INVESTIGATIONS ON ARTIFICIAL DIELECTRICS AT MICROWAVE FREQUENCIES

Part II

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Received on March 2, 1959

Abstract

The expression for the phase change on transmission of an electromagnetic wave incident on a parallel plate type artificial dielectric, as a function of the plate spacing, has been verified experimentally. The theoretical and experimental results agree fairly well for ratios of plate spacing to wavelength from 0.5 to 0.85.

On the basis of a theory due to Woonton[1], the Fresnel type of diffraction for an artificial dielectric with ratio of plate spacing to wavelength of 0.64 has been theoretically investigated. There is close agreement between experimental and theoretical results.

Theoretical

(*a*) Effect of plate spacing on transmission of H_{01} electromagnetic waves through a parallel plate dielectric :—

In deriving[2] an expression for the transmission co-efficient T for a wave incident on the parallel plate dielectric the following assumptions have been made : (1) The array of plates are separated by a constant distance, (2) The metallic plates have infinite conductivity, (3) The plates have zero thickness. The expression for the phase change Φ derived from the expression for the transmission co-efficient T[3] is,

$$\begin{aligned}\Phi/2.303 = n\pi &+ (a/\lambda)[0.8128 + 5.6326\mu] + (a/\lambda)^3[-0.1432 + 1.19\mu - 1.1534\mu^3] \\ &+ (a/\lambda)^5[-0.0672 + 0.777\mu - 1.0326\mu^3 + 0.245\mu^5] \\ &+ (a/\lambda)^7[0.0384 + 0.63\mu - 1.26\mu^3 + 3.528\mu^5 - 0.4032\mu^7] \\ &+ (a/\lambda)^9[0.728\mu^3 - 1.528\mu^5 + 1.1866\mu^7 - 0.3858\mu^9] \\ &+ (a/\lambda)^{11}(1.008\mu^5 - 2.304\mu^7 + 1.584\mu^9 - 0.288\mu^{11}] \\ &+ (a/\lambda)^{13}[1.288\mu^7 - 3.864\mu^9 + 3.864\mu^{11} - 1.288\mu^{13}] \qquad [1]\end{aligned}$$

where 'a' is the plate spacing and μ the refractive index of the parallel plate medium given by the expression

$$\mu = (\lambda/2a)[4a^2/\lambda^2 - 1]^{\frac{1}{2}} \qquad [2]$$

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Fresnel Diffraction in the H plane due to the parallel plate medium :—The basis of the theoretical investigation is the determination of the diffraction due to an aperture which mutilates the field due to a lienear radiator such as a H plane sectoral horn. The field at a distant point P in the XZ (Fig. I) plane *viz.*, the H plane of the linear radiator is given by the expression[1]

$$\Psi_1\left(R_1, \frac{\sin\theta_1}{\lambda}\right) = R.\,P.\text{ of }\left[\frac{1+\cos\theta_1}{R_1}\ \exp.j(\omega t - k\,R_1) \times \right.$$

$$\left. \times \left(\frac{j}{2\lambda}\right)^{\frac{1}{2}} Y \int_{-\infty}^{\infty} F_1(x)\exp.\,(-j2\pi x\gamma_1)\,dx\right] \quad [3]$$

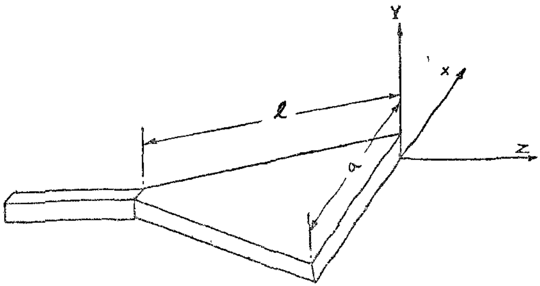

Fig. I

where $F_1(x)$ is the aperture function which gives the field distribution in the aperture, $\gamma_1 = -(\text{Sin}\,\theta_1)/\lambda$, θ_1, and R_1 being the co-ordinates of P with respect to origin O of a right handed system of co-ordinates X, Y, Z; Y being a function of y, is a constant in the expression since only changes in the XZ plane are of interest. The field in terms of a new origin O' on OZ can be evaluated by introducing a term to express the phase retardation involved. By using the Fourier transform technique Woonton obtains an expression for the effect of the mutilating aperture situated at O' (Fig. II) in terms of the angle θ_1' and R_1' the distance of P from O'. The expression is

$$G_{1\,a\,m}(\gamma) = \int_{-\infty}^{\infty} G_1\left(\beta + \frac{\sin\alpha}{\lambda}\right)\frac{\sin\pi c(\beta-\gamma)}{\pi c(\beta-\gamma)}\exp.\,j\pi b\lambda(\beta^2-\gamma^2)\,d\beta \qquad [4]$$

where $G_1(\gamma)$ is the Fourier Transform of $F_1(x)$ and is given by the expression

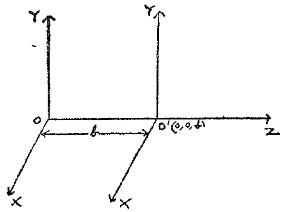

Fig. II

$$G_1(\gamma) = \int_{-\infty}^{\infty} F_1(x) \exp.(-j2\pi\gamma x)\, dx.$$

α is the angle through which the radiator is rotated about a vertical axis in the XZ or H plane, c is the width of the mutilating aperture and $G_{1\alpha_m}(\gamma)$ refers to the effect of the mutilating aperture situated at O' (0, 0, b).

Two cases of diffraction arise, one in which the artificial dielectric composed of individual apertures is moved transverse to the axis of the radiator which is also the axis of the interferometer used in the experimental investigation and the other in which the axis of the dielectric is continuously changed in direction with reference to the axis of the radiator.

Case 1: The artificial dielectric axis and the axis of the interferometer system are parallel in this case. That is $\alpha = 0$ and $\gamma = 0$. Hence the response function for each individual element is given by

$$G_{1m}(0) = c\,\Omega \int_{-\infty}^{\infty} G_1(\beta) \frac{\sin \pi c\beta}{\pi c\beta} \exp. j\pi b\lambda\, \beta^2\, d\beta \tag{5}$$

where Ω is a phase factor which accounts for the phase retardation of one element with respect to another at the receiver; it is obvious that $\Omega = [(\exp. j2\pi c)/\lambda]\,(\sin\theta_1 + \sin\theta_2)$ where θ_1 and θ_2 are the angles made by the line joining the aperture centre to the centre of the radiator, and the centre of the receiving horn respectively, with the axis of the system, *viz.*, the line joining the radiator centre and the centre of the receiving horn, these angles being in the H plane. The integral is expressed as a Fourier transform in exp. $[j2\pi(c/2)\beta]$ as

$$G_{1m}(0) = I_1(c) + I_1(-c)$$

where
$$I_1(c) = \left(\frac{j}{b\lambda}\right)^{\frac{1}{2}} \int_{-\infty}^{\infty} \left[\int_{-\infty}^{x} \exp.\left(\frac{-j\pi z^2}{b\lambda}\right) dz \right] F_1(c/2 - x)\, dx \tag{6}$$

where $F_1(x)$ is the original of the transform $G_1(\beta)$ and is expressed as $F_1(x) \longleftrightarrow G_1(\beta)$ to represent the inverse relationship.

Now $$F_1(x) = F_A(x)\phi(x)$$

where $$F_A(x) = A \text{ for } |x| \geqq a/2$$
$$= 0 \text{ for } |x| > a/2$$

and $\phi(x)$ is a phase factor. For the case considered $\phi(x) = \exp.(-j\pi x^2/\lambda l)$ which is substituted in (6); on integrating with respect to z we have

$$I_1(c) = A\left(\frac{j}{2}\right)\int_{-\infty}^{\infty}\left\{\left(\frac{1-j}{2}\right) + C\left[\left(\frac{2}{b\lambda}\right)^{\frac{1}{2}}x\right] - jS\left[\left(\frac{2}{b\lambda}\right)^{\frac{1}{2}}x\right]\right\} \times$$
$$\times \exp.\left[-j\pi\{c/2 - x\}^2/\lambda l\right]dx \qquad [7]$$

where $C\left[(2/b\lambda)^{\frac{1}{2}}x\right] - jS\left[(2/b\lambda)^{\frac{1}{2}}x\right]$ is given by the asymptotic expansions

$$\frac{(1-j)}{2} + \left(\frac{b\lambda}{2}\right)^{\frac{1}{2}}\frac{1}{\pi x}\exp.\left(\frac{-j\pi x^2}{b\lambda}\right)\left[j - \frac{b\lambda}{2\pi x^2} + \ldots\right] for\ x > 0$$

and $$\frac{-(1-j)}{2} + \left(\frac{b\lambda}{2}\right)^{\frac{1}{2}}\exp.\left(\frac{-j\pi x^2}{b\lambda}\right)\left[j - \frac{b\lambda}{2\pi x^2} + \ldots\right] for\ x < 0$$

In the expressions for $I_1(c)$ and $I_1(-c)$ the appropriate expansions are substituted. By evaluating the integrals of the form

$$I_1(c) = I_1'(c) + I_2'(c) \text{ and } I_1(-c) = I_1'(-c) + I_2'(-c)$$

where $$I_1'(c) = A(j/2)^{\frac{1}{2}}\int_{-\infty}^{\infty}(1-j)\exp. -j\pi\left[(c/2-x)^2\frac{1}{\lambda l}\right]dx$$

$$I_2'(c) = -A(j/2)^{\frac{1}{2}}\left(\frac{b\lambda}{2}\right)^{\frac{1}{2}}\int_{-\infty}^{\infty}\frac{1}{x}\exp.\frac{-j\pi c^2}{4\lambda l}\exp.\left[\frac{-j\pi}{\lambda}\left(\frac{1}{b}+\frac{1}{l}\right)x^2 + \frac{j\pi c}{\lambda l}x\right]dx$$

we have $$G_{1m}(0) = cA(1-j)\left(\frac{\lambda l}{2}\right)^{\frac{1}{2}}\exp.\frac{j2\pi c}{\lambda}(\sin\theta_1 + \sin\theta_2) \qquad [8]$$

Hence the variation is given by exponential term.

To obtain the diffraction characteristic on moving the dielectric transverse to the axis of the system, the individual response functions for each aperture, *viz.* $G_{1m}(0)$ are added up. Each value of $G_{1m}(0)$ must strictly speaking incorporate a different γ *i.e.*, the distance of the aperture from the radiator to take into account the loss in intensity at the aperture. This however is not necessary. The difference obtained between the accurate value of $G_{1m}(0)$ and its value not taking into account the changes in γ will be negligible. Now the shift of the dielectric by a distance equal to the width of n apertures will change the total response function I_0 for all the apertures put together calculated for an axis position of the dielectric, by an amount equal to the change in intensity produced by the n apertures in the new position taken up by them. The new response function therefore is $I_0 - \sum_n G_{1mn}(0)$, where $I_0 = \sum_N G_{1mN}(0)$, N being the total number of apertures in the dielectric. The diffraction effect is shown graphically, normalized with respect to the intensity calculated when the dielectric is on axis, in Fig. IV. Theoretical and experimental results are plotted for comparison.

Case 2:—This case arises when the dielectric axis and the axis of the system are oriented at an angle α with each other. The response function $G_{1\alpha m}(\gamma)$ is then given by

$$G_{1\alpha m}(\gamma) = c\int_{-\infty}^{\infty} G_1\left(\beta + \frac{\sin\alpha}{\lambda}\right)\frac{\sin \pi c(\beta-\gamma)}{\pi c(\beta-\gamma)}\ \exp.\ j\pi b\lambda(\beta^2-\gamma^2)\,d\beta$$

Putting $(\beta-\gamma)=\phi$ and if $\gamma_0 = (\sin\alpha - \sin\theta)/\lambda$ and $\beta + (\sin\alpha)/\lambda = \phi + \gamma_0$, so that

$$G_{1\alpha m}(\gamma) = c\int_{-\infty}^{\infty} G_1(\phi+\gamma_0)\frac{\sin \pi c\phi}{\pi c\phi}\ \exp.\ j\pi b\lambda\{\phi^2 + 2\gamma\phi\}\,d\phi \qquad [9]$$

$$= c'\int_{-\infty}^{\infty} G_1(\phi+\gamma_0)\frac{\exp.\,(j\pi b\lambda\phi^2 + j\pi c'\phi)}{2j\pi c'\phi}\,d\phi$$

$$- c''\int_{-\infty}^{\infty} G_1(\phi+\gamma_0)\frac{\exp.\,(j\pi b\lambda\phi^2 + j\pi c''\phi)}{2j\pi c''\phi}\,d\phi \qquad [10]$$

where $c' = (2b\lambda\gamma/c + 1)$, $c'' = (2b\lambda\gamma/c - 1)$, *i.e.*, $G_{1\alpha m}(\gamma) = I(c') - I(c'')$

Following Woonton again we have

$$I(c') = A\left(\frac{j}{b\lambda}\right)^{\frac{1}{2}} \exp.\,(-j\pi c'\gamma_0)\int_{-\infty}^{\infty}\int_{-\infty}^{x} \exp.\left(\frac{-j\pi z^2}{b\lambda}\right)dz\ \times$$

$$\times \exp.\left[\left(\frac{-j\pi}{\lambda l}\right)(c'/2-x)^2\right]\exp.(j2\pi x\gamma_0)dx.$$

and $$I(c'') = A\,(j/b\lambda)^{\frac{1}{2}}\exp.\,(j\pi c''\gamma_0)\int_{-\infty}^{\infty}\int_{-\infty}^{x}\exp.\left(\frac{-j\pi z^2}{b\lambda}\right)dx\times$$

$$\times \exp.\left[\left(\frac{-j\pi}{\lambda l}\right)(c''/2-x)^2\right]\exp.(j2\pi x\gamma_0)\,dx.$$

On integrating with respect to z and using the asymptotic expansions

$$I(c') = I_1' + I_2'$$

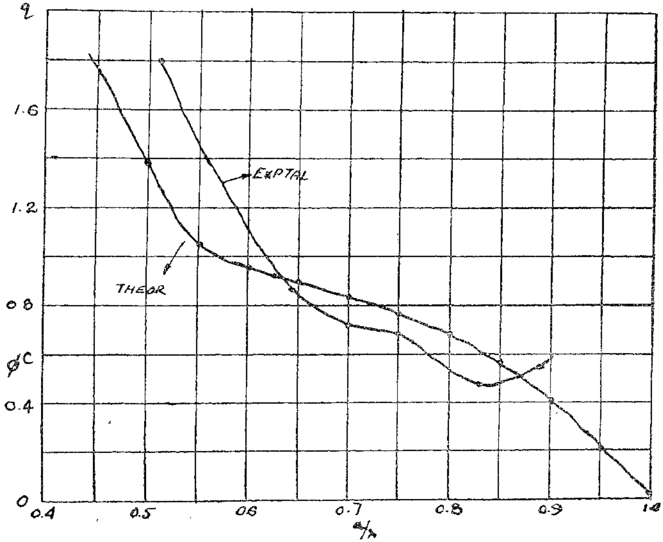

Fig. III

Phase Shift $\sim$ a/λ

where

$$I_1' = A\,(j/2)^{\frac{1}{2}}\,(1-j)\ \text{exp.}\,(-j\pi c'\gamma_0)\int_{-\infty}^{\infty} \text{exp.}\left[\frac{-j\pi}{\lambda l}(c'/2-x)^2 - 2x\gamma_0\right]dx$$

$$I_2' = \frac{-A}{\pi}\left(\frac{jb\lambda}{2}\right)^{\frac{1}{2}} \text{exp.}(-j\pi c'\gamma_0)\int_{-\infty}^{\infty}\frac{1}{x}\ \text{exp.}\left[\frac{-j\pi}{\lambda}\left(\frac{1}{b}+\frac{1}{l}\right)x^2 + \left(\frac{j\pi c'}{\lambda l} - 2\gamma\right)\right]dx$$

With similar expressions for I_1'' and I_2'', being the components of $I(c'')$. By integration we find that

$$I(c') = \left[A\,(j/2)^{\frac{1}{2}}\ \text{exp.}\left\{-j\pi\left(c'\gamma_0 + \frac{c'^2}{4\lambda l}\right)\right\}\right]\left[(1-j)\sqrt{\frac{\pi}{a_1}}\ \text{exp.}\ \frac{b_1^2}{a_1} - \left(\frac{jb\lambda}{4}\right)^{\frac{1}{2}}\right]$$

and

$$I(c'') = \left[A(j/2)^{\frac{1}{2}}\,\text{exp.}\left\{-j\pi\left(c''\gamma_0 + \frac{c''^2}{4\lambda l}\right)\right\}\right]\left[(1-j)\sqrt{\frac{\pi}{a_2}}\ \text{exp.}\ \frac{b_2^2}{a_2} - \left(\frac{jb\lambda}{4}\right)^{\frac{1}{2}}\right]$$

where

$$b_1 = \tfrac{1}{2}\left[\frac{j\pi c'}{\lambda l} - 2\gamma_0\right]; \qquad b_2 = \tfrac{1}{2}\left[\frac{j\pi c''}{\lambda l} - 2\gamma_0\right]; \qquad a_1 = a_2 = \frac{j\pi}{\lambda l}.$$

$$\therefore \quad I = A\,(1-j)^2\sqrt{\frac{j\pi}{2a}}\ \text{exp.}\,(-j\pi)\left[\frac{2b\lambda\gamma\gamma_0}{c} + \frac{b^2\lambda\gamma^2}{c^2 l} + \frac{1}{4\lambda l}\right]\times$$

$$\times\ \text{exp.}\left(\frac{-j\lambda l}{4\pi}\right)\left[\left\{\frac{4j\pi}{\lambda l}\left(\frac{b\lambda\gamma}{c} - \gamma_0\right)^2 - \frac{\pi^2}{\lambda^2 l^2}\right\} - \left(\frac{jb\lambda}{4}\right)^{\frac{1}{2}}\right]\times$$

$$\times\left[-2j\sin\{(\pi+1)\gamma_0 + b(\lambda-\gamma)/cl\}\right]$$

$$\therefore \quad I = K\ \text{exp.} -j\pi\left(\frac{b^2\gamma^2\lambda}{c^2 l} + \frac{1}{4\lambda l}\right)\text{exp.}\left(\frac{-j2\pi\gamma_0\gamma}{c}\right)\left[\frac{b}{c}\left(\lambda + \frac{4}{l}\right)\right]\times$$

$$\times\ \text{exp.}\ \frac{-j\lambda l}{4\pi}\left[\frac{\pi^2}{\lambda^2 l^2}\left(\frac{4b^2\lambda^2\gamma^2}{c^2} - 1\right)(1+j)\left(\frac{b\lambda}{8}\right)^{\frac{1}{2}}\right]\text{exp.}\ \frac{-j\lambda l\gamma_0^2}{\pi}\times$$

$$\times\ \sin\left[(\pi+1)\gamma_0 + b(\lambda-\gamma)/cl\right] \qquad [11]$$

where $K = A\,(1-j)^2\sqrt{(l\lambda/2)}$.

This expression can be used for calculating the individual diffraction pattern for a single aperture, by substituting the experimental values of b, c, λ, l, to obtain a simplified expression involving only γ and γ_0. Since the axis of the dielectric and the axis of the system coincide, $\gamma = 0$ and hence a further simplification arises. In calculating the diffraction due to a number of apertures, the individual diffracted intensity I must be multiplied by a factor exp. $j2\pi c\,(\sin\alpha)$ so as to account for the phase changes between apertures when a rotation through an angle α occurs about an axis in the YZ plane perpendicular to the axis of the dielectric, and c = distance between plates. The simplified expression when $b = 35$ cm., $c = 2$ cm., $l = 28$ cm., $\lambda = 3{\cdot}14$ cm. is

$$I = A \text{ exp. } (-j\,32.8\gamma_0^2) \sin[4.142\,\gamma_0 - 1.42] \text{ exp. } 4\pi\gamma_0 \qquad [12]$$

Experimental

(*a*) The effect of plate spacing on the phase shift produced by an artificial dielectric has been studied with a microwave interferometer by constructing frames for the artificial dielectric of wood, with grooves in each so that the strips could be fixed with them and the spacing kept constant. The spacings

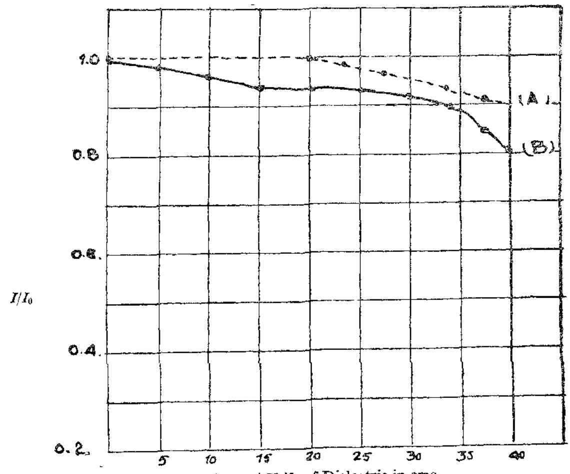

Fig. IV

Horizontal Diffraction Patterns

Distance of Dielectric from Transmitting Horn=35 cms.

(A) Experimental Curve. (B) Theoretical Curve Spacing=2.0 cms.

were 1·6, 2·0, 2·2, 2·35, 2·6 and 2·8 cms. corresponding to values of the ratio spacing/wavelength of 0·51, 0·635, 0·7, 0·77, 0·83 and 0·89. The spacings were accurate to ± 0·05 cm. in each case as measured by calipers. In each case the measured shift Δ_m was noted and the connected shift Δd calculated from the expression

$$\Delta_m - \Delta_d = \lambda/4 - \mu t - \lambda/2\pi \text{ arc tan } \left[\frac{2\mu}{1+\mu^2} \cot \frac{2\pi\mu t}{\lambda}\right]$$

where $t =$ thickness of the dielectric and its refractive index.

A graph showing the experimental and theoretical results is given in Fig. III.

(*b*) The diffraction characteristics of the dielectric have been experimentally obtained with the help of the microwave interferometer for the case of a dielectric of plate spacing 2·0 cm. The theoretical and experimental results for the case of (1) transverse movement (2) rotation of dielectric about a vertical axis are plotted in Figs. IV and V.

The measurements were performed in the same manner and using the same equipment as in Part I (ref. 3). The errors arising due to diffraction have been discussed by Woonton, *et al.*[4]

CONCLUSIONS

(*a*) The results show that the phase shift of an artificial dielectric whose spacing increases within the range $\lambda/2$ to λ, decreases uniformly *i.e.* the ratio phase-shift/thickness of dielectric decreases. This may be compared to the case of a natural dielectric. In the latter, the ratio phase shift/thickness of dielectric is more or less constant over a wide range of thickness.

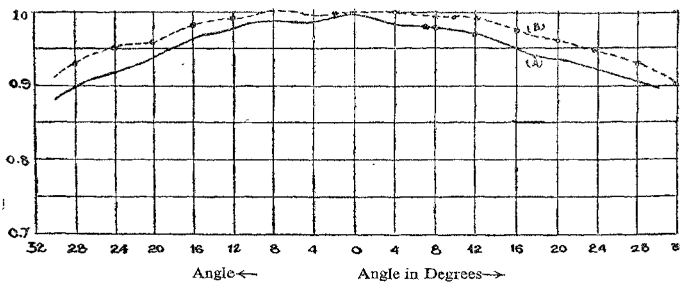

Fig. V

Fresnel Diffraction

Position of Diel.=35 cms. from Horn (=b of Theory). Rotation of Dielectric about Vertical Axis. (A) Experimental Curve. (B) Theoretical Curve. Spacing=2.0 cms.

(*b*) The diffraction measurements show that with a reduction in effective area presented to the source of radiation the intensity of the received radiation decreases more or less uniformly. This is brought out in the theoretical and experimental results.

ACKNOWLEDGMENT

The author expresses his grateful thanks to the Ministry of Education, Govt. of India for awarding a Senior Research Scholarship and to Sri S. K. Chatterjee, Indian Institute of Science for suggesting the problem and helpful guidance and to the Head of the Department of Electrical Communication Engineering, Indian Institute of Science, for giving all facilities during the course of the work.

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