematical processing unit calculates the ratio of response signal (V_B) to the ex

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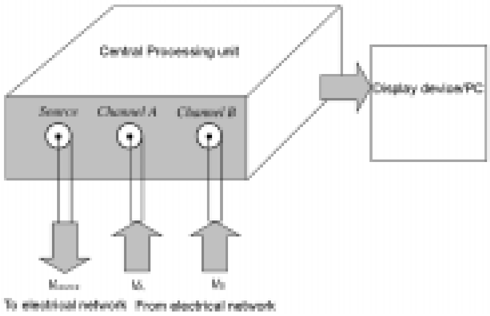

FIG. 1. Block diagram of network analyzer.

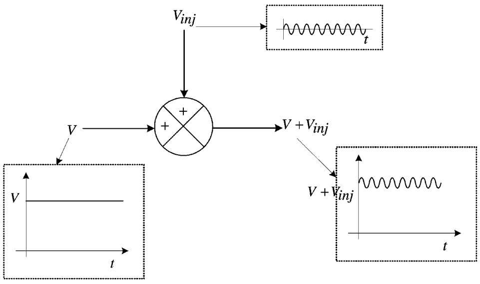

FIG. 2. Block diagram representation of introducing a disturbance in a voltage.

citation signal (V_A). The frequency of the source is varied over the frequency range of concern and the gain is calculated. The display device plots the gain vs frequency.

3. Requirements

A good network analyzer is a very specialized instrument and preferably have the following features:

1. Operating frequency range from < 1 Hz to > 10 MHz.
2. Swept–sine output oscillator with settable magnitude to drive circuit under test.
3. Dual receiver channels to measure input and output of interest at the frequency of the oscillator. The magnitude and phase gains are measured in several formats.
4. High noise rejection with selectable bandwidth of the receiver channels.
5. Direct interface to a computer for post-processing of data.

Some of the commonly available network analyzers are Hewlett Packard frequency response analyzer (Model: HP4194A), AP Instruments (Model: 200), Solartran frequency response analyzer (Model: FRA1250), Powertek frequency response analyzer (Model: GP102) and Venable Industries network analyzer. The work reported in this paper employed the AP Instrument network analyzer (Model: 200). In loop-gain measurement it gives reliable results up to 30 kHz [3].

4. Applications in finding the performance functions of power supplies

The objective of measuring the performance functions is to verify the small-signal analytical models developed for power supply. To determine the performance functions of the power supply it is necessary to perturb the various inputs of the power supply. Some standard procedures are available in the literature to perturb voltage, current and duty cycle [4, 5].

4.1. *Perturbation of voltage and current*

Figure 2 shows the representation of introducing perturbation in the voltage. V represents the voltage to be perturbed and V_{inj} the perturbing voltage (Figs 2 and 3). By adding the perturbing voltage, V_{inj}, it is possible to introduce disturbance in the required voltage (V) (Fig. 2). Figure 3 shows the various circuits to introduce a perturbation in the voltage. The

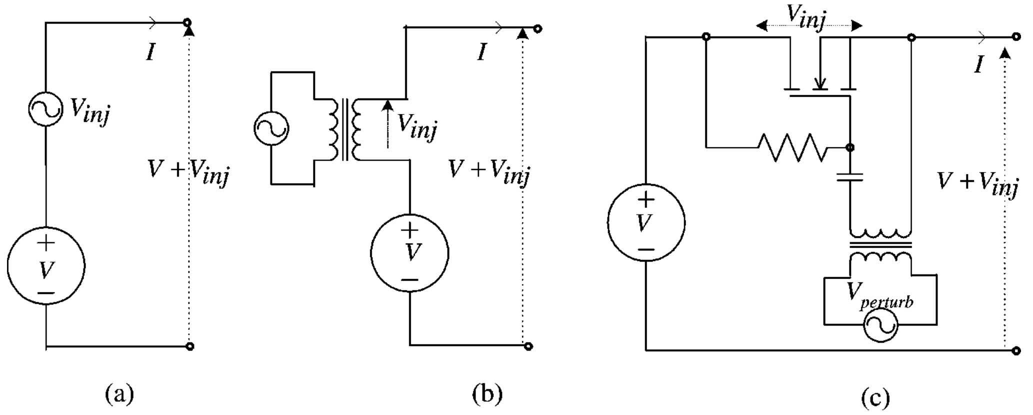

FIG. 3. Circuits used to introduce perturbation in the voltage.

rule to be followed while perturbing any voltage is that the injected voltage needs to be connected in series.

The source of the network analyzer has a 50Ω current limit resistor, and hence the circuit shown in Fig. 3(a) cannot be used for power supplies where the current (*I*) through the voltage source (*V*) is high. Further, the source of the network analyzer is not isolated for the circuit shown in Fig. 3(a).

The network analyzer is isolated through injection isolator in the circuit shown in Fig. 3(b). Usually, current rating of injection isolator is limited. The circuit in Fig. 3(b) cannot be used for the power supplies if the current (*I*) through the voltage source (*V*) is high. Figure 3(c) presents a better circuit to introduce perturbation in the voltage as it can be used even when the current through voltage source (*V*) is high. The MOSFET shown in Fig. 3(c) is operating in the active region, and hence any change in gate-to-source voltage affects drain current and drain-to-source voltage along the load line. This circuit can be used to introduce disturbance in voltage. It is to be noted that the voltage injected has both dc (undesired) and ac (desired) components. Circuits shown in Fig. 4 are used for introducing disturbance in the current and are dual of the circuits shown in Fig. 3.

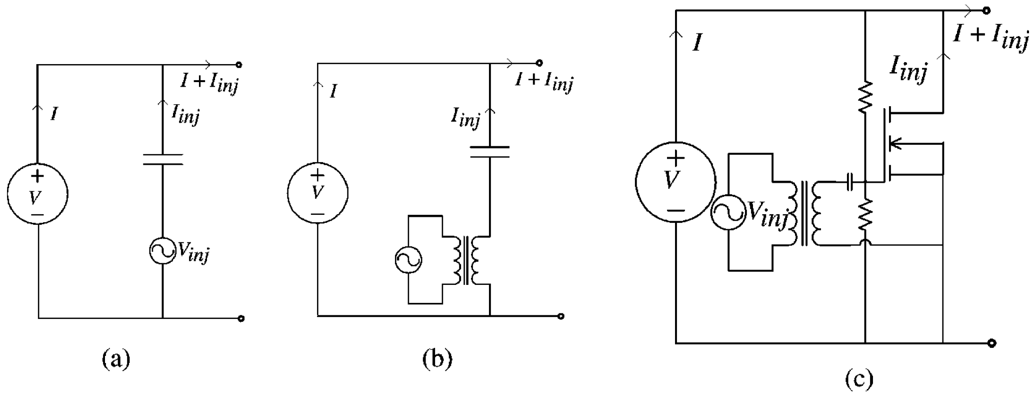

FIG. 4. Circuits used to introduce perturbation in the current.

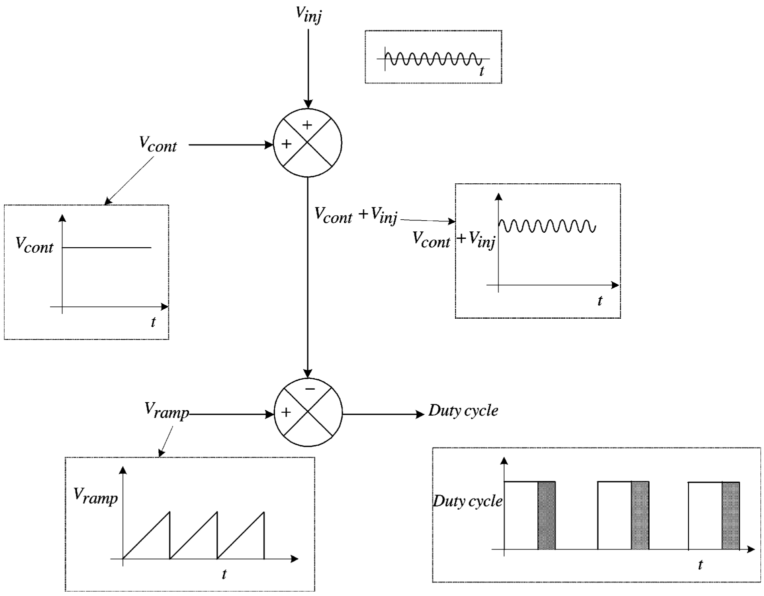

FIG. 5. Block diagram representation of introducing perturbation in the duty cycle.

4.2. *Perturbation in duty cycle*

In PWM converters, the duty cycle is generated by comparing ramp voltage and constant dc voltage; change in duty cycle is obtained by varying the control voltage. To introduce perturbation in the duty cycle, it has to be in control voltage (Fig. 5). V_{ramp} represents the ramp voltage and V_{cont}, the control voltage. The circuit employed is similar to voltage injection as in Fig. 3(b).

5. Small-signal analysis of buck converter

This section deals with the technique to measure the performance functions of buck converter. The details of the buck converter are: Input voltage, $V_{\text{DC}} = 12$ V, Output voltage, $V_0 = 5$ V, Output filter inductor, $L = 300\ \mu$H; Output filter capacitor, $C = 100\ \mu$F; Switching frequency (fs) = 100 kHz.

5.1. *Measurement of audio susceptibility*

Audio susceptibility of the converter quantifies the amount of input voltage variations that reach the output voltage. It is a function of frequency and is defined as,

$$f = \frac{\hat{V}_o}{\hat{V}_g}\ \hat{d}(s) = 0; \quad \hat{i}_z(s) = 0. \tag{1}$$

To measure audio susceptibility, perturbation is introduced in the input voltage of the power supply and its effect is observed in the output voltage. Figure 6 shows the voltage injection point and measurement points for measuring audio susceptibility of buck converter.

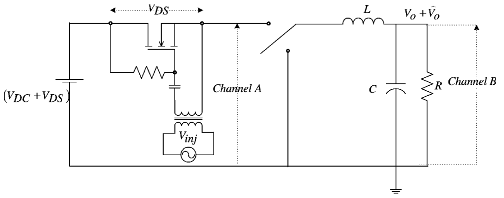

FIG. 6. Buck converter audio susceptibility measurement setup.(In the above plot and in the subsequent plots, analytical results are shown through dotted line and experimental results through continuous line).

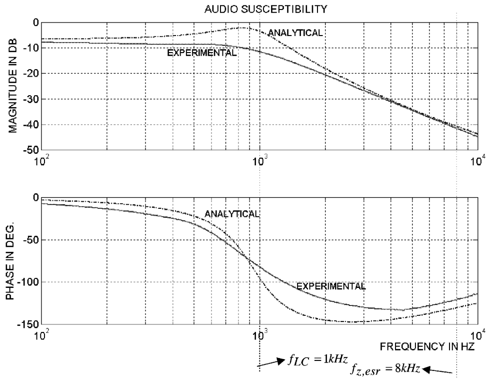

FIG. 7. Estimated and measurement results of buck converter audio susceptibility.

In buck converter, the audio susceptibility is of the form,

$$\frac{\hat{V}_0(s)}{\hat{V}_g(s)}=\left[\frac{DR}{R+R_L}\right]\frac{1+sR_cC}{1+s\left(R_cC+(R\parallel R_L)C+\left(\frac{L}{(R+R_L)}\right)\right)+s^2LC\left(\frac{R+R_c}{R+R_L}\right)}. \tag{2}$$

In eqn (2), non-idealities like ESR of the capacitor, source resistance and resistance of the inductor are included.

Figure 7 shows the experimental and estimated characteristics of audio susceptibility. The second-order pole is about 1 kHz because of the LC filter. The ESR of the filter capacitor is 0.2 Ω. It introduces a zero in the audio susceptibility transfer function.

5.2. *Measurement of input admittance*

Input admittance of the converter relates how the converter interfaces with the source. Input admittance is defined as,

$$Z_{in}=\frac{\hat{i}_g(s)}{\hat{v}_g(s)}\qquad \hat{d}(s)=0;\hat{i}(s)=0. \tag{3}$$

To measure the input admittance, disturbance is introduced in the input voltage and its effect is seen in the input current. Input current is measured through Hall effect current sensor. Figure 8 shows the buck converter input admittance signal injection and measurement points.

In buck converter the input admittance is of the form,

$$\frac{\hat{I}_g(s)}{\hat{V}_g(s)}=\left[\frac{R+R_L}{D^2}\right]\frac{1+s\left(R_cC+(R\parallel R_L)C+\left(\frac{L}{(R+R_L)}\right)\right)+s^2LC\left(\frac{R+R_c}{R+R_L}\right)}{1+s(R+R_c)}. \tag{4}$$

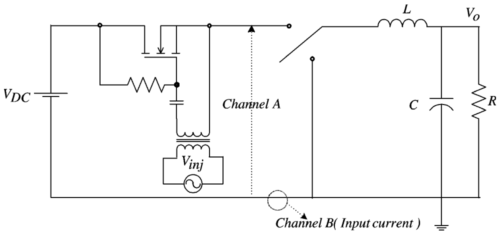

FIG. 8. Buck converter input admittance measurement set-up.

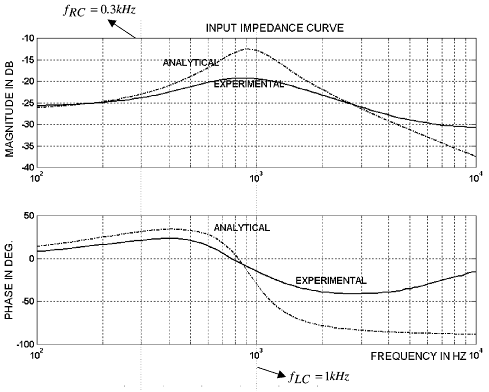

FIG. 9. Estimated and measurement results of buck converter input admittance.

The various non-idealities of the converter are in eqn (4). Figure 9 shows the experimental and estimated characteristics of input admittance. Experimental result validates the analytical model especially the zero (0.3 kHz) and the second order pole (1 kHz). From the experimental result it is clear that the injection circuit introduces some undesirable zero at higher frequencies.

5.3. *Measurement of output impedance*

The output impedance relates the capacity of power supply to cater to dynamic loads. Output impedance is defined as,

$$Z_o = \frac{\hat{V}_o(s)}{\hat{i}_z(s)} \quad \hat{d}(s) = 0; \hat{v}_g(s) = 0. \tag{5}$$

To measure the output impedance a disturbance is introduced in the output load current and its effect is observed in the output voltage. Buck converter output impedance is of the form,

$$\frac{\hat{V}_o(s)}{\hat{I}_z(s)} = [R \| R_c] \frac{\left(\frac{1}{R_c C} + s\right)\left(s + \frac{R_L}{L}\right)}{1 + s\left(R_c C + (R \| R_L)C + \left(\frac{L}{(R+R_L)}\right)\right) + s^2 LC\left(\frac{R+R_c}{R+R_L}\right)}. \tag{6}$$

Figure 10 shows the output impedance signal injection and measurement points for the buck converter. A 1 Ω resistor is used to measure the injected current. The experimental result deviates little from the analytical result due to the non-idealities of the injection circuit (Fig. 11).

5.4. *Measurement of control voltage gain*

The control voltage gain transfer function relates the output voltage and the control duty ratio. Control voltage gain is defined as,

$$G_v = \frac{\hat{V}_o(s)}{\hat{d}(s)} \quad \hat{v}_g(s) = 0; i_z(s) = 0. \tag{7}$$

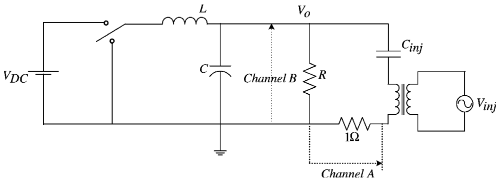

FIG. 10. Buck converter output impedance measurement set-up.

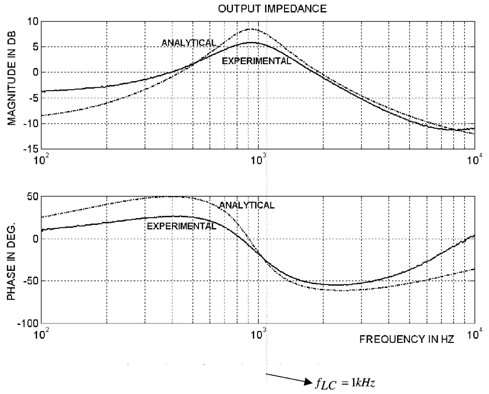

FIG. 11. Estimated and measurement results of buck converter output impedance.

To measure the control voltage gain, perturbation is introduced in the duty cycle and its effect is seen in the output voltage.

The control voltage gain of the buck converter is of the form,

$$\frac{\hat{V}_o(s)}{\hat{d}(s)} = \frac{V_o}{d} \frac{(1+sR_cC)}{\left(1+s\left(R_cC+(R\,||\,R_L)C+\left(\frac{L}{(R+R_L)}\right)\right)+s^2LC\left(\frac{R+R_c}{R+R_L}\right)\right)}. \tag{8}$$

In PWM controller, the control voltage is compared with ramp voltage to produce duty cycle. As discussed earlier, to perturb the duty cycle, disturbance is introduced in the control voltage.

The following equations outline the control voltage gain measurement.

$$\text{Control voltage gain} = \frac{\hat{V}_o(s)}{\hat{d}(s)}; \tag{9}$$

$$\text{i.e. control voltage gain} = \frac{\hat{V}_o(s)}{\hat{V}_c(s)} \times \frac{\hat{V}_c(s)}{\hat{d}(s)}. \tag{10}$$

In eqn (10), $\hat{V}_o(s)/\hat{V}_c(s)$ is the transfer function of output voltage to control voltage and $\hat{V}_C(s)/\hat{d}(s)$, the PWM gain.

Step 1. Measurement of output voltage to control voltage ratio $\left(\frac{\hat{V}_o(s)}{\hat{V}_c(s)}\right)$.

The transfer function between the output and control voltages is measured in Step 1. Figure 12 shows the voltage injection point and the measurement points for Step 1.

Step 2. PWM gain measurement

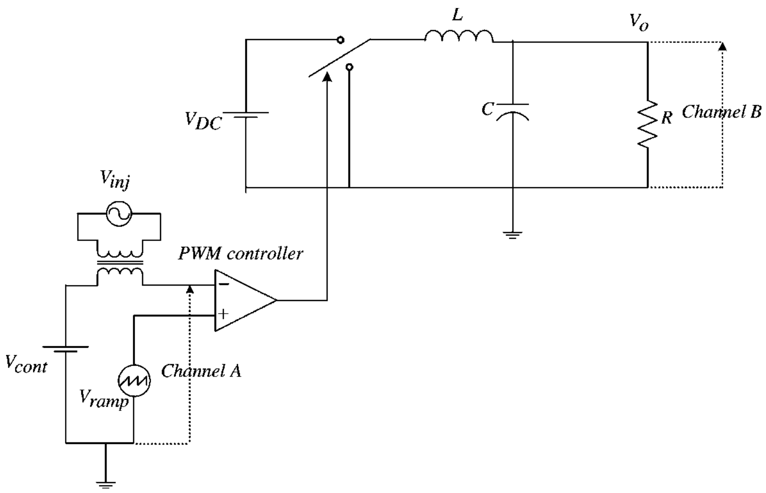

FIG. 12. Buck converter output voltage to control voltage transfer function measurement set-up.

PWM controller gain is calculated in Step 2. Equation (11) is used to calculate the PWM gain.

$$\frac{V_c}{d} = \frac{V_L - V_H}{D_L - D_H}. \tag{11}$$

In eqn (11), D_L and D_H are the duty cycles, and the corresponding control voltages are V_L and V_H. Figure 13 shows the variation of the duty cycle with the control voltage, and Fig. 14, the experimental and estimated results of control voltage gain characteristics. The output LC filter introduces a complex conjugate pole pair. The non-ideality of the output capacitor (ESR) introduces a zero in the transfer function. The frequency response (Fig. 14) clearly indicates this.

5.5. *Measurement of control current gain*

The control current gain transfer function relates the control duty ratio and the input current. Control current gain is defined as,

$$G_i = \frac{\hat{i}_g(s)}{\hat{d}(s)} \quad \hat{v}_g(s) = 0; i_z(s) = 0. \tag{12}$$

To measure the control voltage gain, perturbation is introduced in the duty cycle and its effect is seen in the input current.

Control current gain transfer function is of the form,

$$\frac{\hat{I}_g(s)}{\hat{d}(s)} = \frac{DV_g}{R} + \frac{DV_g(1+sCR)}{R\left(1+s\left(R_cC+(R \parallel R_L)C+\left(\frac{L}{(R+R_L)}\right)\right)+s^2LC\left(\frac{R+R_c}{R+R_L}\right)\right)}. \tag{13}$$

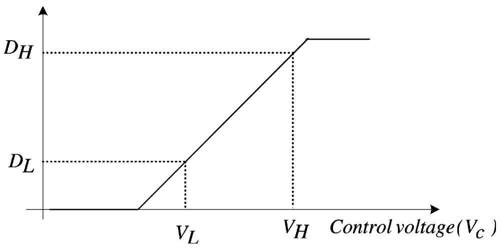

FIG. 13. Variation of duty cycle with control voltage.

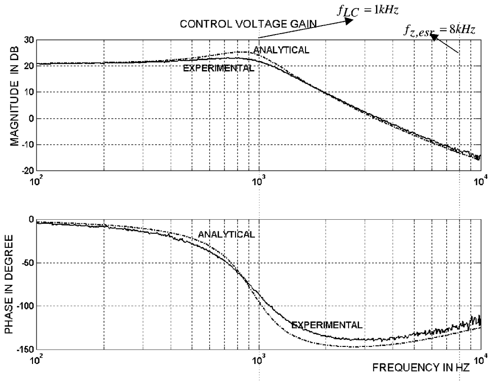

FIG. 14. Estimated and measurement results of buck converter control voltage gain.

The following equations deals with the control voltage gain measurement.

$$\text{Control current gain} = \frac{\hat{I}_g(s)}{\hat{d}(s)} \tag{14}$$

$$\text{Control current gain} = \frac{\hat{I}_g(s)}{\hat{V}_c(s)} \times \frac{\hat{V}_c(s)}{\hat{d}(s)}. \tag{15}$$

In eqn (15), $\hat{I}_g(s)/\hat{V}_c(s)$ is the transfer function of input current to control voltage and $\hat{V}_C(s)/\hat{d}(s)$, the PWM gain.

Like control voltage gain transfer function, control current gain transfer function is also measured in two steps. In Step 1, the transfer function between input current and control voltage is measured. In Step 2, the PWM gain is calculated.

Step 1. Measurement of output voltage to control current ratio $\left(\frac{\hat{I}_g(s)}{\hat{V}_c(s)}\right)$.

The transfer function between the input current and control voltage is measured in Step 1. Figure 15 shows the voltage injection point and measurement point for Step 1. The input current is measured using a Hall effect current sensor.

Step 2. PWM gain measurement is measured as explained earlier.

Figure 16 shows the estimated and experimental results of control current transfer function.

6. Loop-gain measurement technique

Every control loop should be properly designed and validated before shipping power supply. This section outlines the loop-gain measurement technique of the power supply. To measure loop gain of the power supply, the perturbation signal is injected into the loop and allowed to travel throughout the loop.

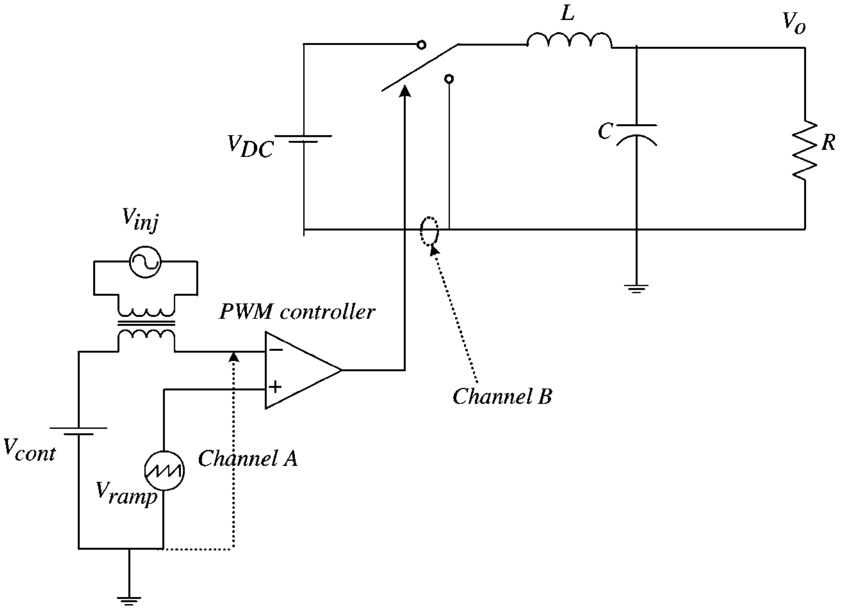

FIG. 15. Buck converter input current to control voltage transfer function measurement set-up.

Figure 17 shows a typical block diagram of isolated power supply. The perturbation signal (V_{inj}) is injected through the injection isolator and the measurement points are shown in the figure.

Without changing the voltage injection it is possible to measure various parameters. Table I shows the voltage measurement points for channels A and B.

Figure 18 shows the measured and estimated loop gain of a push-pull converter. The frequency response analyzer used for measuring the loop gain gives reliable measurement up to 30 kHz. The experimental and estimated results almost match in the region of our interest.

For successful loop-gain measurement, the following points may be kept in mind.

(1) The use of differential probes should be avoided. In the case of isolated power supplies, if possible, short the ground of secondary, primary and control circuitry.

(2) While measuring for switching power supplies, the use of lowest bandwidth results in best noise rejection. For a faster sweep speed, use a higher bandwidth receiver setting.

(3) Loop measurements above 30 kHz can be very difficult due to instrument grounding and high-frequency cross-talk between cables on injected and return channels. This is true of any network analyzer, and great care must be taken for measurements beyond this frequency.

(4) Inject the signal into a low-ripple part of the circuit, if possible.

(5) It is not required to operate the converter at its full operating condition while measuring the loop gain. It is possible to find the optimal operating point by trail and error.

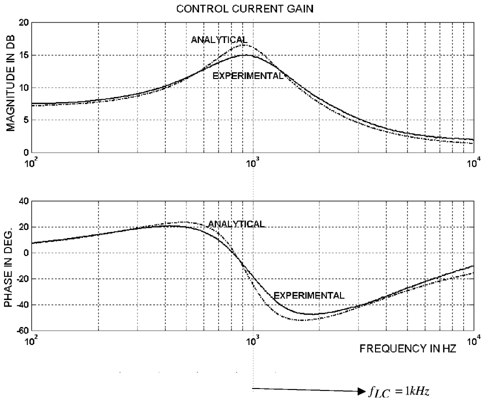

FIG. 16. Estimated and measurement results of buck converter control current gain

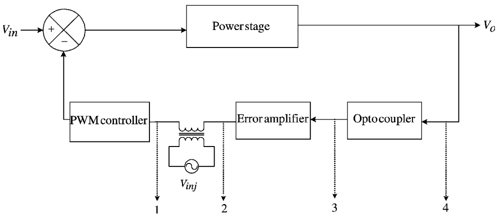

FIG. 17. Power supply block diagram.

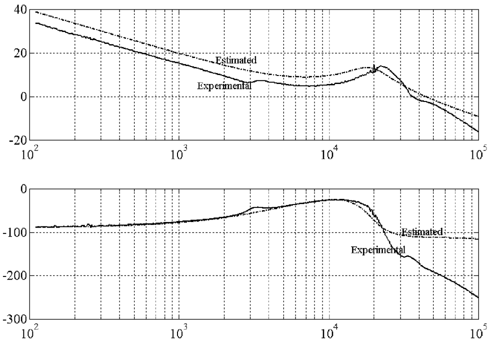

FIG. 18. Measured and estimated loop gain characteristics of a sample push-pull converter.

Table I
Measurement points and measuring parameter

Channel A	Channel B	Measuring parameter
1	2	Loop gain
3	2	Compensation network gain (Error amplifier)
1	4	Control voltage gain
4	3	Voltage sensor gain

7. Conclusion

The paper highlights the measurement techniques for evaluating the performance functions of power supply. These measurement techniques are available in various sources; the paper aims at bringing them together.

Measurement techniques used for input admittance and output impedance do not yield results in accordance with the analytical model. Additional dynamics may be introduced by the injection circuit. This requires further study. Further, these techniques can either be extended or suitably modified for current-mode controlled power supplies, digitally controlled power supplies, ac–dc converters, ac–ac converters and dc–ac converters.

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