

Multidisciplinary design optimization applied to a sounding rocket

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Abstract

Multidisciplinary design optimization (MDO) is an emerging discipline in aerospace engineering. In this paper, MDO is applied to “RX-250-LPN” sounding rocket to optimize its performance. In the MDO of the referred vehicle, three fields have been considered—trajectory, propulsion and aerodynamics. A special design structure matrix is developed to assist data exchange among the fields. This design process uses response surface method (RSM) for multidisciplinary optimization of the rocket. RSM is applied to the design at two stages: propulsion model and the system level. In the propulsion model, RSM determines an approximate mathematical model of the engine output parameters as a function of design variables. At the system level, RSM fits a surface of objective function versus design variables. Finally, an optimization method is applied to the response surface in the system level to find the best result. The application of the multidisciplinary design optimization procedure developed by us increased the accessible altitude (performance index) of the referred sounding rocket by 25%.

Keywords: Multidisciplinary design optimization, sounding rocket, central composite design, response surface method, and equation of motion of a rocket.

1. Introduction

Aerospace vehicles generally require input of design variables from a variety of traditional aerospace fields such as aerodynamics, structure, propulsion, performance, cost and trajectory. Traditional optimization methods cannot always be applied as they use variables from one field only. Multidisciplinary techniques are required for this class of design problems [1]. In other words, multidisciplinary design optimization (MDO) provides a collection of tools and methods that permit the tradeoff between the fields involved in the design process. MDO methods also consider interdisciplinary interactions to achieve better overall system, i.e. MDO is a process that accounts for the effects of interactions of several disciplines [2, 3].

The application of MDO method in the design of launch vehicles has been increasing over recent years as the designers used the method to develop better designs. One of the applications that NASA has been working on for several years is the design of fully reusable launch vehicles (RLV). The design of RLV is a multidisciplinary process which requires analysis of aerodynamics, propulsion, weight, cost, trajectory and configuration. The objec-

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tive of the RLV design is to determine the setting of variables that will minimize the vehicle dry weight [4–7]. The RLVs are classified as single-stage-to-orbit (SSTO) and two-stage-to-orbit [8]. MDO is used in the design of SSTO to select the best configuration with respect to important vehicle parameters like dry weight and operational complexity [9–11]. A special SSTO designed recently is the rocket-powered-combined-cycle single-stage-to-orbit (RBCC–SSTO) [12, 13]. The design of the RBCC-SSTO is a highly multidisciplinary process.

An alternative application of MDO is the optimization of multistage launch vehicle design developed at EADS-LV [14]. EADS-LV has been designing launchers for many years.

Analogous to other launch vehicles, the design of a sounding rocket involves considerable multidisciplinary activity. In this paper, the multidisciplinary design optimization of a sounding rocket is formulated using the response surface method (RSM) for the first time. A sounding rocket is a research rocket that launches equipment into the upper atmosphere on a suborbital trajectory to take measurements and return to the surface. It basically comprises two parts; a solid fuel rocket motor and the payload. The payload is the section which carries the instruments to perform the experiment and send the data back to earth. These rockets allow scientists to conduct investigations at specified times and altitudes.

In this paper, “RX-250-LPN” sounding rocket design is optimized, considering the multidisciplinary nature of the problem by applying RSM. Propulsion, aerodynamics and trajectory are involved in the multidisciplinary design. These fields are modeled, and the data is exchanged. Multidisciplinary analysis is performed for every selected combination of design variables, and the results of the analysis are recorded. Then, RSM is used to fit a surface over the results obtained.

In this work, RSM is applied in two categories: in the propulsion model and at the system level. Regression analysis is then used to determine the response equations. An optimization method is applied to the response surface at system level to find the best combination of design variables such that the maximum altitude of the rocket is obtained.

2. RSM

The optimization of the reference sounding rocket employs an RSM originally developed by Box and Wilson [15]. RSM utilizes central composite design (CCD) to efficiently characterize a parameter space using statistically selected experiments. CCD employs orthogonal arrays from the design of experiment (DOE) theory to study a parameter space with a significantly small number of experiments [16, 17]. Stanley *et al.* [18] summarize an application of Taguchi methods to launch vehicle parametric design. CCD utilizes first-order models augmented with $2n + 1$ additional experiments. CCD is designed to be able to fit a model that captures all of the two variable interactions, the linear terms and the second-order terms [17]. For the CCD of a system with two design variables, consider the following equation which describes the model as a function of design variables:

$$y = \mathbf{b}_0 + \mathbf{b}_1 * A + \mathbf{b}_2 * B + \mathbf{b}_3 * AB + \mathbf{b}_4 * A^2$$

where A and B are design variables and Y is the objective function to be optimized. \mathbf{b} are the coefficients of the equation which determine the effect of each term. The appropriate CCD design is shown in Table I.

Table I
CCD design with two variables

Run	A	B
1	-1	-1
2	-1	1
3	1	-1
4	1	1
5	$-a_1$	0
6	a_1	0
7	0	$-a_2$
8	0	a_2
9	0	0

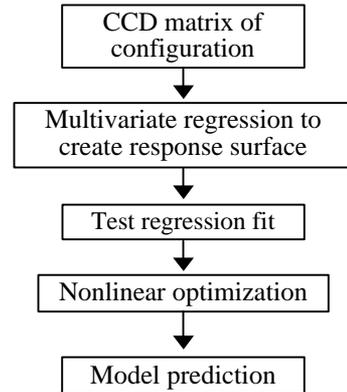


FIG 1. Response surface method.

The value of a_1 and a_2 can be selected arbitrarily. The resulting data is then analyzed using regression analysis techniques to determine the output response surface as a function of the input variables. Afterwards, the resulting generalized response surface equation is statistically analyzed for lack of fit. Subsequent to that, the optimum result and the values of design variables are determined using nonlinear optimization techniques. Finally, the predictive capability of the model is determined [9]. The CCD, regression technique, RSM design and optimization method are shown in Fig. 1.

The RSM optimum is not limited to the best combination of different levels of all the variables. Therefore, RSM allows more accurate solution of optimization problems. A comparison of MDO methods such as the collaborative methods shows that RSM is better than the other methods for fewer number of design variables [4]. It is however not suitable for discrete variables [12].

3. Design problem statement

In this paper, the MDO method is applied to “RX-250-LPN” to optimize the accessible altitude of the rocket. For design optimization of the rocket, several areas should be considered acting interactively. MDO takes into account propulsion, trajectory, and aerodynamic fields in the design of the rocket. The design requires proper consideration of the effects of each field on the vehicle and their interactions. The codes are written in MATLAB m.file and then integrated in a design structure matrix. Once each of the codes is properly set up, one could easily link the inputs and outputs of the three disciplines to each other within the design structure matrix. Several parallel efforts have been and are being undertaken to identify the information framework and design structure matrix for integrated design [19]. The proposed design structure matrix including fields and the flow of data between the fields is shown in Fig. 2.

The design variables are the rocket engine thrust and burning time. Two design variables are changed simultaneously and for each combination of the variables, multidisciplinary analysis is performed and the results are recorded.

The two variables of design vector are fed to the propulsion discipline. The outputs of propulsion discipline are the mass of fuel, the total mass of engine, the length of the en-

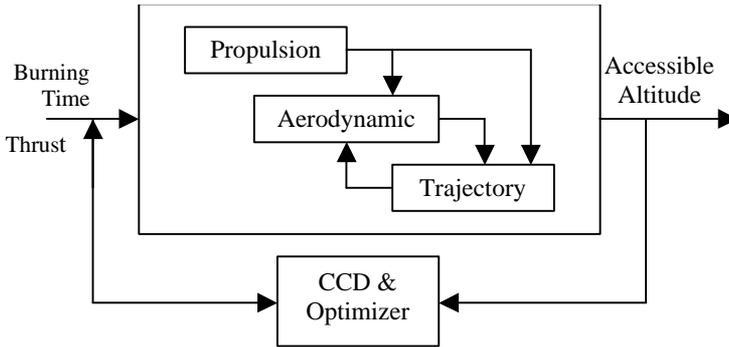


FIG 2. Design structure matrix.

engine, engine's area, the velocity of burnt products and the pressure in the exit area of the engine. The output of propulsion discipline are fed to the aerodynamic and trajectory units.

Aerodynamic field gets variables from propulsion and trajectory fields. The length of the engine and its area, Mach number and angle of attack are input variables to the aerodynamic model. Output variables consist of aerodynamic coefficients and center of pressure. Output variables of the aerodynamic field are fed to trajectory field.

Trajectory discipline receives input variables from propulsion and aerodynamic areas. Velocity of burning product, pressure in the exiting area, the mass of fuel and the total mass of the engine, aerodynamic coefficients and center of pressure are used as input variables to trajectory. Mach number, angle of attack, angular and linear velocity and position of rocket in the reference frame are output variables from trajectory.

Multidisciplinary analysis is done at several selected values of thrust and burning time as per CCD requirements. The analysis proceeds from one area to the next with data flowing from one to the other. In fact, variables are passed from one area to the other. The results of the analysis are recorded, and are used to find a response surface for the design space.

In this paper, response surface is applied at two levels: in the propulsion analysis and at the system level. Therefore, the output variables of the propulsion model can be expressed as an approximate mathematical model of the design variables. In the following, each area and its input and output parameters are described.

3.1. Trajectory

The flight profile of a sounding rocket follows a parabolic trajectory. Subsequent to launch and after the rocket motor uses up its propellant, it separates from the vehicle. The payload continues its journey into space after separation from the motor and begins conducting the experiments. When the experiments are completed, the payload returns to earth.

In this paper, the six-degrees-of-freedom model of the rocket is analyzed [20]. The code of the trajectory analysis is written in MATLAB m.file. The equations of rocket motion are as follows:

$$\begin{aligned}
m \left(\frac{dv_{cx}}{dt} + \mathbf{w}_y v_{cz} - \mathbf{w}_z v_{cy} \right) &= G_x + P_x + X + F_{kx} \\
m \left(\frac{dv_{cy}}{dt} + \mathbf{w}_z v_{cx} - \mathbf{w}_x v_{cz} \right) &= G_y + P_y + Y + F_{ky} \\
m \left(\frac{dv_{cz}}{dt} + \mathbf{w}_x v_{cy} - \mathbf{w}_y v_{cx} \right) &= G_z + P_z + Z + F_{kz} \\
J_x \frac{d\mathbf{w}_x}{dt} + (J_z - J_y) \mathbf{w}_y \mathbf{w}_z &= M_{Px} + M_{Ax} + M_{kx} \\
J_y \frac{d\mathbf{w}_y}{dt} + (J_x - J_z) \mathbf{w}_z \mathbf{w}_x &= M_{Py} + M_{Ay} + M_{ky} \\
J_z \frac{d\mathbf{w}_z}{dt} + (J_y - J_x) \mathbf{w}_x \mathbf{w}_y &= M_{Pz} + M_{Az} + M_{kz}
\end{aligned} \tag{1}$$

These equations of motion are written in the body frame. The following equations are used to convert the position of the rocket from the body frame to the reference frame:

$$\begin{aligned}
\frac{d\mathbf{g}}{dt} &= \mathbf{w}_x + \mathbf{w}_y \sin \mathbf{g} \tan \mathbf{y} + \mathbf{w}_z \cos \mathbf{g} \tan \mathbf{y} \\
\frac{d\mathbf{q}}{dt} &= \mathbf{w}_y \sin \mathbf{g} / \cos \mathbf{y} + \mathbf{w}_z \cos \mathbf{g} / \cos \mathbf{y} \\
\frac{d\mathbf{y}}{dt} &= \mathbf{w}_y \cos \mathbf{g} - \mathbf{w}_z \sin \mathbf{g}
\end{aligned} \tag{2}$$

$$V_x = v_{cx} \cos \mathbf{q} \cos \mathbf{y} + v_{cy} (\cos \mathbf{q} \sin \mathbf{y} \sin \mathbf{g} - \sin \mathbf{q} \cos \mathbf{g}) + v_{cz} (\cos \mathbf{g} \sin \mathbf{y} \cos \mathbf{q} + \sin \mathbf{g} \sin \mathbf{q})$$

$$V_y = v_{cx} \sin \mathbf{q} \cos \mathbf{y} + v_{cy} (\sin \mathbf{q} \sin \mathbf{y} \sin \mathbf{g} + \cos \mathbf{q} \cos \mathbf{g}) + v_{cz} (\sin \mathbf{q} \sin \mathbf{y} \cos \mathbf{g} - \cos \mathbf{q} \sin \mathbf{g})$$

$$V_z = -v_{cx} \sin \mathbf{y} + v_{cy} \cos \mathbf{y} \sin \mathbf{g} + v_{cz} \cos \mathbf{y} \cos \mathbf{g}$$

In the trajectory code, the equations of motion are numerically integrated from an initial to a terminal set of state conditions.

The forces applied to the rocket during the flight consist of thrust, drag, lift, gravity and coriolis forces. The coriolis forces are very small and can be considered negligible. The force applied on the rocket is only the aerodynamic moment and the other forces pass through the center of mass. There is no controlling program in the sounding rocket and the aerodynamic force causes the rotation of the sounding rocket about its center of mass. The thrust forces are calculated as follows:

$$\begin{aligned}
 P_x &= m \cdot V_r + a_{ex} (P_{ex} - p) \\
 P_y &= 0 \\
 P_z &= 0
 \end{aligned} \tag{3}$$

In the above equation, the mass flow (m), velocity of burning product (V_{ex}), engine's area (a_{ex}) and the pressure in the exiting area of engine (P_{ex}) are read from propulsion for any section of engine. p is the atmospheric pressure which is read from the atmospheric model.

The aerodynamic forces are calculated as follows:

$$\begin{aligned}
 X &= -0.5(C_{xa} \cos \alpha - C_{ya} \sin \alpha) \rho a_{ex} v^2 \\
 Y &= -0.5(C_{xa} \sin \alpha + C_{ya} \cos \alpha) \rho a_{ex} v v_{cy} / (\sin \alpha) \\
 Z &= 0
 \end{aligned} \tag{4}$$

C_{xa} and C_{ya} are aerodynamic coefficients which are read from the aerodynamic area during the flight and for any selection of motor. ρ is the density of the atmosphere which is read from atmospheric model.

The pressure and density of atmosphere are read from standard table of atmosphere [20]. Then this data is used to create the atmospheric model. The model is written in MATLAB m.file and the data of the standard table is used to create a polynomial. The order of polynomial determines the accuracy of the model. Therefore, the model can be expressed in polynomials as functions of pressure and density of the atmosphere and in any altitude, and the pressure and density of atmosphere can be calculated from polynomials and sent to the trajectory equations.

By integrating the above equations of motion, the trajectory of rocket can be determined. For example, the trajectory of rocket for $\text{thr} = 12500[\text{N}]$ and burning time = 45.3[s] is shown in Fig. 2. The trajectory is shown at apogee point (Fig. 3).

3.2. Propulsion

The "RX-250-LPN" sounding rocket uses a solid fuel motor. The RSM described earlier is used to model the propulsion discipline. The design variables are the burning time and the thrust of the engine. First, the CCD method was used to statistically select the values of design parameter to be examined to adequately characterize the parameter space. For two design variables, nine combinations are determined, and thus nine experiments must be conducted. A solid propellant motor design software (SPRMD) which is developed at our laboratory is used to calculate the output variables of propulsion model at each design point. Table II shows the output variables obtained for each design point.

The data in Table II is used to create a response surface to obtain the output variables of the propulsion model as a mathematical function of the two input variables. Therefore, in every desired value of thrust and burning time, output variables can be computed. For example, a response surface of burning product's velocity versus design variables can be obtained. Regression techniques are used to fit the response surface over the data in Table II. This surface is shown in Fig. 4.

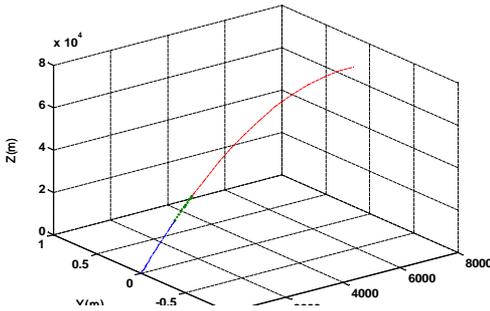


FIG. 3. Trajectory of the rocket at apogee point.

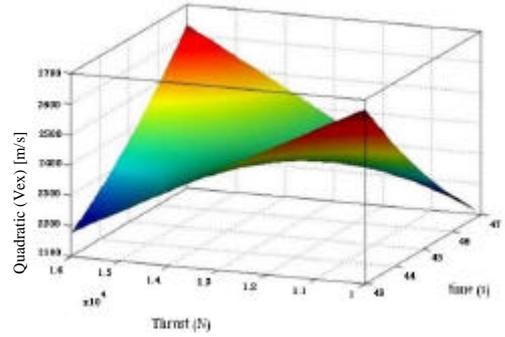


FIG. 4. Response surface of velocity.

The approximate mathematical model of velocity of burning products is as follows:

$$V_{ex} = 2358 + 9.027 \times thr - 27.92 \times t - 0.298 \times thr^2 + 256 \times thr \times t + 38.07 \times t^2 \tag{5}$$

As shown, this model for the velocity of burning product in the exit area of the motor is a function of two input variables individually, the interaction between them and the second order of two input variables. Figure 5 shows the predictive capability of the model. The predictive figure shows the difference between the actual and predicted data and the accuracy of fitting.

The standardized residual of fitting is:

$$\% \text{residual} = \frac{V_{ex}(\text{predicted}) - V_{ex}(\text{actual})}{100} \tag{6}$$

The standardized residual of velocity is show in Fig. 6.

As can be seen from Fig. 6, the maximum residual of fitting is 4%; therefore, the accuracy of fitting response surface is acceptable. The same method is used for the other output variables of propulsion model. The mathematical models of output variables are as follows:

$$P_{ex} = 18735.519 - 41.6318 \times thr + 2146.95 \times t + 2117.93 \times thr^2 - 12767.55 \times thr \times t - 1622.07 \times t^2$$

$$l_{mot} = 3.698 + 0.74 \times thr + 0.1815 \times t - 0.0965 \times thr^2 - 0.275 \times thr \times t - 0.1 \times t^2$$

Table II
Outputs of the motor design code

Thr	t_{burn}	V_r	a_{ex}	P_{ex}	m_{pr}	m_{mot}	l_{mot}
12000	44	2416	0.0397	15147.3	209.75	250.9	3.272
12000	46	2244.9	0.29	24482	237.66	276.9	3.608
14000	44	2367.5	0.0397	17197	243.98	322.52	3.917
14000	46	2367.1	0.0397	18020	255.1	344.39	4.07
10000	45	2363.1	0.0274	15796	185.9	255.54	2.87
16000	45	2360.6	0.0278	17165	263.2	351.6	4.31
13000	43	2404.4	0.0397	16124	218.5	262.1	3.42
13000	45	2406.4	0.0397	17486	238.9	284.2	3.72
13000	47	2405.6	0.0397	17159	228.8	273.9	3.56

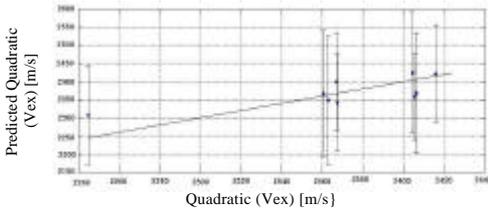


FIG. 5. Difference between actual and predicted data.

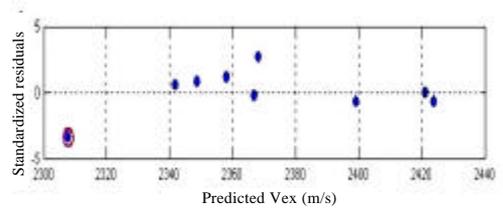


FIG. 6. Residuals of predicted points.

$$a_{ex} = 0.0385 + 0.00162 \times thr - 0.001783 \times t - 0.01098 \times thr^2 + 0.01605 \times thr \times t + 0.000983 \quad (7)$$

$$mmot = 295.33 + 70.54 \times thr + 15.345 \times t - 4.898 \times thr^2 - 6.195 \times thr \times t - 17.98 \times t^2$$

$$mpr = 236.44 + 38.67 \times thr + 13.3 \times t - 11.23 \times thr^2 - 25.185 \times thr \times t - 6.25 \times t^2$$

Hence, for any desired values of thrust and burning time, other engine parameters and their error can be calculated from the above equations. These output variables are fed to the trajectory and aerodynamic fields, which in turn affect the output variables and characteristics of the other areas.

3.3. Aerodynamic

The aerodynamic model calculates the aerodynamic coefficients and the center of pressure of the rocket. These coefficients are fed to trajectory model to calculate the aerodynamic forces and moments. The input variables to the aerodynamic discipline are Mach number, angle of attack and configuration of the rocket. Configuration characteristics of the rocket consist of payload shape, finsets, and motor size. The payload shape and finsets are considered constant, but the size of the motor is varied since the characteristics of engine change. The size of the motor includes the length of motor ($lmot$) and the exiting area of the motor, which are output variables from propulsion. The preliminary configuration (schematic) of the rocket is shown in Fig. 7.

As can be seen from Table II, there are different values of motor length and the exit area. Also, aerodynamic properties have to be estimated with different Mach numbers and angle of attack during the flight. In order to obtain these coefficients for any value of $lmot$ and a_{ex} and any conditions of flight, a 4D interpolation must be performed among different motor lengths, exit areas, angles of attack and Mach numbers.

The aerodynamic model must be analyzed for every combination of $lmot$ and a_{ex} (Table III). Therefore, the experiment on the aerodynamic model had to be performed 27 times to calculate the aerodynamic coefficients for selected configurations in Table III and for different flight conditions. The aerodynamic coefficients are functions of $lmot$, a_{ex} , \mathbf{a} and Mach number.

$$C_{xa}(lmot, a_{ex}, \mathbf{a}, Mach)$$

$$C_{ya}(lmot, a_{ex}, \mathbf{a}, Mach) \quad (8)$$

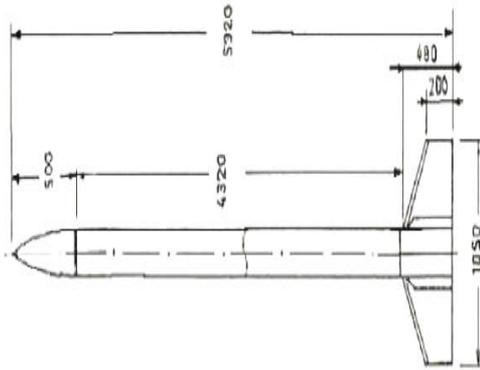


FIG. 7. Rocket configuration.

Table III
Combinations of exit area and length of motor for aerodynamic calculations

Step 1		Step 2		Step 3	
a _{ex}	l _{mot}	a _{ex}	l _{mot}	a _{ex}	l _{mot}
	2.87		2.87		2.87
	3.272		3.272		3.272
	3.42		3.42		3.42
	3.56		3.56		3.56
0.0274	3.608	0.0296	3.608	0.0397	3.608
	3.72		3.72		3.72
	3.917		3.917		3.917
	4.07		4.07		4.07
	4.31		4.31		4.31

For example, for Mach = 1, and $a_{ex} = 0.0274$, the drag coefficients with different angles of attack and motor lengths are as shown in Fig. 8. For L = 3.42 m, the drag coefficients with different motor areas and angles of attack are shown in Fig. 9.

For any selection of motor parameters with any flight condition, these coefficients can be obtained and used as input variables to the trajectory model.

4. Altitude optimization

For selected combination of two design variables in Table II, which are in conformity with CCD experiments, multidisciplinary analysis was performed and the altitude of the rocket was obtained. The results of the analysis are shown in Table IV. The selected values of a_1 and a_2 are relatively large such that a larger design space can be considered.

In order to obtain the best combination of design variables, a response surface must be fitted to the results obtained. The response surface of altitude versus thrust and burning time is shown in Fig. 10.

The approximate mathematical model of the altitude is:

$$H = 6161863 - 161813 \times thr - 437316 \times t + 6000 \times thr^2 - 299835 \times thr \times t + 109386 \times t^2. \quad (9)$$

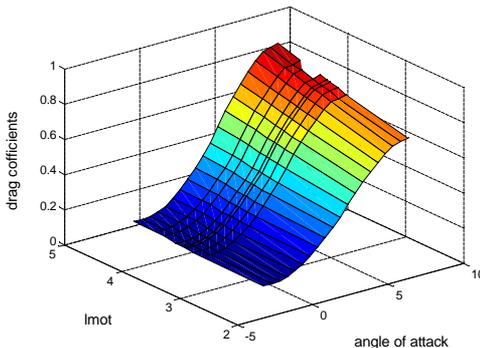


FIG. 8. Drag coefficients for Mach = 1, $d = 0.178$.

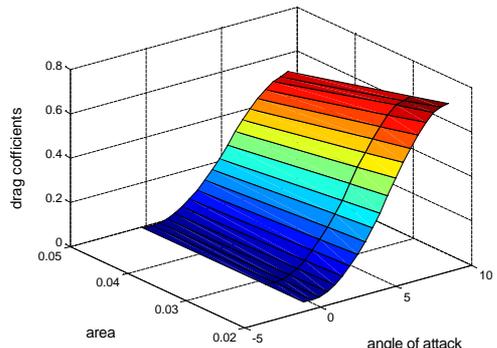


FIG. 9. Drag coefficients for Mach = 1, $l = 3.42$.

Table IV
Altitude of the rocket for selected values of design variables

Thr	<i>t</i>	<i>h</i> (altitude)
12000	44	67,637
12000	46	80,962
14000	44	52,428
14000	46	45,764
10000	45	79,189
16000	45	56,434
13000	43	71,368
13000	45	54,918
13000	47	63,918

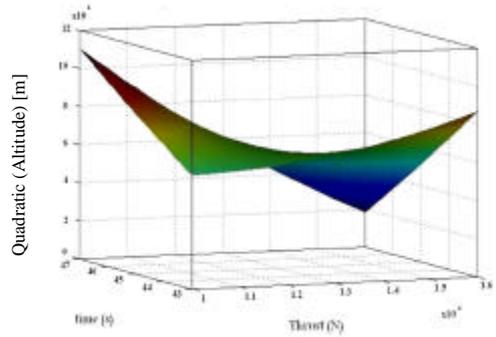


FIG. 10. Response surface of accessible altitude.

From the above equation, the altitude of the rocket can be obtained for any selection of motor with any value of thrust and burning time. The difference between the actual and the predicted altitudes, and the standardized residuals are shown in Figs 11 and 12, respectively.

A useful measure of the quality of a design is its prediction error variance (PEV). The PEV surface is an indicator of how capable the design is in estimating the response of the underlying model (Fig. 13). This shows where the response predictions are the best. This design predicts well in the center and the middle of the faces, but has the highest error in the corners. However, the corner errors are still small; the best way to improve the design is to delete the corner points. As can be seen from the figure, two corner points are deleted, and the error of fitting reduces from 7% to 4%.

After the response surface is determined, an optimization method is employed to determine the maximum point. The optimizer is a gradient method that must be started from an initial estimate. Therefore, an initial value must be guessed for the thrust and burning time. Figure 14 is a contour plot of the response surface, which facilitates guessing the initial value of the design variables.

As can be seen from the figure, the maximum altitude point is near two points: *Thr* = 15500, *T* = 43 and the other point is near *Thr* = 11000, *T* = 46. Therefore, the gradient method applies twice starting from these two points. The results of these two gradient methods are:

$$\begin{cases} thr = 15008, t = 43.6 \Rightarrow H_{\max} = 87,498 \\ thr = 10900, t = 46.4 \Rightarrow H_{\max} = 88,052 \end{cases}$$

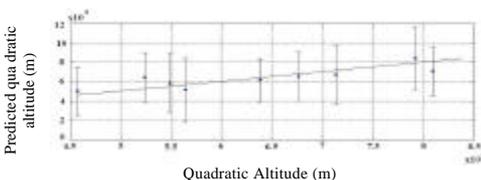


FIG. 11. Predicted altitude.

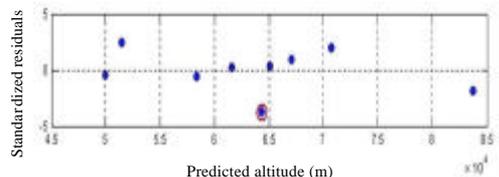


FIG. 12. Residuals of fitting.

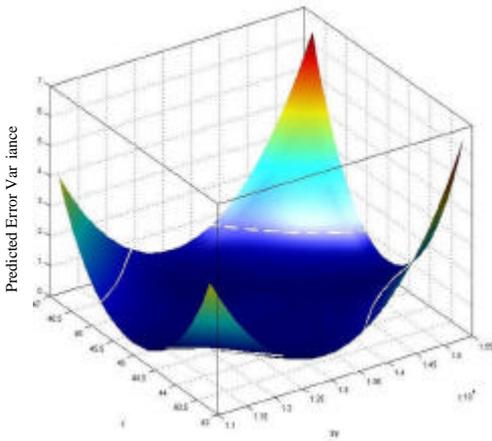


Fig. 13. Variance of predicted error.

By comparing the above results, $Thr = 10900$ and $t = 46.4$ are selected. By this combination of the design variables, the best altitude of the rocket is obtained. The optimum design solution is presented in Table V.

5. Conclusion

Multidisciplinary optimization method is applied to the “RX 250–LPN” sounding rocket to optimize the accessible altitude of the referred vehicle. Three fields—trajectory, propulsion and aerodynamics—contribute to the design. First, analysis tools are provided followed by the complicated engine design code, aerodynamic model for different configurations of the rocket and the trajectory model. These tools are provided by MATLAB and are written as m.file codes. Then the analyzed models are integrated and incorporated in a design structure matrix. This design process uses RSM for multidisciplinary optimization of the referred sounding rocket. RSM is applied at two stages: at propulsion model and at the system level. When the response surface is determined at the system level, an optimization method which is usually a gradient method is used to find the optimum result.

For any selection of motor and consequently any configuration of the motor, the aerodynamics and trajectory codes can be executed to compute the output parameters. Also, by using RSM, the altitude of the rocket can be obtained for every selection of the motor and can be expressed as a function of the design variable.

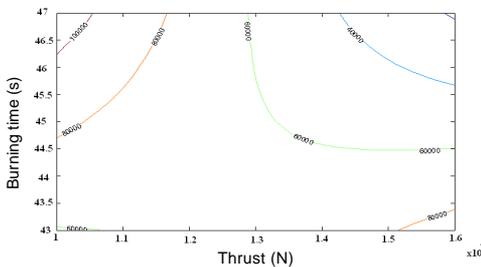


Fig. 14. Contour plot of altitude.

Table V
Optimum design point for motor

<i>thr</i>	10900
<i>t</i>	46.4
<i>lmot</i>	3.3451
<i>P_{ex}</i>	24691
<i>V_r</i>	2225.2
<i>a_{ex}</i>	0.0233
<i>mmot</i>	248.5
<i>mpr</i>	222.56

In the MDO of the rocket, the interaction among the three main fields is considered. By using RSM, the maximum altitude of the rocket trajectory increases from 70,000 to 88,000 m. By varying the motor of the rocket, the configuration of the rocket is varied. The thrust decreases from 12,500 to 10,900 N and the burning time increases from 45.3 to 46.4 s to optimize the altitude of the rocket.

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Nomenclature

m	= Total mass of the rocket
m_{pr}	= Mass of the fuel
m_{mot}	= Total mass of the motor
$\mathbf{n}_{cx}, \mathbf{n}_{ex}, \mathbf{n}_{cx}$	= Velocity of the rocket in body frame
$\mathbf{w}_x, \mathbf{w}_y, \mathbf{w}_z$	= Angular velocity of the rocket in body frame
V_x, V_y, V_z	= Velocity of the rocket in reference frame
x, y, z	= Position of the rocket in reference frame
$\mathbf{g}, \mathbf{q}, \mathbf{y}$	= Angular position of the rocket in reference frame
J_x, J_y, J_z	= Inertial moment of the rocket
G_x, G_y, G_x	= Components of gravity force
P_x, P_y, P_z	= Components of thrust force
X, Y, Z	= Components of aerodynamic force
F_{kx}, F_{ky}, F_{kz}	= Components of Coriolis force
M_{Ax}, M_{Ay}, M_{Az}	= Components of aerodynamic moment
M_{Px}, M_{Py}, M_{Pz}	= Components of thrust moment
M_{kx}, M_{ky}, M_{kz}	= Components of Coriolis moment
\dot{m}	= Mass flow
P_{ex}	= Pressure in the exiting area of engine
V_r	= The velocity of burning product
a_{ex}	= Exit area of motor
p	= Pressure of atmosphere
\mathbf{r}	= Density of atmosphere