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Centralized PI/PID controllers for nonsquare systems with RHP zeros

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Abstract

Two centralized controller-tuning methods, Davison's method [*Multivariable tuning regulators:* The feed forward and robust control of general servo mechanism problem, *IEEE Trans. Auto. Control*, **21**, 35–21 (1976)] and Tanttu and Lieslehto method [A comparative study of some multivariable PI controller tuning methods, in *Intelligent tuning and adaptive control*, Pergamon Press, pp 357–362 (1991)], are extended to nonsquare systems with right half-plane zeros. These methods have been applied to two examples with right half-plane zeros in the individual scalar elements—coupled pilot plant distillation columns (2 outputs and 3 inputs) and a crude distillation process (4 outputs and 5 inputs). Simulation studies have been carried out for these examples for both servo and regulatory problems. For the coupled pilot plant distillation column example, the proposed methods are compared with the robust decentralized controller design method proposed by Loh and Chiu [Robust decentralized control of nonsquare systems, *Chem. Engng Commun.*, **158**, 157–180 (1997)]. The performance of square and nonsquare controllers is compared for the crude distillation process example.

Keywords: Nonsquare system, centralized controller, decentralized controller, RHP zero.

1. Introduction

Processes with unequal number of inputs and outputs often arise in the chemical process industry. Such nonsquare systems may have either more outputs than inputs or more inputs than outputs. Nonsquare systems with more outputs than inputs are generally not desirable, as all of the outputs cannot be maintained at a set point since it is overspecified. The control objective in this case is to minimize the sum of square errors of the outputs with the given fewer inputs. For these systems, robust performance (with no offset) is impossible to achieve due to the presence of an inevitable permanent offset that results in at least one of the outputs [1].

More frequently encountered in the chemical industry are nonsquare systems with more inputs than outputs. Here, better control can be achieved by redesigning the controller eliminating the steady-state offsets. Examples of nonsquare systems are mixing tank process [1], 2×3 system, Shell standard control problem [2], 5×7 system, crude distillation unit [3], 4×5 system, etc. A common approach towards the control of nonsquare processes is to first 'square up' or 'square down' the system through the addition or removal of ap-

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propriate inputs (manipulated variables) or outputs (controlled variables) to obtain a square system matrix. Then the multivariable control design methods can be employed to achieve design specifications, but none of these alternatives is desirable. Adding unnecessary outputs to be measured can be costly, while deleting inputs leaves fewer variables to be automatically manipulated in achieving the desired control. Similarly, reducing the number of measured outputs decreases the amount of feedback information available to the system, and arbitrarliy adding new manipulated inputs can incur unnecessary cost. Hence, if superior performance can be achieved by the original nonsquare system, it is preferable to squaring the system [4].

Loh and Chiu [4] have extended the independent design procedure for robust decentralized controllers proposed by Hovd and Skogestad [5] to nonsquare systems with more inputs than outputs. The proposed design method is applied to a nonsquare mixing tank example, which is a simple system (i.e. there are no RHP zeros and time delays) with the first-order elements. From the simulation results they have concluded that nonsquare systems should be controlled in their original state instead of squaring them by adding or deleting the variables. The design procedure is complicated and the obtained controller is not of the conventional form (i.e. not of PI or PID form).

In multi-input and multi-output system, interaction and location of transmission zero are important. The system with one or more right half-plane transmission (RHPT) zeros is called non-minimum phase system. These RHP zeros impose limitations on stability and controllability of the system. They affect both the amplitude and phase angle. The extra phase lag that is added by the RHP zero contributes to the instability and makes the control difficult. So the controller design for having positive zeros is of greater concern in the present work. A few reported methods are available for non-minimum phase systems to design multivariable square systems, which involve complicated control strategies and lengthy calculations. Though several rigorous methods are available for designing multivariable PI controllers [6], simple methods are preferable. Simple tuning methods are available to design the multivariable centralized controllers for minimum phase system such as (i) Davison's method [8], and (ii) Tanttu and Lieslehto method [7]. Dinesh *et al.* [9] have shown that Davison's method gives good performance for square non-minimum phase system. Shaji and Chidambaram [10] have used the method for the square-down crude distillation, a 4×4 system. In the present work, these two simple methods are extended to nonsquare systems with RHP zeros.

2. Centralized controller design methodology

2.1. *Davison's method*

Davison [7] has proposed a centralized multivariable PI controller tuning method for square systems. Here the proportional and integral gain matrices are given by

$$K_C = d\,[G(S = 0)]^{-1} \quad (1)$$

$$K_I = e\,[G(S = 0)]^{-1} \quad (2)$$

where $[G(S = 0)]^{-1}$ is called the rough tuning matrix, and d and ε are the finetuning parameters, which generally range from 0 to 1. In the present work, this method is extended to nonsquare system. As inverse does not exist for nonsquare system, Moore–Penrose pseudo-inverse is used. For matrix A, Moore–Penrose pseudo-inverse is

$$A^{\dagger} = A^{H} (A^{*} A^{H})^{-1}. \tag{3}$$

A^{H} is the Hermitian matrix of A. So for nonsquare system, PID controller gains are:

$$K_C = d\,[G(S = 0)]^{\dagger} \tag{4a}$$

$$K_I = e\,[G(S = 0)]^{\dagger} \tag{4b}$$

$$K_D = \quad [G(S = 0)]^{\dagger}. \tag{4c}$$

2.1.1. *Calculation of tuning parameters*

The system is not stable for the entire region (0 to 1) of tuning parameters in the Davison's method. In the present method, the tuning parameters (d, e, g) are calculated as follows: First the characteristic equation, det $(I + KG)$, is obtained. For a 2×2 system, characteristic equation can be obtained easily but for higher-order systems packages like *symbolic math* (Matlab toolbox) have to be used. Then, Routh stability criterion is applied to find the range of the tuning parameters for which the system is stable. The system is then simulated by tuning around these values and finetuning parameters are chosen based on performance.

Usually for design of controllers for SISO systems with a positive zero, only the stable invertible portion is considered. That is, the numerator dynamics due to positive zero is neglected. For square MIMO systems, system positive zero should be separated and this should not be considered for the controller design. In the Davison's method, steady-state gain matrix only is considered. As such no change or modification is required for the Davison's method for the system with positive zero. In the present work, we try to check how the Davison's method works for such systems.

2.2. *Tanttu and Lieslehto method*

Morari and Zafiriou [11] have discussed a method for the design of PI controller known as internal model control (IMC). Tanttu and Lieslehto [7] have developed a multivariable PI controller tuning method based on IMC principles. First, PID controller ($k_{c,ij}$) for each of the scalar transfer functions ($g_{p,ij}$) of the process is designed based on the IMC method. For a first-order time delay (FOPTD) system

$$k_{ij} k_{cij} = (2t_{ij} + L_{ij})/2\,l \tag{5a}$$

$$t_{Iij} = (t_{ij} + 0.5 L_{ij}). \tag{5b}$$

Here k_{ij} and L_{ij} are the gain and time delay of an element in the process model for the i^{th} output and j^{th} input. k_{cij} and t_{Iij} are the proportional gain and integral time constant of the IMC controller of the ij^{th} loop. In this method, the same filter constant is used for each of the scalar systems so that there is only one tuning parameter (l). Then the multivariable PI controllers can be designed as follows:

$$R_c = \begin{bmatrix} 1/R_{c11} & 1/R_{c12} \dots & 1/R_{c1n} \\ 1/R_{c21} & 1/R_{c22} \dots & 1/R_{c2n} \\ \dots & \dots & \dots \\ 1/R_{cm1} & 1/R_{cm2} & 1/R_{cmn} \end{bmatrix} \quad (6)$$

$$R_I = \begin{bmatrix} 1/R_{I11} & 1/R_{I12} \dots & 1/R_{I1n} \\ 1/R_{I21} & 1/R_{I22} \dots & 1/R_{I2n} \\ \dots & \dots & \dots \\ 1/R_{Im1} & 1/R_{Im2} & 1/R_{Imn} \end{bmatrix} \quad (7)$$

$$R_D = \begin{bmatrix} 1/R_{D11} & 1/R_{D12} \dots & 1/R_{D1n} \\ 1/R_{D21} & 1/R_{D22} \dots & 1/R_{D2n} \\ \dots & \dots & \dots \\ 1/R_{Dm1} & 1/R_{Dm2} & 1/R_{Dmn} \end{bmatrix} \quad (8)$$

where

$$k_{Iij} = (k_{cij} / t_{Iij}) \quad (9a)$$

and

$$k_{D\,ij} = (k_{cij} * t_{Dij}). \quad (9b)$$

For a square system

$$K_c = [R_c]^{-1} \quad (10a)$$

$$K_I = [R_I]^{-1} \quad (10b)$$

$$K_D = [R_D]^{-1}. \quad (10c)$$

This method is extended to a nonsquare system taking the pseudo-inverse.

$$K_c = [R_c]^{\dagger} \quad (11a)$$

$$K_I = [R_I]^{\dagger} \quad (11b)$$

$$K_D = [R_D]^{\dagger}. \quad (11c)$$

k_{cij}, k_{Iij} and k_{Dij} are calculated from equations given in Morari and Zafiriou [11]. Tuning parameter ($\ddot{e}$) has to be calculated by trial-and-error-method. The lower limit of l is given in literature for individual elements [11]. This limit is taken as the initial value and the final value of the tuning parameter is obtained by trial-and-error method by simulation.

2.3. *Robust decentralized controller (Loh–Chiu method* [4]*)*

The independent procedure for robust decentralized controllers proposed by Hovd and Skogestad [5] is extended by Loh and Chiu [4] to nonsquare system with more inputs than outputs in their work. The controller design equations are:

$$C = Q[I - G_M Q]^{-1} \tag{12}$$

where Q is nonsquare IMC controller.

$$Q = [G_M^{-}]^{\dagger} F. \tag{13}$$

G_M^{-} is the minimum phase part of G_M and F = diag $\{f_I\}_{i\sim k}$ is a low-pass diagonal filter with a steady-state gain of 1.

3. Simulation example 1: Two coupled pilot plant distillation columns

Levien and Morari [12] have discussed the example of two coupled distillation columns in their work. They have considered a square system (3×3). In the present work, the above system having non-minimum phase is considered with 2 outputs and 3 inputs. Here the outputs are mole fraction of ethanol in distillate (y1) and mole fraction of water in bottoms (y2), and manipulated variables are distillate flow rate (u1), steam flow rate (u2), and product fraction from the side column (u3). The system transfer function is given as:

$$G(s) = \begin{bmatrix} \dfrac{0.052e^{-8s}}{19.8s+1} & \dfrac{-0.03(1-15.8s)}{108s^2+63s+1} & \dfrac{0.012(1-47s)}{181s^2+29s+1} \\ \dfrac{0.0725}{890s^2+64s+1} & \dfrac{-0.0029(1-560s)}{293s^2+51s+1} & \dfrac{0.0078}{42.3s+1} \end{bmatrix} \tag{14}$$

3.1. *Davison's method for coupled pilot plant distillation columns*

From the above transfer function matrix, steady-state gain matrix is given by:

$$G(0) = \begin{bmatrix} 0.052 & -0.032 & 0.012 \\ 0.0725 & -0.0029 & 0.0078 \end{bmatrix}. \tag{15}$$

The pseudo-inverse for the above matrix is calculated and substituted in eqns (4a and b) to obtain the proportional and integral gains of PI controllers. The system is stable over the range d = 0.7 to 1 and e = 0 to 0.1. Tuning parameters are obtained by tuning the controller around these values. The overall controller is obtained with d = 1 and e = 0.03. The final PI controller matrix is obtained as:

$$G_c = \begin{bmatrix} -2.0845 + \dfrac{-0.0625}{s} & 15.1612 + \dfrac{0.4548}{s} \\ -33.0046 + \dfrac{-0.9901}{s} & 23.9571 + \dfrac{0.7187}{s} \\ 7.1044 + \dfrac{0.213}{s} & -3.8095 + \dfrac{-0.1142}{s} \end{bmatrix} \tag{16}$$

3.2. *Tanttu and Lieslehto method for coupled pilot plant distillation columns*

Centralized PI controller is designed for the two-coupled distillation columns using the Tanttu and Lieslehto method as given in Section 2.2. Individual IMC settings for the two-coupled distillation column systems, with process model $G(s)$, are found and arranged in matrix form given by:

$$R_c = \begin{bmatrix} \dfrac{l}{457.69} & \dfrac{-31.6+l}{2032.2} & \dfrac{98+l}{2416.7} \\ \dfrac{l}{882.7} & \dfrac{1120+l}{17586.2} & \dfrac{l}{5423.07} \end{bmatrix} \tag{17a}$$

$$R_I = \begin{bmatrix} \dfrac{l}{19.23} & \dfrac{-31.6+l}{32.25} & \dfrac{98+l}{83.3} \\ \dfrac{l}{13.79} & \dfrac{1120+l}{344.82} & \dfrac{l}{128.205} \end{bmatrix}. \tag{17b}$$

Here, l is a tuning parameter. The initial value of l = 13 (l > 1.7*time delay) is used. Final controller settings are found as discussed in Section 2.2. The recommended value for the tuning parameter is l = 15. The final PI controller is obtained as:

$$G_c = \begin{bmatrix} 9.6833+\dfrac{0.3145}{s} & 3.4640+\dfrac{0.1084}{s} \\ -3.1519+\dfrac{-0.1221}{s} & 14.5772+\dfrac{0.2666}{s} \\ 14.0520+\dfrac{0.5107}{s} & 0.1198+\dfrac{0.0390}{s} \end{bmatrix}. \tag{18}$$

In the controller matrix by Tanttu and Lieselhto method, two of the controller setting signs are different from that of Davison’s method [eqns (16) and (18)]. Since system zeros are not defined for nonsquare systems, it is not clear how to separate the positive system zero from the nonsquare transfer function matrix. The presence of such zeros may cause the change of sign in some of the individual controllers. Further research is required in this direction.

3.3. *Decentralized controller method for coupled pilot plant distillation columns*

Loh and Chiu method is applied to the coupled pilot plant distillation columns and the decentralized controller designed. The pairing of the manipulated and controlled variables is obtained using block relative gain (BRG) [1]. The first output variable y_1 is paired with u_2 and y_2 is paired with u_1 and u_3. This paring leads to less interaction and unity BRG. From the pairings the block diagonal model is obtained and given by:

Table I
ISE values for the servo problem for centralized controller: Two coupled distillation columns example

Method	Step change in	ISE values Y_1	Y_2	Sum of ISE
Davison	Y_1	33.04	12.41	45.45
	Y_2	3.11	11.46	14.57
Tanttu & Lieslehto	Y_1	34.04	44.04	78.07
	Y_2	0.4532	52.765	53.218
Loh & Chiu	Y_1	44.99	9.91	54.90
	Y_2	19.64	28.34	47.98

Table II
ISE values of the regulatory problem for two coupled distillation columns–example for disturbances at the input

Method	Step in	ISE values Y_1	Y_2	Sum of ISE	IAE values Y_1	Y_2	Sum of IAE
Davison	V_1	0.0319	0.0465	0.0784	1.79	3.58	4.37
	V_2	0.01	0.0056	0.156	1.18	0.84	2.02
	V_3	0.0075	0.0046	0.0127	1.045	0.847	1.893
Tanttu & Lieslehto	V_1	0.0481	0.0216	0.0698	2.151	2.076	4.227
	V_2	0.0080	0.0248	0.0334	1.254	3.065	4.319
	V_3	0.0092	0.0043	0.0136	1.069	1.227	2.296
Loh & Chiu	V_1	0.041	0.107	0.148	2.67	4.67	7.34
	V_2	0.036	0.021	0.057	2.75	1.54	4.39
	V_3	0.0065	0.0045	0.011	1.03	0.658	1.688

$$G_M(s)=\begin{bmatrix}\dfrac{-0.03\,(1-15.8s)}{108s^2+63s+1} & 0 & 0\\ 0 & \dfrac{0.0725}{890s^2+64s+1} & \dfrac{0.0078}{42.3s+1}\end{bmatrix}. \tag{19}$$

The IMC controller Q is designed by substituting G_M in eqn (12). Here, the first-order filter is used. Simulations are carried out for different values of tuning parameter [4] and the best value is obtained as $e_1 = e_2 = 28$. With this tuning parameter value the final controller is given by eqn (20). From the controller transfer function matrix (Q) it is clear that it is not of the conventional PI/PID form.

$$Q=\begin{bmatrix}\dfrac{108s^2+63s+1}{-0.474s^2-1.314-0.03} & 0\\ 0 & \dfrac{115453s^4+13761s^3+586s^2+10.7s+.0725}{1349s^5+241.4s^4+280.18s^3+22.64s^2+0.6112s+0.0054}\\ 0 & \dfrac{261345s^5+43765s^4+2827s^3+88s^2+1.3s+0.0078}{1349s^5+241.4s^4+280.18s^3+22.64s^2+0.6112s+0.0054}\end{bmatrix} \tag{20}$$

3.4. *Simulation results*

Simulation was carried out for both servo and regulatory problems using SIMULINK. Results are compared using ISE values for both the methods (Tables I and II). The load transfer function matrix for the disturbances is not available. For the regulatory problem, the load transfer function matrix is assumed as that of the process transfer function matrix (i.e. load enters along with the manipulated variable). Davison's method gives the lowest ISE values than the other two methods for both servo and regulatory problems. Decentralized controller gives lower ISE values than Tanttu and Lieslehto method for the servo problem, whereas for the regulatory problem Tanttu and Lieslehto method gives lower ISE values than decentralized controller (Figs 1 and 2). For step change in $y1$ the decentralized controller gives sluggish response than the centralized controller and settling time is more in the case of Tanttu and Lieslehto method, whereas for step change in $y2$, Tanttu and Lieslehto method gives

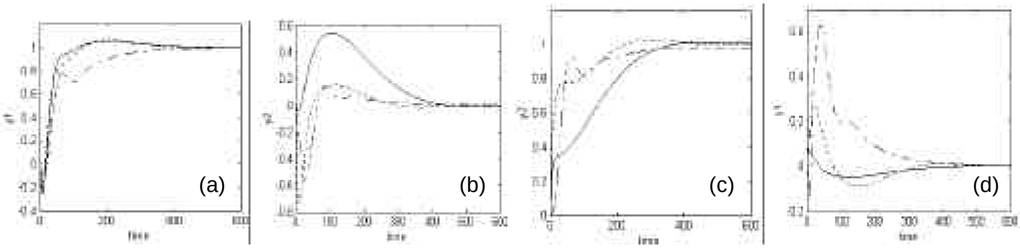

FIG. 1. Performance comparison of the three methods for unit step changes in $y1$ or $y2$ for coupled pilot plant distillation columns. (a) Response, (b) Interaction for step change in $y1$, and (c) Response and (d) Interaction for step change in $y2$. Legend: Solid–Tanttu and Lieslehto method, Dot–Davison's method, Dash dot–Decentralized controller.

sluggish response and the decentralized controller gives more interactions than the centralized controllers.

3.5. *Robustness studies for coupled pilot plant distillation column*

Robustness studies were carried out for this system by increasing the individual element gain by 10%. The same controller setting as previously obtained was used. The performance of the three methods for the perturbed system is shown in Fig. 3. The performance of the centralized controllers is similar to that of the perfect parameter system. The sum of ISE for the perfect parameter system and the perturbed system is compared in Table III. Davison's method gives the ISE values for the perturbed system close to that of the nominal system. It gives more robust performance than the other two methods.

4. Simulation example 2: Crude distillation process

The crude distillation unit lies at the front end of a refinery. This unit performs the initial distillation of the crude oil into several boiling range fractions. The crude is pumped in from storage tanks and, after desalination, is preheated against crude tower products and overhead streams. The crude is then partially vaporized in two parallel fuel gas-fired heaters. The vapor and liquid from the heaters enter the flash zone at the bottom of the crude column. Muske *et al*. [3] have considered the crude distillation unit at Cosmo Oil's Sakai Refinery. They have given the transfer function for three general crudes and average crude. Crude 2 is considered in this work and its transfer function is given in eqn (21). In this example,

Table III
Sum of ISE values for robustness comparison for coupled pilot plant distillation column

Method	Step change in	Sum of ISE: Perfect parameter	Sum of ISE: +10% change in time constant
Davison	Y_1	45.45	44.2
	Y_2	14.57	13.23
Tanttu &	Y_1	78.07	72.8
Lieslehto	Y_2	53.218	47.96
Loh &	Y_1	54.90	55.32
Chiu	Y_2	47.98	48.64

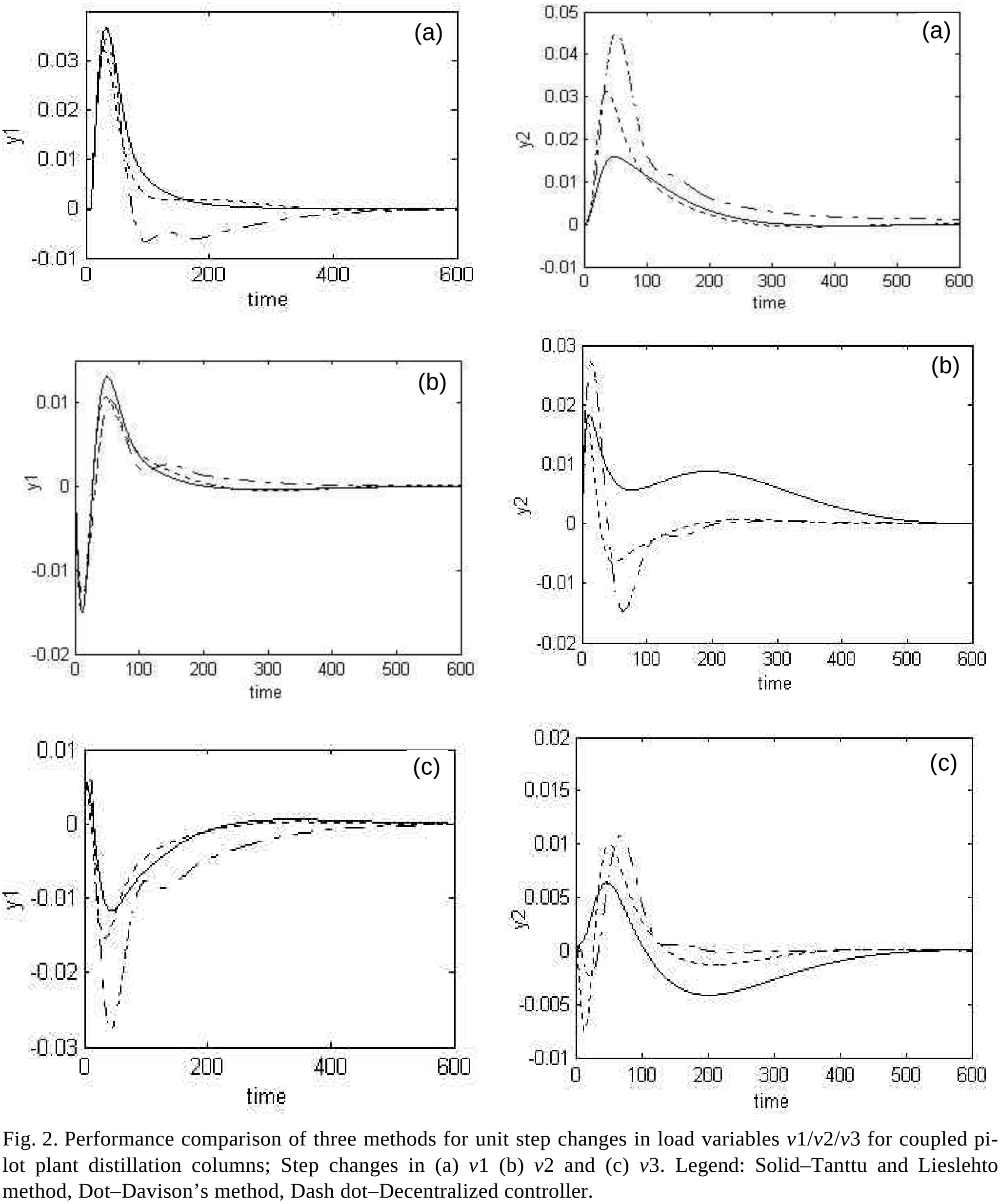

Fig. 2. Performance comparison of three methods for unit step changes in load variables $v1/v2/v3$ for coupled pilot plant distillation columns; Step changes in (a) $v1$ (b) $v2$ and (c) $v3$. Legend: Solid–Tanttu and Lieslehto method, Dot–Davison's method, Dash dot–Decentralized controller.

controlled variables are naphtha/kerosene cutpoint ($y1$), kerosene/LGO cutpoint ($y2$), LGO/HGO cutpoint ($y3$) and measured over flash ($y4$). Manipulated variables are top temperature ($u1$), kerosene yield ($u2$), LGO yield ($u3$), HGO yield ($u4$) and heater outlet temperature ($u5$). For this, process centralized PID controllers are designed using Davison's and Tanttu and Lieslehto methods.

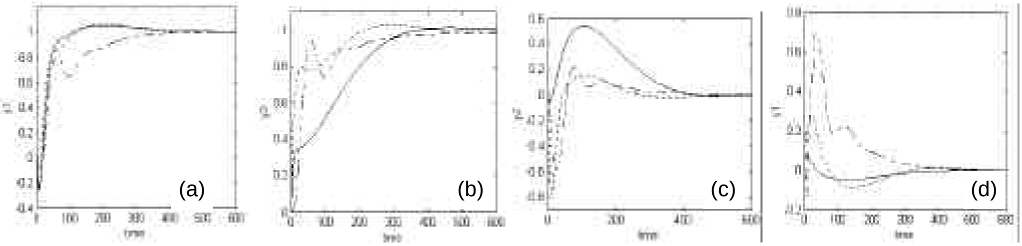

FIG. 3. Performance comparison of the two methods for unit step change in $y1$ or $y2$ for coupled pilot plant distillation column with 10% deviation in each scalar system gain. (a) Response, (b) Interaction for step change in $y1$; and (c) Response, and (d) Interaction for step change in $y2$. Legend: Solid–Tanttu and Lieslehto method, Dot–Davison's method, Dash dot–Decentralized controller.

$$G(s)=\begin{bmatrix} \frac{3.8\,(16s+1)}{140s^2+14s+1} & \frac{2.9e^{-6s}}{10s+1} & 0 & 0 & \frac{-0.73\,(-16s+1)e^{-4s}}{150s^2+20s+1} \\ \frac{3.9\,(4.5s+1)}{96s^2+17s+1} & \frac{6.3}{20s+1} & 0 & 0 & \frac{16se^{-2s}}{(5s+1)\,(14s+1)} \\ \frac{3.8\,(0.8s+1)}{23s^2+13s+1} & \frac{6.1\,(12s+1)e^{-s}}{337s^2+34s+1} & \frac{3.4e^{-2s}}{6.9s+1} & 0 & \frac{22se^{-2s}}{(5s+1)(10s+1)} \\ \frac{-1.62\,(5.3s+1)e^{-s}}{13s^2+13s+1} & \frac{-1.53\,(3.1s+1)}{5.1s^2+7.1s+1} & \frac{-1.3\,(7.6s+1)}{4.7s^2+7.1s+1} & \frac{-0.6e^{-s}}{2s+1} & \frac{0.32\,(-9.1s+1)e^{-s}}{12s^2+15s+1} \end{bmatrix} \quad (21)$$

4.1. *Davison's method for crude distillation process*

For the transfer function matrix, given in eqn (21), steady-state gain matrix is:

$$G(0)=\begin{bmatrix} 3.8 & 2.9 & 0 & 0 & -0.73 \\ 3.9 & 6.3 & 0 & 0 & 0 \\ 3.8 & 6.1 & 3.4 & 0 & 0 \\ -1.62 & -1.5 & -1.3 & -0.6 & 0.32 \end{bmatrix}. \quad (22)$$

For this matrix the pseudo-inverse is calculated and substituted in eqn (11) to get proportional, integral and derivative gains. As discussed in Section 2.1.1 tuning parameters are calculated. Simulation is carried out for the designed controller and the tuning parameter values obtained are: $d = 1$, $e = 0.22$, $g = 0.9$. Controller with these tuning parameter values gives better performance and minimum ISE values. The final controller transfer function is given by eqn (23).

$$G_C=\begin{bmatrix} 0.443+\frac{0.133}{s}+0.399s & -0.201-\frac{0.060}{s}-0.181s & 0.023+\frac{0.007}{s}+0.021s & 0.061+\frac{0.018}{s}+0.055s \\ -0.274-\frac{0.082}{s}-0.247s & 0.283+\frac{0.085}{s}+0.255s & -0.015-\frac{0.005}{s}-0.014s & -0.037-\frac{0.011}{s}-0.033s \\ -0.003-\frac{0.0009}{s}-0.003s & -0.284-\frac{0.085}{s}-0.256s & 0.294+\frac{0.088}{s}+0.265s & -0.0004-\frac{0.00012}{s}-0.0003s \\ -0.586-\frac{0.176}{s}-0.527s & -0.491-\frac{0.147}{s}-0.442s & -0.294-\frac{0.088}{s}-0.265s & -1.647-\frac{0.494}{s}-1.482s \\ -0.153-\frac{0.046}{s}-0.138s & 0.078+\frac{0.023}{s}+0.007s & 0.064+\frac{0.019}{s}+0.058s & 0.166+\frac{0.049}{s}+0.149s \end{bmatrix} \quad (23)$$

5.2. *Tanttu and Lieslehto method*

For the crude distillation problem, a centralized PID controller is designed using the Tanttu and Lieslehto method as discussed in Section 2.2. The initial value of the tuning parameter is $l = 11$ ($l > 1.7$*time delay). Simulation studies are carried out for the system with different tuning parameter values and the recommended value is $l = 30$. The controller with this tuning parameter value gives better performance and lower ISE values. The final PID controller matrix with this l value is given by:

$$G_C = \begin{bmatrix} 0.143+\frac{0.011}{s}+0.689s & -0.071-\frac{0.004}{s}+0.36s & 0.021+\frac{0.002}{s} & 0.027+\frac{0.004}{s} \\ -0.072-\frac{0.005}{s}-0.453s & 0.129+\frac{0.007}{s}-0.519s & -0.019-\frac{0.002}{s}-0.202s & -0.025-\frac{0.004}{s} \\ -0.037-\frac{0.00002}{s}-0.075 & -0.032-\frac{0.009}{s}-0.556s & 0.068+\frac{0.009}{s}-0.215s & -0.013-\frac{0.00007}{s}-0.121s \\ -0.005-\frac{0.021}{s} & -0.032+\frac{0.017}{s} & -0.043-\frac{0.021}{s} & -0.118-\frac{0.055}{s} \\ -0.191-\frac{0.008}{s}-1.959s & 0.171+\frac{0.004}{s}+1.309s & 0.024+\frac{0.002}{s} & 0.032+\frac{0.005}{s} \end{bmatrix} \tag{24}$$

Tanttu and Lieslehto method is based on IMC method which gives only PI controller [eqn (24)] for the FOPTD transfer function models. IMC method gives PID controller settings for higher-order systems, whereas in Davison's method we can calculate K_D as well as get PID controllers too [eqn (23)].

4.3. *Simulation results*

Crude distillation is a large-scale problem with 4 outputs and 5 inputs. For this example, centralized controllers are designed using two proposed methods. Simulations are carried out on the two designed controllers for both the servo and regulatory problems. These are compared based on the ISE values given in Tables IV and V for servo and regulatory problems, respectively. ISE values show that the Davison's method gives good response and low ISE values than

Table IV
ISE values of the servo problem for centralized controller crude distillation column example

Method	Step in	ISE values Y_1	Y_2	Y_3	Y_4	Sum of ISE
Davison's	Y_1	3.194	0.134	0.792	0.066	4.185
	Y_2	1.033	7.128	1.758	0.264	10.18
	Y_3	0.061	0.071	5.031	0.239	5.403
	Y_4	0.074	0.088	0.142	3.153	3.457
Tanttu & Lieslehto	Y_1	13.57	0.769	1.262	0.022	15.62
	Y_2	0.254	18.0	1.264	0.016	19.54
	Y_3	0.078	0.068	17.28	0.915	17.82
	Y_4	0.377	0.366	0.376	16.57	17.69

Table V
ISE values for the regulatory problem for the crude distillation column example with disturbances at the input

Method	Step in	ISE values Y_1	Y_2	Y_3	Y_4	Sum of ISE
Davison's	V_1	17.23	17.20	15.02	2.70	52.17
	V_2	10.22	31.57	23.16	2.22	67.17
	V_3	0.098	0.108	13.69	2.143	16.04
	V_4	0.021	0.025	0.038	0.371	0.457
	V_5	0.796	2.23	4.428	0.079	6.534
Tanttu & Lieslehto	V_1	204.1	133.9	125.4	27.7	491.2
	V_2	8.98	392.3	324.8	22.43	827.5
	V_3	0.128	0.200	148.1	21.98	170.4
	V_4	0.134	0.131	0.131	5.239	5.636
	V_5	6.121	3.148	7.211	1.108	17.58

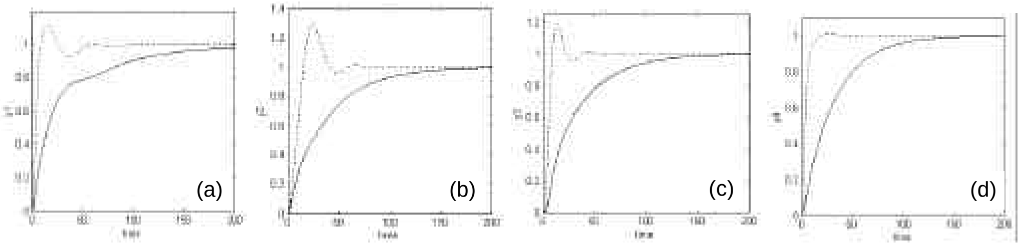

FIG. 4. Performance comparison of Davison's method and Tanttu and Lieslehto method for unit step changes in (a) $y1$, (b) $y2$, (c) $y3$ or (d) $y4$ for crude distillation process. Legend: Solid–Tanttu and Lieslehto method, Dot–Davison's method.

the Tanttu and Lieslehto method. ISE values of the Tanttu and Lieslehto method are about 2–3 times, compared to Davison's method for the servo problem, whereas for the regulatory problem still larger ISE values are obtained in Tanttu and Lieslehto method. Simulation results are given in Fig. 4 for servo problem for step changes in all the controlled variables each at a time. The settling time for the Tanttu and Lieslehto method is more than that of Davison's method. The Tanttu and Lieslehto method gives sluggish response.

4.4. *Robustness studies for the crude distillation process*

Robustness studies are carried out for the crude distillation process by increasing the individual element process gain by 10% using the same controller settings. The sum of ISE values is given in Table VI for both the perfect parameter and the perturbed systems for step change in set points. From the table it is clear that the Davison's method gives ISE values closer to that of perfect parameter system and hence this method is more robust than the Tanttu and Lieslehto method.

4.5. *Comparison of controllers for square and nonsquare crude distillation process*

The crude distillation process transfer function model is given in eqn (21). The columns indicate manipulated variables and rows controlled variables. From the transfer function matrix, it is clear that the fourth manipulated variable is effecting only the fourth output variable. So the fourth manipulated variable is kept constant so that the transfer function matrix be-

Table VI
Comparison of ISE values for perfect parameter system and perturbed system for crude distillation process

Method	Step in	Sum of ISE	
		for perfect parameter system	+10% change in process gain
Davison's	Y_1	4.185	4.03
	Y_2	10.18	9.999
	Y_3	5.403	5.32
	Y_4	3.457	3.27
Tanttu & Lieslehto	Y_1	15.62	14.262
	Y_2	19.54	17.994
	Y_3	17.82	16.18
	Y_4	17.69	16.23

Table VII
ISE values for the square and nonsquare systems

Step change in	Sum of ISE	
	for nonsquare system	for square system
Y_1	4.185	8.93
Y_2	10.18	19.78
Y_3	5.403	15.16
Y_4	3.457	37.69

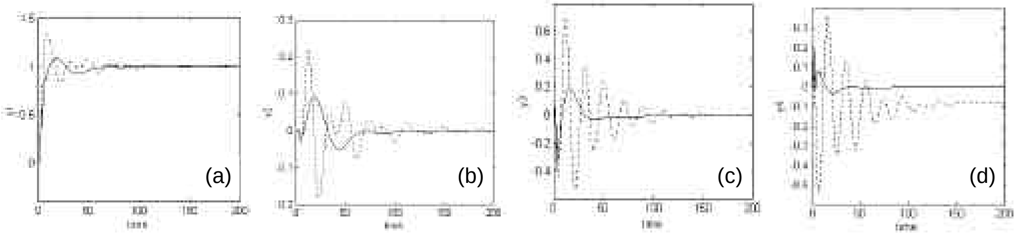

FIG. 5. Performance comparison of the square and nonsquare controllers for step change in *y*1 for crude distillation example. (a) Response of *y*1, Interaction in (b) *y*2, (c) *y*3, and (d) *y*4. Legend: solid–nonsquare controller, dot–square controller.

comes square (4×4 system). For this square transfer function matrix centralized controller is designed using the Davison's method. The controller is given by:

$$GS_c = \begin{bmatrix} 2.038+\frac{1.223}{s}+1.834 & -1.53-\frac{0.918}{s}-1.377s & 1.778+\frac{1.066}{s}+1.6s & 4.651+\frac{2.790}{s}+4.18s \\ -1.261-\frac{0.757}{s}-1.135s & 1.106+\frac{0.663}{s}+0.995s & -1.100-\frac{0.660}{s}-0.990s & -2.878-\frac{1.727}{s}-2.590s \\ -0.014-\frac{0.0086}{s}-0.012s & -0.2741-\frac{0.164}{s}-0.246s & 0..281+\frac{0.169}{s}+0.253s & -0.032-\frac{0.019}{s}-0.0293s \\ 4.228+\frac{2.536}{s}+3.805s & -3.572-\frac{2.143}{s}-3.215s & 4.882+\frac{2.929}{s}+4.394s & 12.77+\frac{7.662}{s}+11.49s \end{bmatrix} \tag{25}$$

The performance of the square controller is compared with that of the nonsquare controller designed previously (Fig. 5). The square controller gives oscillatory response and large settling time. The sum of ISE values for the square and nonsquare controllers for servo problem is also compared (Table VII). For square controller, it is two times higher than the nonsquare controller for step changes in *y*1 or *y*2 or *y*3 and three times higher for step change in *y*4.

5. Conclusion

The simple centralized controller-tuning methods, Davison's method and, Tanttu and Lieslehto method, are extended to nonsquare systems with right half-plane zeros. The proposed methods are applied to two examples: coupled pilot plant distillation columns and a crude distillation unit. Simulations are carried out for both servo and regulatory problems. ISE values are given. For the crude distillation process the two centralized controller methods are compared. Davison's method gives better performance and less settling time than Tanttu and Lieslehto method. Tanttu and Lieslehto method gives sluggish response. ISE values of the Tanttu and Lieslehto method are ~2–3 times as compared to Davison's method for the servo problem. The performances of the square and nonsquare controllers is compared. Improved performance is observed in the nonsquare controller.

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Nomenclature

K_c	Proportional gain matrix of the centralized controller
K_I	Integral gain matrix of the centralized controller
k_{cij}	Proportional gain of a SISO IMC controller
C	Decentralized controller in Loh and Chiu method
Q	Decentralized IMC controller Loh and Chiu method
G_M	Block diagonal model of the actual plant
$G^{-}{}_{M}$	Minimum phase part of the block diagonal model
G_c	Centralized nonsquare PI/PID controller
GS_c	Centralized square PI/PID controller
F	Low-pass diagonal filter
y_1, y_2	System outputs
u_1, u_2, u_3	Manipulated variables
V_1, V_2,V_3	Load variables

Greek letters

d	Tuning parameter for the proportional gain in the Davison's method
e	Tuning parameter for the integral gain in the Davison's method
g	Tuning parameter for the differential gain in the Davison's method
l	Tuning parameter and filter time constant in the Tanttu and Lieslehto method
e$_1$, *e*$_2$	Tuning parameters in the Loh and Chiu method

Superscripts

†	Moore–Penrose pseudo-inverse
H	Hermitian operator