

PROPAGATION OF MICROWAVES ON A SINGLE WIRE

Part III

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ABSTRACT

Theoretical calculations for the radial field spread around a bare copper wire supporting the Sommerfeld wave, as a function of the percentage of power flow p and, also as a function of the wavelength of excitation, for 50%, 75% and 90% power flow, have been made. The attenuation constant of the Sommerfeld surface wave line as a function of wire radius a for different values of wavelength has been calculated. Calculations have also been made for the radius of different constant power contours round a Harms-Goubau dielectric-coated surface wave line, as a function of the dielectric constant, dielectric coating thickness and radius of the supporting wire. An expression for the ratio of radii of constant power contour, with and without dielectric coating, for different percentages of power flow as a function of dielectric coating thickness has been derived. The variation of this ratio, at 3.45 cms. wavelengths, for a dielectric constant of 2.0, shows that the shrinkage of the radial field occurs rapidly up to a coating thickness of 0.01 cm. and then slowly up to 0.03 cm. But beyond a coating thickness of 0.03 cm. the shrinkage of the field is not appreciable.

INTRODUCTION

The work described here is in continuation of the work reported earlier^{1, 2}. The object of the paper is to present graphically theoretical results for a bare copper surface wave line for variations of

(i) the parameters $|\eta|$ and $|\xi|$ involved in the power flow equation, with the wavelength of excitation λ_0 , for different values of the radius a of the line;

(ii) the radial spread of the electric field with respect to the percentages of power flow p , for various values of wire radius ranging from 1.57×10^{-2} cm. to 35×10^{-2} cm.;

(iii) the radius of the area ρ_{pc} , around the wire within which 50%, 75% and 90% of the power is propagated, as a function of wavelength λ_0 , for different values of a ;

(iv) the attenuation constant α , for different values of a at $\lambda_0 = 4.0$ cm., 3.2 cm. and 1.25 cm.;

(v) the axial propagation constant h for different values of a at $\lambda_0 = 4.0$ cm.;

The purpose of the investigation is also to present results for a dielectric-coated copper wire surface wave line for the variation of

(i) the function $F(\gamma' \rho)$ involved in the power flow equation as a function of its argument, and

(ii) the radius ρ_{pd} for different constant power contours, as a function of the dielectric constant ϵ , dielectric coating thickness $(a' - a)$ and wire radius a , where a' is the radius of the dielectric-coated wire.

The object is also to derive expressions for the ratio ρ_{pd}/ρ_{pc} when the argument of the function $F(\gamma' \rho)$ is small and large. The ratio indicates directly the degree and nature of the radial field shrinkage as affected by the thickness of the dielectric-coating.

SOMMERFELD SURFACE WAVE LINE

(i) *Field Components*: The field components of the Sommerfeld wave in cylindrical co-ordinates are as follows³:

$$\begin{aligned} E_r &= A(h/\gamma)Z_1(\gamma r) \cdot \exp[j\omega t - hz] \\ E_z &= A \cdot Z_0(\gamma r) \cdot \exp[j\omega t - hz] \\ H_\theta &= jA \cdot (k^2/\omega\mu_0\gamma) \cdot Z_1(\gamma r) \cdot \exp[j\omega t - hz] \end{aligned} \quad [1]$$

where, Z_0 and Z_1 represent cylinder functions involving the first and second kind Bessel functions;

γ is the radial propagation constant,

h is the axial propagation constant,

A is the excitation constant,

and $\gamma^2 = k^2 + h^2$.

(ii) *Radial Field Spread*: By using equation [1] with proper boundary conditions, the following relation⁴ is obtained:

$$\xi \ln \xi = \eta \quad [2]$$

which reduces to the following relation, which determines the radial field spread:

$$\gamma_2 = \frac{1.12}{a} \cdot (|\xi|)^{1/2} \exp\left\{j\left(\frac{\theta + \pi}{2}\right)\right\} \quad [3]$$

where γ_2 = value of γ in the region outside the wire,

$$\xi = (-j0.89\gamma_2 a)^2 = |\xi| \exp(j\theta) \quad [3a]$$

$$\eta = 2 \cdot (0.89)^2 \cdot \frac{k^2 a}{k_1} \cdot \exp\left(\frac{j 3\pi}{4}\right) = |\eta| \exp\left(\frac{j 3\pi}{4}\right) \quad [3b]$$

$$\theta = -\frac{1}{4}\pi \left(1 - \frac{1}{\ln|\xi| + 1}\right) \quad [3c]$$

k = the free space wave number

and $k_1 = (\omega\mu_0\sigma_c)^{\frac{1}{2}} \exp(-j\pi/4)$, where σ_c represents the conductivity of the line.

For a bare copper wire line immersed in air,

$$|\eta| = 1.70 \times 10^{-3} \times a \lambda_0^{-3/2} \quad [3d]$$

Figure I shows the variation of $|\eta|$ and $|\xi|$ with λ_0 for values of $a = 0.05, 0.10, 0.15, 0.20$ and 0.25 cm. Figure II shows the variation of $|\xi|$ with respect to $|\eta|$ for different values of m , where m represents the decade value of $|\eta|$. Figure III represents the variation of $|\eta|$ with respect to θ . A plot of the radial decay factor $\gamma_2 (= A + jB)$ in the complex plane is shown in Figure IV, where A is the attenuation constant and B , the phase constant in the radial direction. The circles on the graph show the value of the radius of the wire for which the real and imaginary parts of γ_2 have been calculated.

(iii) *Percentage of Power Flow*: By using Poynting vector and the proper field components (equation [1]), Goubau⁴ has derived the following relation for the percentage of power flow within a certain radial distance ρ from the surface wave line:

$$\frac{N_\rho}{N} \approx 1 + \frac{2 \ln(\rho/a)}{\ln 2.2 |\xi|} \quad [4]$$

where N_ρ = power contained in the area beyond the radial distance ρ ,

$$= \operatorname{Re} \left[2\pi \int_\rho^\infty r E_r H_\phi^* \cdot dr \right]$$

N = total power contained within the area around the surface wave line,

$$= \operatorname{Re} \left[2\pi \int_a^\infty r E_r H_\phi^* \cdot dr \right]$$

The above equation [4] has been derived by assuming $|\gamma\rho| < 0.3$ and hence using the small argument representation for the Hankel functions appearing in the expressions for N_ρ and N . An accurate calculation of N_ρ/N requires evaluation of the Hankel functions of complex argument. However, the small argument approximations are satisfied on the ground, that $|\gamma\rho| < 0.3$ covers

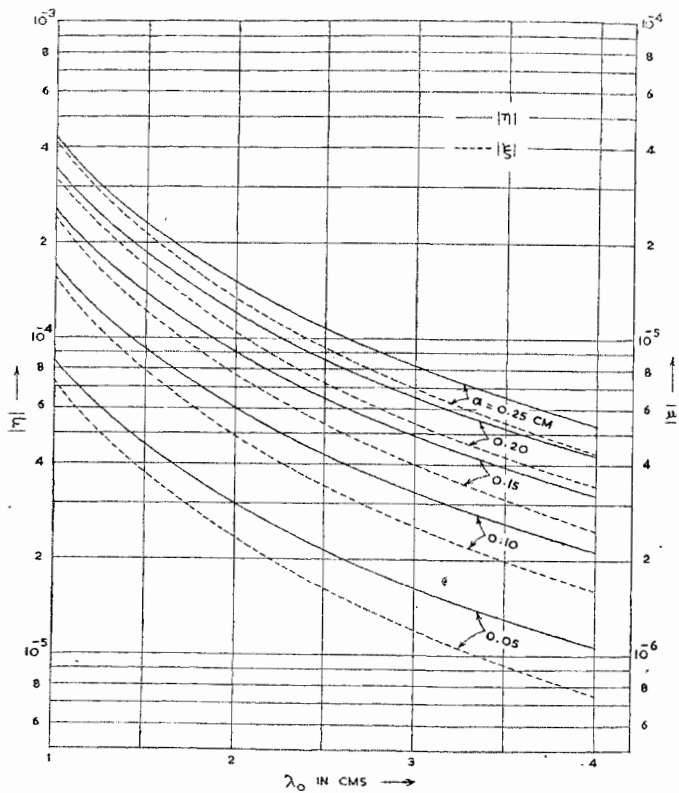


FIG. I

Variation of $|\eta|$ and $|\xi|$ with the wavelength of excitation λ_0 , for different values of the radius a of the Sommerfeld line

an area round the wire which contains the major portion of the energy transmitted along the surface wave line. It has also been assumed in the above derivation that

$$\theta \cot \theta \approx \pi/4.$$

The values of $|\eta|$ and $|\xi|$ are obtained, for values of the wire radii ranging from 35×10^{-2} to 1.57×10^{-2} cm., from equation [3d] and Figure II respectively, at 3.2 cm. wavelength. Substituting this in equation [4], the values of radial distance ρ as a function of the percentage of power flow $[100(N_p/N)]$, for different wire radii are obtained. The results are plotted in Figures V and VI. The radius of the area within which 50%, 75% and 90% of the power is propagated is obtained in a similar way, for wavelengths ranging from 1.0 cm. to 4.0 cms. and for different wire radii and the results are plotted in Figure VII.

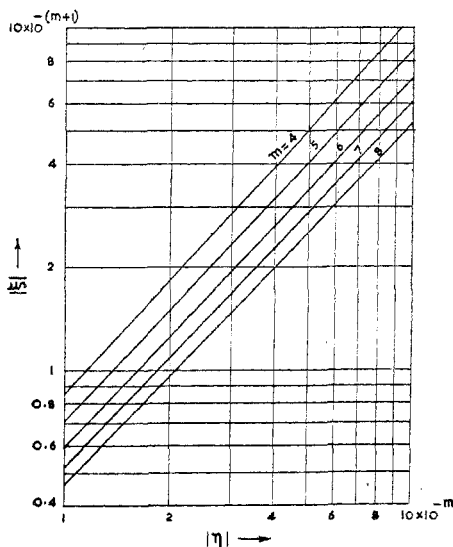


FIG. II

Relation between $|\eta|$ and $|\xi|$ for different values of m

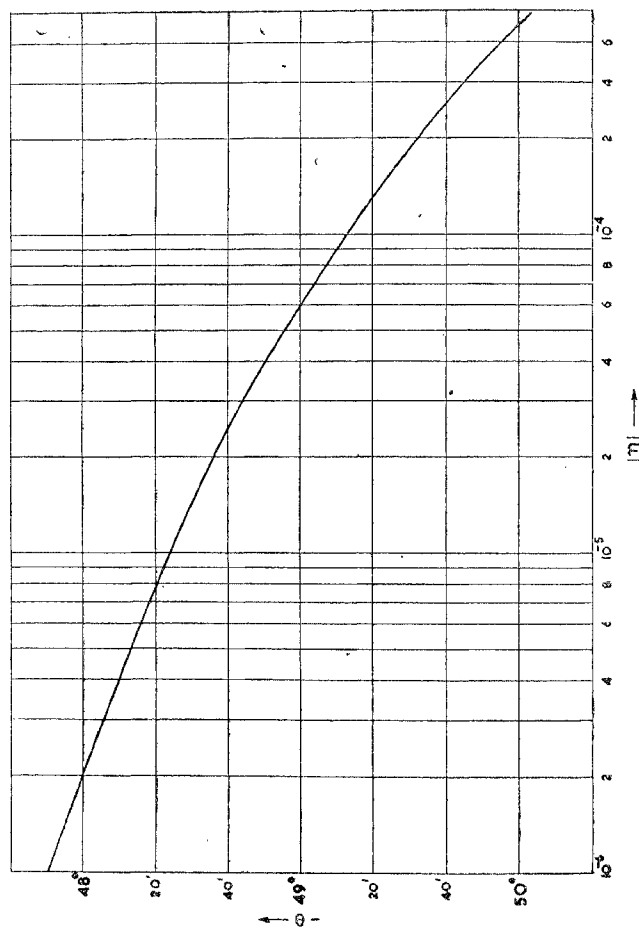


FIG. III
Relation between $|\eta|$ and the phase angle θ

(iv) *Attenuation Constant*: The axial propagation constant of the Sommerfeld surface wave is determined from the relation

$$h^2 = \gamma_2^2 - k^2$$

which yields $h \cong jk - j\gamma_2^2/2k$, neglecting higher order terms other than those of second order involving γ_2^2/k^2 , as $\gamma_2^2/k^2 \ll 1$.

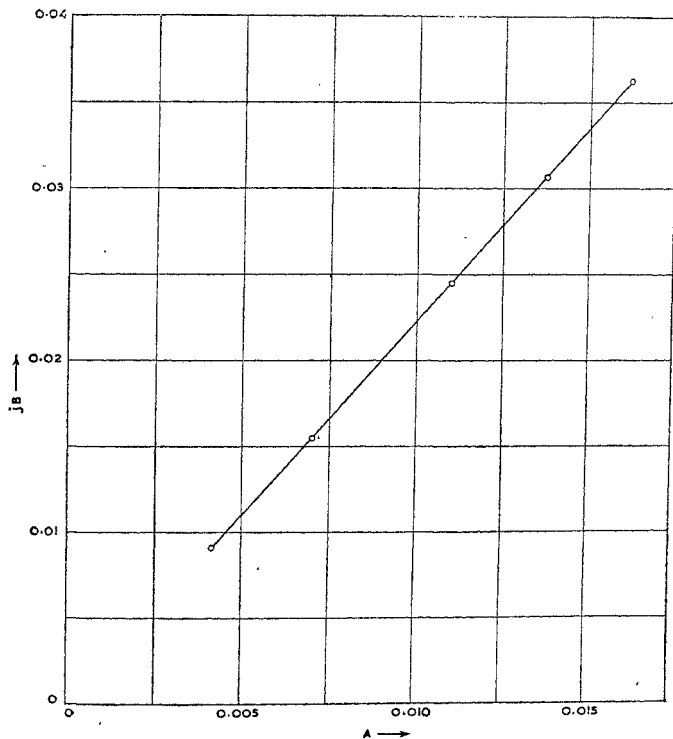


FIG. IV

Radial decay factor γ_2 plotted in the complex plane.

The circles on the graph show the value of the radius of the wire in cms. $\lambda_0=3.20$ cms.

$$\text{As } \gamma_2^2 = \frac{1.25}{a^2} \cdot (|\xi|) \exp [j(\pi + \theta)],$$

the above expression for h reduces to

$$h = jk + j \frac{1.25 (|\xi|)}{2k a^2} [\cos \theta + j \sin \theta] \quad [5]$$

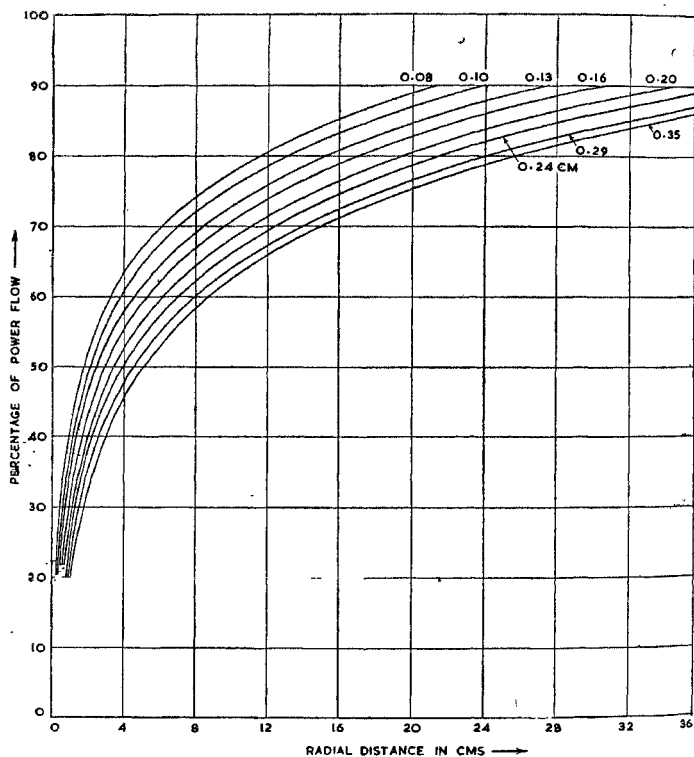


FIG. V

Radial field spread as a function of the percentage of power flow, for different values of radius of the Sommerfeld line. $\lambda_0 = 3.20$ cms.

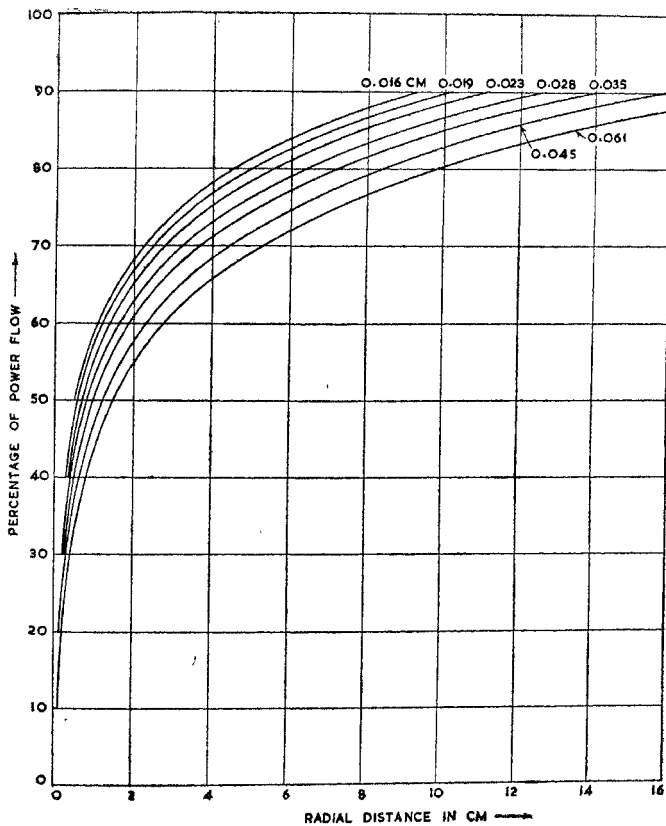


FIG. VI

Radial field spread as a function of the percentage of power flow, for different values of radius of the Sommerfeld line. $\lambda_0 = 3.20$

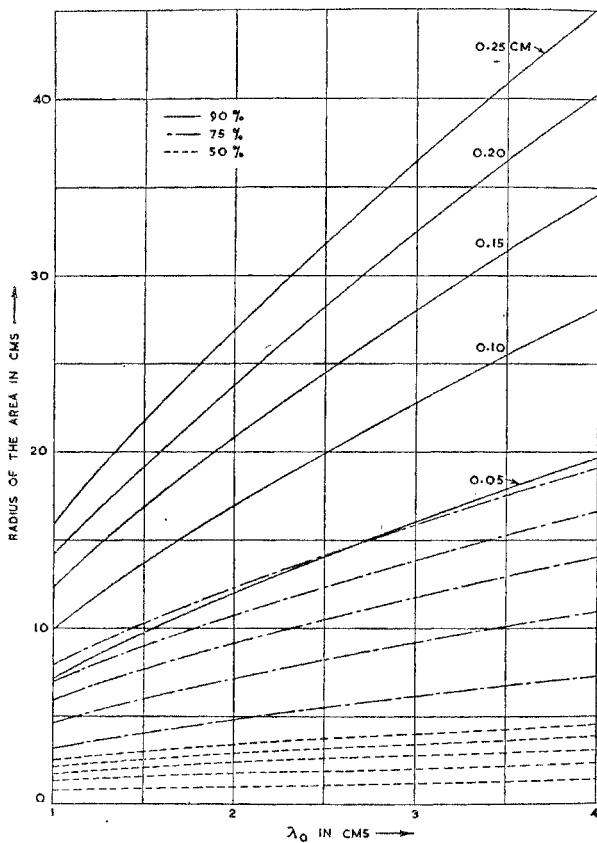


FIG. VII

Radius of the area around the wire within which 50%, 75% and 90% of the power is propagated, as a function of the wavelength λ_0 , for different values of wire radii.

But $h = \alpha + j\beta$. Hence the attenuation constant α and the phase constant β are given by the following expressions :

$$\alpha = -\frac{0.63 \left| \xi \right| \sin \theta}{ka^2} = \frac{-87 \left| \xi \right| \lambda_0 \sin \theta}{a^2} \text{ db/metre} \quad [6]$$

where λ_0 and a are in cms.

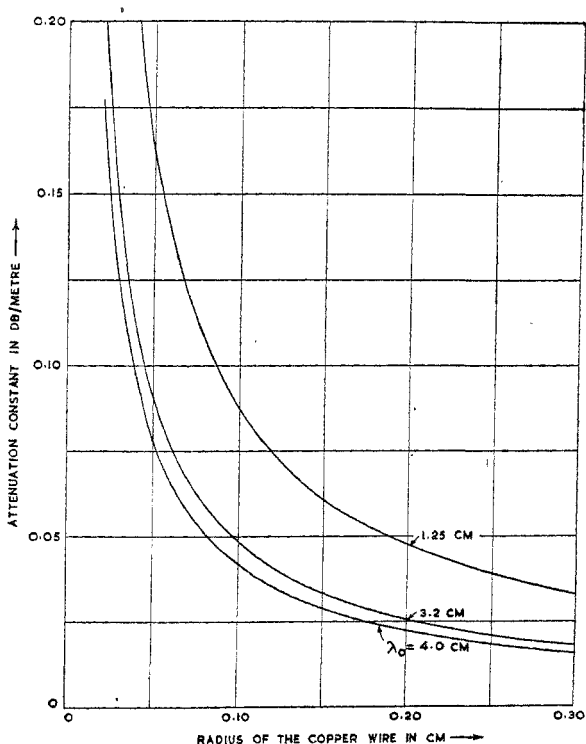


FIG. VIII

Attenuation constant of the Sommerfeld line with respect to the radius of the line, for different wavelengths.

$$\beta = \left[k + \frac{0.63 |\xi| \cos \theta}{k a^2} \right] \text{ cm}^{-1} \quad [7]$$

The phase velocity is given by the following expression :

$$v_p = C \left[1 - \frac{0.63 |\xi| \cos \theta}{k^2 a^2} \right] \text{ metres/sec.} \quad [8]$$

It may be mentioned that, $|\xi|$ being very small, the second term in equation [8] contributes very little to the magnitude of v_p . Hence, the phase velocity of the wave on the Sommerfeld line differs very little from the free space velocity C . This is the reason why a large field spread is associated with the Sommerfeld line. On the other hand, a small value of $|\xi|$ makes α very small.

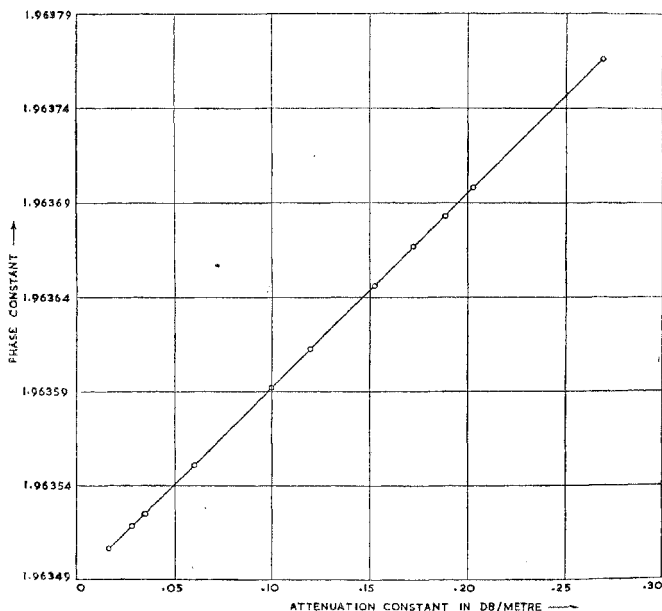


FIG. IX

Axial propagation constant h plotted in the complex plane. The circles on the graph represent different wire radii.

For instance, for a wire of radius 0.13 cm., excited at 3.2 cms. wavelength, α (Sommerfeld line) = 0.04 db/metre, which is almost half of the value for a X-band rectangular waveguide supporting the dominant mode. The attenuation constants for wires of different radii have been calculated for different wavelengths and some of the results are shown in Figure VIII. Figure IX shows a plot of h in the complex plane for values of a varying from 1.57×10^{-2} cm. to 35×10^{-2} cm., at $\lambda_0 = 3.2$ cms. The circles on the graph represents the values of a for which the real and imaginary parts of h have been calculated.

HARMS-GOUBAU SURFACE WAVE LINE

(i) *Field Components*: The field components of the Harms-Goubau wave in cylindrical co-ordinates are

$$\begin{aligned} E_r &= A (h/\gamma) \cdot Z_1(\gamma r) \exp. (j\omega t - hz) \\ E_z &= A Z_0(\gamma r) \exp. (j\omega t - hz) \\ H_\phi &= j A (k^2/\omega\mu_0\gamma) Z_1(\gamma r) \exp. (j\omega t - hz). \end{aligned} \quad [9]$$

where, the cylinder functions Z_0 and Z_1 are given by the following relations:

$$\begin{aligned} Z_0(\gamma r) &= J_0(\gamma r) + b N_0(\gamma r) \\ Z_1(\gamma r) &= J_1(\gamma r) + b N_1(\gamma r) \end{aligned}$$

and the value of b as obtained from the boundary conditions on the surface wave line is

$$b = -\frac{J_0(\gamma_d a)}{N_0(\gamma_d a)}$$

where γ_d refers to the value of γ inside the dielectric layer.

(ii) *Radial Field Spread*: The radial field spread in a Sommerfeld line is reduced by coating the line with a dielectric^{5,4}. By using the field components (equation in 9), imposing proper boundary conditions and matching the transverse wave impedance (E_z/H_ϕ) at the surface ($r = a'$) of the dielectric coated wire, the following equation for the radial field spread in a Harms-Goubau line is obtained:

$$\left[\gamma'^2 \ln 0.89 \gamma' a' - \frac{\gamma'^2}{\epsilon} \ln \frac{a'}{a} \right] = \left(\frac{1}{\epsilon} - 1 \right) \left(\frac{2\pi}{\lambda_0} \right)^2 \ln \frac{a'}{a} \quad [10]$$

where γ' represents the radial decay factor of the surface wave, ϵ represents the dielectric constant of the coating material and a' is the radius of the dielectric coated wire. The above equation [10] has been derived on the assumption that $(a' - a) \ll 1$; i.e., the thickness of the dielectric coating is very small compared to the wire radius a .

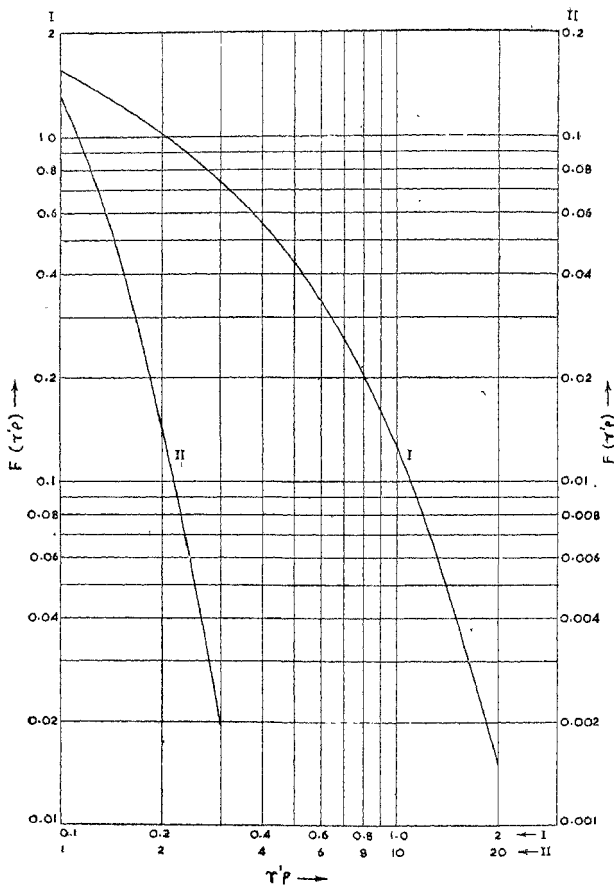


FIG. X

 $F(\gamma'\rho)$ vs $\gamma'\rho$

Curves I and II are calculated for the relation given by equation 12.

(iii) *Percentage of Power Flow*: The power transmitted outside a cylinder of radius ρ round the dielectric coated wire is⁴

$$\Lambda \rho = 2\pi AA^c \cdot \left(\frac{\epsilon_0}{\mu_0}\right)^{1/2} \cdot \frac{hk}{\gamma'^2} \cdot F(\gamma' \rho) \quad [11]$$

where $F(\gamma' \rho)$

$$= (\gamma' \rho)^2 \{ - (2/\gamma' \rho) \cdot j H_0^{(1)}(j \gamma' \rho) \cdot H_1^{(1)}(j \gamma' \rho) - [H_0^{(1)}(j \gamma' \rho)]^2 - [H_1^{(1)}(j \gamma' \rho)]^2 \} \quad [12]$$

If $\gamma' \rho < 0.1$, by using small argument approximations for the Hankel functions $H_0^{(1)}$ and $H_1^{(1)}$, the above expression [12] is simplified to

$$F(\gamma' \rho) \simeq (8/\pi^2) [-\ln(0.89\gamma' \rho) - 0.5]. \quad [13]$$

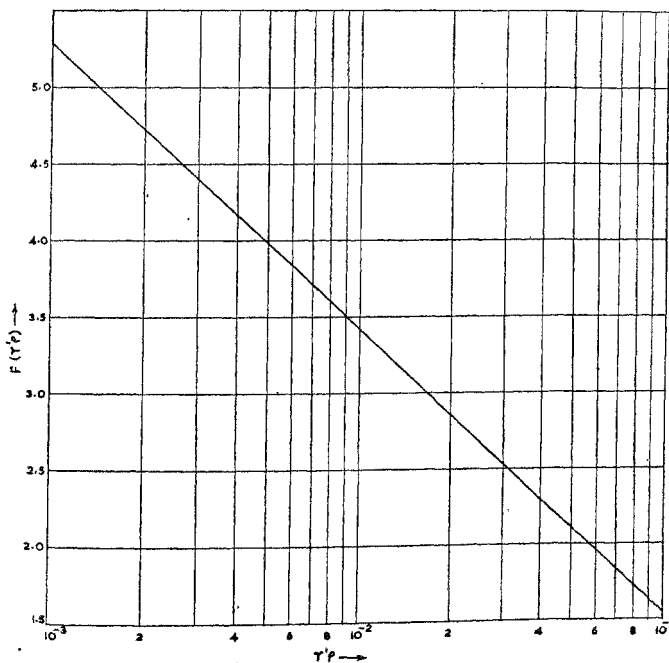


FIG. XI

$F(\gamma' \rho)$ vs $\gamma' \rho$, as calculated for the small argument approximation given by equation 13.

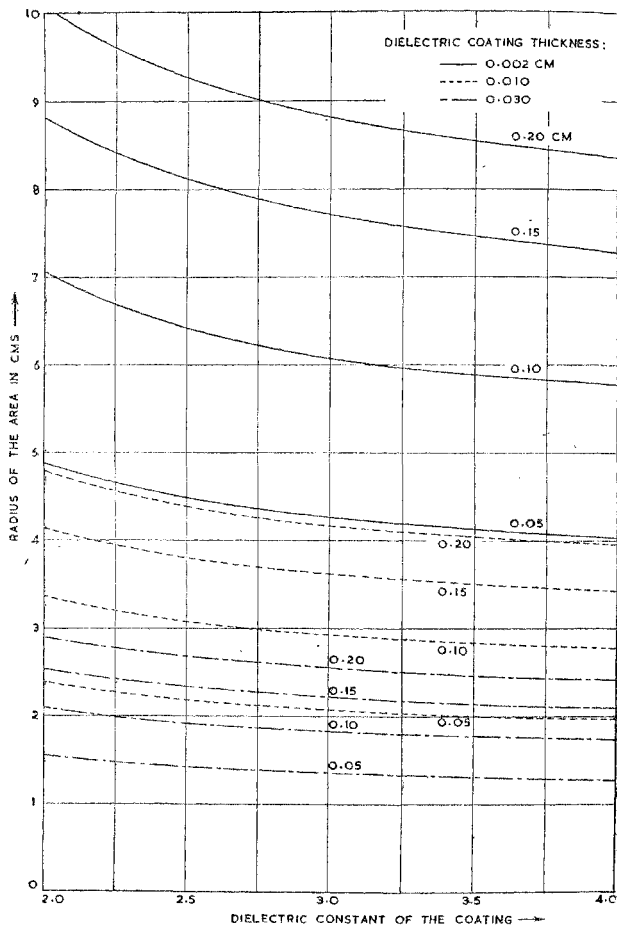


FIG. XII

Radius of the area around the wire within which 90% of the power is propagated, as a function of the dielectric constant of the coating, for different values of wire radius. $\lambda_0 = 3.45$ cms.

If $\gamma' \rho \gg 0.1$, the following asymptotic representation of the Hankel functions can be used in equation [12]:

$$\begin{aligned} H_0^{(1)}(j\gamma' \rho) &= \sqrt{2/j\pi\gamma' \rho} \cdot \exp.[-(\gamma' \rho + j\pi/4)] \\ H_1^{(1)}(j\gamma' \rho) &= \sqrt{2/j\pi\gamma' \rho} \cdot \exp.[-(\gamma' \rho + j3\pi/4)] \end{aligned} \quad [14]$$

Hence equation [12] reduces to

$$F(\gamma' \rho) = (4/\pi) \cdot \exp.(-2\gamma' \rho) \text{ for } \gamma' \rho \gg 0.1. \quad [15]$$

The total power transmitted along the line, obtained by putting $\rho = a'$ in equation [11], is

$$N_{\rho=a'} = 2\pi A \cdot A^0 \cdot \left(\frac{\epsilon_0}{\mu_0}\right)^{1/2} \cdot \left(\frac{hk}{\gamma'^2}\right) \cdot F(\gamma' a') \quad [16]$$

So, the circle of radius ρ_{pd} within which a certain percentage p of the total power of the surface wave travels can be found from the following expression

$$p = 1 - \frac{F(\gamma' \rho_{pd})}{F(\gamma' a')} \quad [17]$$

which is obtained from equations [11] and [16]. The nature of variation of the function $F(\gamma' \rho)$ with respect to $(\gamma' \rho)$ as given by the equation [12] is shown graphically in Figure X. The variation of $F(\gamma' \rho)$ with respect to $(\gamma' \rho)$ as given by the small argument approximation (equation [13]) is shown in Fig. XI. The variation of the radius of the area (ρ_{pd}) around the wire within which 90 per cent of the power is propagated as a function of the dielectric constant of the coating for different values of wire radii as obtained from equation [17] is shown in Figure XII. The variation of ρ_{pd} for 90 per cent, 75 per cent and 50 per cent power flow with thickness of dielectric coating, for $\epsilon = 2.0$ and $\epsilon = 4.0$ and $a = 0.2$ cm is shown in Figure XIII. Figure XIV is given for comparison of ρ_{pd} for the same percentages of power flow and $\epsilon = 2.0$ and $\epsilon = 4.0$, but with a much thinner wire of radius $a = 0.05$ cm.

RADIUS OF CONSTANT PERCENTAGE OF POWER CONTOUR

A comparison of the two lines as regards the field spread can be made from the ratio of the radius of the area around the line containing a certain percentage of power. The above discussion gives no doubt a comparative value of the field shrinkage achieved by dielectric coating. It is however considered worthwhile to derive an expression for the ratio of the two radii for a constant percentage of power flow, which will enable a direct evaluation of the field shrinkage as a function of the dielectric coating.

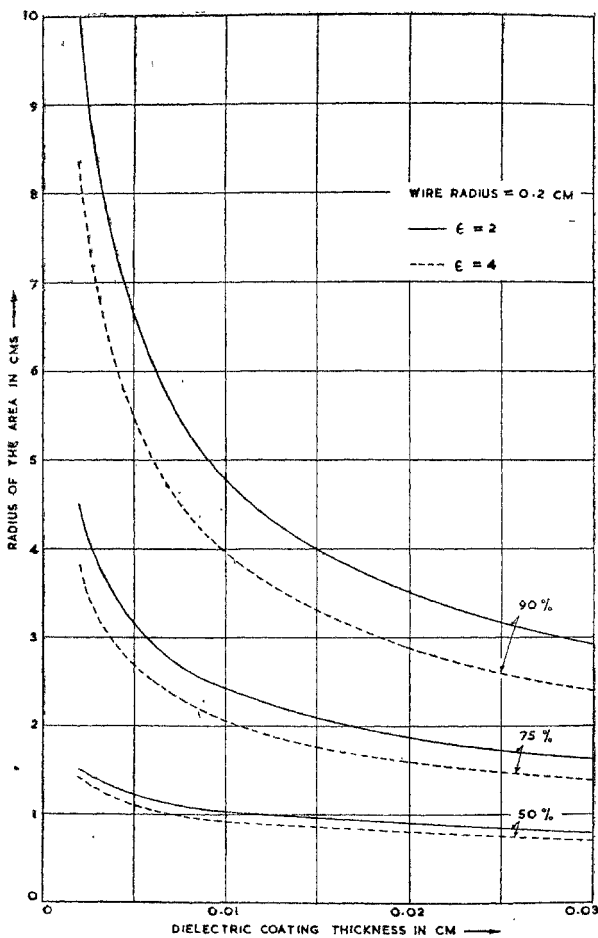


FIG. XIII

Radius of the area around the wire within which 50%, 75% and 90% of the power is propagated as a function of thickness of dielectric coating. $\lambda_0 = 3.45$ cms.

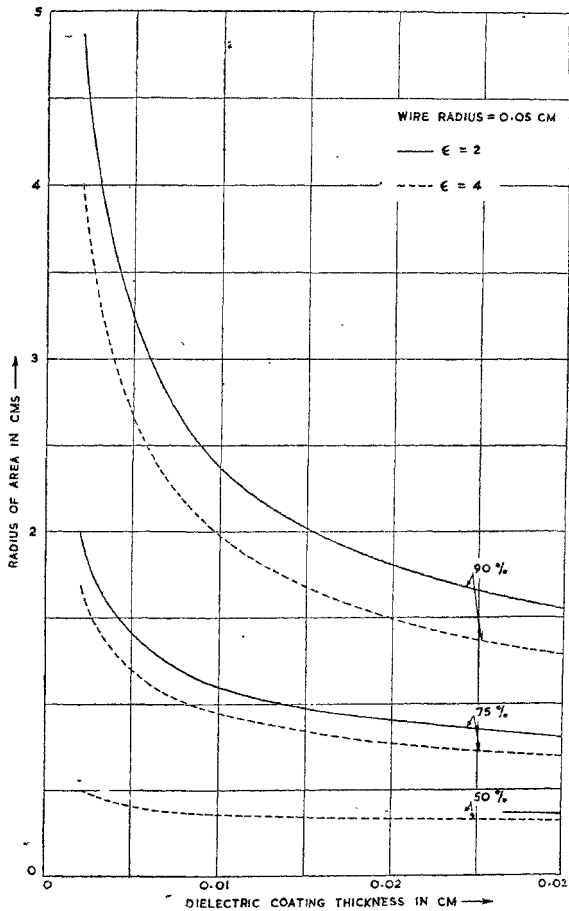


FIG. XIV

Radius of the area around the wire within which 50%, 75% and 90% of the power is propagated as a function of thickness of dielectric coating. $\lambda_0 = 3.45$ cms.

(i) *Sommerfeld Line*: For a bare copper wire line, the radius ρ_{pc} within which a certain percentage p of the total power is contained is given from equation [4] as

$$p = 1 - \frac{N_p}{N} = - \frac{2 \ln(\rho_{pc}/a)}{\ln(2.2 |\xi|)} \quad [18]$$

$$\text{which yields} \quad \rho_{pc} = a / [(2.2)^{p/2} (|\xi|)^{p/2}] \quad [19]$$

where $|\xi|$ is found from [3a] and [3c] as follows:

$$|\xi| = \exp. \{ [\ln 0.89 + \ln \gamma_2 a - j3\pi/8 - \frac{1}{2}] \pm (1/8\sqrt{2}) [\sqrt{(r+x)} - j\sqrt{(r-x)}] \} \quad [20]$$

$$\text{where} \quad r = \sqrt{(x^2 + y^2)} \quad [20a]$$

$$x = (16 - 9\pi^2) + 64 (\ln 0.89 + \ln \gamma_2 a) \\ + 64 \{ (\ln 0.89)^2 + (\ln \gamma_2 a)^2 + 2 \ln 0.89 \ln \gamma_2 a \} \quad [20b]$$

$$\text{and} \quad y = 48\pi (\ln 0.89 + \ln \gamma_2 a) + 40\pi. \quad [20c]$$

So, ρ_{pc} obtained from equations [19] and [20] are given by the following relation

$$\rho_{pc} = a \div (2.2)^{p/2} \langle \exp \{ \ln 0.89 \gamma_2 a - j3\pi/8 - \frac{1}{2} \\ \pm (1/8\sqrt{2}) [\sqrt{(r+x)} - j\sqrt{(r-x)}] \} \rangle^{p/2} \quad [21]$$

(ii) *Harms-Goubau Line*: In the case of a dielectric coated wire, the radius within which a certain percentage p of the total power is contained is given by the relation [17].

Case I: $\gamma' a' < 0.1$; Equation [17] reduces to

$$1 - p = \frac{F(\gamma' \rho_{pd})}{F(\gamma' a')} = \frac{\ln 0.89 \gamma' \rho_{pd} + 0.5}{\ln 0.89 \gamma' a' + 0.5} \quad [22]$$

$$\text{which yields} \quad \rho_{pd} = a' / [\exp(\ln 0.89 \gamma' a')^p \cdot \exp(p/2)] \quad [23]$$

Case II: $\gamma' a' > 0.1$

$$1 - p = \frac{\exp.(2\gamma' a')}{\exp.(2\gamma' \rho_{pd})} \quad [24]$$

$$\text{which yields} \quad \rho_{pd} = a' - \frac{\ln(1-p)}{2\gamma'} \quad [25]$$

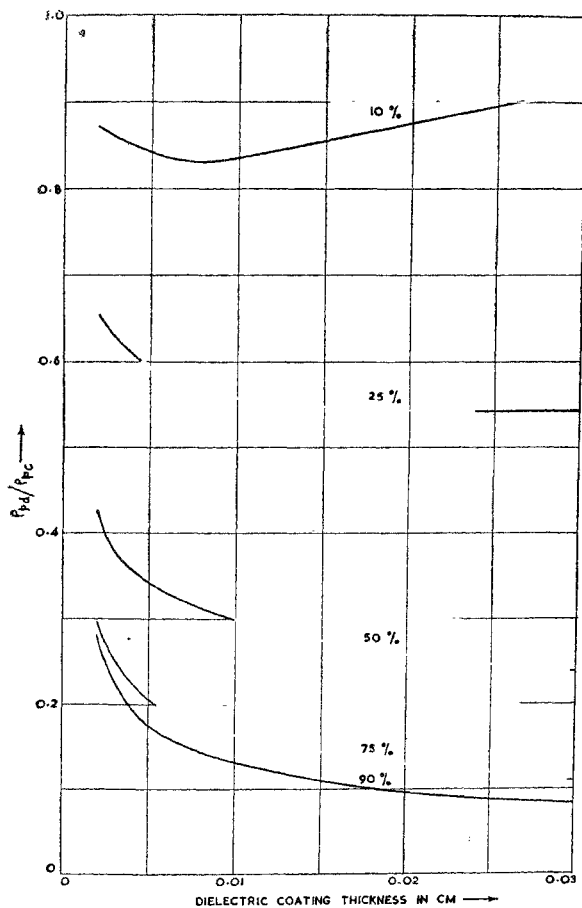


FIG. XV

Ratio of the area of constant percentage power contour for the Harms-Goubau line to that of the Sommerfeld line as a function of thickness of dielectric coating. Wire radius $a=0.10$ cm; Wavelength $\lambda_0=3.45$ cms; Dielectric constant $\epsilon=2.0$

(iii) *Comparison of the Sommerfeld and Harms-Goubau Line*: The extent of the field reduction by coating the Sommerfeld line with a dielectric can be judged from the following ratio of the radii of constant power flow:

Case I: $\gamma' a' < 0.1$

$$\rho_{pd}/\rho_{pc} = (a'/X)[(2.2)^{p/2}(Y)^{p/2}/a] \quad [26]$$

where

$$X = \exp(\ln 0.89 \gamma' a')^p \exp(p/2)$$

$$Y = \exp\left\{\ln 0.89 \gamma_2 a - j 3\pi/8 - \frac{1}{2} \pm (1/8\sqrt{2})[\sqrt{(r+x)} - \sqrt{(r-x)}]\right\}$$

Case II: $\gamma' a' \gg 0.1$

$$\frac{\rho_{pd}}{\rho_{pc}} = \frac{(2.2)^{p/2} \langle \exp\{ \ln 0.89 \gamma_2 a - j 3\pi/8 - \frac{1}{2} \pm (1/8\sqrt{2})[\sqrt{(r+x)} - \sqrt{(r-x)}] \} \rangle^{p/2}}{a [a' - \ln(1-p)/2 \gamma']^p} \quad [27]$$

Figure XV shows a plot of ρ_{pd}/ρ_{pc} for $\epsilon = 2.0$, $a = 0.10$ cm, $\lambda_0 = 3.45$ cm, with respect to the thickness of the dielectric coating for different values of p varying from 10 per cent to 90 per cent. It is observed that the radius of the area containing 25 per cent to 90 per cent of the power flow decreases rapidly up to a dielectric coating thickness of 0.01 cm. and comparatively slowly till $(a' - a) = 0.25$ cm. Thereafter, the rate of decrease is not significant. In the case of $p = 10$ per cent curve, the rate of decrease is not as fast as the other curves and the rising nature of the curve with increasing $(a' - a)$ after 0.008 cm. still remains to be explained.

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