

Short Communication

A hybrid of microwave circuit theory and BEM to analyze irregular coaxial resonators

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Abstract

A field circuit model of the structure of the irregular coaxial resonator (ICR) based on microwave circuit theory is presented. All its circuit parameters are determined by the boundary-element method (BEM). Through this method, a complex three-dimensional electromagnetic problem is converted into two-dimensional electromagnetic and equivalent microwave circuit problems. Numerical results show good accuracy fit for engineering application. The procedure is general and can be used to analyze arbitrary irregular coaxial cavities also.

Keywords: Coaxial resonators, microwave equivalent circuit, boundary-element method.

1. Introduction

Most cyclotron cavities (CC) are irregular coaxial resonators (ICR). The analysis of irregular coaxial resonators is inherently a three-dimensional (3D) electromagnetic boundary value problem (EBVP). It is rather a time- and energy-consuming task even using available commercial software.

To effectively analyze ICR, a novel method which combines the microwave equivalent circuit and BEM method is presented in the paper. Using the new method, a 3D EBVP problem is converted into two-dimensional (2D) EBVP and transmission line circuit problems. All circuit parameters can be determined by using BEM to solve the 2D EBVP. The method has good efficiency and accuracy as only 2D EBVP and circuit analysis are involved.

2. Microwave equivalent circuit of irregular coaxial resonator

A symmetrical half CC structure, shown in Fig. 1, is taken as an example, where accelerating electrodes, usually called Dees, are located at the center of the structure. High-frequency power source is applied to the tuner and is used to fine-tune the resonance frequencies and is structured as a circular parallel plate capacitor. One plate of this capacitor is electrically connected to Dees, while the other is connected to the outer wall of the cavity. It can be

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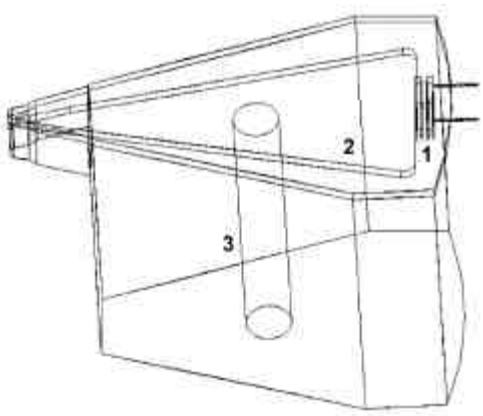


FIG. 1. An irregular coaxial resonant cavity. 1: Tuner, 2: Dees, and 3: Inner pole.

seen that the main transmission line of the cavity is an irregular transverse electromagnetic (TEM) coaxial waveguide comprising an inner pole and an outer shield, while the Dees and the junction of the sections also affect the resonance, in addition to the tuner. Thus the fundamental block can be divided into sections of waveguide, waveguide discontinuity, and lumped components. Each block is modeled on the microwave theory.

For a general waveguide to be modeled, its transverse modal electric and magnetic fields can be expressed [1] as

$$E_T = V(z)e(t) \quad (1)$$

$$H_T = I(z)h(t) \quad (2)$$

where t is a 2D vector and $t = x\vec{x}_0 + y\vec{y}_0$ for Cartesian coordinates. Transverse modal functions, $e(t)$ and $h(t)$, are governed by the following Helmholtz equations

$$(\nabla_T^2 + k_T^2)e(t) = 0 \quad (3)$$

$$(\nabla_T^2 + k_T^2)h(t) = 0 \quad (4)$$

where $\nabla_T^2 = \partial/\partial x^2 + \partial/\partial y^2$, $k_T^2 = k_x^2 + k_y^2$ and modal voltage $V(z)$ and current $I(z)$ are governed by the following transmission line equations

$$\frac{dV(z)}{dz} = -ZI(z) \quad (5)$$

$$\frac{dI(z)}{dz} = -YI(z) \quad (6)$$

where k_x , k_y , and k_z are wave vector components in x -, y -, and z -directions, respectively. Z and Y are modal impedance and admittance, respectively. Therefore, a general waveguide can be represented by a field transmission line depicted by two characteristic parameters k_z and Z (or Y).

Since the impedance of equivalent transmission line formed by the inner pole and the outer shield and the other equivalent transmission line formed by Dees and outer shield do

not equal each other, the connection results in discontinuity. Because of the higher-order modes excited nearby, discontinuity is associated with 3D electromagnetic fields. Scattering parameters should be used to describe the case [2]. The analysis of irregular, coaxial discontinuities [3, 4] or those in similar TEM situation [5] are still problems under exploration. However, when all possible higher-order modes are of E-type, the effect of discontinuity is equivalent to a capacitance, or the so-called step capacitance C_s . Its value, C_s , depends on the dimensions of outer and different inner conductors and engineering formulation is available for a canonical coaxial waveguide discontinuity [6].

The tuners are lumped components here and are relatively easily modeled as parallel capacity plate. The equivalent microwave circuit, which consists of the lumped tuner, transmission line and its discontinuity, of the total half CC structure is as shown in Fig. 2.

3. Model parameters determined

Parameters k_{z1} and $Z1$, shown in Fig. 2, for the waveguide 1 formed by the inner pole and the outer shield and k_{z2} and $Z2$ for the waveguide 2 formed by the Dees and outer shield can be determined by solving 2D Helmholtz equations with respect to the boundary shown in Figs 3(a) and (b), respectively. Since both the waveguides are operating with the required TEM mode, it is known that $k_T = 0$ and $k_{z1} = k_{z2} = k_0$, and $Z1$ and $Z2$ can simply be determined by Laplace equation

$$\nabla_T^2 e(t) = 0 \quad (\text{or } \nabla_T^2 h(t) = 0) \quad (7)$$

with respect to the corresponding boundary conditions. While focusing on fields and resonance, the perfect conductor boundary conditions can be used.

One end of the coaxial TEM waveguide 1 is short-circuited and the other is connected physically to the accelerating electrodes, or Dees, as shown in Fig. 1. It is incorrect to connect two transmission lines in the equivalent circuit directly because of the existing discontinuity between them. Strictly speaking, a scattering matrix needs to take the effect of step into account. For the first-order approximation, only the impact to the lowest TEM mode is considered. The so-called step capacitance C_s denotes discontinuity, while the capacity C_a represents the effect of the lumped tuner. The driven terminals are denoted with cross signs in Fig. 2.

After transforming waveguides 1 and 2 to equivalent canonical coaxial TEM transmission lines, step capacitance C_s can be directly calculated by a known formula [6] to get rid of any 3D EBVP. The transformation may have different criteria; here the impedance equivalence is considered.

It is rather simple now to analyze the circuit finally obtained, since one has to deal with only a circuit problem, instead of a field one. All field aspects are summed up into circuit parameters $Z1$, $Z2$ and C_s . As mentioned earlier, $Z1$ and $Z2$ can be evaluated by solving different 2D Laplace equations using any mature numerical technique; here, BEM [7] is used. The corresponding formulae to solve the impedance parameters are as follows:

$$Z = \frac{1}{cv}; \quad (8)$$

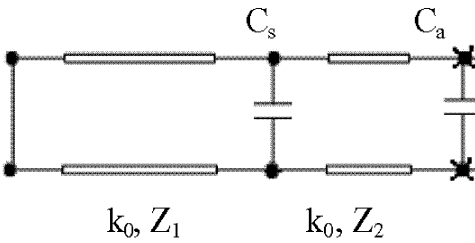


Table I
Calculated and measured resonant frequencies

Gap of the tuner (mm)	Resonant frequency (MHz)	
	Calculated	Measured
10	64.991	62.2
11	65.644	63.3
13	66.675	64.9

FIG. 2. Circuit model for cyclotron cavity structure.

$$c = \frac{q}{V}. \tag{9}$$

In (8), v is the velocity of light and distributed capacity is denoted by c in (8) and (9). By solving a 2D general static electromagnetic boundary-value problem, the distributed capacity of the waveguides 1 and 2 can be easily obtained.

4. Numerical results and discussion

The computer program is very concise and hence the whole frequency fine-tune characteristics can be obtained easily. The key parameters of the example, shown in Fig. 1, are as follows: the half length of the irregular coaxial cavity is 550.0 mm; the radius of the inner pole is 40 mm; the depth of the two Dees is 30.0 mm with a gap of 30 mm between each of them; the angle of acceleration gap is about 6 degrees; the diameter of the tuner is 130 mm. The characteristic parameters of the equivalent circuit extracted by the BEM are as follows: the step capacitance is $C_s = 14.778$ pF; the characteristic impedances are $Z1 = 96.539$ ohm and $Z2 = 8.457$ ohm. The tuner is regarded as parallel plate capacitance. The resonant frequency is varied along with a gap of two Dees.

The calculated resonant frequency of the method presented in this paper and the measured results are compared in Table I. The measured results are obtained from the real cyclotron device. When the gap of the tuner is 10, 11 and 13 mm, the calculated resonant frequency is close to the operating data all the time showing good engineering accuracy.

Table I shows that increase in the gap length of the tuner leads to higher resonant frequency. This interesting phenomena is in conformity with the fact that increase in gap

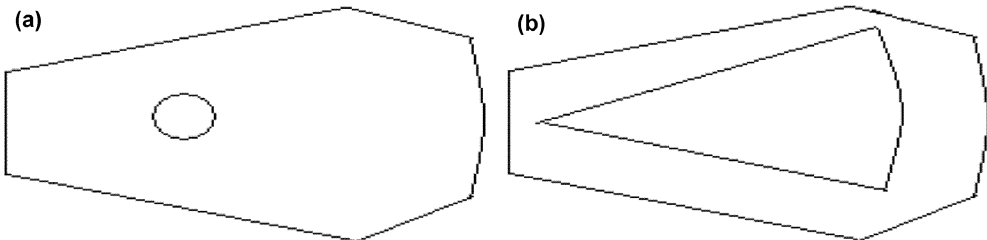


FIG. 3(a). Inner pole and shield waveguide cross-section, and (b) Dees and outer shield waveguide cross-section.

length of the tuner leads to decrease in its capacitance. The equivalent circuit in Fig. 2 shows this reduction will result in higher resonance frequency.

5. Conclusions

This treatment reduces a 3D EBVP problem into a 2D 1D circuit problem. The problems encountered in longitudinal discretization and pseudo mode are overcome and compare favourably with commercial software in efficiency and accuracy. The procedure presented can be extended to more complex irregular coaxial resonators.

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