A COMPARATIVE STUDY OF CYLINDRICAL, ELLIPTICAL AND PRISMATIC FORMS OF ELECTRONIC TUBES

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ABSTRACT

In a previous study of the design shapes of an electronic diode, the prismatic form was shown to be superior to the cylindrical form, from the standpoint of the design parameter, K_d, defined as the ratio of the volume of the vessel to the surface area of the cathode.[2] In the present paper the study is extended to include the condition of space-charge saturation in the device. To facilitate analysis, a third form of tube is considered, in addition to the cylindrical and prismatic forms. This consists of a flat strip cathode surrounded by an anode of thin elliptical cross-section. By the technique of conformal transformation,[3, 4] equivalence is set up between a diode having the elliptical shape and a cylindrical diode. The equivalent structures possessing the same electrostatic capacitance per unit length and giving approximately the same space charge saturated current are then compared as to their design parameters. The thin elliptical diode is found to have a decisively better design parameter than the equivalent cylindrical diode. A further comparison of equivalent elliptical and prismatic diodes shows the latter to be slightly superior to the former. The elliptical form helps to set up equivalence between the prismatic and cylindrical forms of diodes. From the point of view of physical construction, it is likely to provide a more stable structure than the prismatic shape.

1. INTRODUCTION

From the concept of the 'design parameter' of an electronic tube, and from general considerations relating to the economy and techniques of production of

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electronic tubes with thermionic cathodes, a thin flat prismatic structure appears to be a superier alternative to the conventional cylindrical form.[1, 2]   To ensure mechanical strength in a practical design, the shape of the vessel, same as that of the anode, may have to be rounded instead of being of a strictly prismatic form.   In the present study, a thin elliptical coss-section of vessel and anode, surrounding a thin flat strip of cathode, is viewed as one form of practical realization of the prismatic structure.   For such a tube, under the condition that the anode and the cathode are confocal ellipses, the latter chosen to be a degenerate one, it is possible through conformal transformation to find an equivalent cylindrical diode.[3, 4]   This process yields two different tube structures that have the same value of electrostatic capacitance per unit length.   The reason for this is that a conformal transformation preserves the orthogonality of the field and potential lines.

We are then in a position to use a principle put forward by G. D. O'Neill, in the analysis of space-charge-limited currents from two equivalent tube structures, one consisting of a cylindrical cathode and two planar anodes and the other a pair of concentric cylindrical electrodes.[4, 5]   O'Neill's postulate, found by him to be in good agreement with experiment, is that, to a first approximation, it may be assumed that the space-charge saturated currents will be the same in both tube forms provided the cathode has the same electrostatic capacitance per unit length with respect to the anode in either case.

We have a tube structure consisting of a thin flat strip of cathode and elliptical anode, and its equivalent made up of a cylindrical cathode surrounded by a cylindrical anode.   If the cathode surface areas are kept equal, we have, following O'Neill, two tubes having the same space-charge-free capacitance per unit length and giving the same space-charge-saturated current.   The electrostatic capacitance and perveance $(P = I_a/V_a^{3/2})$ of the elliptical diode thus become known.   The design parameters may now be compared for two tubes that have approximately the same electrical characteristics but different geometrical shapes.   The same principle is extended to compare the elliptical diode with the prismatic diode, again keeping the cathode surface areas and the electrostatic capacitances the same in both cases.

2.   CAPACITANCE AND PERVEANCE
OF A TUBE HAVING A FLAT CATHODE AND AN ELLIPTICAL ANODE

Figure I shows a form of tube consisting of an elliptic anode of high eccentricity and a cathode strip which may be visualized as a confocal degenerate ellipse. We should now choose a suitable conformal transformation that changes this geometry to some familiar symmetrical geometry, both giving the same space-charge-free capacitance per unit length.

One such transformation is

$$W = Z + l^2/Z,   \tag{1}$$

where $W = u + jv$, $Z = x + jy$ and l is a real quantity. This converts ellipses in the W-plane to circles in the Z-plane.
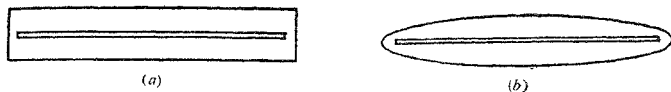

(a)   Fig. I   (b)

(a) Tube with prismatic vessel and anode   (b) Tube with elliptical vessel and anode

Let the width of the cathode strip be $4l$ and let the major and minor axes of the elliptical anode be $2a$ and $2b$ respectively. In order to keep the cathode area per unit length of the tube the same in both cases, we use the following transformation

$$W = \tfrac{2}{8}\,\pi\,Z + l^2/(\tfrac{2}{8}\,\pi\,Z) \tag{2}$$

There results in the Z-plane a cylindrical diode consisting of a cathode cylinder of radius $\rho_c = \tfrac{8}{2}\,\pi\,l$ and an anode cylinder of radius $\rho_p = (a+b)/\tfrac{1}{2}\pi$. The

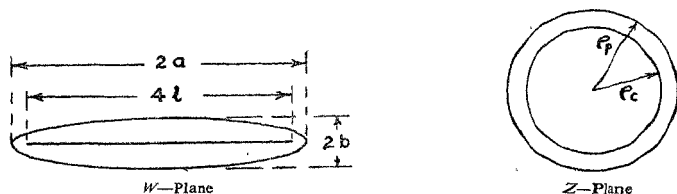W—Plane Z—Plane

Fig. II

Conformal transformation of elliptic diode structure to cylindrical diode

elliptical tube and its equivalent cylindrical tube are shown in Fig II. By the equality of space-charge-free capacitances, we have

$$C = \frac{2\,\pi\,\xi_0}{\ln\,(\rho_p/\rho_c)} = \frac{2\,\pi\,\xi_0}{\ln\,[(a+b)\,(\pi/2)^{-1}/(8\,l/2\,\pi)]} = \frac{2\,\pi\,\xi_0}{\ln\,[(a+b)/2\,l]}$$

$$= \frac{2\,\pi\,\xi_0}{\ln\,\{[a + \sqrt{(a^2 - 4\,l^2)}]/2\,l\}} \quad \frac{\text{farads}}{\text{meter}}, \tag{3}$$

where ξ_0 is the dielectric constant of free evacuated space.

The space-charge-saturated current for the cyclindrical tube is given by[3, 4, 6]

$$\frac{I_a}{h} \cong 2\pi\, 2.33 \times 10^{-6}\, \frac{V_a^{3/2}}{\rho_p\, \beta_p^2} \quad \frac{\text{amp}}{\text{cm}}\,, \tag{4}$$

where $\beta_p = f(\rho_p/\rho_c)$, and h is the active length of the tube. In terms of the dimensions of the equivalent elliptic tube,

$$\frac{I_a}{h} \cong 2\,\pi\, 2.33 \times 10^{-6} \frac{V_a^{3/2}}{[(a+b)/(\pi/2)]\,\beta_p^2} \quad \frac{\text{amp}}{\text{cm}}\,, \tag{5}$$

where $\qquad \beta_p = u - 2u^2/5 + 11\,u^3/120 - 47u^4/3300 + \cdots \tag{6}$

and $\qquad\qquad u = \ln\,(\rho_p/\rho_c) = \ln\,[(a+b)/2\,l] \tag{7}$

The perveance of the elliptic diode, then, becomes

$$P = \frac{I_a}{V_a^{3/2}} = \frac{2\pi\, 2.33 \times 10^{-6}\, h}{[a+\sqrt{(a^2-4\,l^2)}]\beta_p^2/\tfrac{1}{2}\pi} \quad \frac{\text{amp}}{\text{volt}^{3/2}} \tag{8}$$

3. Electrostatic Capacitance and Perveance of the Three Tube Forms

We have the following expressions for the electrostatic capacitance and perveance, for the three geometries, in terms of the essential dimensions of the tubes.

(a) *Prismatic form*

$$C_p = \frac{\xi_0}{r}\, S \quad \text{farads}\,; \qquad P_p = \frac{2.33 \times 10^{-6}}{r^2}\, S \quad \frac{\text{amp}}{\text{volt}^{3/2}} \tag{9}$$

(b) *Cylindrical form*

$$C_c = \frac{2\pi\xi_0}{\ln\,(R_p/R_c)}\, h \quad \text{farads}\,; \qquad P_c = \frac{2\pi\, 2.33 \times 10^{-6}}{R_p\, \beta_p^2}\, h \quad \frac{\text{amp}}{\text{volt}^{3/2}} \tag{10}$$

(c) *Elliptical form*

$$C_e = \frac{2\,\pi\,\xi_0}{\ln\,\{[a+\sqrt{(a^2-4\,l^2)}]/2l\}}\, h \qquad \text{farads}$$

and $\qquad\qquad P_c = \dfrac{2\,\pi\, 2.33 \times 10^{-6}}{[a+\sqrt{(a^2-4\,l^2)}]/\tfrac{1}{2}\,\pi \cdot \beta_p^2}\, h \quad \dfrac{\text{amp}}{\text{volt}^{3/2}} \tag{11}$

In all these expressions, all lengths are in meters, and $\xi_0 = 10^{-9}/36\pi$ farads/meter.

4. Design Parameters of the Three Tube Forms

The design parameter, K_d, or the 'construction quality' of the tube is expressed as[1,2]

$$K_d = V_{\text{vessel}}/ S_{\text{cathode}} \tag{12}$$

If, in the tube forms under consideration, the anode and the envelope coincide, we obtain the following expressions for the design parameters of equivalent cylindrical and elliptical tubes.

$$K_{dc} = (a + b)^2/2 \, \pi \, l \qquad \text{for the cylindrical diode} \tag{13}$$

and $\qquad K_{de} = \pi \, ab/8 \, l \qquad \text{for the elliptical diode} \tag{14}$

The ratio of the design parameters is

$$K_{de}/K_{dc} = \pi^2 \, ab/4 \, (a + b)^2 \tag{15}$$

If b is very small relative to a

$$K_{de}/K_{dc} \cong \pi^2 \, b/4 \, a \tag{16}$$

To compare the elliptical diode with the prismatic form, we must find the condition of equivalence of electrostatic capacitances through the equality of

$$C_e = \frac{2 \, \pi \xi_0}{\ln \left[(a + b)/2 \, l \right]} h \cong \frac{2 \, \pi \xi_0}{b/2 \, l} h, \quad \text{if } b << a, \tag{17}$$

with $\qquad C_p = \xi_0 \, S/r = \xi_0 \, 8 \, l h/r \tag{18}$

The cathode surface areas are equal.   We get

$$r = 2 \, b/\pi \tag{19}$$

$$K_{dp} = \frac{V_{\text{vessel}}}{S_{\text{cathode}}} = \frac{2 \, r \cdot 2 \, a \cdot h}{8 \, l h} = \frac{a}{2 \, l} \, r = \frac{ab}{\pi \, l} \tag{20}$$

The ratio of the design parameters of equivalent elliptic and prismatic diodes is

$$K_{dp}/K_{de} = 8/\pi^2 \tag{21}$$

and the ratio of the design parameters of equivalent cylindrical and prismatic diodes is

$$K_{dp}/K_{dc} = 2 \, b/a \tag{22}$$

5. Conclusion

For the same electrostatic capacitance, and to a first degree of approxima-
tion, for the same space-charge-limited current, it has been shown that the elliptic
form of tube possesses a better design-parameter than the cylindrical form, in the
ratio $\pi^3 b/4a$. The prismatic shape takes the improvement further by a factor
$8/\pi^2$. The improvement is considerable if b can be made very small in relation
to a. This emphasizes the importance of the thin prismatic shape.

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