J. Indian Inst. Sci., Sept.–Oct. 2004, **84**, 155–161

Design of FIR filters using variable window families: A comparative study

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Received on April 07, 2003; Revised on April 21, 2004.

Abstract

In this paper, we report the design equations for FIR filter synthesis by using some recently reported combinational window functions. The performance of these filters is compared with FIR filters designed by using Kaiser window (prolate spheroidal wave function based). We also suggest some corrections in the set of design equations for FIR filter synthesis by using raised cosine window. The variable window functions used in this study other than Kaiser and raised cosine window provide filters with better side lobe fall off rate. This feature can be utilized for better suppression of images associated with interpolators.

Keywords: Window function, FIR filter, window shape parameter, stop band attenuation, normalized window width parameter.

1. Introduction

Linear-phase, inherent-stability, negligible-quantization noise and efficient implementation in multirate digital signal processing (DSP) systems are the features of FIR filters, making them preferred choice over IIR filters. Three following common methods are used to design FIR filters: (i) window, (ii) sampling, and (iii) optimal polynomial [1]. For filters which are ripple-free (< 1%) in the pass band, window method gives results equal to (or slightly better than) any other method [2].

This paper gives a set of design equations for the synthesis of FIR filters with variable windows. For variable window method, the difficulties arise from the determination of the required value of the filter length and the window shape parameter to achieve the desired transition bandwidth and stopband attenuation. This paper provides the relationship between minimum stopband attenuation (ATT) and the window shape parameter, and ATT and the normalized width parameter for the five variable windows.

In this work, filter design using raised cosine (RC) family [3] of window functions is revisited with corrections to the expressions relating to window shape parameter (B) and ATT. Combinational window families [4] are designed by combining a lag window and a

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data window in a linear manner. Bartlett–Hanning (BH) [5], Papoulis-$Cos^4(p\,t)$ (PC4) [6] and Parzen–$Cos^6(p\,t)$ (PC6) [7] are examples of combinational window functions with high side lobe fall of rate (SLFOR) [8]. The SLFOR for BH window is -12dB/octave. For PC6 window SLFOR varies between –24 and –30 dB/octave. For PC4 window this figure varies between –24 and –42 dB/octave. Design equations for these combinational window FIR filters have been established. Finally, a comparative study of the five such classes of FIR filters has been reported. Also, the frequency spectrum of high SLFOR filters is compared with that of the Kaiser window-based filters [9].

2. Filter design relationship

Designing FIR filters using window functions involves the following empirical design equations [10]:

(i) Equation-defining relationship between ATT and window shape parameter ($a/B/b/\tilde{a}/d$ ::Kaiser/RC/BH/PC6/PC4). This equation provides the value of window shape parameter for the desired stopband attenuation.

(ii) Equation-defining relationship between ATT and normalized window width parameter (D) [3]. From this equation, parameter D is obtained for the specified value of attenuation. Filter order is then calculated using the equation:

$$N = \left(\frac{\mathrm{D}}{\Delta F}\right) + 1$$

where ΔF is the specified value of normalized transition bandwidth.

2.1. *Design relationship for Kaiser window filter* [9]

$$a = \begin{cases} 0 & \text{for ATT} \le 21 \\ 0.5842\,(\mathrm{ATT}-21)^{0.4} + 0.07886(\mathrm{ATT}-21) & \text{for } 21 < \mathrm{ATT} \le 50 \\ 0.1102\,(\mathrm{ATT}-8.7) & \text{for ATT} > 50 \end{cases} \tag{1}$$

$$\mathrm{D} = \begin{cases} 0.9222 & \text{for ATT} \le 21 \\ \dfrac{\mathrm{ATT}-7.95}{14.36} & \text{for ATT} > 21 \end{cases} \tag{2}$$

2.2. *Design relationship for RC window* [3]

Variations in the normalized window width parameter (D) and window shape parameter (B) with ATT for this window are plotted in Figs 1 and 2, respectively.

Using Fig. 1, the following relationship between D and ATT is obtained:

$$\mathrm{D} = \begin{cases} 0.0278994\,(\mathrm{ATT}+72.7793) & \text{for } 45.3521 < \mathrm{ATT} \le 49.8970 \\ 0.0527306\,(\mathrm{ATT}+15.2315) & \text{for } 49.8970 < \mathrm{ATT} \le 61.9382 \\ 0.0297673\,(\mathrm{ATT}+74.32) & \text{for } 61.9382 < \mathrm{ATT} \le 70.4576 \end{cases} \tag{3}$$

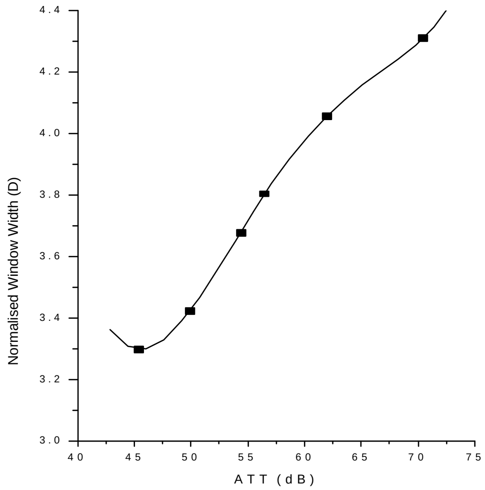

FIG. 1. Plot for ATT vs D (RC window).

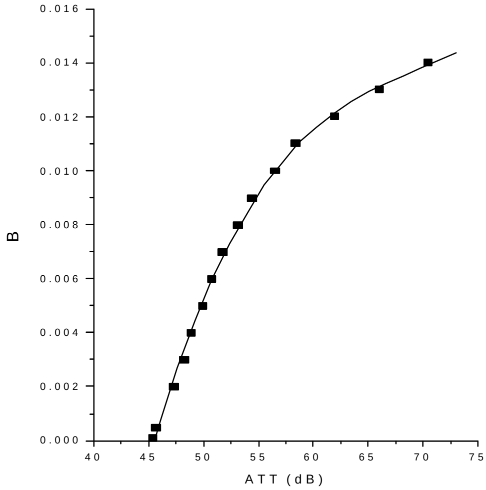

FIG. 2. Plot for ATT vs B (RC window).

Also, the relationship between D and ATT can be defined by a single equation using a fourth degree polynomial as:

$$D = a + (b*\text{ATT}) + (c*\text{ATT}^2) + (d*\text{ATT}^3) + (e*\text{ATT}^4) \quad \text{for } 45.3521 < \text{ATT} \leq 70.4576, \tag{4}$$

where $a = 99.0643$, $b = -6.68503$, $c = 0.171591$, $d = -0.00192193$, and $e = 7.98039*10^{-6}$.

Similarly, using Fig. 2, the following relationship between B and ATT is obtained:

$$\text{B} = \begin{cases} 0.00105166\,(\text{ATT} - 45.2293) & \text{for} \quad 45.3521 < \text{ATT} \leq 54.4249 \\ 0.000500926\,(\text{ATT} - 36.4768) & \text{for} \quad 54.4249 < \text{ATT} \leq 58.4164 \\ 0.00024807\,(\text{ATT} - 13.8192) & \text{for} \quad 58.4164 < \text{ATT} \leq 70.4576 \end{cases} \tag{5}$$

Also, the relationship between B and ATT can be defined by a single equation using third-degree polynomial as:

$$\text{B} = a + (b*\text{ATT}) + (c*\text{ATT}^2) + (d*\text{ATT}^3) \quad \text{for } 45.3521 < \text{ATT} \leq 70.4576, \tag{6}$$

where $a = -0.22874$, $b = 0.0100973$, $c = -0.000142371$ and $d = 6.80417*10^{-7}$.

Earlier work by Prabhu and Bagan [3] gives

$$B = 0.005143\,(\text{ATT} - 52) \quad \text{for } 52 < \text{ATT} < 69.5. \tag{7}$$

This relationship does not satisfy the tabulated results which they have used to obtain the above expression. Also, in an example it has been stated that for attenuation of 63.96 dB, the value of B is 0.0138. But the above expression gives the value of B equal to 0.0615 for attenuation of 63.96. Hence, filter designed using this relationship does not provide the desired frequency response.

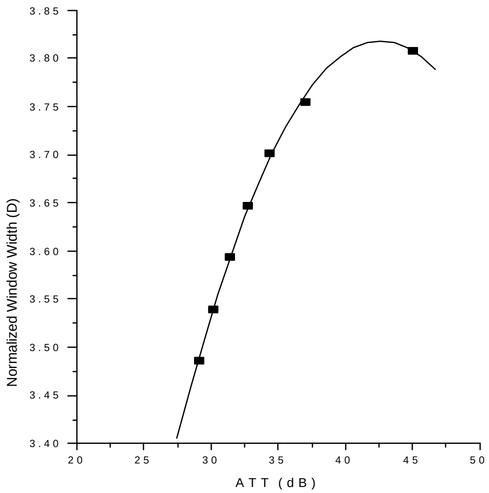

FIG. 3. Plot for ATT vs D (BH window).

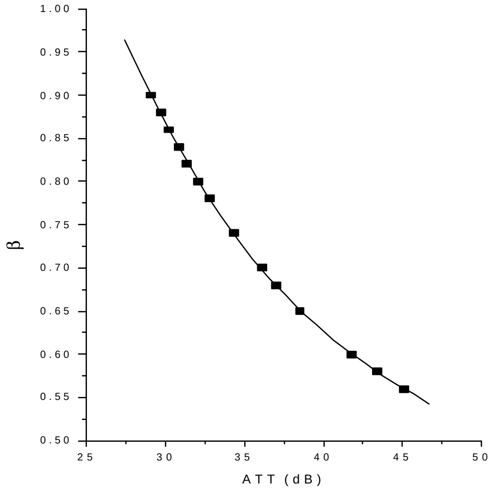

FIG. 4. Plot for ATT vs *b* (BH window).

2.3. *Design relationship for BH window* [5]

Variations in the normalized window width parameter (D) and window shape parameter (*b*) with ATT are plotted in Figs 3 and 4, respectively.

Using Fig. 3, the following relationship between D and ATT is obtained.

$$D = a + (b*\text{ATT}) + (c*\text{ATT}^2) \quad \text{for} \quad 29.064 \le 34.344, \tag{8}$$

where $a = 0.542754$, $b = 0.15387$ and $c = -0.00180691$.

Similarly, using Fig. 4, the following relationship between *b* and ATT is obtained.

$$b = a + (b*\text{ATT}) + (c*\text{ATT}^2) + (d*\text{ATT}^3) \quad \text{for} \quad 29.064 \le \text{ATT} \le 45, \tag{9}$$

where $a = 3.60779$, $b = -0.167517$, $c = 0.00317774$ and $d = -2.13298*10^{-5}$.

2.4. *Design relationship for PC6 window* [7]

Variations in the normalized window width parameter (D) and in the window shape parameter (*g*) with ATT are plotted in Figs 5 and 6, respectively. Using Fig. 5, the following relationship between D and ATT is obtained.

$$D = a + (b*\text{ATT}) + (c*\text{ATT}^2) \tag{10}$$

where

$a = 1.82892$, $b = -0.0275481$, and $c = 0.00157699$, for $30.319 \le \text{ATT} \le 43.598$
$a = 1.67702$, $b = 0.0450205$, and $c = 0$, for $43.598 < \text{ATT} \le 49.437$
$a = 85.4738$, $b = -3.41969$, and $c = 0.035784$, for $49.437 < \text{ATT} \le 57.485$
$a = -8.60006$, $b = 0.477004$, and $c = -0.00355655$, for $57.485 < \text{ATT} \le 68.689$.

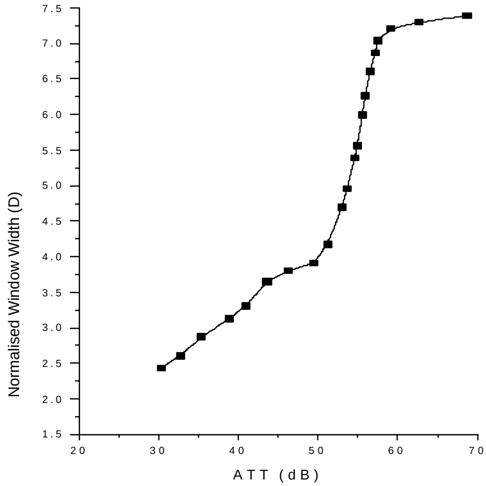

FIG. 5. Plot for ATT vs D (PC6 window).

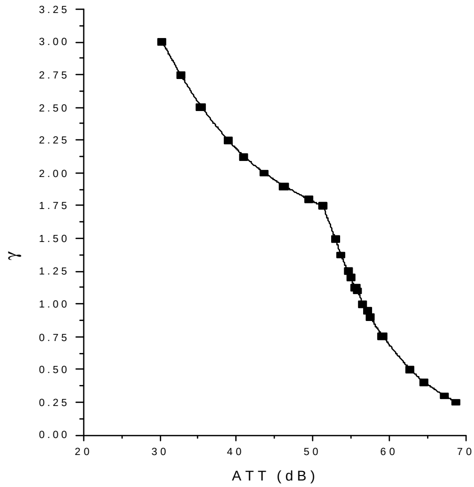

FIG. 6. Plot for ATT vs *g* (PC6 window).

Similarly, using Fig. 6, the following relationship between ã and ATT is obtained:

$$= a + (b * \text{ATT}) + (c * \text{ATT}^2) \tag{11}$$

where $a = 8.15414$, $b = -0.236709$, and $c = 0.00218617$, for $30.319 \leq \text{ATT} \leq 51.251$
$a = 21.3669$, $b = -0.605789$, $c = 0.00434808$, for $51.251 < \text{ATT} \leq 68.689$.

2.5. *Design relationship for PC4 window* [6]

Variations in the normalized window width parameter (D) and window shape parameter (***d***) with ATT are plotted in Figs 7 and 8. Using Fig. 7, the following relationship between D and ATT is obtained:

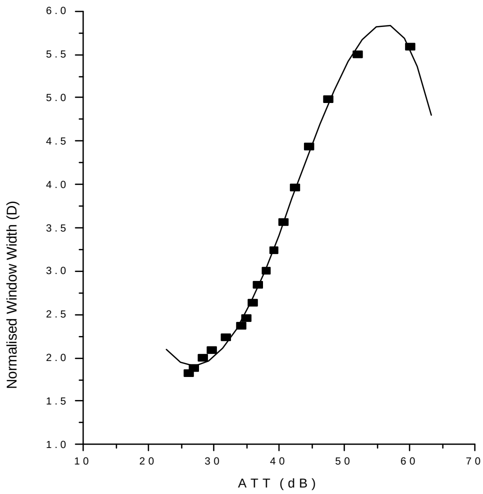

FIG. 7. Plot for ATT vs D (PC4 window).

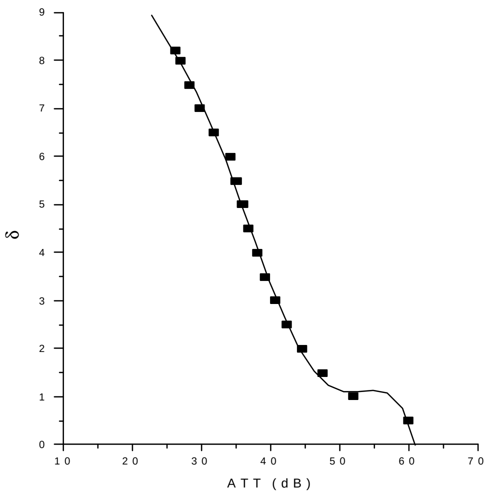

FIG. 8. Plot for ATT vs ***d*** (PC4 window).

Table I
Comparative performance of filters

ATT (dB)	*F*	N Kaiser	RC	BH	PC6	PC4
40	0.15	17	Attenuation	27	23	25
	0.10	25	out of range	41	35	37
	0.05	47		78	67	71
45	0.15	19	23	29	27	31
	0.10	27	35	43	39	47
	0.05	53	67	81	77	91
50	0.15	21	25	Attenuation	29	37
	0.10	31	37	out of range	41	55
	0.05	61	71		81	109
55	0.15	23	27	-do-	39	39
	0.10	35	39		59	57
	0.05	67	77		115	113
60	0.15	27	29	-do-	51	41
	0.10	39	41		75	61
	0.05	75	81		147	119
65	0.15	29	29	-do-	51 }	Attenuation
	0.10	41	43		75 }	out of
	0.05	81	85		149 }	range
69	0.15	31	31	-do-	Attenuation	-do-
	0.10	45	45		out of range	
	0.05	87	87			

$$\mathrm{D} = a + (b*\mathrm{ATT}) + (c*\mathrm{ATT}^2) + (d*\mathrm{ATT}^3) + (e*\mathrm{ATT}^4) \quad \text{for } 26.187 < \mathrm{ATT} \leq 61.076, \tag{12}$$

where a = 8.728537, b = –0.412899, c = – 0.000713, d = 0.000355 and e = –0.000004.

Similarly, using Fig. 8, the following relationship between ***d*** and ATT is obtained.

$$\boldsymbol{d} = a + (b*\mathrm{ATT}) + (c*\mathrm{ATT}^2) + (d*\mathrm{ATT}^3) + (e*\mathrm{ATT}^4) + (f*\mathrm{ATT}^5) \quad \text{for } 26.187 < \mathrm{ATT} \leq 61.076, \tag{13}$$

where a = 83.409, b = –10.9793, c = 0.644612, d = –0.0185287, e = 0.000253473, and f = –1.32148 $*10^{-6}$.

3. Conclusion

Table I provides a comparative performance analysis of these filters. Kaiser window-based filters require least order for a given set of specifications. However, the RC, BH, PC4 and PC6 families have the advantage of being very simple in form. The modified zeroth-order Bessel family needs the same order as the RC family in the vicinity of ATT = 69 dB. Due to higher value of SLFOR, combinational window family filters exhibit a greater descend in the stopband attenuation. Figure 9 shows a comparative frequency response analysis between Kaiser, PC4 and PC6 window filters. Thus, combinational window filters provide a better suppression in the stopbands than Kaiser window filters; however, a higher filter order is required. This feature of combinational window filters can be employed for image

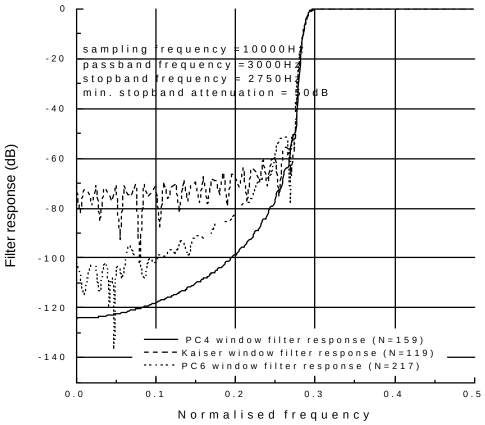

FIG. 9. Comparative plot for high pass filter.

suppression in multirate signal processing. Also, these high SLFOR filters can be used for restricting spectral interaction to occur only between adjacent pairs of filters in quadrature mirror filter design [11].

Acknowledgement

The authors thank the reviewers for guidance and the editorial board to bring the paper to a presentable shape.

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