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A simple method of tuning cascade controllers

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Abstract

A simple method is proposed to design PID controllers for a series cascade control system. The method is based on matching the coefficients of corresponding powers of q and q^2 in the numerator to a_1 and a_2 times that in the denominator of the closed loop transfer function model for a servo problem. This method can be used only when the inner and outer loop transfer functions are known. If these functions are not known, then an identification step needs to be carried out. The method is first applied to design a proportional (P) controller for the inner loop and then to a proportional plus integral (PI) controller for the outer loop. The performance of the proposed controllers is evaluated for an FOPTD model of the inner loop and outer loop process transfer function models. The response and robustness due to perturbation in model parameters are evaluated and compared with the methods of Krishnaswamy and Rangaiah (When to use cascade control, *Ind. Engng Chem. Res.*, **29**, 2163–2166 (1990)) and Lee *et al*. (PID controller tuning to obtain closed loop response for cascade control scheme, *Ind. Engng Chem. Res.*, **37**, 1859–1865 (1998)). The proposed method gives better servo and regulatory performance.

Keywords: Cascade controllers, process control.

1. Introduction

Cascade control is one of the most popular structures for process control. A cascade control system consists of a primary controller and a secondary controller (Fig. 1). Cascade control scheme is used to improve the dynamics response of the closed loop system when the disturbance enters the inner loop or disturbances are present in the manipulated variable. The frequency response method [1] is usually employed to design such controllers. The method involves trial and error graphical method. Krishnaswamy and Rangaiah [2] have proposed a tuning chart that predicts the primary controller settings by minimizing ITAE criterion due to load disturbances on the secondary loop. Lee *et al*. [3] have proposed a synthesis method of designing series cascade controllers. The method consists of first finding the ideal controllers that give desired closed loop responses and then finding the PID approximation of the ideal controllers by Maclaurin series. Their method is shown to be better than that of the IATE method. However, the method has two tuning parameters (closed loop and outer loop time constants) whose values are to be selected by trial and error method. In the present work, a simple method is proposed for designing cascade controllers. The proposed method is an extension of the method of Chidambaram and Padmasree [4, 5] for a single loop feedback system to a cascade control system and is based on servo problem. Most of the meth-

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ods in the literature, like IMC, Pole placement, synthesis, etc. are based on the servo problem. The servo problem needs only $y(s)/u(s)$ information, while controller based on regulatory problem requires which load variable is considered (inner or outer loop) and its transfer function model.

2. The proposed method

Let us consider the transfer function model of the process for the inner loop (Fig. 1) as a FOPTD (first-order plus time delay) model:

$$y_2(s)/u(s) = [k_{p2}\exp(-L_2 s)/(t_2 s + 1)]. \tag{1}$$

The transfer function of the process in the outer loop is also an FOPTD model:

$$y_1(s)/y_2(s) = [k_{p1}\exp(-L_1 s)/(t_1 s + 1)]. \tag{2}$$

Usually, we consider a simple proportional (P) controller for the inner loop and a proportional plus integral (PI) controller for the outer loop. This combination gives a good performance and only three tuning parameters are to be calculated [3]. We first design inner loop controller. The closed loop transfer function model for the inner loop is given by

$$y_2/y_{2r} = K_2\exp(-eq)/[q + 1 + K_2\exp(-eq)], \tag{3}$$

where

$$K_2 = k_{c2}k_{p2};\ e = L_2/t_2,\ q = t_2 s. \tag{4}$$

The time delay system responds only after the time delay (L), hence we cannot alter response between time $t = 0$ and L. In eqn (3), we need not consider further, the term exp $(-eq)$ in the numerator as it shifts only the time axis. To make the degree of the polynomial in q in the numerator same as that of the denominator, the followings steps were carried out:

In the denominator exp $(-eq)$ is rewritten as exp $(-0.5\,eq)$/exp $(0.5\,eq)$. Then the term exp $(0.5\,eq)$ is taken into the numerator and also to the first term in the denominator. By substituting the Taylor's series approximation for the delay terms exp $(0.5\,eq)$ and exp $(-0.5\,eq)$, we get

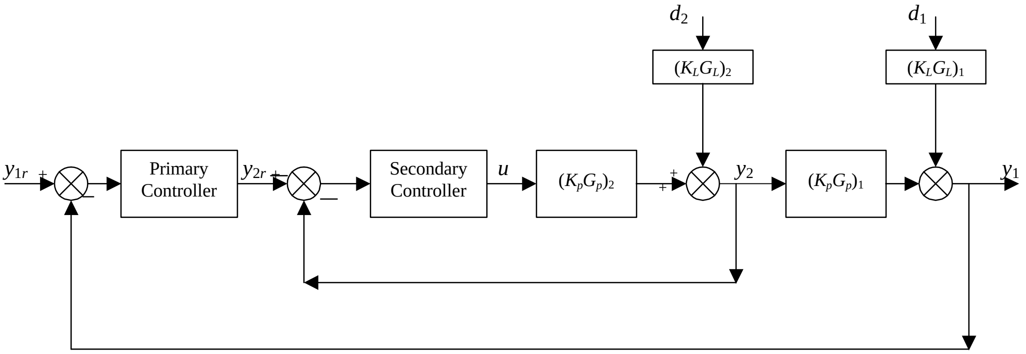

FIG. 1. Series cascade control scheme.

$$y_2/y_{2r} = K_2[1 + 0.5\ eq + 0.125\ e^2q^2 + \ldots]/[(q + 1)(1 + 0.5\ eq + 0.125\ e^2q^2 + \ldots.) + K_2(1-0.5\ eq + 0.125\ e^2q^2 - \ldots)]. \quad (5)$$

Since the objective of the control system is to make y follow y_r, the numerator polynomial can be made equal to denominator polynomial. For the controller design, depending upon the type of controller, we equate the corresponding coefficient of q or q^2 or q^3 in the numerator to that of the denominator. This means that for proportional controller we equate the coefficient of q only. For design of PI controller, the coefficients of q and q^2 are related to that of the denominator. For the design of PID controllers, the coefficients of q, q^2 and q^3 are to be related. Hence, accordingly, proper number terms in the Taylor's series approximation for exp (0.5 eq) and exp (–0.5 eq) are considered.

Since a proportional controller is used, there is bound to be an offset. Hence the coefficient of q^0 in the numerator cannot be equated to the denominator. By equating the coefficient of q in the numerator to that in the denominator, controller gain is calculated as:

$$k_{c2} = [(1/e) + 0.5]/k_{p2}. \quad (6)$$

Similar procedure is adopted to design the controller for the outer loop. Now, we have PI controller in the outer loop. The closed loop transfer function for the outer loop is obtained as:

$$y_1/y_{1r} = R_1\exp(-hs)/R_2 \quad (7)$$

$$R_1 = (P_1q + P_2)\exp(0.5yq) \quad (8)$$

$$R_2 = [(q + 1)\exp(0.5\ eq) + K_2\exp(-0.5\ eq)][fq^2 + q]\exp(0.5\ hq) + (P_1q + P_2)\exp(-0.5\ hq)\exp(0.5\ eq)] \quad (9)$$

where

$$f = t_1/t_2;\ t_I^* = t_I/t_2;\ l = L_1/t_2;\ h = (e + l);\ y = h + e \quad (10)$$

$$K_1 = k_{c1}k_{p1};\ K_2 = k_{c2}k_{p2};\ P_1 = K_1K_2;\ P_2 = P_1/t_I^*. \quad (11)$$

In order to make the degree of the numerator equal to that of the denominator, we make use of the Taylor's series approximation for the exponential term. Since we equate the coefficients of q and q^2 in the numerator to that in the denominator, it is sufficient to retain the terms in the Taylor's series only up to the power of q^2.

Let us now consider the proposed method with two tuning parameters (a_1 and a_2). It is to be noted that there are no tuning parameters for the inner loop. Similar to the procedure carried out for designing the inner loop controller, on equating the coefficient of q, q^2 in the numerator to a_1 and a_2 times that of the corresponding coefficient in the denominator in eqn (7), we get:

$$P_1A_1 + P_2A_2 = \alpha_1(1 + K_2) \quad (12)$$

$$P_1A_3 + P_2A_4 = a_2A_5 \quad (13)$$

where $A_1 = (1-a_1)$

$$A_2 = 0.5\ (y + l\,a_1)$$

$$A_3 = 0.5\ (y + l\,a_2)$$

$$A_4 = 0.125\ (y^2 - l^2 a_2)$$

$$A_5 = (f(1 + K_2) + 1 + 0.5\ e - 0.5\ eK_2 + 0.5\ h(1 + K_2)). \quad (14)$$

Once P_1 and P_2 are calculated, then k_{c1} and t_I can be calculated from eqn (11) as:

$$k_{c1} = P_1/(k_{c2}k_{p2}k_{p1}); \quad (15)$$

$$t_I = (k_{c1}k_{p1}k_{c2}k_{p2})\,t_2/P_2. \quad (16)$$

3. Simulation results

Let us consider the example system considered by Lee *et al*. [3] with the transfer functions models as:

$$k_{p1}G_{p1} = \exp(-10\ s)/(100\ s + 1); \quad (17)$$

$$k_{p2}G_{p2} = 2.0\ \exp\ (-2\ s)/(20\ s + 1). \quad (18)$$

Lee *et al.* have proposed the settings of *P* for the inner loop and PI for the outer loop as $k_{c2} = 3.44$, $k_{c1} = 5.83$ and $t_{I1} = 105$, respectively. Krishnaswamy and Rangaiah [2] have proposed the corresponding settings as $k_{c2} = 2.978$, $k_{c1} = 7.3$ and $t_{I1} = 200$. There are no tuning parameters for the inner loop, but the outer loop has two. It is to be noted that for single-loop FOPTD system, Padmasree and Chidambaram [4] have shown that equating corresponding coefficients (i.e. making $a_1 = 1.0$ and $a_2 = 1.0$) give a good result. We call this method as method-1. Thus, equating $a_1 = 1.0$ and $a_2 = 1.0$ for the above example gives the settings as $k_{c2} = 5.25$, $k_{c1} = 9.666$ and $t_{I1} = 105.91$ (Table I). The servo responses for a unit step change in the set point of y_{1r} are shown in Fig. 2. Similarly, the regulatory responses for a unit step change in the load (entering in the secondary loop at the outlet of process) are compared in Fig. 3. The error comparisons of different methods under perfect parameters are shown in Table II. However, it is found in the present work that such an approach gives a better regulatory performance than that of the servo response. In Tables I and II, all the performance measures such as ISE, IAE and ITAE are given. However, since the response takes a considerably longer time, the ITAE measure will be appropriate here for comparison.

The first method in which the controller settings are calculated based on $a_1 = 1.0$ and $a_2 = 1.0$ gives an oscillatory servo response. This may be due to tight settings. Hence, a

Table I
P/PI Controller settings

Controller parameters	Proposed method-1	Krishnaswamy and Rangaiah [2]	Lee *et al.* [3]
k_{c2}	5.25	2.978	3.44
k_{c1}	9.6667	7.3	5.83
t_I	105.91	200	105

For the proposed method: $a_1 = 1.0$ and $a_2 = 1.0$.

Table II
Performance comparison of different methods under perfect parameters

Error	Proposed method-1		Krishnaswamy and Rangaiah [2]		Lee *et al.* [3]	
	Servo	Regulatory	Servo	Regulatory	Servo	Regulatory
ISE	22.194	0.0133	20.19	0.065	19.94	0.044
IAE	38.959	1.1258	38.48	4.1602	27.37	2.5954
ITAE	1590	99.3511	2920	651.1177	593.73	257.767

For controller settings refer to Table I.

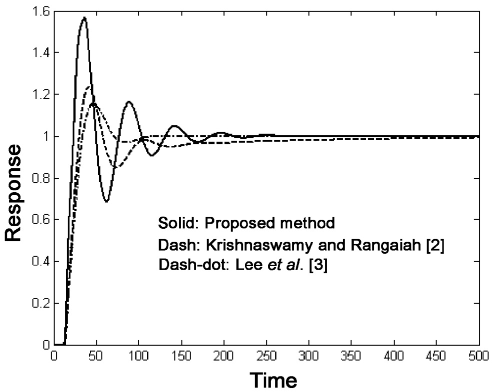

FIG. 2. Comparison of different methods for servo response in y_1 using PI controller for outer loop and P controller for inner loop, under perfect parameters ($a_1 = 1.0$ and $a_2 = 1.0$).

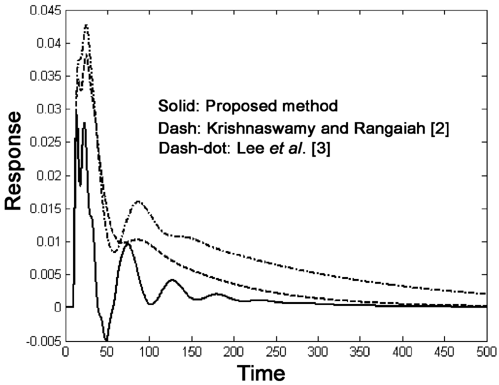

FIG. 3. Comparison of different methods for regulatory response in y_1 for a disturbance in the inner loop. ($a_1 = 1.0$ and $a_2 = 1.0$).

second method is proposed. The values of a_1 and a_2 are chosen to provide a good performance. The guidelines for a_1 and a_2 are studied especially for the case of FOPTD model. Extensive simulation studies are carried out to find the best ratio of a_2/a_1 in the sense of ITAE performance. The ratio a_2/a_1 is varied from 0.3 to 0.7 and that of a_1 from 0.5 to 1.2. It is observed that at $a_2/a_1 = 0.4$ and $a_1 = 0.9$ for the servo response in y_1, the lowest ITAE is obtained. We call this modified method as method-2. For the outer loop, since the values of these parameters (a_1, a_2/a_1) vary between 0.5 and 1, it may be easier to tune these parameters to calculate the controller settings.

It is found by simulation that for the present case study, use of $a_1 = 0.9$ and $a_2 = 0.4\ a_1$ (giving $k_{c2} = 5.25$, $k_{c1} = 4.68$ and $t_{I1} = 104.3$) gives the best result. From eqn (6), it is seen that k_{c2} does not depend upon a_2. But a_2 affects k_{c1}. Therefore, the effect of reduction of the primary controller gain due to changes in a_1 and a_2 is observed on the outer loop only. In general a_2 affects both k_{c1} and t_I. However, for the particular values of $a_2/a_1 = 0.4$ and $a_1 = 0.9$, the value of t_I is not affected. Basically we first design the inner loop controller. Then the outer loop controller is designed.

The controller settings by different methods are given in Table III. The servo responses for a unit step change in the set point of y_{1r} are shown in Fig. 4. Similarly, regulatory re-

Table III
Controller settings when $a_1 = 0.9$ and $a_2 = 0.4\ a_1$

Controller parameters	Proposed method-2
k_{c2}	5.25
k_{c1}	4.68
t_I	104.3

Table IV
Performance comparisons of different methods under perfect parameters

Error	Proposed method-2		Krishnaswamy and Rangaiah [2]		Lee *et al.* [3]	
	Servo	Regulatory	Servo	Regulatory	Servo	Regulatory
ISE	20.05	0.025	20.19	0.065	19.94	0.044
IAE	25.99	2.104	38.48	4.1602	27.37	2.5954
ITAE	495.11	215.41	2920	651.1177	593.73	257.767

Controller settings are given in Table III for the present method.

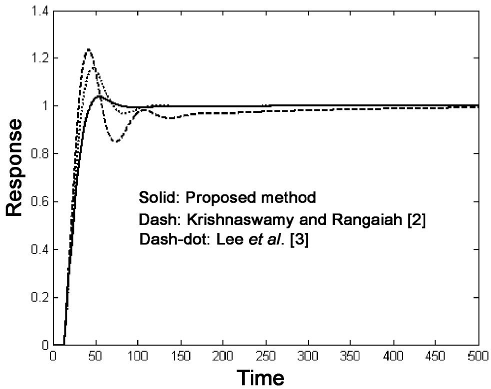

FIG. 4. Comparison of different methods for servo response in y_1 using PI controller for outer loop and P controller for inner loop, under perfect parameters.

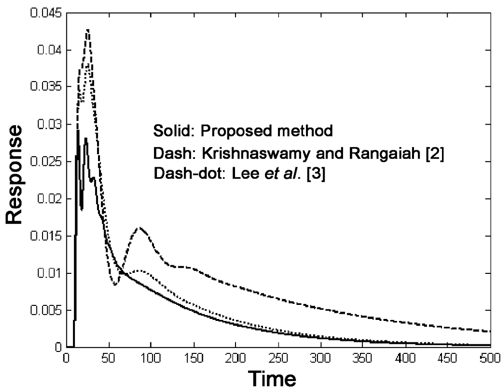

FIG. 5. Comparison of different methods for regulatory response in y_1 for a disturbance in the inner loop under perfect parameters.

sponses for a unit step change in the load (entering the secondary loop at the outlet of process) are compared in Fig. 5. The initial oscillations in the manipulated variable response are also observed for the method of Lee *et al*. Table IV gives performance comparison of the controlled system. The proposed method gives the best result. This method can be used only when the inner and the outer loop transfer functions are known. If these transfer functions are not known, then an identification step needs to be carried out.

The robustness of the proposed controller is evaluated for 20% perturbation of inner loop process gain (in simulation the process gain used is 1.2 times of the value used in designing the controller). Figures 6–11 show the servo and the regulatory responses. Table V(a) shows performance comparison of the controlled system under uncertainty in k_{p2}. The slightly improved performance of the controller under perturbation in k_{p2} is obtained for the other controller design methods also (refer to Tables VI(a) and IV). The gain margin calculations (Table VII) show that all the three methods use a higher value of the gain margin.

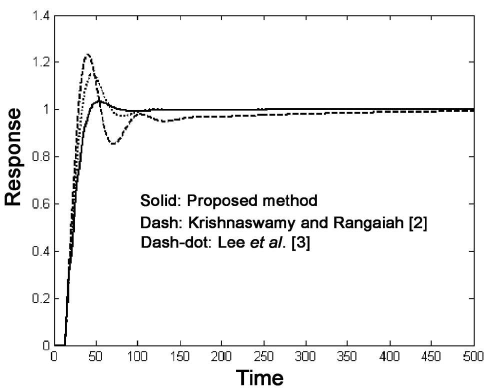

FIG. 6. Servo response in y_1 using PI controller for the outer loop and P controller for the inner loop, with 20% uncertainty in k_{p2} in the process.

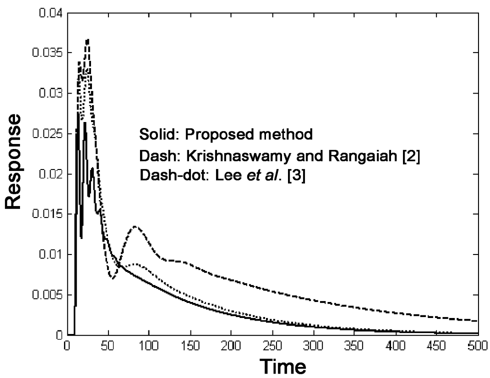

FIG. 7. Regulatory response in y_1 for a disturbance in the inner loop with 20% uncertainty in k_{p2} in the process.

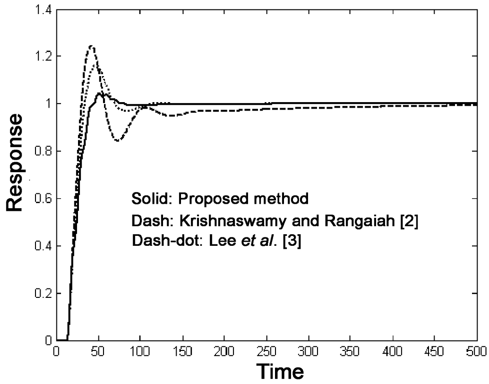

FIG. 8. Servo response in y_1 using PI controller for the outer loop and P controller for the inner loop, with 20% uncertainty in L_2 in the process.

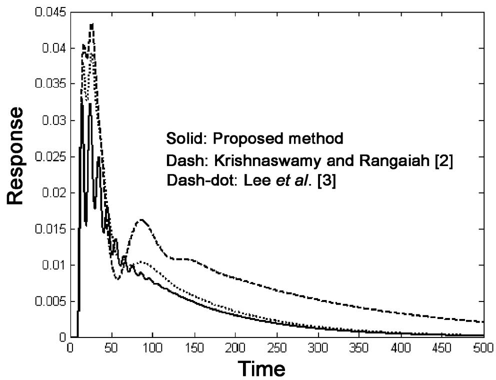

FIG. 9. Regulatory response in y_1 for a disturbance in the inner loop with 20% uncertainty in L_2 in the process.

Hence, under this condition, the perturbed system will give a better performance. The present method (Method-2) gives the best performance.

Simulation result shows that the present method is also robust (refer to Tables V(b) and V(c)) for uncertainty in time delay and separately in the time constant. In general, a feedback loop tries to reduce the sensitivity of the model parameters on the closed loop performance. In cascade control systems, because of the additional feedback loop, the perturbation in the inner loop process gain and time delay will not have significant effect on the servo and regulatory problems.

The proposed controller is also evaluated for 20% perturbation of outer loop process gain. Similar robustness behavior is also observed for a perturbation in the outer loop process gain (Table VI(a)), separately in the process time delay (refer to Table VI(b)) and in the time constant (refer to Table VI(c)). For all the three design methods, the perturbation or

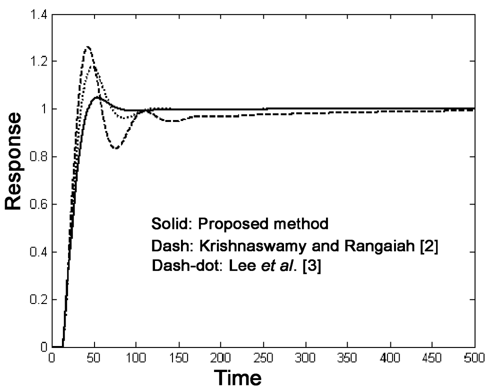

FIG. 10. Servo response in y_1 using PI controller for the outer loop and P controller for the inner loop, with 20% uncertainty in t_2 in the process.

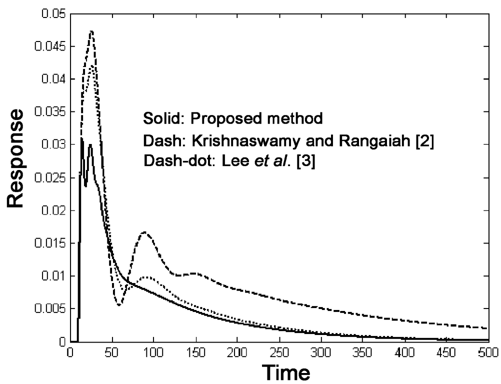

FIG. 11. Regulatory response in y_1 for a disturbance in the inner loop with 20% uncertainty in t_2 in the process.

Table V(a)
Performance comparison of the controlled system under uncertainty in k_{p2} 20% high (k_{p2} = 2.4 in the process; controller designed for k_{p2} = 2)

Error	Proposed method-2		Krishnaswamy and Rangaiah [2]		Lee *et al.* [3]	
	Servo	Regulatory	Servo	Regulatory	Servo	Regulatory
ISE	19.65	0.0183	19.57	0.0456	19.37	0.0309
IAE	25.45	1.7535	37.397	3.4690	26.46	2.1629
ITAE	476.16	178.92	2830.8	541.3174	555.54	213.9037

Table VI(a)
Performance comparison of the controlled system under uncertainty in k_{p1} 20% high in the process (k_{p1} = 1.2 and a_1 = 0.9 and a_2 = 0.4 a_1)

Error	Proposed method-2		Krishnaswamy and Rangaiah [2]		Lee *et al.* [3]	
	Servo	Regulatory	Servo	Regulatory	Servo	Regulatory
ISE	18.88	0.0292	21.38	0.0764	19.80	0.0515
IAE	25.33	2.105	40.82	4.1789	29.36	2.5963
ITAE	495.83	207.33	2764.5	641.125	741.79	249.32

Table V(b)
Performance comparison of the controlled system under uncertainty in L_2 20% high (L_2 = 2.4 in the process and controller designed for L_2 = 2)

Error	Proposed method-2		Krishnaswamy and Rangaiah [2]		Lee *et al.* [3]	
	Servo	Regulatory	Servo	Regulatory	Servo	Regulatory
ISE	20.14	0.0266	20.4728	0.0659	20.1053	0.0446
IAE	25.97	2.1043	38.8687	4.1611	27.5322	2.5955
ITAE	493.44	214.61	2935.8	649.8758	599.3067	256.77

Table VI(b)
Performance comparison of the controlled system under uncertainty in L_1 20% high in the process (L_1 = 12 and a_1 = 0.9 and a_2 = 0.4 a_1)

Error	Proposed method-2		Krishnaswamy and Rangaiah [2]		Lee *et al.* [3]	
	Servo	Regulatory	Servo	Regulatory	Servo	Regulatory
ISE	22.15	0.0271	23.86	0.0698	22.76	0.0473
IAE	29.51	2.1043	44.6626	4.161	33.09	2.5955
ITAE	650.53	215.4813	3230.9	651.70	911.15	257.82

Table V(c)
Performance comparison of the controlled system under uncertainty in t_2 20% high (t_2 = 24 in the process and controller designed for t_2 = 20)

Error	Proposed method-2		Krishnaswamy and Rangaiah [2]		Lee *et al.* [3]	
	Servo	Regulatory	Servo	Regulatory	Servo	Regulatory
ISE	20.39	0.0283	20.9848	0.0722	20.5430	0.0496
IAE	26.51	2.1051	39.8310	4.1694	28.6465	2.5965
ITAE	516.37	207.45	2976.5	639.3069	657.3665	247.94

Table VI(c)
Performance comparison of the controlled system under uncertainty in t_1 20% high in the process (t_1 = 120 and a_1 = 0.9 and a_2 = 0.4 a_1)

Error	Proposed method-2	Krishnaswamy and Rangaiah [2]			Lee *et al.* [3]	
	Servo	Regulatory	Servo	Regulatory	Servo	Regulatory
ISE	21.74	0.0242	20.36	0.0599	21.02	0.0404
IAE	30.81	2.1075	35.30	4.1685	29.44	2.5987
ITAE	928.44	217.5	2405.5	658.37	811.68	259.821

variation in the outer loop parameters (k_{p1} or L_1 or t_1) does not affect the regulatory response. The ITAE is considered here for performance evaluation. In the present method, the variation in time delay has much less effect on the performance than that of the variation in the time constant. The proposed method gives an improved performance than that of the other methods.

4. Stability analysis

The stability of the given controllers can be checked by calculating the gain and phase margin. Ho *et al.* [6] reported a method of calculating a phase margin and gain margin of well-known PID tuning formulas. The settings, which give larger phase margin and gain margin

Table VII
Gain margin (A_m) and phase margin (f_m) with inner loop P controller and outer loop PI controller

Method	Proposed method-2	Krishnaswamy and Rangaiah [2]	Lee *et al*. [3]
A_m	2.9587	2.3337	2.695
f_m(°)	60.89	55.98	57.78

are preferred. The outer loop process and controller transfer functions are denoted by $G_p(s)$ and $G_c(s)$, respectively. The loop transfer function is given by

$$G_p(s)G_c(s) = k_{c1}(1 + (1/t_I s))((k_{c2}k_{p2}\exp(-L_2 s))/(ts + 1 + k_{c2}k_{p2}\exp(-L_2 s)))(k_{p1}\exp(-L_1 s))/(t_1 s + 1) \quad (19)$$

The frequency at which the Nyquist curve has a phase of '$-p$ (phase cross over frequency, w_p)' is obtained by solving the following equation:

$$\tan^{-1}(-1/t_I w_p) - L_1 w_p - L_2 w_p + \tan^{-1}(-t_1 w_p) + \tan^{-1}((t_2 w_p - k_2\sin(L_2 w_p))/(1 + k_2\cos(L_2 w_p))) + p = 0 \quad (20)$$

The gain margin is obtained by the following equation:

$$A_m = \left(\frac{t_I w_p}{k_1 k_2}\right) \frac{(1+t_1^2 w_p^2)^{0.5}}{(1+t_I^2 w_p^2)^{0.5}} \frac{(((1+k_2\cos(L_2 w_P))^2 + (t_2 w_P - k_2\sin(L_2 w_P))^2)}{(((1+k_2\cos(L_2 w_P))^2 + (t_2 w_P - k_2\sin(L_2 w_P))^2)^{0.5}}. \quad (21)$$

The frequency at which the Nyquist curve has amplitude of 1 is known as gain cross over frequency (w_g), and is obtained by solving the following equation:

$$k_1 k_2 = t_I w_g \quad \frac{(1+t_1^2 w_g^2)^{0.5}}{(1+t_I^2 w_g^2)^{0.5}} \frac{(((1+k_2\cos(L_2 w_g))^2 + (t_2 w_g - k_2\sin(L_2 w_g))^2)}{(((1+k_2\cos(L_2 w_g))^2 + (t_2 w_g - k_2\sin(L_2 w_g))^2)^{0.5}}. \quad (22)$$

The phase margin is given by the following equation:

$$f_m = \tan^{-1}(-1/t_I w_g) - L_1 w_g - L_2 w_g + \tan^{-1}(-t_1 w_g) + \tan^{-1}((t_2 w_g - k_2\sin(L_2 w_g))/(1 + k_2\cos(L_2 w_g))) + p = 0. \quad (23)$$

In the present work, the phase margin and the gain margin are calculated for the system $k_{p1}G_{p1} = \exp(-10\ s)/(100\ s + 1)$ and $k_{p2}G_{p2} = 2.0\ \exp(-2\ s)/(20\ s + 1)$ with the controller designed by different methods (Table VII). Controller designed by the present method gives the largest phase margin and hence is more stable and robust than the other methods. These are also shown by simulation study of the closed loop response as discussed earlier.

6. Conclusion

A simple method, by relating the coefficients of q, q^2 of the numerator to that of the denominator of the closed loop transfer function model, is proposed to design a series cascade control system. The servo and regulatory responses of the method are better when compared to the method of Lee *et al*. [3] and Krishnaswamy and Rangaiah [2]. The present method has two tuning parameters with the range 0.2–1.3. It gives the best robust performance when there is an uncertainty in the model parameters. The stability of the proposed controller is also analyzed theoretically.

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Nomenclature

A_1 to A_5	defined by eqn (12)
A_m	Gain margin
d_1	disturbance entering outer loop
d_2	disturbance entering inner loop
k_{c1}, t_I	outer loop PI controller settings
k_{c2}	inner loop controller gain
$(k_p G_p)_1$	outer loop transfer function model
$(k_p G_p)_2$	inner loop transfer function model
$(k_L G_L)_1$	transfer function for load disturbance in the outer loop
$(k_L G_L)_2$	transfer function for load disturbance in the inner loop
K_1	$= k_{p1} k_{c1}$
K_2	$= k_{p2} k_{c2}$
P_1	$= K_1 K_2$
P_2	$= P_1 / t_I^*$
a_1 and a_2 tuning parameters	
e	$= L_1 / t_2$
l	$= L_2 / t_2$
f	$= t_1 / t_2$
t_I^*	$= t_I / t_2$
h	$= (e + l)$
y	$= h + e$
w_p	Phase cross-over frequency
w_g	Gain cross-over frequency
f_m	Phase margin