# DIELECTRIC-COATED SPHERICALLY-TIPPED CONDUCTING CONE ANTENNA EXCITED IN THE UNSYMMETRIC HYBRID MODE AT MICROWAVE FREQUENCIES 

R. Chatteriee and T. S. Vedavathy<br>(Department of Electrical Communicanon Emsineerny, Induan hasware of Sitcha Ronquane 5(0012, Inda

Received on May 16,1975 and in tevised fomm on Aupusi 25.195


#### Abstract

An approximate theory has been derived for the radiated field and gain of the dielectric-couted splurically-thped conducting cone antenna excited in the whymmetric hybrid mode, assuning a given distribution of swfuse currents on the antenna. The calculated radiation patterns and gain of a large number of antennas of varying dimensions have been veriffed by experiment in the $X$-band. The measured input impedance of the antennas has been reported.


## 1. Introduction

In 1950, Schorr and Beck [1] studied the problem of electromagnetic radiation from a conducting conical horn. Significant contributions have been made by Felsen [2-6] to the problem of scattering of electromagnetic waves by conducting conical structures and also to the problem of radition from tapered surface wave antennas [7]. The radiation characteristics of a semi-infinite conducting cone antenna, as well as that of a finite cone has been studied by Adachi [8,9], as review of the radiation characteristics of the conical structures has been given by Wait [10]. As far as the authors are aware of, there has been no work on dielectric-coated conducting coneantennas till 1964. In 1964, Yeh [11] has introduced the use of Sommerfeld's complex-order wave functions in deriving theoretically the radiation characteristics of a dielectric-coated spherically-tipped semi-infinite conducting cone excited in the symmetric TM mode, but there is no experimental verification. Chatterjee [12] has made a rigorous theoretical analysis of the electromagnetic boundary-value problem of the dielectric-coated spherically-tipped semi-infinite conducting cone, and has shown that symmetric TE, symmetric TM and symmetric and unsymmetric hybrid modes
can be supported by such a structure, but that unsymmetric TE or TM modes cannot be supporied. Subsequently the radiation characteristics of finite length dielectric-coated spherically-tipped conducting cone antennas have been studied theoretically by an approximate method by the authors and verifed experimentally [13, 14].

In this paper an approximate theory for the radiated field and gain of the finite length dielectric-coated spherically-tipped conducting antenna excited in the unsynmetric hybrid mode has been derived, assuming a given distribution of surface currents on the antenna. The calculated radiation patterns and gain of a large number of antennas of varying dimensions as well as the assumed surface current distribution have been verified by experiment. The measured input impedance of the antennas has also been reported,

## 2. Geometry of the Structure

Figure 1 shows the geometry of the structure. The antenna consists of a dielectric-coated spherically-tipped conducting cone $A^{\prime} A O B B$ '. This


Fig. 1
antenna is excited by a mode transformer which is a $T E_{11}$ mode circular cylindrical waveguide. The $T E_{11}$ mode in the circular metal waveguide is transformed into the first unsymmetric hybrid mode in the dielectric-coated spherically-tipped conducting cone. The spherical polar coordinates of any point on the stracture are desiguated ( $r^{\prime}, \theta^{\prime}, \phi^{\prime}$ ), while those of a distant point are designated $(r, \theta, \phi)$.
3. Estimate of the Surface Currents on the Antenna

The determination of the radiated field requires a knowledge of the field components and hence the surface currents on the surface of the antenna. The field distributions on the surface of the antenna are determined by utilizing the field components of the $T E_{11}$ mode in the circular cylindrical conducting waveguide which acts as a launcher and also from a knowledge of the near field variations along the coordinate directions $r^{\prime}, \theta^{\prime}, \phi^{\prime}$ obtained by measurements with the help of appropriate field probes.

The components of the electromagnetic field inside a hollow metal circular waveguide progpagating the dominant $T E_{x 1}$ mode are:

$$
\begin{align*}
& H_{z}=C_{1} J_{1}(h \rho) \cos \phi \exp j(\omega t-\bar{\beta} z)  \tag{1}\\
& H_{\rho}=-j{ }_{h}=-\bar{\beta} C_{1} J_{1}^{\prime}(h \rho) \cos \phi \exp j(\omega t-\bar{\beta} z)  \tag{2}\\
& H_{\phi^{\prime}}=j \overline{h_{1}^{\prime 2}} C_{1} J_{1}(h \rho) \sin \phi \exp j(\omega t-\bar{\beta} z)  \tag{3}\\
& E_{\rho^{\prime}}=\frac{\omega \mu_{0}}{\bar{\beta}} H_{\phi^{\prime}}  \tag{4}\\
& E_{\phi^{\prime}}=-\bar{\beta} \mu_{0} H_{\rho}  \tag{5}\\
& E_{z}=0 \tag{6}
\end{align*}
$$

where

$$
\begin{align*}
& h=\begin{array}{c}
1 \cdot 84, \\
a
\end{array}, \quad a \text { ' being the radius of the cylinder, and } \\
& \bar{\beta}=\sqrt{ } \omega^{2} \mu_{0} \epsilon_{0}-\overline{h^{2}} \text {. } \tag{7}
\end{align*}
$$

The measurements of the near field of the antenna show that all the field components vary as $\cos \phi$ or $\sin \phi$ (Fig. 2). Hence thefield components inside the cylindrical waveguide as given by equations (1) to (6) may be approximately assumed to be transformed to $H_{r^{\prime}}, H_{\theta^{\prime}}, H_{\phi^{\prime}}, E_{r^{\prime}} E_{\theta^{\prime}}$ and $E_{\phi^{\prime}}$ in the dielectric coated spherically-tipped conducting cone according to the following transformation:

$$
\begin{align*}
& H_{\gamma}=H_{z} \cos \theta+H_{\rho} \sin \theta  \tag{8}\\
& H_{\theta^{\prime}}=-H_{z} \sin \theta+H_{p} \cos \theta  \tag{9}\\
& H_{\phi^{\prime}}=H_{\phi} \tag{10}
\end{align*}
$$

and similar equations for $E_{\tau^{\prime}}, E_{\theta^{\prime}}$, and $E_{\phi^{\prime}}$.
The tangential electric field components on the inner surface of the conducting cylindrical waveguide is zero. Hence it is assumed thai the tangential electric field components on the outer surface (iii) of the dieelectric coating (Fig. 1) are negligible compared to the tangential magnetic field components. The tangential electric field components on the spherical metal tip (i) are zero. Hence it is assumed that only the surface electric currents due to the tangential magnetic fields on the surface of the antenna contribute to the radiated field at a distant point. The spherical tip (i) being very small compared to the cone, the contribution of surface electric currents on it to the radiated field are neglected in comparison with the contribution of the surface electric currents on surface (iii) of the dielectric-coated metal cone. Thus
only $\quad \vec{u}_{\phi^{\prime}} J_{\phi^{\prime}}=-\vec{n} \times \vec{u}_{r^{\prime}}, H_{r^{\prime}}$
and $\quad \vec{u}_{r^{\prime}} J_{r^{\prime}}=-\vec{n} \times \vec{u}_{\phi^{\prime}} H_{\phi^{\prime}}$
are considered to be the predominant components of surface electric current on the surface (iii) which contribute to the radiated field.

From equations (1) to (10), $H_{r^{\prime}} \alpha \cos \phi^{\prime}$ and $H_{\phi^{*}} \alpha \sin \phi^{\prime}$, and thus $\left|H_{\mathbf{r}^{\prime}}\right|$ and hence $\left|J_{\phi^{\prime}}\right|$ may be taken as $\alpha R e \exp \left(j \phi^{\prime}\right)$, and $\left|H_{\phi^{\prime}}\right|$ and hence $\left|J_{r^{\prime}}\right|$ may be taken as $\alpha I_{m} \exp \left(j \phi^{\prime}\right)$.

The following assumption for the variation of the field components $H_{r^{\prime}}$ and $H_{\phi^{\prime}}$ with the space coordinates and time and hence for the surface currents $J_{\phi^{\prime}}$ and $J_{r^{\prime}}$ respectively can be made;

$$
\begin{align*}
J_{\phi^{\prime}}= & -H_{r^{\prime}}=K_{1} \exp _{k_{2} r^{\prime}}(j \omega t) \\
& \left.+R \exp \left(-j k_{2} r^{\prime}\right)\right] \operatorname{Re}\left(j k_{2} r^{\prime}\right)  \tag{11}\\
J_{3^{\prime}}= & \left.-H_{\phi^{\prime}}=-\frac{K_{2} \exp \left(j \phi^{\prime}\right)}{k_{2} r^{\prime}}\right)\left[\exp \left(j k_{2} r^{\prime}\right)\right. \\
& \left.+R \exp \left(-j k_{\mathbf{1}} r^{\prime}\right)\right] I m \exp \left(j \phi^{\prime}\right) \tag{12}
\end{align*}
$$

where the constants $K_{1}$ and $K_{2}$ are determined from equations (8) and (9), knowing the field components $H_{\rho}, H_{\phi^{\prime}}, H z$ inside the cylindrical conducting waveguide which excites the dielectric-coated spherically-tipped conducting pons.


Fis 2 Weriation of power ( $H_{r}^{2}$ ) with $\phi$ in the vieinty of the seriol $\cdots \cos ^{2} \phi$ (Theoretical), $\ldots H_{r}^{2}$ Experimentoil

The nature of variation of the field components as given by equations 11) and (12) are justified by experimental results (Figs. 3 and 4).
$k_{2}=2 \pi / \lambda_{g}$, where $\lambda_{g}$ is the measured guide wavelength in the dielectrucoated spherically-tipped conducting cone and is twice the average distance


Fio. 3. Standmg wave pattern along the surface of the aerial,
$\rightarrow$ Calculated.

-     - -- Experimental:


FIG. 4. Standing wave pattern along the surface of the aerial.
between maxima or mmima of the standing-wave pattern on the surface of the sirture. $R$ is the masured reflection coefficient in the structure and it is evaluated by using the fomula

$$
R=\frac{V S W R-1}{V S W R+1}
$$

where $V S W R$ is the average $V S W R$ as obained from the curves shown in Figs. 3 and 4. Equations (11) and (12) show that $J_{\phi}$ and $J_{r}$ both consist of two travelling spherical waves one in the positive $r^{\prime}$ direction and the other in the negative $r^{\prime}$ direction.

## 4. Theoretical Study of the Radiated Field

In the derivation of the expressions for the components of the radiated field at a distant point $P$ due to $J_{\phi}$ and $J_{\tau^{\prime}}$, it will be convenient to assume that $J_{\phi^{\prime}}$ and $J_{r^{\prime}}$ vary as $\exp \left(J \phi^{\prime}\right)$ and in the final expressions the real or imaginary parts may be laken as the case may be.

The distance $r^{\prime \prime}$ (Fig. 1) of the distant point $P(r, \theta, \phi)$ from a typical surface current element $d \Sigma$ on the conical surface (iii) situated at a point ( $r^{\prime}, \theta^{\prime}, \phi^{\prime}$ ) is given by

$$
\begin{align*}
r^{\prime \prime} & =r-r^{\prime} \cos \theta_{1} \cos \theta-r^{\prime} \sin \theta, \sin \theta \cos \left(\phi-\phi^{\prime}\right) \\
& =r-z \cos \theta-\rho \sin \theta \cos \left(\phi-\phi^{\prime}\right) \tag{13}
\end{align*}
$$

where

$$
d \Sigma=r^{\prime} \sin \theta d \phi^{\prime} d r^{\prime}
$$

Using Schelkunoff's Equivalence Principle, the magnetic and electric vector potential $\vec{A}$ and $\vec{F}$ at the distant point $P$ are given by

$$
\begin{align*}
& \vec{A}=\frac{1}{4 \pi} \int_{\Sigma} \frac{\vec{I} \exp \left(-j k_{0} r^{\prime \prime}\right)}{r^{\prime \prime}} d \Sigma  \tag{14}\\
& \vec{F}=\frac{1}{4 \pi} \int_{\Sigma} \frac{\vec{M} \exp \left(-j k_{0} r^{\prime \prime}\right)}{r^{\prime \prime}} d \Sigma \tag{15}
\end{align*}
$$

where $\vec{J}$ and $\vec{M}$ are the surface currents on surface $\Sigma$. Figure 5 shows the surface $\Sigma$ which consists of surfaces (i), (iii) and the outer surface of cylindrical conducting waveguide together with a very large sphere. Only the components $J_{\phi^{\prime}}$ and $J_{r^{\prime}}$ of the surface electric current on surface (iii) are


FIG. 5. Surface $\Sigma$ used in the application of Schelkunoff's equivalence principle.
predominant, while the magnetic currents are negligible. Since surface (i) is very small compared to surface (iii), the contribution of the surface currents on (i) is negligible. The contribution of the surface currents on the rest of the surface $\Sigma$ may be neglected.

Due to the surface current $J_{\phi^{\prime}}$ on surface (iii) as given by equation (11), the $x$ and $y$ components of the magnetic vector potential $\vec{A}_{1}$ at the distant point $P$ are given by

$$
\begin{align*}
& A_{x_{1}}=K_{1} \exp (j \omega t) \int_{\phi^{\prime}=0}^{2_{0}^{2 \pi}} \int_{r^{\pi}=0}^{r_{0}} \frac{\exp \left(-j k_{0} r^{\prime \prime}\right)}{r^{\prime \prime}} \\
& \times\left[\frac{\exp \left(j k_{2} r^{\prime}\right) \pm R \exp \left(-j k_{2} r^{\prime}\right)}{\overrightarrow{k_{2}} r^{\prime}}\right] \\
& \times \exp \left(j \phi^{\prime}\right)\left(\sin \phi^{\prime}\right) r^{\prime} \sin \theta_{1} d \phi^{\prime} d r^{\prime} \\
& =-K_{1} \exp \left\{j\left(\omega t-k_{0} r\right)\right\} \sin \theta_{1} \\
& \times \int_{r^{2}=a}^{n \theta} 2 \pi j\left[\dot{J}_{2}\left(k_{0} r^{\prime} \sin \theta_{j} \sin \theta\right) \exp (j 2 \dot{\phi})\right. \\
& \left.+J_{0}\left(k_{0} r^{\prime} \sin \theta_{1} \sin \theta\right)\right]\left[\exp \left\{\left(j k_{0} r^{\prime}\right)\left(\frac{k_{2}}{k_{0}}+\cos \theta_{1} \cos \theta\right)\right\}\right. \\
& \left.+R \exp \left\{\left(-j k_{0} r^{\prime}\right)\left(\frac{k_{2}}{k_{0}}-\cos \theta_{1} \cos \theta\right)\right\}\right] d r^{\prime}  \tag{16}\\
& A_{\boldsymbol{Y}_{1}}=K_{1} \exp (j \omega t) \int_{\phi^{\prime=}}^{2 \pi} \int_{r^{\prime}=a}^{r_{0}} \frac{\exp \left(-j k_{0} r^{\prime \prime}\right)}{r^{\prime \prime}} \\
& \times\left[\exp \left(j k_{2} r^{\prime}\right)+R \exp \left(-j k_{2} r^{\prime}\right)\right] \\
& \times \exp \left(j \phi^{\prime}\right)\left(\cos \phi^{\prime}\right) r^{\prime} \sin \theta_{1} d \phi^{\prime} d r^{\prime} \\
& =\frac{K_{1} \exp \left\{j\left(\omega t-k_{\theta} r\right)\right\} \sin \theta_{1}}{\overline{k_{2} r}} \\
& \times \int_{r^{\prime}=a}^{r_{0}} 2 \pi\left[-\exp (2 j \phi) J_{2}\left(k_{0} r^{\prime} \sin ^{\prime} \theta_{1} \sin \theta\right)\right. \\
& \left.+J_{0}\left(k_{0} r^{\prime} \sin \theta_{1} \sin \theta\right)\right]\left[\exp \left\{\left(j k_{0} r^{\prime}\right)\left(\frac{k_{2}}{\widetilde{k}_{0}}+\cos \theta_{1} \cos \theta\right)\right\}\right. \\
& +R \exp \left\{( - j k _ { 0 } r ^ { \prime } ) \left(\begin{array}{l}
\left.\left.\left.k_{2}-\cos \theta_{1} \cos \theta\right)\right\}\right] d r^{\prime}, ~ \\
\tilde{k}_{0}
\end{array}\right.\right. \tag{17}
\end{align*}
$$

where

$$
\begin{aligned}
& k_{0}=\frac{2 \pi}{\lambda_{0}}=\omega \sqrt{\mu, \epsilon_{0}}, \lambda_{0} \text { being the free-space wavelength } \\
& k_{2}=2 \pi, \lambda_{g} \text { being the measured guide wavelength } \\
& r_{0}=\text { finite length of the dielectric-coated spherically-tipped conduct- } \\
& \text { ing cone antenna } \\
& a=r a d i u s \text { of the spherical conducting tip. }
\end{aligned}
$$

$\because$ The $\theta$ and $\phi$ components of the electric field intensity $\vec{E}_{1}$ at the distant point are given by

$$
\begin{align*}
& E_{\theta_{1}}=\left(E_{x_{2}} \cos \phi+E_{y_{1}} \sin \phi\right) \cos \theta  \tag{18}\\
& E=\left(-E_{x_{1}} \sin \phi+E_{y_{i}} \cos \phi\right) \tag{19}
\end{align*}
$$

where

$$
\begin{align*}
& E_{x_{1}}=-j \omega \mu A_{x_{x}} \\
& E_{y 1}=-j \omega \mu A_{y_{1}} \tag{20}
\end{align*}
$$

Because $\left|J_{\phi^{\prime}}\right|=\left|H_{r^{\prime}}\right| \propto \cos \phi^{\prime}=\operatorname{Re} \exp \left(j \phi^{\prime}\right)$, taking the real parts of the final expressions, $E_{\theta_{1}}$ and $E_{\phi}$, are given by

$$
\begin{align*}
E_{\theta_{1}}= & \frac{K_{1}^{\prime} \exp \left\{j\left(\omega t-k_{0} r^{\prime}\right)\right\} \sin \theta_{1}}{r} \\
& \times \int_{r^{\prime}}^{r} \cos \theta\left[J_{0}\left(k_{0} r^{\prime} \sin \theta_{1} \sin \theta\right) \sin \phi\right. \\
& \left.-\sin (3 \phi) J_{2}\left(k_{0} r^{\prime} \sin \theta_{1} \sin \theta\right)\right] \\
& \times\left[\exp \left\{\left(j k_{0} r^{\prime}\right)\left(\frac{k_{2}}{k_{0}}+\cos \theta_{1} \cos \theta\right)\right\}\right. \\
& \left.+R \exp \left\{\left(-j k_{0} r^{\prime}\right)\left(\frac{k_{2}}{k_{0}}-\cos \theta_{1} \cos \theta\right)\right\}\right] d r^{1}  \tag{21}\\
E_{\phi_{1}}= & \frac{K_{1}^{\prime} \exp \left\{j\left(\omega t-k_{0} r\right)\right\} \sin \theta_{1}}{r}
\end{align*}
$$

$$
\times \int_{r=0}^{f_{0}}\left[J_{0}\left(k_{0} r^{\prime} \sin \theta_{1} \sin \theta\right) \cos \theta\right.
$$

$$
\left.-\cos (3 \phi) J_{2}\left(k_{0} r^{\prime} \sin \theta_{1} \sin \theta\right)\right]
$$

$$
\times\left[\exp \left\{\left(j k_{0} r^{\prime}\right)\left(\frac{k_{2}}{k_{0}}+\cos \theta_{1} \cos \theta\right)\right\}\right.
$$

$$
\begin{equation*}
\left.4 R \exp \left\{\left(-j k_{0} r^{\prime}\right)\left(\frac{k_{2}}{k_{0}}-\cos \theta_{1} \cos \theta\right)\right\}\right] d r^{*} \tag{22}
\end{equation*}
$$

### 4.4. 80

where

$$
\begin{equation*}
K_{\imath}^{\ell}=\frac{-j \omega_{\mu} 2 \pi K_{1}}{k_{2}} \tag{23}
\end{equation*}
$$

Now, considering the component $J_{r_{1}}$ of the surface electric current density on surface (iii), $\left|J_{r^{\prime}}\right| \equiv\left|H_{\phi^{\prime}}\right|$ and $\left|H_{\phi^{\prime}}\right| \propto \sin \phi^{\prime}=\operatorname{lm} \exp \left(j \phi^{\prime}\right)$. At diametrically opposite points $A$ and $A^{\prime}$ on the circle which is a cross-section of the conical surface (iii) $\theta=\theta_{1}$ (see Fig. 1), $J_{r}$ is equal in magnitude but opposite in direction. Because of this fact, the components $J_{r^{\prime}}$ at these diametrically opposite points subtract in the $z$ direction but add in the $\rho$ direction in the $x-y$ plane. Hence it is necessary to consider only the contribution of the $\rho$ component $J_{r^{\prime}} \sin \theta_{1}$ of $J_{r^{\prime}}$ in calculating the far field.

The components $E_{\theta,}$ and $E_{\phi,}$ of the electric field intensity $\vec{E}_{2}$ due to $J_{r}$, at the distant point $P$ are then given by

$$
\begin{align*}
E_{\theta_{2}}= & \frac{K_{2}^{\prime}}{r} \exp \left\{j\left(\omega t-k_{0} r\right)\right\} \sin ^{2} \theta_{1} \int_{r^{\prime}=\pi}^{r_{0}} \cos \theta^{\prime} \\
& {\left[-\sin (3 \phi) J_{2}\left(k_{0} r^{\prime} \sin \theta_{1} \sin \theta\right)-\sin \phi J_{0}\left(k_{0} r^{\prime} \sin \theta_{1} \sin \theta\right)\right] } \\
& {\left[\exp \left\{\left(j k_{0} r^{\prime}\right)\left(\frac{k_{2}}{k_{0}}+\cos \theta_{1} \cos \theta\right)\right\}+R \exp \left\{\left(-j k_{0} r^{\prime}\right)\right.\right.} \\
& \left.\left.\times\left(\frac{k_{2}}{k_{0}}-\cos \theta_{1} \cos \theta\right)\right\}\right] d r^{\prime} \tag{24}
\end{align*}
$$

and

$$
\begin{align*}
E_{\phi_{\mathrm{y}}}= & \frac{K_{2}^{\prime}}{} \exp \left\{j\left(\omega t-k_{0} r\right)\right\} \sin ^{2} \theta_{1} \int_{0}^{\gamma_{0}} \\
& \times\left[-\cos (3 \phi) J_{2}\left(k_{0} r^{\prime} \sin \theta_{1} \sin \theta\right)-\cos \phi J_{0}\left(k_{0} r^{\prime} \sin \theta_{1} \sin \theta\right)\right] \\
& \times\left[\exp \left\{\left(j k_{0} r^{\prime}\right)\left(\begin{array}{l}
k_{2} \\
k_{0}
\end{array}+\cos \theta_{1} \cos \theta\right)\right\}+R \exp \left\{( - j k _ { 0 } r ^ { \prime } ) \left(\frac{k_{2}}{k_{0}}\right.\right.\right. \\
& \left.\left.\left.-\cos \theta_{1} \cos \theta\right)\right\}\right] d r^{\prime} \tag{25}
\end{align*}
$$

where

$$
\begin{equation*}
K_{2}^{1}=-j \omega \mu 2 \pi K_{2} \tag{26}
\end{equation*}
$$

The total electric field intensity $E$ has $\theta$ and $\phi$ components given by

$$
\begin{equation*}
\boldsymbol{E}_{\theta}=E_{\theta_{1}}+E_{\theta_{3}} \tag{27}
\end{equation*}
$$

and

$$
E_{\phi}=E^{\phi}+E_{\phi_{\phi}}
$$

5. Radiation Patterns in the Principal $\phi=0^{\circ}$ and $\pi=90^{\circ}$ Planes In $\phi=0^{a}$ Plane,

$$
E_{\theta_{1}}=E_{\theta_{2}}=0
$$

Hence the normalized radiation power pattern is given by

$$
\left|\begin{array}{c}
E  \tag{28}\\
E_{\max }
\end{array}\right|^{2}=\left\lvert\, \begin{aligned}
& \left.f_{2}(\theta)\right|^{2} \\
& \left.f_{2}(\theta)\right|_{\max } ^{2}
\end{aligned}\right.
$$

where

$$
\begin{align*}
& \begin{aligned}
& f_{2}(\theta)= \sin \theta_{1} \int_{r^{n}}^{r_{0}}\left[J_{0}\left(k_{0} r^{\prime} \sin \theta_{1} \sin \theta\right)\right. \\
&\left.-J_{2}\left(k_{0} r^{\prime} \sin \theta_{1} \sin \theta\right)\right]\left[\operatorname { e x p } \left\{\left(j k_{0} r^{\prime}\left(\frac{k_{2}}{k_{0}}+\cos \theta_{1} \cos \theta\right)\right\}\right.\right. \\
&+R \exp \left\{\left(-j k_{0} r^{\prime}\right)\left(\frac{k_{2}}{k_{0}}-\cos \theta_{1} \cos \theta\right)\right\} d r^{\prime} \\
& 90^{\circ} \text { Plane, } \\
& E_{\phi_{k}}=E_{\phi_{*}}=0 .
\end{aligned}
\end{align*}
$$

In $\phi=90^{\circ}$ Plane,

Hence the normalized radiation power pattern is gven by

$$
\begin{equation*}
|E E|^{2} \left\lvert\, E_{\max }^{2}=\frac{\left|f_{1}(\theta)\right|^{2}}{\left|f_{2}(\theta)\right|_{\max }^{2}}\right. \tag{30}
\end{equation*}
$$

where $f_{1}(\theta)$ is given by

$$
\begin{align*}
f_{1}(\theta)= & \sin \theta_{1} \int_{r_{0}}^{r_{0}}\left[J_{0}\left(k_{0} r^{\prime} \sin \theta_{1} \sin \theta\right)+J_{2}\left(k_{0} r^{\prime} \sin \theta_{1} \sin \theta\right)\right] \\
& \times\left[\exp \left(j k_{0} r^{\prime}\right)\left(\frac{k_{2}}{k_{0}}+\cos \theta_{1} \cos \theta\right)+R \exp \left(-j k_{0} r^{\prime}\right)\right. \\
& \left.\times\left(\frac{k_{2}}{k_{0}}+\cos \theta_{1} \cos \theta\right)\right] \mathrm{dr}^{\prime} \tag{31}
\end{align*}
$$

## 6. Total Power Radiated and Directive Gain

The total power radiated is

$$
W=\int_{\phi=0}^{2 \pi} \int_{\theta=0}^{\pi} \frac{\left(E_{\theta}^{2}+E_{\phi}^{2}\right)}{\eta} r^{2} \sin \theta d \theta d \phi
$$

where

$$
\eta_{1}^{7}=\frac{\mu_{0}}{\epsilon_{0}}
$$

and the directive gain of the antenna is given by

$$
\begin{equation*}
G=\frac{4 \pi\left[r^{2}\left(E_{\theta}^{2}+E_{\phi}^{2}\right)\right]_{\max }}{\eta W} \tag{33}
\end{equation*}
$$

The final expressions for $W$ and $G$ are given by

$$
\begin{align*}
& W=\frac{C_{1}{ }^{2}}{r^{2}} \frac{\pi}{\eta} \int_{\theta=0}^{\pi}\left[\left\{\left|f_{3}(\theta)\right|\right\}^{2}+\left\{\left|f_{4}(\theta)\right|\right\}^{2}\right] \\
& \left(1+\cos ^{2} \theta\right) \sin \theta d \theta \tag{34}
\end{align*}
$$

and

$$
\begin{equation*}
G=\frac{4\left[\left|f_{2}(\theta)\right|\right]_{\max x+\theta=0}^{2}}{\int_{-\infty}^{\pi}\left\{\left|f_{3}(\theta)\right|^{2}+\left|f_{4}(\theta)\right|^{2}\right\}\left(1+\cos ^{2} \theta\right) \sin d \theta} \tag{35}
\end{equation*}
$$

where

$$
\begin{align*}
f_{\mathbf{3}}(\theta)= & \sin \theta_{1} \int_{r_{1}=0}^{f_{0}}\left[J_{0}\left(k_{\theta} r^{\prime} \sin \theta_{1} \sin \theta\right)\right] \\
& \times\left[\operatorname { e x p } \left\{\left(j k_{\theta} r^{\prime}\right)\left(\frac{k_{2}}{k_{0}}+\cos \theta_{1} \cos \theta\right)\right.\right. \\
& \left.+R \exp \left\{\left(-j k_{0} r^{\prime}\right)\left(\frac{k_{2}}{k_{0}}+\cos \theta_{1} \cos \theta\right)\right\}\right] d r^{\prime} \tag{36}
\end{align*}
$$

and

$$
\begin{align*}
f_{4}(\theta)= & \sin \theta_{1} \int_{r^{\prime}=a}^{0}\left[J_{0}\left(k_{0}^{\prime} r^{\prime} \sin \theta_{1} \sin \theta\right)\right] \\
& \times\left[\exp \left\{\left(j k_{0} r^{\prime}\right)\left(\frac{k_{2}}{k_{0}}+\cos \theta_{1} \cos \theta\right)\right\}\right. \\
& \left.+R \exp \left\{\left(-j k_{0} r^{\prime}\right)\left(\frac{k_{2}}{k_{0}}-\cos \theta_{1} \cos \theta\right)\right\}\right] d r^{\prime} \tag{37}
\end{align*}
$$

## 7. Numerical Computations of the Radiation Patterns and

 Directive GainThe radiation patterns in the $\phi=0^{\circ}$ and $\pi=90^{\circ}$ planes and the gain have been numerically computed for several antennas of varying dimensions using equations (28), (30) and (35). The results are given in Figsd 6 to 12 together with the experimental ourves:


Fig. 6. Caloulated and measured H-piane radiation pattern of the delectric coated metal cone anterma.

## 8. Experimental Investigations

The following experimental investigations on the antennas have been carried out as a function of the length and taper angles:
(a) Study of the near field components of the electric and magnetic field.
(b) Study of the radiation patterns.
(c) Measurement of the gain of the antennas by the (i) reflection [15]. and (ii) comparison methods.
I.I.Sc. -4
(d) Study of the impedance trransforming characteristics of the mode transducer used to launch the HE mode on the antenna by measurement of scattering coefficients [16].
(e) Study of the input impedance of the antennas.

Some of the results of the experimental investigations are given in Figs. 6 to 15 .


Fig. 7.
EXPERIMENTAL

- THEORETICAL

9. Discussion of the Theoretical and Experimental Results

## (a) Near Field

The near field measurements made on a number of antennas to study the variation of the field components verify
(i) the assumption that the field components vary as $\cos \phi^{\prime}$ (Fig. 2)
(ii) the assumptions given by equations (11) and (12) is approximately correct (Figs. 3 and 4).

Dielectric-Coated Spherically-Tipped Conducting Cone Antenna


Fig. 8. Calculated and observed beam widths of the major lobe - Calculated, .... Experimental.
(iii) The hybrid HE mode is excited (as shown by the existence of all the field components $E_{r^{\prime}}, E_{\theta^{\prime}}, E_{\phi^{\prime}}, H_{r^{\prime}}, H_{\theta^{\prime}}, H_{\phi^{\prime}}$.
(b) Radiation Patterns

Figures 6 to 12 show the theoretical and experimental results of the radiation patterns. It may be observed that:


Fig. 9. Caiculated and observed positions and relative inter ities of the side lotcs. - Calculated, .... Observed ist lobe, $\times \times$ Observed 2nd lobe.
(1) The positions of the major lobe and the first two side lobes which could be experimentally observed agree fairly well with theory.
(ii) There is fair agreement between the theoretical and experimental beam widths and positions of the nulls.


Fxc. 10. Calculated and observed positions and relative intensities of the side lobes.
(iii) The first side lobe is 14 to 20 dbs and the second side lobe is 20 to 30 dbs below the major lobe, while the higher order side lobes are more han 25 db below the major lobe. It may therefore be said that this type If antenna yields a strong axial major lobe and well suppressed side lobes;


Fro. 11. Caleulated and observed positions and relative intensities of the side lober.




4. EAPERIMENTAL -THEORETICAL

$$
\text { ( ) } 2 N D \text { SIDE LOBE } \quad x \times \text { IST SIDE TOBE }
$$

Fig. 12
(c) Gain

Figures 13 and 14 show the calculated and experimental values of the gains of the antennas. It may be observed that:
(i) For longer antennas the theoretical gain agrees with the experimental values obtained by both methods.
(ii) For shorter antennas the theoretical gain agrees better with the experimental value obtained by the comparison method than by the reflection method. This fis possibly |due to the fact that due to high reflection coefficient the reflection method cannot be expected to give correct results.
(d) Figures 15 and 16 show a plot of the impedance characteristics of the antenna as a function of length. The almost constant input impedance characteristics of the antenna having different taper angles and different lengths shows that the antenna possesses broad-band characteristics,


Fig. 13. Calculated and observed gain of the aerials.
-. Theoretical
. . . Experineatal (Reflection method)
$\times \times$ Experimental (Comparison method)

## 10. CONCLUSIONS

As a result of the theoretical and experimental investigations on the dielectric-coated spherically-tipped conducting cone antennas it may be concluded that:
(i) The antenna excited in the unsymmetric hybrid mode behaves like an end fire antenna having a narrow major lobe and a large number of minor lobes of very low intensities.


Fic. 14. Calculated and observed gain of the arials.

- Theoretical
.... Experimental (Reflection method)
$\times \times$ Experinentaĩ (Comparison method)
(ii) The beam width of the major lobe can be minimized and the gain of the antennas maximized by proper choice of the angles $\theta_{0}$ and $\theta_{1}$ of the cones for a given length $r_{0}$ of the antema.
(iii) The antenna has broad-band characteristics as shown by the radiation and impedance characteristics.

More details of the work are contained in [17].


Fry 15 Variation of relative input impedance $\sqrt{R_{i}^{2}+x_{1}^{2}}$, the resistive
port $R_{f}$ and the reactive part $x_{1}$ $=\sqrt{R_{1}^{2}+\bar{x}_{1}^{2}},+$ Resistive port $H_{1}, x \times$ Reactive port $x_{1}$


Fig 16 Variation of ralative input impedance $\sqrt{R_{2}^{2}+x_{2}^{2}}$, the resistive
part $R_{2}$ and the reactive part $x_{2}$

$-\sqrt{R_{2}^{2}+x_{2}^{2}}, \quad$ Resistive part $R_{2} . x \times$ Reactive port $x_{2}$

## ACKNOWLEDGEMENTS

The authors wish to express their gratitude to Dr. S. Dhawan, Director of the Indian Institite of Science, Bangalore, for providing facilities for carrying out the investigations. They are also grateful to the Council of Scientific and Industrial Research for the sanction of a research scheme on the above subject.

## REFERENCES

[1] Schorr, M. G. and Beck, F. J.
${ }^{4}[2]$ Feisen, L. B.
[3] Felsen, 工 B.
[4] Felsen, L. B. . Plane-wave scattering by small-angle cones, R.R.E. Trants, Voll. AP-5, p. 121.
[5] Felsen, L. B. .. Radiation from ring soutces m the presence of a semiinfinite cone, I.E.E.E. Trans., 1959, Vol. AP-7, p. 168.
[6] Felsen, L. B. .. Electromagnctic properties of vedge and cone surfaces with a linearly varying surface impedance, IEE.E. Traps., 1959, Vol. AP-7. Sp. Suppl., p. S231.
.. Radiation from taperen surface wave antenna, T.E.E.E. Trans., 1960, Vol. AP-8, p. 577.

A theoretical analysis of semi-nfinte conical antenna, f.E.E.E. Trans. 1960, Vot. AP-8, p. 534.

The fimite conical antenna, Y.E.E.E. Trans, 1959, Vol. AP-7 Sp. Suppl., p. S406.

- Electromagnetic Radiotion from Conical Structures. Chap, 12, pp. 483, Antenna Theory, Part I, Ecited by R. E. Collin and F. J. Zucker, McgrawiHill Book Co., N.Y.
. An application of sommorfeld's complex ozder wave. functions to an antenna problem, Journ. Math. Phys., 1964,5 (3), 344.
.. Radiation from a dielectric-coated, spherically-tripped perfectly conducting conts, Journ. Ind. Inst. Sct., 1970, 52 (1), 48.

Dielectric-coated sphericallymtipped metal cone antennas excited in unsymmetric mode at microwave frequencies, Journ. Asia Electronics Union, No. 2, Serial No, 16, 1972, pp. 45-49.
[14] Chatterjee, R. and Vedavathy, T. S.
[15] Barlow, H. M. and Cuhen, A. I.
[16] Deschamps, O. A.
[17] Vedavathy, T. S.

Dielectric-coated ipherically-fipped metal cone antennas excited 1sk wisymmetric hybric mode at microwave fiequencies, 2-11 D3, Proc. Internationtal Symposium on Antennas and Propagation held at Tohokn University, Sendai, Japan, September. 1-3. 1971.

Microntave Measurements, Consiable and Co., Lta., Eondon.
.. Determination of reflection coefficient and imsertion loss of a wavegride junction, Joum. Appl. Phys., 1953. 24, (8), pp. 1006.
.. Dielctruccoated Spheric.lly-tipped Metal Cone Aerials Exeited in the Unsymmetric IHyarid Mode, A thesis submitted for the PhD. degree of the Indian Institite of Scionce, Bangalore, India, 1972.

# Calendar of events: Conferences/Symposia at the Indian Institute of Science Campus 

| Sl. Name of the School | Period | Sponsoring Department <br> of the Institute |
| :--- | :---: | :---: |
| No. | November to <br> December 1975 | Chemical Engineering |
| 1. Lecture Course on Cavitation | 24th November to | School of Automation |
| 2. QIP-"Organization and Logical Design | 6 December 1975 |  |

On the basis of the information received by the Editorial Office on 30th October 1975.

