

# Numerical investigations and validation of hyperbolic heat conduction model applied to fast precooling of a slab food product

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Received on December 6, 2005; Revised on November 2, 2006.

## Abstract

One-dimensional hyperbolic heat-conduction equation with heat generation in fast precooling process of slab food product has been developed. The internal heat generation due to respiration is a nonlinear term in temperature and depends on the type of a given product. The equation obtained is then solved numerically using finite-difference techniques. A parametric study is conducted to illustrate the influence of the heat-flux relaxation time and the Biot number by comparison of the solution between the hyperbolic and the Fourier conduction models. The calculated results show that the hyperbolic model must be applied for higher values of relaxation time. Also, increasing the Biot number tends to quicken the cooling process and this will lower the value of the applicable relaxation time.

**Keywords:** Hyperbolic conduction model, fast precooling, food, heat generation, numerical analysis.

## 1. Introduction

When food is stored at ambient temperature, moisture loss and wilting take place. Precooling rapidly removes heat immediately after harvest or slaughtering so that the temperature is quickly brought to cold storage level. This enables in selecting smaller-size heat-transfer equipment for cold storage. Different types of precooling techniques are used to lower the temperature of food to the required value, such as air-, ice- or hydro-cooling [1, 2]. In view of the heat and mass transfer, the precooling process occurs both due to convective heat transfer and moisture evaporation when air blast cooling is considered. For packed foods with impermeable packing containers, cooling is found to be effected only by convective heat transfer. A number of investigators have discussed the precooling of food. Major works have been reported by Gaffney *et al.* [3], Gowda *et al.* [4], Ansari and Afaq [5], Ansari [6], and Mokhtar and Abbas [7].

Mathematical models suggested for predicting food precooling characteristics are based on Fourier's theory of heat conduction, which yields a parabolic equation for the temperature field. This equation implies an infinite propagation speed of the thermal signal.

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Rastegar *et al.* [8], Kaminski [9] and Mitra *et al.* [10] have shown that for heat transfer in materials with nonhomogeneous inner structures, such as biological materials, the heat flux equilibrates to the imposed temperature gradient via a relaxation phenomenon characterized by a thermal relaxation or thermal characteristic time. This is the time necessary for accumulating the thermal energy required for propagative heat transfer to a particular point in the medium. Kaminski [9] estimates this thermal relaxation time for meat products to be in the order of 20–30 s and for materials such as glass ballotini, sand, H acid, etc. in 10–50 s range. Mitra *et al.* [10] have measured a value of  $15.5 \pm 2.1$  s in processed bologna meat. Such large values of the characteristic time suggest that in heat conduction processes that occur for time periods of the order of thermal relaxation time, significant non-Fourier behavior is expected. Furthermore, in situations dealing with transient heat flow in extremely short periods of time or at very low temperatures, the classical heat-diffusion theory breaks down, because the wave nature of heat propagation becomes dominant. In turn, understanding such bioheat transfer processes may result in some interesting applications in food heat treatment and conservation, laser applications in medicine and many others.

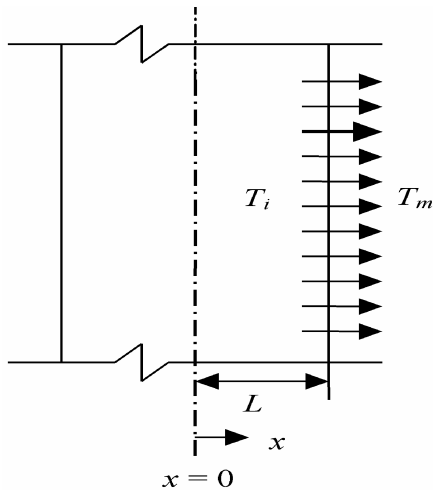
This heat-conduction model has been tested experimentally by Peshkov [11] to ensure the existence of thermal waves using superfluid liquid helium near absolute zero. Mitra *et al.* [10] showed experimentally that transient heat conduction in biological materials (i.e. meat) is accurately described by a hyperbolic heat transfer model rather than the parabolic Fourier model. Lu *et al.* [12] have simulated thermal wave propagation in biological tissue by the dual reciprocity boundary element method. A preliminary survey on the mechanisms of the wave-like behavior of heat transfer in living tissue was conducted by Liu [13]. Cattaneo [14] and Vernotte [15] have proposed a modified constitutive equation for the heat-flux density to account for the finite propagation speed of the thermal wave. This equation yields the hyperbolic wave energy equation where the temperature field propagates with finite velocity.

In this work, the hyperbolic conduction model is applied to simulate fast precooling process of slab-shaped food product; the internal heat generation due to respiration of the food is also included. The pure convective heat transfer at the boundary as well as with simultaneous heat and moisture transfer are considered. The governing equations were solved numerically by a MacCormack predictor–corrector finite-difference method. The effect of Biot number and thermal relaxation time on the center temperature is analyzed.

## 2. Problem formulation

Consider an infinite slab of a food product with half width ( $L$ ) and  $x$ , the axial coordinate, is taken as the slab center line. Figure 1 shows the schematic diagram and the coordinate system of the problem under consideration. The initial slab-shaped food temperature  $T_i$  is higher than the cold medium temperature  $T_m$ . In the non-Fourier model, to account for the finite propagation speed of the thermal wave, the heat flux has the form suggested by Cattaneo [14] and Vernotte [15] as:

$$q_x + \tau \frac{\partial q_x}{\partial t} = -k \frac{\partial T}{\partial x}, \quad (1)$$



**Table I**  
**Constants in eqn (3) in the range 0–27°C [3]**

Food product	A	B
Apple	$4.59 \times 10^{-6}$	2.66
Peaches	$1.37 \times 10^{-6}$	3.88
Brussels sprouts	$4.87 \times 10^{-6}$	2.47

FIG. 1. Physical model and coordinate system.

where  $q_x$  is the heat flux in  $x$  direction,  $T$ , the temperature,  $t$ , the time,  $k$ , the thermal conductivity, and  $\tau$ , the thermal relaxation time. In eqn (1), heat flux relies on both its rate of change and the temperature gradient. Introducing this modified heat-flux equation into the equation of energy conservation yields:

$$\rho C_p \frac{\partial t}{\partial t} = -\frac{\partial q_x}{\partial x} + q''' \tag{2}$$

Eliminating  $q$  between eqns (1) and (2) leads to the hyperbolic heat conduction (HHC) equation as:

$$\tau \frac{\partial^2 T}{\partial t^2} + \frac{\partial T}{\partial t} = \alpha \frac{\partial^2 T}{\partial x^2} + \frac{q'''}{\rho c} \tag{3}$$

Equation (3) represents the temperature field which propagates with finite speed  $v = \sqrt{\alpha/\tau}$ , where  $\alpha$  is the thermal diffusivity and  $q'''$ , the internal heat respiration of the food per unit volume. The heat respiration is defined by nonlinear temperature-dependent equations as given in Gaffney *et al.* [3].

$$q''' = \rho A(T + 17.8)^B \tag{4}$$

where  $T$  is the food product temperature in °C, and  $A$  and  $B$  are constants for a given food product. Table I gives these constants for foods indicated at a temperature range of 0–27°C.

For the food geometry studied, and when the food is initially at a uniform temperature and under the assumption that symmetrical cooling occurs, the initial and center boundary conditions can be written as:

$$\begin{aligned} T(X, 0) = T_i, \quad q_x(x, 0) = 0 \quad \text{at} \quad t = 0, \quad 0 \leq x \leq L \\ q_x(0, t) = 0 \quad \text{at} \quad t > 0, \quad x = 0. \end{aligned} \tag{5}$$

The pure convective boundary condition is defined at the surface as:

$$q_x(L, t) + \tau \frac{\partial q_x(L, t)}{\partial t} = -k \frac{\partial T(L, t)}{\partial x} = h(T(L, t) - T_m) \text{ at } t > 0, x = L. \tag{6}$$

It is more convenient to transform the governing equations, (1) and (2), into a dimensionless form by introducing the following dimensionless variables

$$\left. \begin{aligned} \theta &= \frac{T - T_m}{T_i - T_m}, & X &= \frac{x}{L}, & t^* &= \frac{\alpha}{L^2} t, & \phi_x &= \frac{q_x L}{kk(T_i - T_m)} \\ \tau^* &= \frac{\tau \alpha}{L^2}, & Q''' &= \frac{L^2}{k(T_i - T_m)} q''' \end{aligned} \right\} \tag{7}$$

The set of equations, (1) and (2), is expressed in terms of the dimensionless variables (7) as:

$$\frac{\partial \theta}{\partial t^*} + \frac{\partial \phi_x}{\partial X} - Q''' = 0, \tag{8}$$

$$\tau^* \frac{\partial \phi_x}{\partial t^*} + \frac{\partial \theta}{\partial X} + \phi_x = 0. \tag{9}$$

The governing equations, (8) and (9), are subject to the following initial and boundary conditions:

$$\begin{aligned} \theta(X, 0) = 1, \phi_x(X, 0) = 0 & \text{ at } t^* = 0, \quad 0 \leq X \leq 1 \\ \phi_x(0, t^*) = 0 & \text{ at } t^* > 0, \quad X = 0 \\ \frac{\partial \phi_x(1, t^*)}{\partial t^*} + \phi_x(1, t^*) = Bi\theta & \text{ at } t^* > 0, \quad X = 1 \end{aligned} \tag{9}$$

where  $Bi$  is the Biot number ( $Bi = hL/k$ ), and  $Q'''$ , the dimensionless heat generation due to food respiration. By using the nonlinear representation of heat generation which is given in eqn (4),  $Q'''$  can be written as:

$$Q''' = \frac{\rho L^2}{k \Delta T} A(\theta \Delta T + T_m + 17.8)^B. \tag{10}$$

The surface boundary condition defined in eqn (6) can be modified to involve the effect of simultaneous heat and moisture transfer. This boundary condition is improved and used by Ansari [6]. In this paper, the improved boundary condition will be used. It is given as:

$$\frac{\partial \phi_x(1, t^*)}{\partial t^*} + \phi_x(1, t^*) = 1.75Bi \theta \text{ at } t^* > 0, X = 1. \tag{11}$$

It is worth noting that eqn (11) is an approximation since the latent heat transfer is not directly proportional to the temperature difference.

### 3. Solution methodology

The governing equations, (8) and (9), subjected to the initial and boundary conditions (10), are solved numerically by the MacCormack predictor–corrector method [16]. This method

is an explicit finite-difference method of second-order accuracy and is very effective in solving problem with a moving discontinuity. Equations (8) and (9) can be written in the vector form as:

$$\frac{\partial \mathbf{E}}{\partial t^*} + \frac{\partial \mathbf{F}}{\partial X} + \mathbf{H} = 0, \tag{12}$$

where

$$\mathbf{E} = \begin{bmatrix} \theta \\ \phi_x \end{bmatrix}, \mathbf{F} = \begin{bmatrix} \phi_x \\ \theta \end{bmatrix}, \mathbf{H} = \begin{bmatrix} -Q''' \\ \phi_x \end{bmatrix}. \tag{13}$$

Using the MacCormack corrector–predictor technique, the finite-difference formula of eqn (12) can be written as

Predictor:

$$\hat{\mathbf{E}}_i^{n+1} = \mathbf{E}_i^n - \frac{\Delta t^*}{\Delta X} (\mathbf{F}_{i+1}^n - \mathbf{F}_i^n) - \Delta t^* \mathbf{H}_i^n. \tag{14a}$$

Corrector:

$$\mathbf{E}_i^{n+1} = \frac{1}{2} \left[ \mathbf{E}_i^n + \hat{\mathbf{E}}_i^{n+1} - \frac{\Delta t^*}{\Delta X} (\hat{\mathbf{F}}_{i+1}^{n+1} - \hat{\mathbf{F}}_i^{n+1}) - \Delta t^* \hat{\mathbf{H}}_i^{n+1} \right], \tag{14b}$$

where subscript  $i$  denotes the spatial grid point, superscript  $n$ , the time interval level and the hat the predicted value at the next time level ( $n + 1$ ). The finite-difference scheme given in eqn (14) is stable for  $\Delta t^* / \Delta X \leq 1$  [16]. Most calculations were conducted using 1000 space steps and 1500 time intervals. In the early stages of the solution,  $t^* \leq \Delta t^*$ , the time interval was taken 20 times shorter than the basic ones to reduce the oscillation of the solutions.

Equations (8) and (9) without heat generation ( $Q''' = 0$ ) can be solved semi-analytically by the Laplace transformation method. Eliminating  $\phi_x$  between eqns (8) and (9) yields:

$$\tau^* \frac{\partial^2 \theta}{\partial t^{*2}} + \frac{\partial \theta}{\partial t^*} = \frac{\partial^2 \theta}{\partial X^2} + Q'''. \tag{15}$$

The initial and boundary conditions associated with the equation set (8)–(9) are:

$$\begin{aligned} \theta = 1, \quad \frac{\partial \theta}{\partial t^*} = 0 \quad \text{at} \quad t^* = 0, \quad 0 \leq X \leq 1 \\ \frac{\partial \theta}{\partial X} = 0 \quad \text{at} \quad t^* > 0, \quad X = 0 \\ \frac{\partial \theta}{\partial X} = -1.75Bi \theta \quad \text{at} \quad t^* > 0, \quad X = 1. \end{aligned} \tag{16}$$

With the notation that  $L\{\theta(X, t^*)\} = W(X, S)$ , eqn (15) can be written as:

$$\frac{d^2 W}{dX^2} - (\tau^* S^2 + S)W = -(1 + S). \tag{17}$$

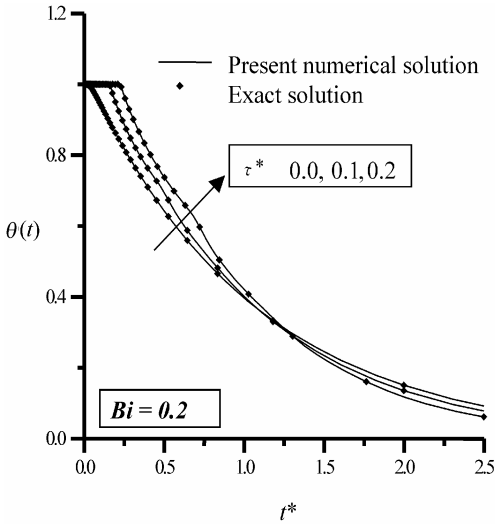


FIG. 2. Current numerical results and exact solution of the center temperature history at  $Bi = 0.2$ .

The Laplace transform of the boundary conditions becomes:

$$\begin{aligned}
 W(0, S) &= 0 \\
 \frac{dW}{dX}(1, S) &= -1.75BiW.
 \end{aligned}
 \tag{18}$$

The solution of eqn (13) is:

$$W(X, S) = C_1 e^{mX} + C_2 e^{-mX} + \frac{1}{S},
 \tag{19}$$

where  $C_1, C_2$  are the constants of integration and  $m = \sqrt{\tau^* S^2 + S}$ .

Applying the boundary conditions in eqn (18) to determine the constants of integration and using the Riemann-sum approximation as:

$$\theta_k(X, t^*) \cong \frac{e^{\lambda t^*}}{t^*} \left[ \frac{1}{2} W_k(X, \lambda) + \text{Re} \sum_{n=1}^N W_k \left( X, \lambda + \frac{in\pi}{t^*} (-1)^n \right) \right].
 \tag{20}$$

In eqn (20),  $\text{Re}$  represents the real part of the summation and  $i = \sqrt{-1}$ . The quantity  $\lambda t^* = 5.1$  gives the most satisfactory results [17]. Equation (20) yields the exact temperature distribution.

The computer code developed in this work was validated by comparing the results obtained by the present numerical method with the exact solution given in eqn (20). Figure 2 shows the exact solution of temperature history at the center of the food slab along with the output of the computer code. The results show excellent agreement with the exact solution without heat generation; this has established confidence in the numerical results reported here.

#### 4. Results and discussion

Precooling methods investigated previously considered the Fourier conduction model despite the fact that the duration of precooling was the theme of such studies [4, 5]. Decreasing the duration of precooling means that the method is effective over other methods.

In this paper, we study the application of the hyperbolic conduction model in fast precooling of food product, which occurs in a short period of time. This purpose is achieved by comparing the transient center temperature distribution obtained by the hyperbolic and Fourier conduction models.

To investigate the influence of the hyperbolic conduction model on the fast precooling of slab food product, physical parameters which appear in the governing equation and boundary conditions need to be studied well. The parameters are the dimensionless heat-flux relaxation time  $\tau^*$  and  $Bi$ . Heat generation due to food respiration is considered a parameter varying with temperature. Constants in eqn (10) are assigned such values for the purpose of numerical calculation. Constants A and B in eqn (10) depend upon the type of product and the values are consistent (Table I) with those given by Gaffney *et al.* [3]. In this investigation, we consider apple as the food product. Thus, after assigning the numerical value of apple from Table I in eqn (10), it becomes:

$$Q''' = 1.52e - 5(30\theta + 12.8)^2. \quad (21)$$

The heat generation given by eqn (10) is the nonlinear term and can be seen in the governing equations. It depends mainly on the food product type. This study is focused on the application of non-Fourier heat conduction in fast precooling of a given food product (apple). For different food products, a similar analysis is conducted with different constants (A and B).

Figure 3(a) shows a set of dimensionless center temperature variation curves with dimensionless time.  $Bi$  is equal to 0.2 and the heat-flux relaxation time has the values  $\tau^* = 0.1, 0.2, 0.3, 0.4, 0.5, 1.0, 2.0, 5.0$ . Clearly, this figure exhibits a transient behavior typical to rapid cooling of bodies subjected to surface cooling and provides a comparison between the Fourier and the hyperbolic conduction models. It can be seen that the applicability of the hyperbolic conduction model depends on the value of the relaxation time. The new variation term of heat diffusion in relaxation process is the added term in the Fourier conduction model. If a zero value is assigned to it, the two models become the same. For higher values of  $\tau^*$ , wave characteristics of the conduction model appear. At that point, the presently used model, Fourier conduction model, is not widely applicable. In Fig. 3(b), and for the case of precooling considered here, the cutoff value which makes a noticeable difference between the models studied is  $\tau^* = 1$ . Above this value, hyperbolic conduction model must be considered.

Figure 3(b) shows another set of center temperature distribution at  $Bi = 2$ , which means that the Biot number will increase the cooling effect at the boundary and decrease the duration of the cooling process. The steady-state time reached ends the precooling process and emphasizes the effect of the Biot number on the applicability of the hyperbolic model for higher values. The appearance of the wavy conduction model is noticed at a lower value of relaxation time ( $\tau^* = 0.1$ ) (Fig. 3(b)).

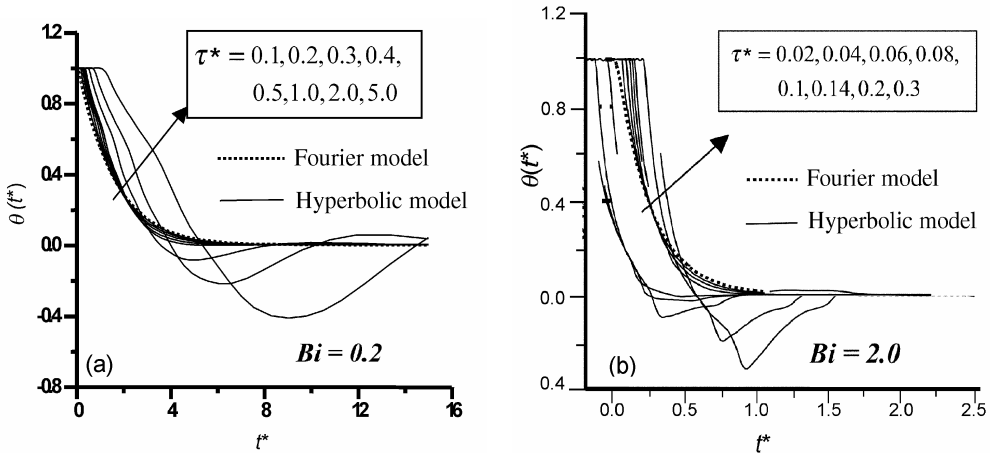


FIG. 3. Center temperature history at (a)  $Bi = 0.2$  and (b)  $Bi = 2.0$ .

## 5. Conclusions

In this paper, a novel hyperbolic heat-conduction equation with heat generation is applied to simulate fast precooling process of slab food product. The internal heat generation due to respiration is a nonlinear term in temperature and depends on the type of food product and conditions. It was found that the hyperbolic conduction model should be considered in fast precooling of food products because of the higher values of their relaxation time. The effect of the Biot number is to fasten the cooling process, and to emphasize on the applicability of the hyperbolic conduction model even for lower values of relaxation time.

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## Nomenclature

$A, B$	=	Constants
$Bi$	=	Biot number
$c$	=	Specific heat of the food [J/kg K]
$h$	=	Surface film conductance [W/m <sup>2</sup> K]
$k$	=	Thermal conductivity of product [W/mK]
$L$	=	Half length of the slab product [m]
$q$	=	Heat flux density [W/m <sup>2</sup> ]
$q^m$	=	Heat of respiration of the food [W/m <sup>3</sup> ]
$Q^m$	=	Dimensionless heat of respiration
$t$	=	Time [s]
$T$	=	Temperature [K]
$\Delta T$	=	Temperature difference ( $T_i - T_m$ )
$x$	=	Distance from center [m]
$X$	=	Dimensionless coordinate

### Greek letters

$\alpha$	=	Thermal diffusivity [m <sup>2</sup> /s]
$\phi_x$	=	Dimensionless heat flux in the $x$ -direction
$\theta$	=	Dimensionless temperature
$\rho$	=	Food density [kg/m <sup>3</sup> ]
$\tau$	=	Thermal relaxation time [s]
$\tau^*$	=	Dimensionless thermal relaxation time

### Subscripts

$i$	=	Initial
$m$	=	Cooling medium

### Superscripts

*	=	Dimensionless quantity
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