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Comparison of methods of minimization of cogging torque in wind generators using FE analysis

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Abstract

Most small wind turbines use permanent magnet (PM) generators. Small wind turbines are usually self-starting and require very simple controls. The cogging torque, which is generated by the interaction between the magnetized pole of PMs and the salient poles of the armature core, affects the self-start ability and results in undesirable mechanical vibrations and noises. Thus, minimization of cogging torque, which is of utmost importance in improving the operation of small wind turbines is investigated in the present work. In this paper, three design options are investigated to minimize cogging torque: uniformity of air gap, pole width, and skewing. Although the design improvement is intended for small wind turbines it is also applicable to larger wind turbines. A software package based on 2D finite-element analysis is used for the analysis of cogging torque.

Keywords: Finite-element method (FEM), wind turbine, small generator, permanent magnet, remote application, cogging torque.

1. Introduction

In the case of a directly driven wind generator, the removal of gear helps to increase its efficiency slightly but not reliability [1]. In modern days, small wind turbines are used for water or oil pumping, battery charging and power generation. Small wind turbines are attractive alternatives to conventional sources of energy in places of high wind potential and when power requirement is small. They can be used in many remote or dedicated applications, such as weather stations, remote cabins, campsites, boats, etc. Small wind turbines are also used in grid-connected mode. Considering the market potential, many improvements in the technology can pay for themselves through superior quality, lower maintenance, and higher reliability and in particular the quiet operation of the machine.

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The cogging torque of the permanent magnet (PM) wind generators is generated from the interaction of permanent magnet MMF harmonics and air-gap permeance harmonics due to slotted structure of stator core. It manifests itself by the tendency of a rotor to align in a number of stable positions even when the machine is unexcited, and results in a pulsating torque, which does not contribute to net effective torque. These torque ripples result in undesirable mechanical vibration and noise. For this reason, the cogging torque should be kept small in the design process for the purpose of constant speed operation and high precision position control [2, 3].

2. System configuration

Based on the location of the PM, there are two types of small wind turbine, PM generators. One is outside rotating PM generator and the other inside rotating PM generator. In outside rotating generator, magnet poles rotate outside the armature winding, i.e. just opposite to conventional PM motors. The inside rotating generator is similar to conventional PM motors, in which the rotor, i.e. permanent magnet poles, rotates inside the armature winding. Both configurations have been widely used, and arguments can be made that for one being better than the other, but with proper design, both can be optimized [3].

Many methods to reduce cogging torque have been presented in earlier literature, including pole shifting, uneven distribution of stator slots, stator tooth notching, and others. Some papers predict cogging torque using analytical or MMF diagram approach [1, 4]. But these methods are time-consuming and moreover give only approximate results. This paper investigates the inside rotating PM generator configuration using finite-element analysis. This investigation is based on an existing PM generator and is not intended for the design of a new generator. Therefore, only potential modifications were investigated (i.e. stator skewing and magnet geometry). A software package based on finite-element analysis is used to predict and quantify the cogging torque in the design process. Finite-element method, one of the numerical methods, with the advent of computer technology, overcomes the difficulties of all the conventional methods [3]. The dimensional aspects determining the cogging torque of a PM generator are investigated. In order to obtain an exact torque profile, the finite-element method (FEM) is used considering the rotation of rotor and the saturation of silicon steel. Such a procedure leads fast convergence of the optimization algorithm to an accurate and stable solution [5].

2.1. *Cogging torque analysis*

The cogging torque [1] is given by the equation

$$T = -\frac{\partial}{\partial \theta} \int_0^{2\pi} F(\phi).A(\phi - \theta) d\phi \,, \tag{1}$$

where θ is the relative angle between the PM and the armature core. In PM wind generators, the exciting field and the magnetization functions are defined, respectively, as the magnetic energy density and magnetic flux density along the air gap when the armature core has no winding slots [1].

The exciting field function $F(\phi)$ of PM generator can be expressed as follows using Fourier transformations [1, 6]:

$$F(\phi) = B_i^2(\phi)/2\mu_0 = X_0 + \sum_{k=1}^{\infty} X_k \sin(kP\phi + x_{kp})$$

$$X_0 = (\pi - q)B_0^2/(2\pi\mu_0); \quad X_K = -\sin(kq)B_0^2/(2\pi\mu_0), \tag{2}$$

where P = number of poles, and $x_{kp} = \Pi/2$.

On the other hand, the armature function $A(\phi)$ is defined as the product of the volume function, $v(\Phi)$, which represents the volume of air gap per unit angle, and the square of the permeance function defined as

$$\omega(\phi) = B_g(\phi)/B_i(\phi), \tag{3}$$

where $B_g(\Phi)$ is the magnetic flux density in the air gap when the armature core has winding slots. Since the volume function can be approximated as a constant function, the armature function $A(\phi)$ of the generator can be Fourier-transformed to be written as follows:

$$A(\phi) = \omega^2(\phi).\upsilon(\phi) = Y_0 + \sum_{k=1}^{\infty} Y_k \sin(kS\phi + y_{ks}), \tag{4}$$

$$Y_0 = (\pi - p)A_m/\pi Y_K = -2\sin(kp)A_m/(k\pi)$$

where S is the number of slot of armature core, p is one half of the slot opening, A_m is the volume per unit angle and $y_{ks} = \Pi/2$.

Substituting eqns (2–4) in the torque eqn (1), the equation for cogging torque [1] can be written as

$$T = \pi \sum_{n=1}^{\infty} nGX_{nG}Y_{nG} \sin(nG\theta + x_{nG} - y_{ng}), \tag{5}$$

where G is the LCM (least common multiplier) of P and S and θ is the relative angle between the PM and the armature core. From eqns (2–5), it is clear that the lower harmonic components of the cogging torque are bigger than those of the higher-order harmonic components.

3. Estimation of cogging torque

The aerodynamic power [7] generated by the wind turbine is:

$$W = \frac{1}{2}\rho ACpV^3, \tag{6}$$

where ρ = density of air, A = swept area of the blade, C_p = performance coefficient, and V = wind speed.

An important parameter for governing wind turbine performance is tip-speed ratio (TSR), which is given by

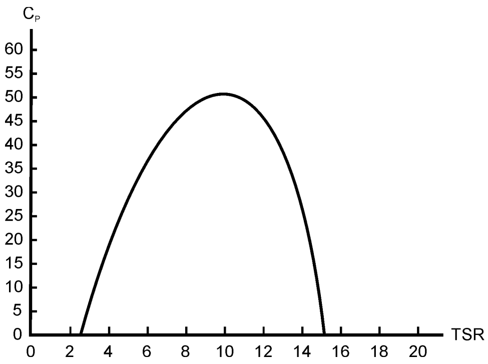

FIG. 1. A typical C_p curve for wind generator.

$$\lambda = \frac{\omega_r R}{V}, \tag{7}$$

where R is the radius of the rotor of wind turbine. Wind turbine operation is governed by its TSR [7, 8]. A larger wind turbine operates at a lower rpm and hence its TSR is high, as is clear from eqn (7). As an example, a 300-kW wind turbine operates at about 50–60 rpm [7]. On the other hand, a 10-kW wind turbine operates at about 300–500 rpm rated and hence has low TSR. The level of aerodynamic power generated by the wind turbine is decided by its performance coefficient (*Cp*), which can be related to its TSR. A typical *Cp* vs TSR curve [7] is shown in Fig. 1.

3.1. *Starting condition of wind turbine*

The rotor speed (ω_p) is very low during start-up of wind generator. Hence, as shown by eqn (7), the TSR (λ) is also low. From Fig. 1, it can be seen that the resulting *Cp* is very low at low TSR, thereby resulting in low aerodynamic power. Therefore, it is desirable that during start-up the cogging torque of the PM generator should be low enough so that the aerodynamic power can overcome it as with a large cogging torque during start-up, the wind turbine may never come out of stall mode and may never start.

3.2. *Running condition*

Because of shorter blades and lower mass, small wind turbines typically have lower rotor inertia as compared with large turbines. Thus, the cogging torque produced dominates in small wind turbine systems over the damping effects of inertia. Also, during low wind speeds, when the rotor rotational speed is low, the kinetic energy stored in the rotor may not be large enough to overcome the cogging torque. So the dominant cogging torque may cause noise and mechanical vibration in small wind generator and if not properly designed, such vibrations may lead to blade failure.

3.3. *Uniform air gap vs nonuniform air gap*

In direct-driven wind generators, the rotor rpm is low, thereby requiring large number of poles to operate these devices. Generally, rotational speed of a direct-driven electric genera-

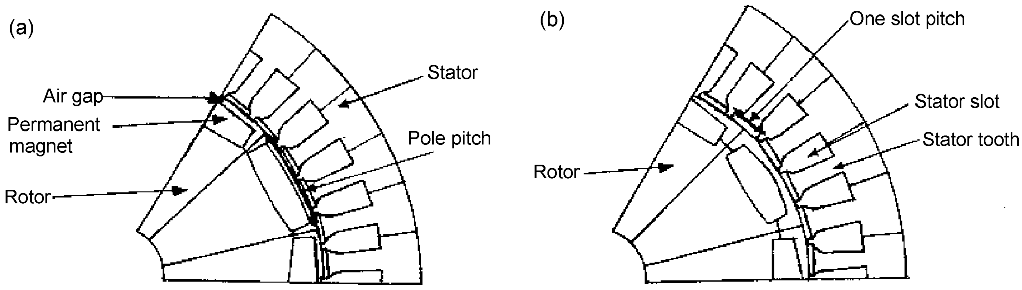

FIG. 2(a). Permanent magnet with uniform air-gap. (b) Permanent magnet with nonuniform (bread loaf) air gap.

tor for a wind turbine is lower than conventional electric machines. Due to large number of poles in direct-driven wind generators, the width of the magnet pole is very short. Thus the rotor has almost uniform air gap. The magnet can also be shaped to get a uniform air gap. But the cogging torque can be minimized if the shape of the magnetic pole is such shaped as to create a nonuniform air gap. This shape is that of a loaf of bread. The basic idea in this method is to modify the pole shape of the armature core so that the LCM of *P* and *S* in eqn (5) should be higher [6]. A comparison between uniform air gap and bread-loaf-type permanent magnet surface is illustrated in Fig. 2(a) and (b), respectively. The shape of the nonuniform pole surface (i.e. bread loaf type) can be optimized to reduce the cogging torque [8, 9].

In Fig. 3, the normalized cogging torque is plotted against rotor angle both for uniform and nonuniform air gaps (bread-loaf magnet shape). The rotor is rotated in a fraction of the slot pitch and the resulting torque is calculated. It is apparent from the plot that the cogging torque can be reduced by about 50% by shaping the magnet nonuniform.

3.4. *Pole width to pole pitch ratio*

Cogging torque for a PM wind generator varies if the width of the pole is changed. For a particular width of the pole the cogging torque is minimized. In the present work, the width

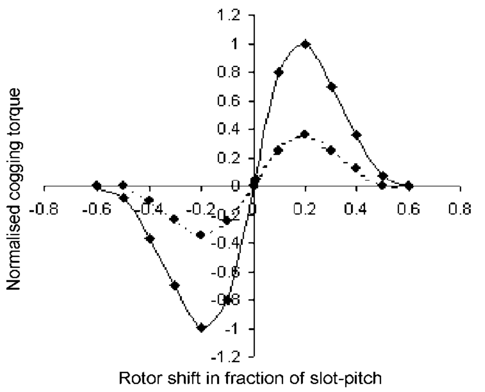

FIG. 3. Plot of normalized cogging torque against rotor shift. —◆— Uniform air-gap; ---◆--- Nonuniform air gap.

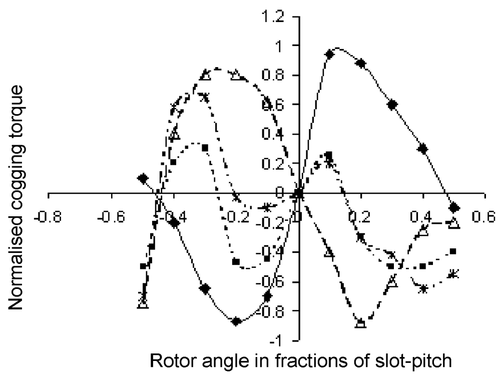

FIG. 4. Cogging torque for different PM width. —♦— Original PM width; ---■--- 0.9 of PM width; – –*– – 0.88 of PM width; – – Δ – – 0.85 of PM width.

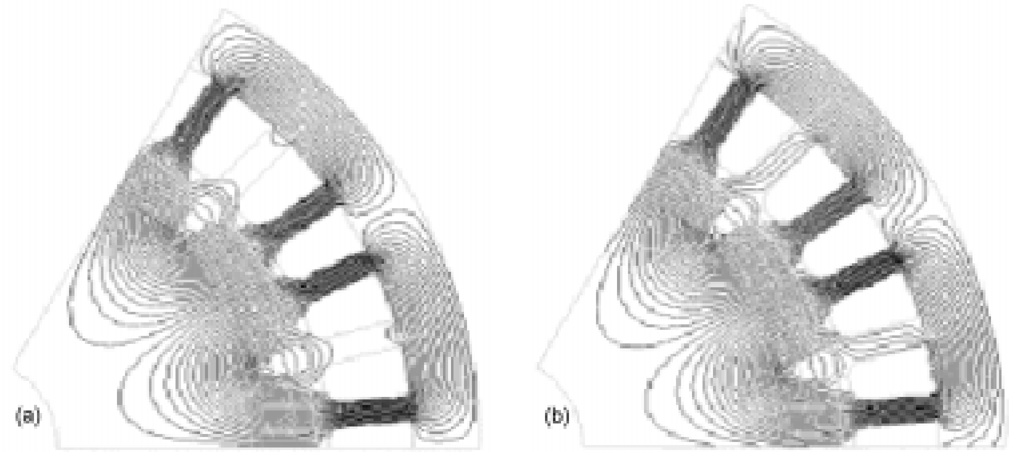

FIG. 5. Flux lines for (a) minimum, and (b) maximum cogging torque positions.

of the magnet pole is truncated with respect to the original width while the original shape of the pole surface and the design of stator lamination are kept unchanged for the entire investigation of cogging torque. The width of the pole is normalized to the original width of the pole and the cogging torque is computed for different rotor angles in fraction of slot pitch while maintaining the pole pitch and keeping the shape of the pole surface unchanged. Figure 4 shows the variation of cogging torque with the rotor angle as the width of the magnet pole is truncated with respect to the original width. This plot clearly shows that for a particular width of the pole, known as optimum width, the cogging torque is minimized. It is worth noting that as the width of the pole is close to the optimum width, the waveform of cogging torque changes. The frequency of the cogging torque doubles at optimum width. As the width is further reduced below the optimum width, the cogging torque starts increasing in magnitude again with the frequency back to the frequency of the cogging torque at the original width. As an illustration, the flux lines in the generator for two different rotor positions are presented in Fig. 5(a). In Fig. 5(a), the flux lines are for minimum cogging torque position and are symmetrically distributed. Figure 5(b) represents the flux lines for maximum cogging torque position.

3.5. *Effect of skewing*

Skewing the stator stack or skewing the magnet pole in PM generators also results in the reduction of cogging torque. A perfect skew can nearly eliminate cogging torque. However, skewing adds complexity to the manufacturing process and results in additional cost of the machine. Skewing the stator may complicate the winding installation, reduce the effective slot area, and increase the conductor length thereby increasing the stator resistance, whereas skewing the magnet requires to be shaped properly, resulting in additional manufacturing cost. Usually a full skew of one slot pitch is implemented to reduce the cogging torque. One slot pitch is an arc covering one slot and one tooth of the stator. The cogging torque is calculated by skewing stator of the generator in fractions of the rotor slot pitch (fractional skew) with original permanent magnet width.

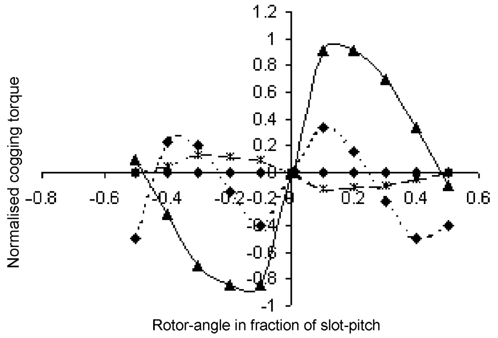

FIG. 6. Cogging torque for different skewing angles and for optimized PM width. —♦— Full skew; – –*– – Fractional skew; —▲— Original condition; ---◆--- With optimized width.

Figure 6 shows the variation of cogging torque with rotor angle in a fraction of slot pitch when the stator of the generator is skewed in a fraction of the rotor slot pitch. Different skewing degrees are presented with nonuniform air gap. The cogging torque is normalized to the peak cogging torque of the original width without skew. A full skew of one slot pitch is presented as the reference. Fractional pitch covers only 88% of the full skew. For comparison, the cogging torque with optimized magnet width is also presented on the same graph. The graph shows that the skewing angle plays an important role in reducing the cogging torque. The closer the skewing angle to a full skew, the smaller the resulting cogging torque. It is clear that controlling the magnet width does not eliminate the cogging torque completely. There is a finite residual cogging torque with optimized magnet width whereas a perfect skew can nearly eliminate cogging torque.

4. Conclusions

This paper presents an overview of different methods to reduce the cogging torque of small wind turbine generators and the importance of minimizing cogging torque and is not an exhaustive study of all the available cogging reduction options. Finite-element analysis is used to quantify the cogging torque in the design process and the dimensional aspects determining the cogging torque of a PM generator are investigated. From the analysis it is concluded that the nonuniform i.e. bread-loaf-type pole shape of the permanent magnet results in reduction of cogging torque of PM wind generators. It is also apparent that cogging torque is reduced as the pole arc to pole pitch ratio is varied and for an optimum pole width cogging torque is minimum. While skewing can potentially eliminate cogging, other design approaches may be lower in cost and easier to manufacture. These approaches may result in a finite residual cogging torque whereas a perfect skew can nearly eliminate cogging torque.

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Symbols

A = Swept area of turbine, m^2
A_m = Volume per unit angle
B = Magnetic flux density, Wb/m^2
C_p = Performance coefficient of wind turbine
P = Number of poles
S = Number of slots of armature core
T = Cogging torque, Nm
W = Aerodynamic power generated by the turbine, Watts
$A(\Phi)$ = Armature function
$F(\Phi)$ = Exciting field function
$\upsilon(\Phi)$ = Volume function
$\omega(\Phi)$ = Permeance function

Greek letters

ρ = Density of air
λ = Tip speed ratio (TSR)