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THE NORMAL RULE OF "LEADING AND LAGGING  
VECTORS" AND DEVIATIONS FROM THIS RULE  
IN AN AXIAL TURBOMACHINE STAGE

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ABSTRACT

In early papers<sup>1-3</sup> the normal rule of "leading and lagging vectors" describing the operational regime of a turbomachine stage has been formulated and used. The rule refers to the behaviour of the vectors  $e$ ,  $w$  while they turn in the plane of velocity hodographs. The normal rule has to be examined for its possible deviations, which characterise operational process of a turbine or compressor stage weakened or strengthened intentionally or otherwise. It is necessary to keep to the normal rule of leading and lagging vectors while designing a turbomachine stage, although the deviations can manifest themselves under conditions of variable regime even in the stage designed for a certain regime subject to the normal rule. The possible deviations of the normal rule are discussed in this paper.

NOMENCLATURE

$c$	absolute velocity	$\alpha_1, \beta_1$	angles $c_1$ and $w_1$ , respectively make with $u$
$c$	absolute velocity vector		
$u$	circumferential velocity	$\alpha_2, \beta_2$	angles $c_2$ and $w_2$ , respectively make with the negative $u$
$u$	circumferential velocity vector		
$w$	relative velocity	$\gamma$	any angle
$w$	relative velocity vector		

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*Subscripts*: 1 - inlet to blade. 2 - exit to blade. *c* - compressor. *t* - turbine. *u* - circumferential component.

1. *Normal rule of "Leading and Lagging Vectors"*: In the plane of velocity hodographs, of turbomachine stage, as the flow progresses through the stage, *c* leading *w* characterises a compressor stage while *c* lagging behind *w* is characteristic of a turbine stage. This is the *normal case of a proper turbomachine stage* where both *c* and *w* are turning in the same direction.

2. *Possible deviations from the "normal rule of leading and lagging vectors."* Apart from the normal case theoretically four other cases can arise:

- i. *c* is turning, *w* does not turn but changes in magnitude only.
- ii. *c* changes in magnitude only, *w* is turning.
- iii. *c* and *w* turn away from each other.
- and iv. *c* and *w* turn towards each other.

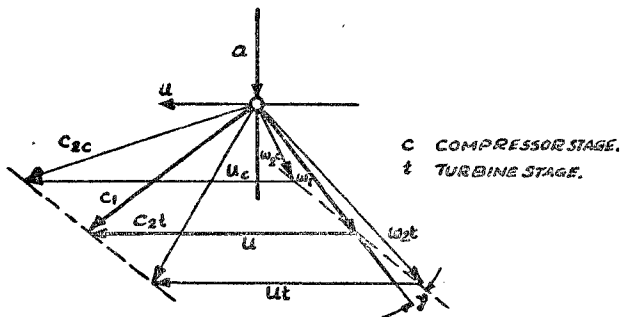


Fig. I

Fig. I represents the velocity triangles of a typical axial turbomachine stage. The flow is admitted from above and *u* is directed to the left. Taking the direction of  $w_1$  as the reference, *w* hodograph can be represented by any curve, in particular any straight line at an angle  $(\beta_1 + \gamma)$  to the *u* axis.  $\gamma$  can then have any value, positive, negative or zero. With  $\gamma > 0$ , if *w* turns anticlockwise (Fig. 1), *w* increases in magnitude ( $w_{2t} > w_1$ ). *c* also turns anticlockwise with increase in magnitude ( $c_{2t} > c_1$ ). *c* is lagging behind *w* and turbomachine stage is an axial turbine.

Again with  $\gamma > 0$ , if *w* turns clockwise its magnitude decreases ( $w_{2c} < w_1$ ). *c* also turns clockwise in the normal manner with decrease in magnitude ( $c_{2c} < c_1$ ). The stage is an axial compressor.

The inlet and exit triangles fix the extreme points of  $w$ ,  $c$  hodographs. The hodographs may take the form of any curve. But as regards the nett resulting effect, the two extreme points only are of importance and it is immaterial whatever be the manner of transition from the initial to the final points of the hodographs. It is, thus, possible to determine the complete effect of interaction between the equivalent stream and mathematical profile (i.e., between flow and blades) by replacing any curvilinear hodograph by a straight line joining the two extreme points. This straight line may be called a final substitute hodograph  $c$  or  $w$  respectively.

### 2.1. $c$ turning, $w$ changing in magnitude only.

In the axial turbomachine stage  $\gamma = 0$  gives the condition for  $c$  turning and  $w$  changing in magnitude only (Fig. II).

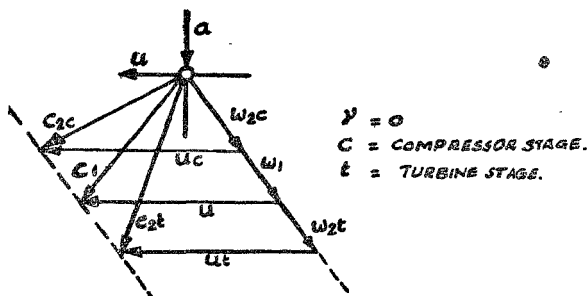


Fig. II

2.1(a) When  $w$  increases in magnitude ( $w_{2t} > w_1$ ),  $c$  turns anticlockwise and the stage is an axial turbine with a flat mathematical blade profile.

2.1(b) When  $w$  decreases in magnitude ( $w_{2t} < w_1$ ),  $c$  turns clockwise and the stage is an axial compressor with a flat mathematical blade profile.

### 2.2. $w$ turning, $c$ changing in magnitude only.

Fig. III represents the velocity triangles of such a stage and  $(\beta_1 + \gamma) = \alpha_1$ ,  $\gamma$  being negative.

2.2(a) When  $c$  increases in magnitude ( $c_{2c} > c_1$ ),  $w$  turns clockwise and the stage is an axial compressor.

2.2(b) When  $c$  decreases in magnitude ( $c_{2t} > c_1$ ),  $w$  turns anti-clockwise resulting in an axial turbine stage.

2.3.  $c$  and  $w$  turn away from each other.

Fig. IV represents this condition, when  $\gamma$  is negative ( $\gamma < 0$ ).  $c$  turns clockwise,  $w$  turns anticlockwise and an axial compressor stage (but obviously with a normal turbine mathematical blade profile) results.

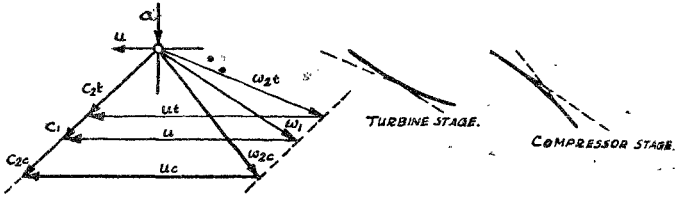


Fig. III

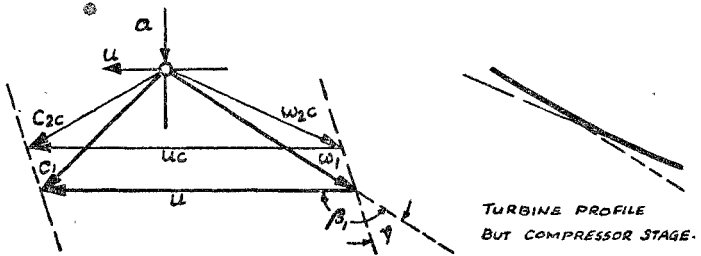


Fig. IV

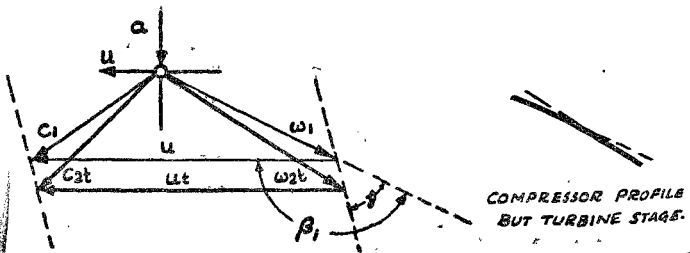


Fig. V

2.4.  $c$  and  $w$  turn towards each other.

As in the case of 2.3, under the conditions  $c$  and  $w$  turn towards each other,  $\gamma$  is again negative ( $\gamma < 0$ ).

Fig. V represents the velocity triangles.  $c$  turns anti-clockwise,  $w$  turns clockwise and a turbine stage results,

3. Particular case:  $\gamma$  is negative and  $(\beta_1 + \gamma) = 90^\circ$ .

Under the conditions  $(\beta_1 + \gamma) = 90^\circ$ , there is no power transfer in the turbomachine stage, and the stage is mechanically transparent in the circumferential direction even though the mathematical blade profile is not flat. Fig. VI represents the velocity triangles.

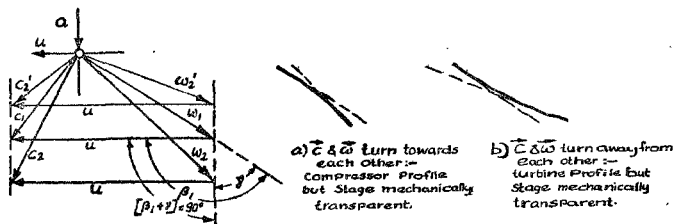


Fig. VI

3(a)  $c$  and  $w$  turning towards each other, gives a compressor blade profile but the stage is mechanically transparent.

3(b)  $c$  and  $w$  turning away from each other results in a turbine blade profile but the axial stage is mechanically transparent.

CONCLUSION

It has been shown that apart from the normal rule of leading and lagging vectors, particular cases exist. In axial turbomachine stage, these particular cases are characterised by contrary interaction between the direction of turn and change in magnitude of vectors  $c$  and  $w$ . If  $w$  increases in magnitude, it generally denotes turbine effect. To increase this turbine effect, it is necessary to turn  $w$  in the proper manner following the normal rule of leading and lagging vectors. When flow is admitted from above, and  $u$  is directed to the left in the scheme of the stage,  $w$  has to be turned anticlockwise in case of an axial turbine stage. Such a turn of  $w$  gives intensified turbine effect which is expressed by change of  $c_u$  from  $c_{1u}$  to  $c_{2u}$ ;  $c_{2u}$  being less than  $c_{1u}$ . Taking apart one of these effects (for instance removing turn of  $w$  but retaining only its increase in magnitude), weakens the turbine effect, though  $w$  remains the leading vector.

Having dealt with the two contrary effects in an axial turbomachine stage, it is possible to convert it into a stage of contrary purpose. For instance in Fig. IV, the direction of turn of  $w$  (expressed by blade shape), is aimed to realise a turbine stage. However the change in magnitude of  $w$  (at expense of changing radial sizes of blading) is adopted to create compressor effect, suited for a compressor stage. As a result  $c_{2u} > c_{1u}$ , compressor effect predominates and the stage is an axial compressor though the mathematical blade profile has the contour of a turbine profile. Since in this case,  $w$  and  $c$  are turning away from each other ( $w$ , anticlockwise and  $c$ , clockwise), distinguishing as leading or lagging vectors is not possible.

In the design of a turbomachine stage, turbine or compressor, the normal rule of leading and lagging vectors must be used to build the proper stage. It means that it should not be aimed at weakening the respective effect. For instance in the design of an impulse turbine stage,  $w$  decreases in magnitude due to losses (friction, waves, vortices etc.). To reduce the effect of these losses, the turn of  $w$  is normally increased making  $\beta_2 < \beta_1$ . The proper change in the magnitude of  $w$  can be accomplished by changing the radial size of blade from inlet to exit.

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