

PID Controller tuning of cascade control systems using genetic algorithm

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Abstract

A simple genetic algorithm is applied for tuning of PID controllers for the cascade control systems. A methodology for selecting the search region is proposed using Ziegler–Nichols tuning method. Stability and robustness criteria are ensured in the selection of the search region, enabling the method to be applicable to online tuning. The inner and outer loops are tuned simultaneously, making the method applicable without disturbing the control strategy and ensuring overall optimal solution. The sum of integral absolute error values of the regulatory response is used as the objective function. The performance of this method is compared with the random search method of Lee *et al.* and Luus and Jaakola (LJ) algorithm. The results show the superiority of genetic algorithm (GA) over the other methods. The effect of unstable domain in the region of search is studied for both GA and LJ.

Keywords: Genetic algorithms, Luus and Jaakola algorithm, Lee *et al.*'s method, cascade control, controller tuning, global optimum.

1. Introduction

Cascade control is used extensively in many chemical processes such as chemical reactors, distillation columns, fire heaters and heat exchangers [1]. Cascade control systems improve the performance of single loops by considering the disturbances associated with the process. Cascade control scheme utilizes two control loops. The secondary (or) inner loop is embedded with a primary (or) outer loop. Very few methods are available in the literature for tuning of cascade control systems. Lee *et al.* [1] have proposed a method which enables to tune the P/PI (Inner loop P and outer loop PI) and PID/PID (Inner loop PID and outer loop PID) controllers both for the inner loop and the outer loop simultaneously. The method consists of first finding the ideal controller by Maclaurin Series. They compared their method with the frequency response method [3] and the ITAE minimization method of Krishnaswamy *et al.* [4]. Lee *et al.* method gives better performance than the other methods. In the present methodology, genetic algorithm (GA) is used for tuning of P/PI and PID/PID controllers for both inner and outer loops simultaneously. This method has the advantage of selecting the performance criteria especially by the user, which is not found in Lee *et al.*'s method. The tuning problem is a nonlinear one. GA [5–10] proved to be very effective in finding global optima. Genetic algorithms are search-and-optimization proce-

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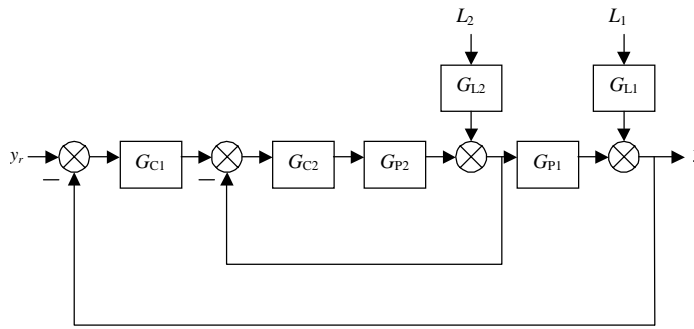


FIG. 1. Block diagram of a cascade control system.

dures that are motivated by the principles of natural genetics and natural selection. The powerful capability of genetic algorithm in locating the global optimal solution is used in the design of controllers. The present method is compared with the method of Luus and Jakola (LJ), which is a random search method, and widely used in finding optima when the search region is nonlinear. Simulation studies were carried out on two examples to demonstrate the procedure and to compare its performance with those of other tuning methods.

2. Objective function evaluation

The sum of the integral of absolute errors (IAE) of the regulatory problem is used as the objective function to be minimized by GA. Figure 1 shows the block diagram of a cascade control system.

In the figure, L_1 , L_2 are the loads, G_{L1} , G_{L2} are the load transfer functions, G_{P1} , G_{P2} are the process transfer functions, and G_{C1} , G_{C2} are the controller transfer functions. The performance of a single loop control system is usually evaluated by integral absolute error (IAE). Since two controllers settings are to be obtained, the objective function should reflect the performance of the two loops. Hence the function is selected as follows:

$$J = IAE1 + IAE2. \quad (1)$$

J represents the sum of IAE values of the regulatory problem, IAE1 is the IAE value for a unit step change in load 1 (L_1) and IAE2, the IAE value for a unit step change in load 2 (L_2). P/PI (P controller in the inner loop and PI controller in the outer loop) and PID/PID control modes are considered separately for the cascade control system. The optimal control parameters are found subject to the minimization of the above objective function using GA and LJ algorithms separately. The optimum controllers obtained in the regulatory problem are used in the servo problem to study the servo response of the systems.

3. Simulation parameters used in algorithms

A simple genetic algorithm is used in this work. The binary alphabet and the Gray coding is used for the encoding of controller parameters. A generation gap of 0.9, linear rank-based fitness assignment with a selective pressure of two, proportionate selection with stochastic universal sampling, simple (single-point) crossover and fitness-based reinsertion are used.

The size of the population is chosen to be 80 in accordance with the experimental studies of Grefenstette [5], so that the search space is attacked at many points simultaneously, thus resulting in faster convergence to the global optimum (which is a minimum in this case). The initially and randomly selected population are left to evolve for 100 generations, after which no significant change is found in the objective function value. So this is used as termination criteria for the algorithm [10]. Since the solution is a region and not a single point, the parameters obtained in the 100th generation are used for comparison. Twelve-bit string element is used for the encoding of each of the controller parameters. The crossover and mutation probabilities are chosen to be 0.45 and 0.02, respectively. The genetic algorithm MATLAB tool box developed by Chipperfield *et al.* [6] (available freely on the web) was used in the present study.

The random search optimization method of Luus and Jaakola is chosen to compare with GA. LJ algorithm is used in many chemical engineering problems and is found to be effective in locating global optima. The following parameters are used in the LJ algorithm.

Number of inner loop iterations: 9
Number of outer loop iterations: 201
Reduction parameter (ϵ): 0.98

4. Design procedure

The following methodology is used in the selection of search region for the genetic algorithm. The P/PI controller settings are selected by the open loop Ziegler–Nichols method [11] based on the first-order plus time delay (FOPTD) model. The inner loop is first tuned by Ziegler–Nichols tuning method ($K_{c,des}$, $T_{i,des}$ and $T_{d,des}$). With tuned inner loop, the outer loop is then tuned using Z–N tuning method. The ranges of the parameters for the inner- and outer-loop controllers are chosen as follows: For the P/PI control mode, the maximum value of K_c is chosen as $1.3 K_{c,des}$ and the minimum value of T_i is chosen as $T_{i,des}/1.3$ in the genetic algorithm. For the PID/PID control mode, the ranges of the controller parameters are chosen based on the ratio of delay to time constant of the primary process. For example-1, the primary process transfer function has L/T_p , (where L is dead time and T_p , the time constant) 0.1, the maximum value of K_c is chosen as $K_{c,des}/1.3$, the minimum value of T_i is chosen as $1.3T_{i,des}$ and the maximum value of T_d as $T_{d,des}/1.3$. For example-2, the primary process transfer function has L/T_p , 0.045 (nearly), the maximum value of K_c is chosen as $K_{c,des}$, the minimum value of T_i is chosen as $T_{i,des}$ and the maximum value of T_d as $T_{d,des}$. For convenience in representing the ranges, the parameters are encoded as K_c , K_c/T_i and $K_c T_d$ in the algorithm. Thus the region of instability is eliminated from the search domain. The same search region is used for the LJ algorithm with half of the range as the initial guess, so that the random points are distributed in the entire range. For the genetic algorithm, only the search region is required whereas LJ algorithm requires both the search region and the initial guess.

5. Example 1

The FOPTD model of Lee *et al.* [1] is considered in this example. The processes of the primary and secondary loops and the disturbances entering in them are given below.

Table I
Calculated Z-N settings for examples-1 and -2

Example	Loop	PI settings		PID settings		
		k_c	T_i	k_c	T_i	T_d
1	Inner	5.0	∞	5.0941	3.84	0.96
	Outer	7.7647	20.575	7.7647	20.575	5.1437
2	Inner	1.5333	∞	0.1547	103.75	25.938
	Outer	0.1547	103.75	1.5333	5.5	1.375

Table II
Controller settings range (P/PI) for GA and LJ algorithm for examples-1 and -2

Example	Loop	k_c	$k_T = k_c/T_i$
1	Inner	(0, 5.850)	(∞)
	Outer	(0, 9.4250)	(0, 0.2406)
2	Inner	(0, 1.7290)	(∞)
	Outer	(0, 0.1872)	(0, 0.0011)

$$G_{p1} = \frac{e^{-10s}}{100s+1}, \quad G_{p2} = \frac{2e^{-2s}}{20s+1}, \quad G_{L1} = \frac{e^{-10s}}{10s+1}, \quad G_{L2} = 1. \quad (2)$$

Two control structures are studied with this example: P/PI control mode (the inner loop controller is proportional and the outer loop controller is PI) and PID/PID control mode (both inner and outer loop controllers are PID).

5.1. P/PI control strategy

This type of control strategy is used often in the chemical industry. The offset produced by the inner loop proportional controller is eliminated by the outer loop PI controller. The search region is selected as explained in the design procedure. The simulation is carried out up to 250 min. The Z-N settings are given in Table I. The corresponding ranges for the GA and LJ are calculated as explained in Section 4 and are given in Table II. The initial guesses for the LJ algorithm are taken as follows.

$$K_{c2} = 2.9250, \quad K_{c1} = 4.7125, \quad K_{c1}/T_{i1} = 0.1203. \quad (3)$$

GA and LJ are used to search the optimum control parameters in this region. The controller settings are given in Table III along with those of Lee *et al.*'s method and the response curves for the unit step change in loads L1 and L2 are given in Fig. 2. As shown in the figure, GA gives better performance over those of Lee *et al.*'s method and the LJ algorithm. The servo response is studied with the same controller settings (obtained using the sum of IAE values (Table IV) of the regulatory problem as the objective function) (Fig. 3). Even though the servo performance is not included in the objective function, GA gives the best performance. GA response overshoots a little but has less raise time. Lee *et al.*'s method gives better performance than the LJ algorithm.

Table III
Controller settings for the P/PI mode for example-1 and the values of the objective function

Method	Inner loop	Outer loop	Objective function
Lee <i>et al.</i>	$K_{c2} = 3.444$	$K_{c1} = 5.83, T_{i1} = 105$	28.767
LJ algorithm	$K_{c2} = 2.8511$	$K_{c1} = 4.6386, T_{i1} = 99.99130.345$	
GA	$K_{c2} = 5.85$	$K_{c1} = 6.7552, T_{i1} = 100.5922.24$	

Table IV
Comparison of IAE values for a step change in set point for P/PI mode for example-1 (calculated up to 250 min with sample time of 1 min)

Method	IAE values
Lee <i>et al.</i>	28.7670
LJ	29.8795
GA	27.4769

Table V
Controller settings range (PI/PID) for both GA and LJ algorithms

Example	Loop	$(k_{c,\min}, k_{c,\max})$	$(k_{i,\min}, k_{i,\max})$	$(k_{d,\min}, k_{d,\max})$
1	Inner	(0, 3.9185)	(0, 0.7849)	(0, 2.8937)
	Outer	(0, 5.9728)	(0, 0.2233)	(0, 23.633)
2	Inner	(0, 0.1547)	(0, 0.00149)	(0, 4.0126)
	Outer	(0, 1.5333)	(0, 0.2788)	(0, 2.1083)

5.2. PID/PID control strategy

The search region for the PID/PID control mode of example-1 is calculated using the procedure explained in Section 4. First the Z–N settings (Table I) are found using the Z–N tuning method. The simulation is carried out up to 250 min. The ranges for the GA and LJ algorithm are given in Table V. The initial guesses for the LJ algorithm are assumed as:

$$K_{c2,\text{des}} = 1.9592, \quad K_{c2,\text{des}}/T_{i2,\text{des}} = 0.39248, \quad K_{c2,\text{des}}T_{d2,\text{des}} = 1.4469, \quad K_{c1,\text{des}} = 2.9864, \\ K_{c1,\text{des}}/T_{i1,\text{des}} = 0.11166, \quad K_{c1,\text{des}}T_{d1,\text{des}} = 11.817. \quad (4)$$

The optimal control parameters for the PID/PID control mode by both the methods are given in Table VI along with those of Lee *et al.*'s settings and the regulatory response curves are shown in Fig. 4. As shown in Fig. 4 and Table VI, GA performs better than the Lee *et al.*'s method and the LJ algorithm. The servo response by all the three methods is shown in Fig. 5 and the IAE values are tabulated in Table VII. The Lee *et al.*'s method gives better performance over that of LJ algorithm, while GA shows the best performance among all the methods.

6. Example-2

The higher-order system taken by Lee *et al.* [1] is considered in this example. The transfer function model of the system is given here.

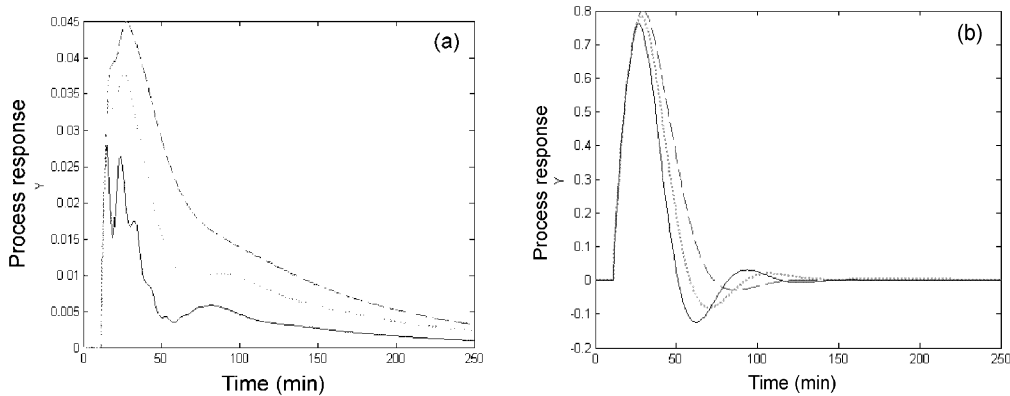


FIG. 2. Closed loop response for a step change in (a) load L2 and (b) response for a step change in load L1 for the P/PI mode of example-1. Legend: dotted line: Lee *et al.*'s method; dash line: LJ method; thick line: GA.

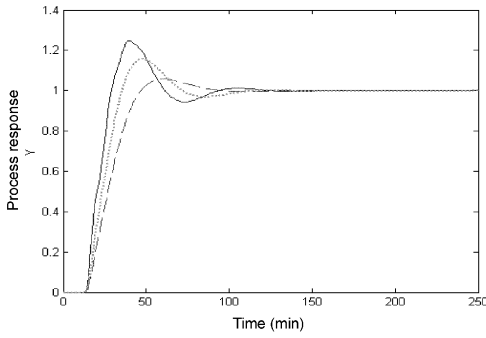


FIG. 3. Closed loop response for a unit step change in set point for P/PI mode of example-1. Legend: as in Fig. 2.

$$G_{p1} = \frac{10(-5s+1)e^{-5s}}{(30s+1)^3(10s+1)^2}, G_{p2} = \frac{3e^{-3s}}{13.3s+1},$$

$$G_{L1} = \frac{e^{-10s}}{100s^2 + 20s + 1}, G_{L2} = \frac{1}{100s + 1} \tag{5}$$

The two control modes, P/PI and PID/PID, are studied.

6.1. P/PI control strategy

The search region is selected as explained in Section 4. First, the stable region is found using the Z–N technique for both the primary and the secondary controllers. The simulation is carried out up to 2500 min. The Z–N settings are given in Table I. The corresponding ranges for the GA and LJ algorithm are calculated as discussed in Section 4 and are given in Table II. The halves of the ranges are taken as initial guesses for the LJ algorithm and are as below.

$$K_{c2} = 0.8645, \quad K_{c1} = 0.0936, \quad K_{c1}/T_{i1} = 5.5000e-004. \tag{6}$$

The optimal control settings calculated by the GA and LJ algorithms are given in Table VIII along with Lee *et al.*'s settings and the response curves for the regulatory problem are shown in Fig. 5. It can be seen (Fig. 5 and Table VIII) that GA gives best performance than

Table VI
Controller settings for the PID/PID mode for example-1 and the values of the objective function

Method	Inner loop	Outer loop	Objective function
Lee <i>et al.</i>	$K_{c2} = 3.444, T_{i2} = 20.666,$ $T_{d2} = 0.6451$	$K_{c1} = 5.83, T_{i1} = 105,$ $T_{d2} = 4.8$	18.9564
LJ algorithm	$K_{c2} = 1.8802, T_{i2} = 5.9974,$ $T_{d2} = 0.7275$	$K_{c1} = 2.9074, T_{i1} = 88.963,$ $T_{d1} = 4.0373$	35.441
GA	$K_{c2} = 3.9089, T_{i2} = 4.9797,$ $T_{d2} = 0.03597$	$K_{c1} = 5.9728, T_{i1} = 101.41,$ $T_{d1} = 1.5528$	17.368

Table VII
Comparison of IAE values for a step change in set point for example-1 for PID/PID control mode (Calculated up to 250 min with sample time of 1 min)

Method	IAE values
Lee <i>et al.</i>	29.0945
LJ	38.3195
GA	22.9105

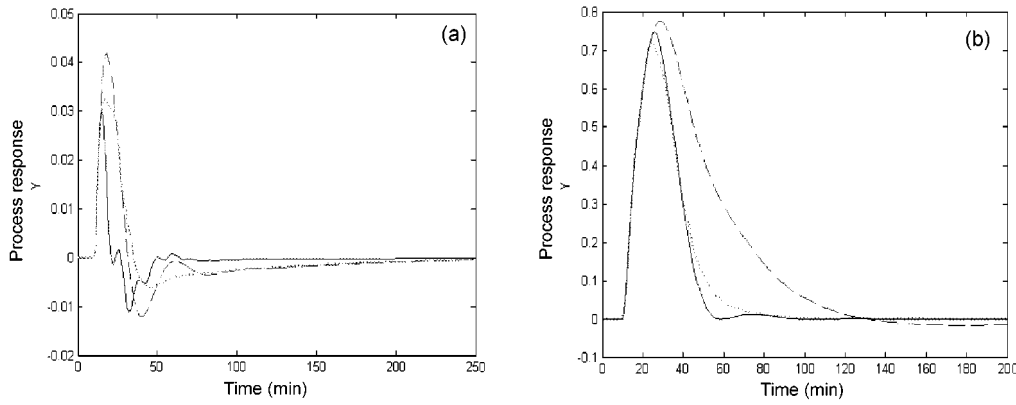


FIG. 4. Closed loop response for a step change in loads (a) L2, and (b) L1 of example-1 for PID/PID control mode. Legend as in Fig. 2.

the other methods. The performance of LJ algorithm is in between GA and the Lee *et al.*'s method. The servo response is shown in Fig. 6 and the IAE values are tabulated in Table IX. GA slightly overshoots the Lee. *et al.*'s method but has a small rise time.

6.2. PID/PID control strategy

The search region for the PID/PID control mode of example-2 is calculated using the procedure explained in Section 4. First the Z–N settings are found using the Z–N tuning method. The simulation is carried out for 2500 min. The Z–N settings for the inner and outer loop controllers are shown in Table I. The corresponding ranges for the GA and LJ algorithm are shown in Table V. The initial guesses for the LJ algorithm are taken as:

$$K_{c2,des} = 0.07735, \quad K_{c2,des}/T_{i2,des} = 0.000745, \quad K_{c2,des}T_{d2,des} = 2.0063,$$

$$K_{c1,des} = 0.7666, \quad K_{c1,des}/T_{i1,des} = 0.1394, \quad K_{c1,des}T_{d1,des} = 1.0541. \quad (7)$$

GA and the LJ algorithm are used to search the optimum control parameters in this region. The control settings are given in Table X along with those of Lee *et al.*'s settings and the response curves for the regulatory problem are shown in Fig. 7. As shown in Fig. 7 and Table X, GA gives better performance over Lee *et al.*'s method and the LJ algorithm. The

Table VIII
Controller settings for the P/PI mode for example-2 and the values of the objective function

Method	Inner loop	Outer loop	Objective function
Lee <i>et al.</i>	$K_{c2} = 0.883$	$K_{c1} = 0.09, T_{i1} = 90.53$	518.29
LJ algorithm	$K_{c2} = 0.8654$	$K_{c1} = 0.09453, T_{i1} = 63.778$	415.73
Genetic algorithms	$K_{c2} = 1.729$	$K_{c1} = 0.09956, T_{i1} = 90.515$	305.44

Table IX
Comparison of IAE values for a step change in set point (calculated up to 2500 min with a sample time of 1 min) for example-2 with P/PI mode

Method	IAE values
Lee <i>et al.</i>	140.2216
LJ	155.7818
GA	133.7004

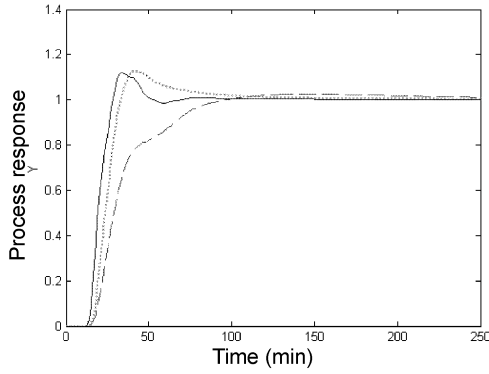


FIG. 5. The closed loop response for a unit step change in set point of example-1 for PID/PID control mode. Legend: as in Fig. 2.

servo response is shown in Fig. 8 and the IAE values are given in Table XI. The servo and the regulatory performances of LJ algorithm are in between the GA and Lee *et al.*'s method, while GA gives the best performance. The robustness of the controller settings by all the methods is verified with a perturbation of +20% in the time delay of G_{p1} , G_{p2} and G_{L1} . All the methods give stable responses with GA giving the best performance.

7. Effect of unstable domain in the region of search

The effect of unstable domain in the region of search of the controller settings is studied for example-1 with P/PI mode. The search region is chosen as follows without taking Z-N settings into consideration.

$$K_{c2} = (0, 10), \quad K_{c1} = (0, 10) \quad \text{and} \quad K_{c1}/T_{i1} = (0, 10). \tag{8}$$

GA is used to search for the optimum controller parameters in this range. It converges to the settings which have an oscillatory closed loop performance for a unit step change in load L1 as shown in Fig. 10.

The response is highly sensitive to modeling errors. Even a +10% perturbation in the process delay makes the system unstable. LJ algorithm is applied in the same search region and the initial guesses are assumed as follows.

Table X
Controller settings for the PID/PID mode for example 2 and the values of the objective function

Method	Settings		Objective function
	Inner loop	Outer loop	
Lee <i>et al.</i>	$K_{c2} = 0.883, T_{i2} = 14.5, T_{d2} = 1.117$	$K_{c1} = 0.09, T_{i1} = 90.53, T_{d1} = 18.2$	147.04
LJ algorithm	$K_{c2} = 1.4566, T_{i2} = 5.4999, T_{d2} = 1.3751$	$K_{c1} = 0.1469, T_{i1} = 103.75, T_{d1} = 25.939$	89.033
GA	$K_{c2} = 1.5168, T_{i2} = 5.4408, T_{d2} = 0.054648$	$K_{c1} = 0.1261, T_{i1} = 84.595, T_{d1} = 31.811$	81.633

Table XI
Comparison of IAE values for a step change in set point (calculated up to 2500 min with a sample time of 1 min) for example-2 with PID/PID mode

Method	IAE values
Lee <i>et al.</i>	119.3226
LJ	105.3321
GA	100.3622

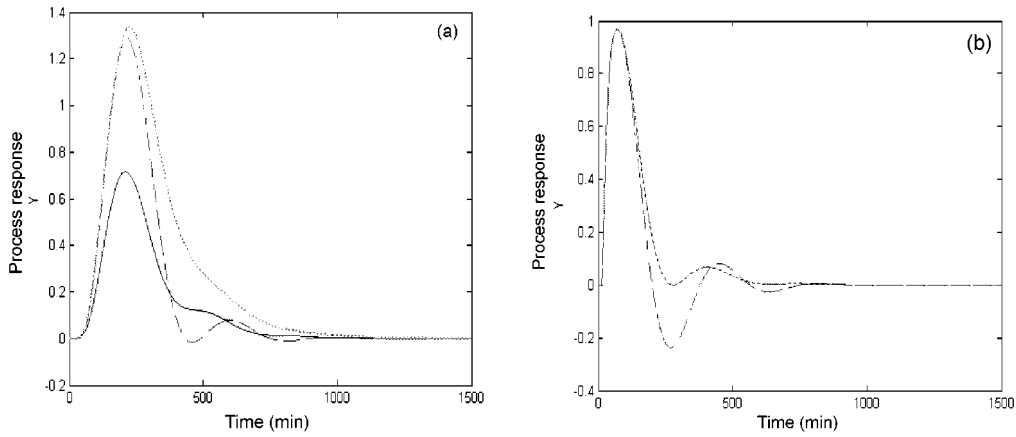


FIG. 6. Closed loop response for a unit step change in (a) load L2, and (b) load L1 for example-2 with P/PI mode. Legend: as in Fig. 2.

$$K_{c2} = 5, \quad K_{c1} = 5, \quad K_{c1}/T_{i1} = 5. \tag{9}$$

The algorithm failed to converge. If the Z–N settings are taken as initial guess then the algorithm converged to the better control parameters than in Table III. This shows that the LJ algorithm is sensitive to the initial guesses. To check the stability of the method, the following criteria are employed: If a step change in any input variable makes the output variable uncontrollable, then the system is unstable, otherwise it is stable. The robustness criteria used in the present case is as follows: If a 20% change in the process time delay or process gain makes the system unstable then the controller settings are not robust. Otherwise the controller settings are robust.

The effect of initial guess on the LJ algorithm is examined by considering example-1. In Section 5, the search region (the lower and upper bounds of the controller settings) is selected. According to Section 4, the initial guess is fixed (i.e. the mid-point of the search region). In this section, the performance of the LJ algorithm is verified by taking Z–N settings as the initial guess. It is observed that the LJ algorithm with Z–N settings as initial guess gives a better response than the other initial guess for the P/PI mode. The settings are

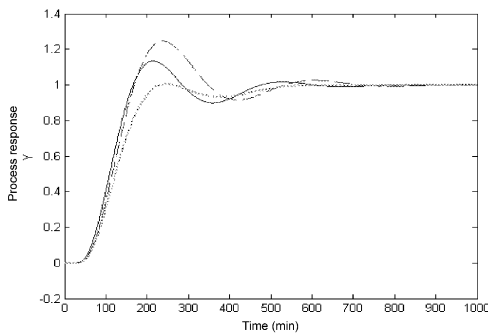


FIG. 7. Closed loop response for a unit step change in set point of example-2 for P/PI mode. Legend is the same as in Fig. 2.

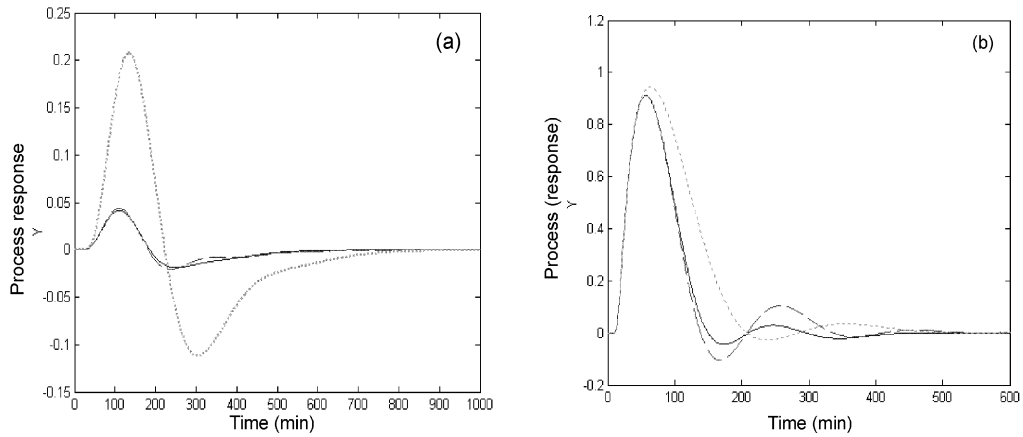


FIG. 8. Closed loop response for a step change in loads (a) L2, and (b) L1 for example-2 with PID/PID mode. Legend: as in Fig. 2.

also robust by both the guesses for P/PI mode. It is found that the LJ algorithm with mid-point of the range as initial guess gives a sluggish response than the Z-N settings as the initial guess for the PID/PID mode. Both approaches give the same objective function values. The settings of the LJ with Z-N values as the initial guess goes to unstable region with +20% perturbations in the time delay of G_{p1} , G_{p2} and G_{L1} , whereas the LJ with mid-point of the range as initial guess gives robust settings for the PID/PID control mode.

The LJ method with Z-N settings as the initial guess sometimes gives better performance than the LJ method with mid-point of the range (as suggested in section 4) as initial guess, but does not guarantee robustness. So it is always safe to fix the range of the controller parameters as suggested in Section 4 than fixing Z-N values as the initial guess. If the stability criterion is included in the objective function formulation, then the LJ algorithm with Z-N settings as initial guess can be used safely.

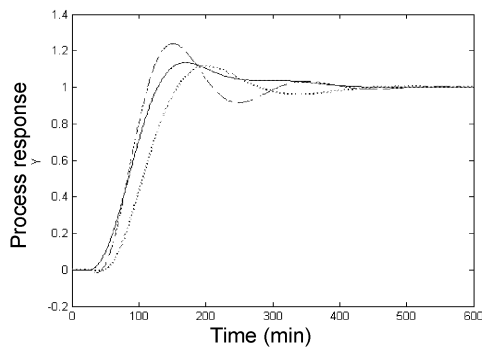


FIG. 9. Closed loop response for a unit step change in the set point of example-2 with PID/PID mode. Legend: as in Fig 2.

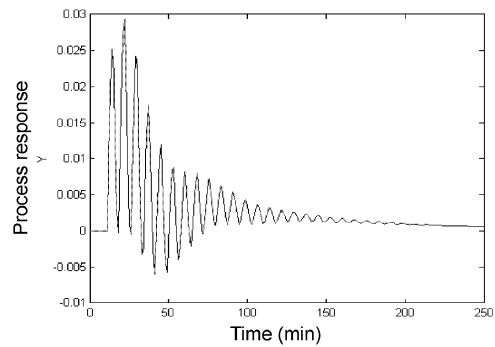


FIG. 10. Closed loop response for a unit step change in load L1 for example-1 with P/PI settings from GA.

9. Conclusions

A simple genetic algorithm and a random search method of Luus and Jaakola algorithm are used for tuning of cascade control systems. Search region is selected by eliminating the unstable domain using the open loop Ziegler–Nichols tuning method. The integral absolute error of the regulatory response is used as performance criteria. The proposed method is compared with that of the Lee *et al.*'s method. Two different examples used by Lee *et al.* with two different control strategies, i.e. P/PI and PID/PID are studied. GA gives a better performance over the other methods. The effect of the unstable domain in the search region is also examined using the first example with P/PI control mode. GA converges to the controller settings, which are not robust. So it is suggested that the unstable domain should be eliminated from the search region to guarantee robustness. It is suggested that the mid-point of the stable search region should be used as initial guess for the LJ algorithm to achieve robustness. It is concluded that GAs are very effective in optimal tuning of the cascade control systems. The simultaneous tuning of both the inner and outer loops makes the method easily applicable.

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Nomenclature

- G_{c1} = Transfer function of the controller in the inner loop
 G_{c2} = Transfer function of the controller in the outer loop
 G_{L1} = Load transfer function in the inner loop
 G_{L2} = Load transfer function in the outer loop

G_{p2}	=	Transfer function of the process in the inner loop
G_{p3}	=	Transfer function of the process in the outer loop
K_{cdes}	=	Proportional gain calculated from Z–N tuning method
T_{ides}	=	Integral time calculated from Z–N tuning method
T_{ddes}	=	Derivation time calculated from Z–N tuning method
K_c	=	Proportional gain obtained from GA or LJ
K_I	=	K_c/T_i
K_d	=	$K_c T_d$
T_i	=	Integral time obtained from GA or LJ
T_d	=	Derivation time obtained from GA or LJ
Y	=	Output variable of the outer loop
y_r	=	Set point of y