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## A STUDY OF THE TRANSIENT STABILITY PROBLEM

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### ABSTRACT

This paper presents a simple graphical method of determining the critical clearing angle making use of the fundamental stability theorems of Lagrange and Liapounoff for a conservative system. A graphical construction is also given to find the data for the swing curve without numerical computation. The methods proposed are illustrated by two typical examples and the results obtained are compared with those of the existing methods.

### INTRODUCTION

The differential equation characterising the dynamic behaviour of a synchronous machine, known as the swing equation, is<sup>3,4</sup>

$$M \frac{d^2\delta}{dt^2} = P_i - P_a \quad [1]$$

under the usual assumptions of constant input, no damping and constant voltage behind transient reactance, where

$M$  = inertia constant

$P_i$  = shaft power input corrected for rotational losses

$P_a = P_m \sin \delta$  = electrical power output corrected for electrical losses

$P_m$  = amplitude of the power angle curve

$\delta$  = rotor angle with respect to a synchronously rotating reference.

By introducing a new variable, modified time  $T$  defined by the equation

$$T = t \sqrt{[(\pi/180)(P_m/M)]}$$

equation 1 reduces to

$$d^2\delta/dT^2 = P - \sin \delta \quad [2]$$

where  $\delta$  is in radians and  $P = P_i/P_m$ . In the following analysis, the swing equation will be made use of in the form of equation 2 only.

Let the swing equation during fault and that after the fault is cleared be respectively

$$d^2\delta/dT_1^2 = P_1 - \sin \delta \quad [3]$$

$$d^2\delta/dT_2^2 = P_2 - \sin \delta \quad [4]$$

where  $T_1 = t \sqrt{[(\pi/180)(P_{m_1}/M)]}$ ,  $T_2 = t \sqrt{[(\pi/180)(P_{m_2}/M)]}$ ,  $P_1 = P_i/P_{m_1}$  and  $P_2 = P_i/P_{m_2}$ ,  $P_{m_1}$  and  $P_{m_2}$  being respectively the amplitudes of the power angle curve during the fault and after clearing. The post fault swing equation 4 describes the motions of an autonomous conservative system with a nonlinear restoring force  $F(\delta) = \sin \delta - P_2$ . Setting  $d\delta/dT_2 = \omega_1$ , equation 4 becomes

$$d\omega_1/d\delta = (P_2 - \sin \delta)/\omega_1 \quad [5]$$

with the initial conditions,  $\delta = \delta_0$ ,  $\omega_1 = \omega_{10}$  at  $T_2 = 0$ . Separating variables and integrating, equation 5 becomes

$$\frac{\omega_1^2}{2} + \int_0^\delta F(\delta) d\delta = \frac{\omega_{10}^2}{2} + \int_0^{\delta_0} F(\delta) d\delta = E \quad [6]$$

where  $E$  is the total energy of the system. The quantity  $V(\delta) = \int_0^\delta F(\delta) d\delta$  re-

presents the work done by  $F(\delta)$  and so it is the potential energy.  $\omega_1^2/2$  represents the kinetic energy. In other words, equation 6 means: kinetic energy + Potential energy = constant, thus expressing the law of conservation of energy. For a conservative system there are two fundamental stability theorems<sup>2</sup>: the theorem of Lagrange which states, "If the potential energy is a minimum at the state of equilibrium, the equilibrium is stable", and the converse theorem of Liapounoff, "If the potential energy is not a minimum at the state of equilibrium, then the equilibrium is unstable". For  $0 < \delta < \pi$ , the potential energy  $V(\delta)$  is a minimum at  $\delta = \sin^{-1} P_2$  and a maximum at  $\delta = \pi - \sin^{-1} P_2$ . The first equilibrium point of equation 5 is located at  $\delta = \sin^{-1} P_2$  and the second equilibrium

point at  $\delta = \pi - \sin^{-1} P_2$ . Hence by virtue of the theorems stated above, the first equilibrium point is a stable one known as a vortex and the second an unstable one known as a saddle. Solving equation 6 for  $\omega_1$  we get

$$\omega_1 = \sqrt{2[E - V(\delta)]} \quad [7]$$

In the  $(\delta, \omega_1)$  plane, called the phase plane, equation 7 specifies a definite phase trajectory for a definite value of the total energy  $E$ . For  $E = V_{max}(\delta)$ , equation 7 describes the particular phase trajectory, known as the separatrix, passing through the unstable equilibrium point. The critical clearing point is located on the separatrix, its co-ordinates being fixed by initial conditions. The initial conditions are given by the velocity versus displacement curve of the sustained fault swing equation with proper adjustment of time scale. Setting  $\omega = d\delta/dT_1$  in equation 3 and integrating there results

$$\frac{\omega^2}{2} = \int_{\delta(0)}^{\delta} (P_1 - \sin \delta) d\delta \quad [8]$$

where  $\delta(0)$  is the rotor angle at the instant of fault inception. The locus of initial conditions is given by

$$\frac{\omega_{10}^2}{2} = K^2 \frac{\omega^2}{2} \quad [9]$$

where  $K = dT_1/dT_2$ . The critical clearing angle can be obtained by superposing the  $\omega_{10} V \delta$  curve on the phase portrait for the post fault swing equation, the point of its intersection with the separatrix giving the critical clearing angle. The following alternative procedure results in a simple and elegant graphical construction. The equation of the separatrix curve is

$$\omega_1^2/2 = V_{max} - V(\delta). \quad [10]$$

Equating (9) and (10) a transcendental equation of the type  $f(\delta) = A\delta + B + \cos \delta = 0$  results. The solution of this equation can be effected graphically by plotting the straight line  $(-A\delta - B)$  and the  $\cos \delta$  curve, the intersection between the two giving the critical clearing angle  $\delta_c$ . A refined value  $\delta_{c1}$  of  $\delta_c$  can be obtained by means of the Newton-Raphson formula

$$\delta_{c1} = \delta_c - f(\delta_c)/f'(\delta_c). \quad [11]$$

To summarise, the procedure for finding the critical clearing angle by the methods discussed above involves the following steps.

*Method 1*

(a) The separatrix curve given by equation 10 is sketched in the  $(\delta, \omega)$  plane.

(b) The locus of initial conditions given by equation 9 is also drawn on the same graph sheet. The abscissa of the intersection point between the two curves gives the critical clearing angle.

*Method 2*

(a) Equating (9) and (10) a transcendental equation of the form  $A\delta + B + \cos \delta = 0$  is found. If the approximate solution of this equation found graphically is  $\delta_c$ , a better approximation  $\delta_{c1}$ , can be obtained by using equation 11.

*Time corresponding to the critical clearing angle.* Let the  $\omega$  Vs  $\delta$  curve given by (8) of the sustained fault swing equation 3 be drawn. The increment in  $T_1$  needed to traverse the increment  $\Delta\delta$  is<sup>1,5</sup>

$$\Delta T_1 = \Delta\delta / \omega_{av} \quad [12]$$

where  $\omega_{av} = (\omega_1 + \omega_2)/2$ ,  $\omega_1$  and  $\omega_2$  being the values of  $\omega$  at the beginning and end of the increment  $\Delta\delta$ . If  $\Delta T_1$  is small, it is nearly true that

$$\omega_{av} = \omega(0) + \Delta\omega/2 \quad [13]$$

where  $\omega(0)$  is the value of  $\omega$  at the beginning of the increment  $\Delta T_1$  and  $\Delta\omega$  is the change in  $\omega$  during this increment. A combination of the equations 12 and 13 gives

$$\Delta\omega = (2/\Delta T_1) \Delta\delta - 2\omega(0). \quad [14]$$

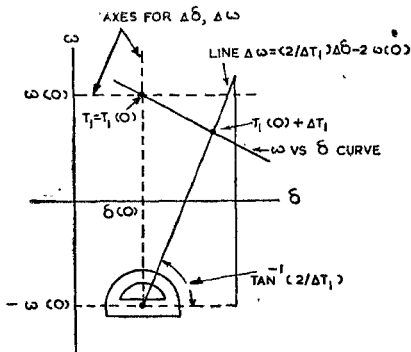


FIG. I

Graphical Construction for finding time

If a fixed value of  $\Delta T_1$  is chosen, equation 14 represents a straight line of slope  $(2/\Delta T_1)$  and  $\Delta\omega$  intercept of  $-2\omega(0)$  where  $\Delta\omega$  and  $\Delta\delta$  are measured from  $\omega(0)$  and  $\delta(0)$  existing at the beginning of the increment. The intersection of this line with the  $\omega$  Vs  $\delta$  curve locates the point satisfying simultaneously the original differential equation 3 and also equation 14. The process of finding time can be mechanized as shown in Fig. 1. The point  $[\delta(0), \omega(0)]$  at  $T_1 = T_1(0)$  is first located. A protractor is then put at the point  $[\delta(0), -\omega(0)]$  with its horizontal and vertical lines aligned with the coordinate axes. The intersection between the  $\omega$  Vs  $\delta$  curve and the line with the slope  $(2/\Delta T_1)$  locates the point  $T_1(0) + \Delta T_1$ . The process can be repeated locating rather quickly a series of points equally spaced in time. The methods presented above will now be illustrated by two examples.

#### Example 1.

A 25 MVA, 60 cycle water wheel generator delivers 20 MW over a double circuit transmission line to a large metropolitan system which may be regarded as an infinite bus. A 3-phase fault occurs at the middle of one of the transmission lines which is subsequently cleared. Find the critical clearing angle and time.<sup>3</sup>

#### DATA :

$$\text{Initial angle, } \delta(0) = 18.1^\circ$$

$$\text{Input power, } P_i = 0.8 \text{ P.u.}$$

$$\text{Inertial constant, } M = 2.56 \times 10^{-4} \text{ p.u.}$$

$$\text{Prefault power angle equation} = 2.58 \sin \delta$$

$$\text{Power angle equation during fault} = 0.936 \sin \delta$$

$$\text{Post fault power angle equation} = 2.06 \sin \delta$$

$$T_1 = 8t, T_2 = 11.4t, K = 0.702$$

#### Method 1

The potential energy  $V(\delta)$  for the post fault swing equation is

$$V(\delta) = \int_0^{\delta} (\sin \delta - 0.388) d\delta = 1 - \cos \delta - 0.388 \delta.$$

The maximum value of  $V(\delta)$ ,  $V_{max}(\delta) = 0.86$  and it occurs at  $\delta = 157.2^\circ$ . The equation for the separatrix curve is

$$\omega_1 = \sqrt{2 [0.86 - V(\delta)]}.$$

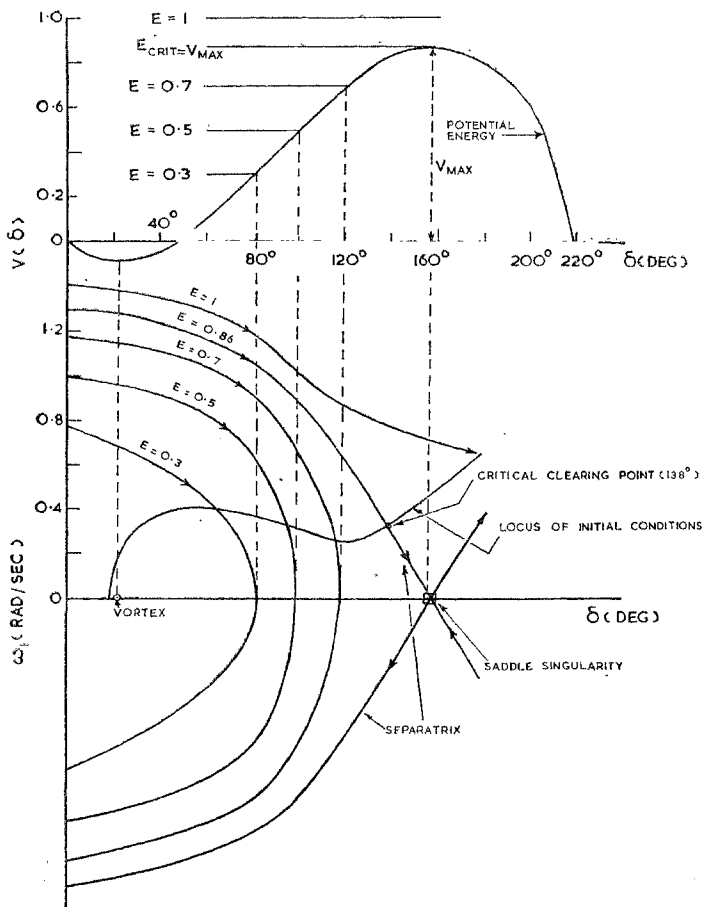


FIG. II

Phase Trajectories for the post fault swing equation of example

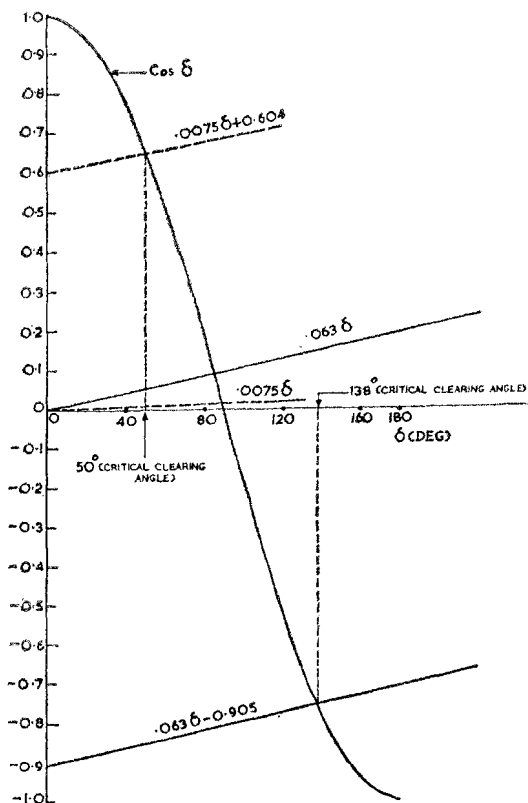


FIG. III  
Finding Critical Clearing Angle

The locus of initial conditions is given by

$$\omega_{10} = 0.702 \omega$$

where 
$$\omega = \sqrt{(1.71 \delta + 2 \cos \delta - 2.44)}. \quad [15]$$

The intersection of the separatrix curve and the locus of initial conditions occurs at  $\delta = 138^\circ$ , which is the critical clearing angle. The details of the procedure are shown in Fig. 2.

### Method 2

Equating the expressions for  $\omega_1$  and  $\omega_{10}$  we get

$$0.063 \delta - 0.905 = \cos \delta.$$

The solution of this equation obtained graphically in Fig. III is  $\delta_c = 138^\circ$ . A better value  $\delta_{c_1}$  is obtained using equation 11.

$$\delta_{c_1} = 2.41 + 0.01/0.7321 = 2.4237 \text{ radians or } 138.9^\circ.$$

This value of critical clearing angle compares favourably with  $139^\circ$  obtained by the equal area method.

The graphical construction for finding time from the  $\omega$  Vs  $\delta$  curve given by equation 15 is clearly illustrated in Fig. IV. The increment  $\Delta T_1$  is chosen as 0.4 sec. corresponding to a value for  $\Delta t$  equal to 0.05 sec. Thus the angle to be used in the graphical construction is

$$\tan^{-1}(2/0.4) = \tan^{-1}5 = 78.7^\circ.$$

In using the graphical construction indicated in this paper either same scale is to be used on both the  $\delta$  and  $\omega$  axes or, if the scales on the two axes are different, its effect on the angle  $\tan^{-1}(2/\Delta T_1)$  must be properly taken care of as follows:

Let 1 unit of  $\delta$  be represented by  $\delta_x$  mm

Let 1 unit of  $\omega$  be represented by  $\omega_y$  mm

Let  $\Delta T_1 = h$  secs.

Then the angle to be used in the graphical construction is  $\tan^{-1}[(2/h)(\delta_x/\omega_y)]$ . In Fig. IV the scales on both the axes are chosen equal. The times corresponding to various angles are shown across the positive half of the  $\omega$  Vs  $\delta$  curve. The first point corresponding to 0.11 sec. ( $28^\circ$ ) is located using the formula given by equation 12, for it is very difficult to locate intersection point between the line



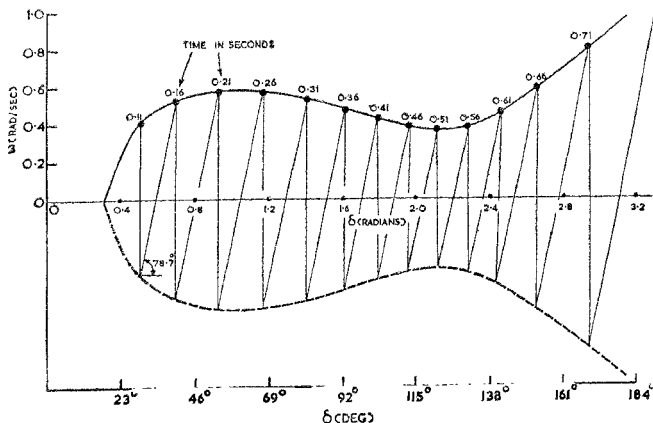


FIG. IV  
Time scale on the  $\omega$  Vs  $\delta$  curve

with the slope 5 and the initial portion of the  $\omega$  Vs  $\delta$  curve. Thereafter graphical construction is used to find time. By interpolation, the time corresponding to the angle of  $139^\circ$  is found as 0.62 sec, which is the critical clearing time. The critical clearing time obtained by step-by-step method II<sup>3,4</sup> is 0.61 sec. The swing curves obtained by the two methods are shown in Fig. V.

#### Example 2

A generator supplies power through parallel high voltage transmission lines to a large metropolitan system considered as a infinite bus. A 3-phase fault occurs on one of the transmission lines which is subsequently cleared. Find the critical clearing angle and time.<sup>4</sup>

#### DATA:

Initial angle, $\delta(0)$	= $35.2^\circ$
Input power, $P_1$	= $1 p.u$
Inertial constant, $M$	= $2.78 \times 10^{-4} p.u$
Prefault power angle equation	= $1.735 \sin \delta$
Power angle equation during fault	= $0.42 \sin \delta$
Post fault power angle equation	= $1.25 \sin \delta$
$T_1 = 5.15 t, T_2 = 8.86 t, K = d T_1 / d T_2$	= $0.582$

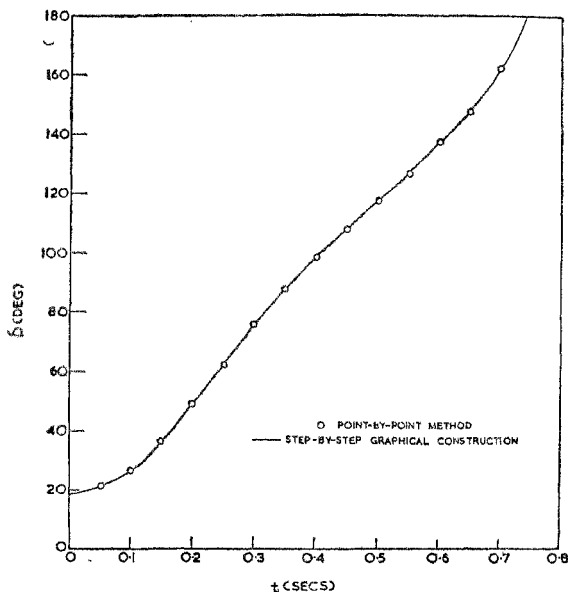


FIG. V  
Swing Curve for Example 1 with Sustained Fault

The potential energy  $V(\delta)$  for the post fault swing equation is

$$V(\delta) = \int_0^{\delta} (\sin \delta - 0.8) d\delta = 1 - \cos \delta - 0.8 \delta.$$

The maximum value of  $V(\delta)$ ,  $V_{max}(\delta) = -0.17$  and it occurs  $\delta = 126.8^\circ$ . The equation for the separatrix curve is

$$\omega_1 = \sqrt{2[-0.17 - V(\delta)]}$$

and that for the locus of initial conditions is

$$\omega_{10} = 0.582 \omega \text{ where } \omega = \sqrt{(4.76 \delta + 2 \cos \delta - 4.55)}.$$

The intersection of the separatrix curve and the locus of initial conditions occurs at  $\delta = 52^\circ$ , which is the critical clearing angle. The details of the procedure are shown in Fig. VI.

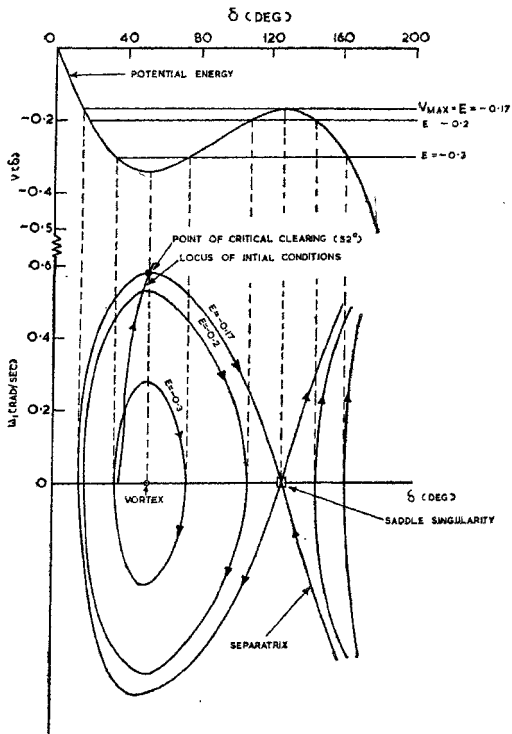


FIG. VI

Phase portrait for the post fault swing Equation of Example 2

*Method 2*

Equating the expressions for  $\omega_1$  and  $\omega_{10}$  we get

$$\cos \delta = 0.0075 \delta + 0.604.$$

The solution of this equation obtained graphically in Fig. III is  $\delta_c = 50^\circ$ . A better value of  $\delta_c$  is obtained using equation 11 as  $\delta_{c1} = 52^\circ$ , which compares favourably with the value of  $51.6^\circ$  obtained by the equal area method. Using the graphical construction illustrated clearly in the previous problem, the time corresponding to the critical clearing angle of  $52^\circ$  is found as 0.11 sec. Step-by-step method II also gives 0.11 sec.

## DISCUSSION

The method presented in this paper identifies the critical switching angle as a point on the separatrix curve, which separates the region of stable motions from that of unstable motions. In addition, the method described in the paper introduces the analysis of the potential energy, stored in the generator rotor, as a tool in the determination of the critical switching angle. This interpretation is made possible by the fundamental stability theorems of Lagrange and Liapounoff for a conservative system. The fundamental difference between the topological method presented here and the conventional equal area method is that the former is based on the concept of energy, while the latter is based on the concept of power; in the former, the critical clearing angle is obtained by equalising the maximum value of the potential energy with the total initial energy and in the latter by equalising the area representing acceleration power with that representing deceleration power. While, in the phase plane method, it is clearly shown that the various possible motions of the system take place along paths of constant energy, this is not placed in evidence in the equal area method. Nevertheless, the two methods complement each other bringing out the important fact that the stability of a nonlinear system, for a given type of excitation, is dependent on the initial conditions unlike a linear system which is either stable or unstable, the driving function and initial conditions having no effect on stability.

The graphical construction given in this paper finds time increments by assuming a constant average velocity during a small angular increment  $\Delta \delta$ , while the point-by-point method II<sup>3,4</sup> finds the incremental angles during a small time interval  $\Delta t$ , assuming the velocity to be constant at the value computed at the middle of  $\Delta t$ : the former method finds time increments from angular increments whereas the latter method finds angular increments from time increments. There is good agreement between the results obtained by the two methods even though they proceed on different lines. If an analytical solution is to be found for the swing equation, it is necessary first of all to approximate  $\sin \delta$  by a polynomial in  $\delta$ . Reference 1 approximates  $\sin \delta$  by  $a \delta + b \delta^3$  such

that the integral square error is a minimum. Minimization of the integral square error essentially means that emphasis is placed on the errors according to the square of the error magnitude. In other words, the approximation resulting from the minimization of the integral square error attempts to cut down large errors at the cost of many small errors. Other types of error criteria may be chosen, but their mathematical treatment becomes very difficult, if not impossible. Therefore, analytical solutions are of little value.

#### CONCLUSIONS

The authors believe that the cosine  $\delta$  curve method, based on the energy concept, for finding the critical clearing angle in conjunction with the step-by-step graphical method for finding time form a good combination for the solution of the transient stability problem.

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