PROPAGATION OF MICROWAVES ON A SINGLE WIRE

Part IV

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Abstract

A theoretical study of the reduction in phase velocity and radial field spread of the Harms-Goubau line, as a function of the wire radius, coating thickness and dielectric constant for different wavelengths, has been carried out. It is observed that the dielectric coating thickness has more control over the spread of the field in the radial direction than the dielectric constant of the coating. The percentage of the total power flowing in the dielectric coating, as a function of the coating thickness and dielectric constant for three wavelengths has been calculated. These results when compared with that of the Sommerfeld line show clearly the superiority of the Harms-Goubau line as a surface wave guide.

Introduction

In an earlier communication,[1] the effect of the wire radius on the field spread and propagation parameters for the Sommerfeld and Harms-Goubau line was reported. The object of the present paper is to study the variation of the phase velocity and radial field spread, with respect to the wire radius in the case of Sommerfeld line and wire radius, dielectric coating thickness and dielectric constant in the case of the Harms-Goubau line respectively. It is found that a dielectric coating thickness of even 0.002 cm reduces the field spread to more than half its value without the dielectric coating. To obtain the same reduction in field spread for the Sommerfeld line, the wavelength of excitation has to be reduced to one third its original value.

Having found the superiority of the Harms-Goubau line over that of the Sommerfeld line, it is of interest to calculate the percentage of the total power flowing in the dielectric coating. Detailed calculations have been done to determine this percentage of the power, as a function of the dielectric coating thickness and dielectric constant for wavelengths $\lambda_0 = 4.00$, 3.45 and 1.25 cms and wire radius $a = 0.05$, 0.10, 0.15 and 0.20 cm. It is obserded that increasing the dielectric coating thickness increases the percentage of power flowing in the coating, where as, it decreases as the dielectric constant of the coating is increased. If we consider cases, where the reduction in phase velocity is less than 4%, and the dielectric constant of the coating $\epsilon > 2.0$, the power flowing in the coating is less than 8% of the total power propagated.

RADIAL DECAY FAGTOR

(i) *Sommerfeld line*:—The radial decay factor γ_2, as derived from the boundary conditions is given by[2]

$$\gamma_2 = \frac{1.123}{a}(|\xi|)^{1/2}.\exp(j\tfrac{1}{2}\theta) \qquad [1]$$

where $$|\xi|\,ln\,|\xi| = -|\eta|$$

and $|\eta| = 1.70 \times 10^{-3} \times a\lambda_0^{-3/2}$ for a copper wire immersed in air. a is the radius of the wire and λ_0, the free space wavelength. The real part of γ_2, as derived from equation [1] is

$$\mathrm{Re}\,[\gamma_2] = [1.123(|\xi|)^{1/2}.\cos\tfrac{1}{2}\theta]/a. \qquad [2]$$

The real part of the decay factor is plotted as a function of the wire radius for different wavelengths in Fig. I. It is observed from the graphs that the decay factor does not increase appreciably, within the wavelength range $\lambda_0 = 4.00$ cms to 3.2 cms. Secondly, for the wavelength region plotted, the rate of increase of the value of the decay factor is not much for wire radii $a = 0.30$ cm to 0.10 cm. However, it is much faster as the radius of the line decreases below 0.10 cm. The slow rate of increase of the decay factor is responsible for the slow rate of reduction of radial field spread in the Sommerfeld line.

(ii) *Harms-Goubau Line*:—Using the field components as reported elsewhere,[1] imposing the boundary condition that the tangential component of the electric field on the surface of a perfect conductor is zero and matching the transverse impedance $Ez/H\phi$ at the inrerface between the dielectric coating and air, the following equation is obtained.[3,4]

$$\frac{\omega\mu_0\gamma_d}{k_d^2}.\frac{J_0(\gamma_d a')N_0(\gamma_d a) - J_0(\gamma_d a)N_0(\gamma_d a')}{J_1(\gamma_d a')N_0(\gamma_d a) - J_0(\gamma_d a)N_1(\gamma_d a')}$$

$$= [\surd(\mu_0/\epsilon_0)]\,[(\gamma'^2 a'/k)\,ln\,0.89\,\gamma' a'] \qquad [3]$$

where γ_d and k_d refer to the value of the decay factor and wave number inside the dielectric and γ', the value of the decay factor outside the dielectric, *i.e.*, in air. In the above expression, the ratio of the Hankel Functions $[H_0^{(1)}(j\gamma' a')/H_1^{(1)}(j\gamma' a')]$ has been taken to be equal to $j\,ln\,(0.89\gamma' a')$, as a small argument approximation.

If we consider wires, whose radii are very small compared with the free space wavelength λ_0 and the dielectric coating thickness very small or of the same order of magnitude as the wire radius, the small argument approximate relations are used for the Bessel and Neumann functions and hence equation [3] reduces to

$$\gamma'^2\{ln(0.89\,\gamma' a') - (1/\epsilon)\,ln(a'/a)\} = (1/\epsilon - 1)(2\pi/\lambda_0)^2\,ln(a'/a) \qquad [4]$$

where ϵ represents the relative dielectric constant of the coating.

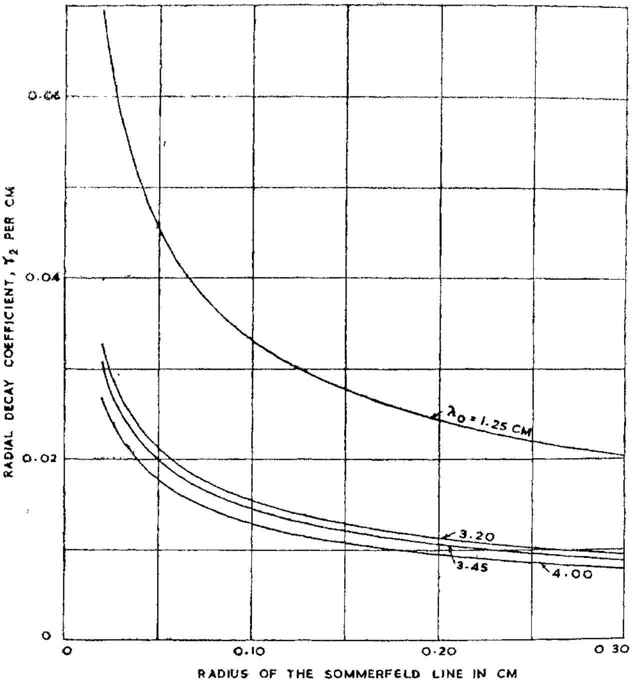

FIG. I

Radial decay factor, as a function of the wire radius, for different wavelengths

The radial decay factor γ', as calculated from the above expression, is plotted in Fig. II as a function of ϵ for different values of λ_0 and two different values of dielectric coating thickness $a'' = 0.002$ cm. and 0.01 cm. The variation

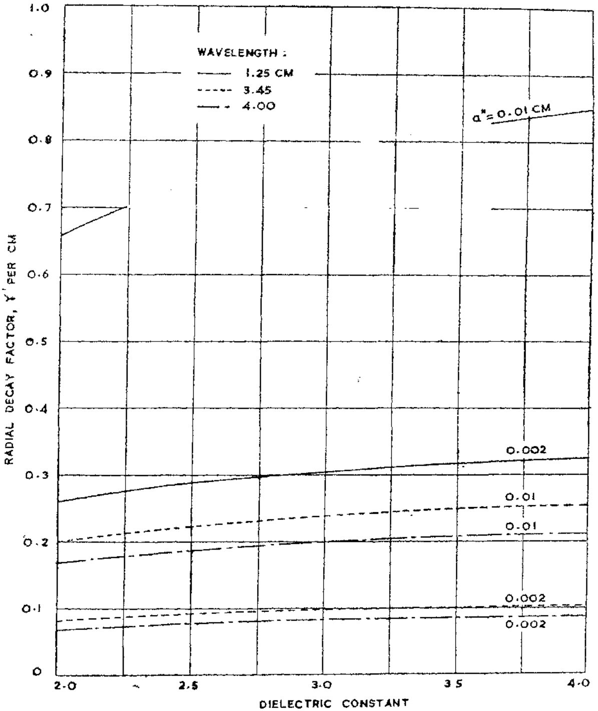

Fig. II

Radial decay factor of the Harms-Goubau line as a function of the dielectric constant of the coating for different wavelengths and coating thicknesses, Wire radius $a = 0.10$ cm.

of γ' as a function of the dielectric coating thickness for different wavelengths and two different value of the dielectric constant $\epsilon = 2.0$ and 4.0, is presented in Fig. III. A study of these two sets of graphs show that the rate of increase of γ' with increase in the value a'' is faster than the rate of increase with the increase in value of ϵ. In other words, the coating thickness is more effective, in increasing γ', than the dielectric constant. For example, increasing a'' from 0.01 cm to 0.02 cm at $\lambda_0 = 3.45$ cms and $\epsilon = 2.0$, the value of γ' is increased by a factor 1.5, where as increasing ϵ from 2.0 to 4.0 at the same wavelength with $a'' = 0.01$ cm, increases γ' from 0.20 to 0.25 only.

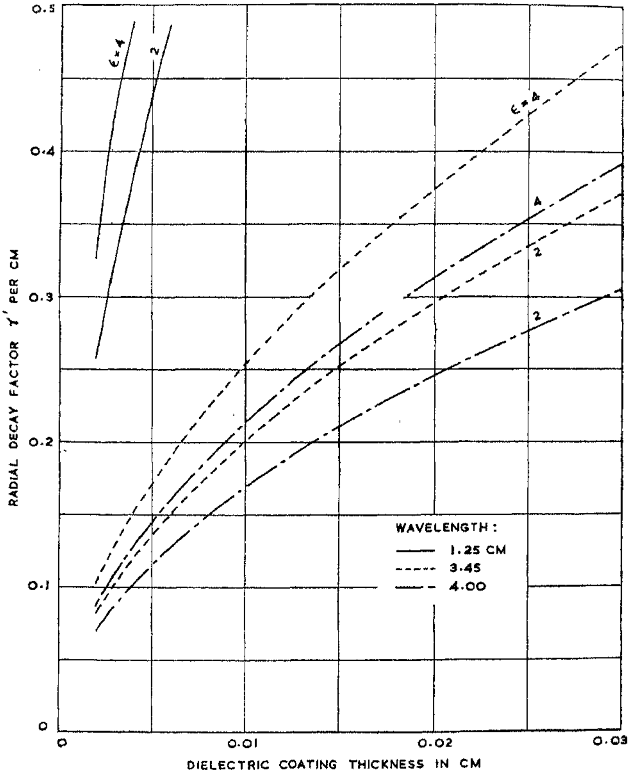

FIG. III

Radial Decay factor as a function of the dielectric coating thickness, for different wavelengths. Wire radius $a = 0.10$ cm.

The percentages of power flow contained within a certain radial distance from the transmission line, as calculated for both the Sommerfeld and Harms-Goubau line, are represented graphically for the sake of comparison in Fig. IV. The curve $a'' = 0$ corresponds to the Sommerfeld line. The radius of the wire $a = 0.10$ cm and the free space wave length $\lambda_0 = 4.00$ cms and the dielectric constant $\epsilon = 2.40$ for the Harms-Goubau line. The variation of the radius of the area

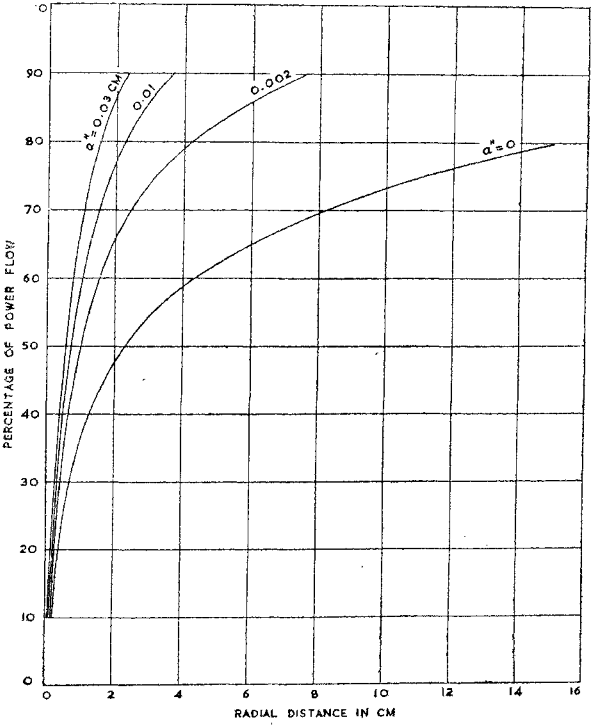

FIG. IV

Radial field spread as a function of the percentage of power flow, for different dielectric coating thicknesses a''. Wire radius $a=0.10$ cm $\cdot$ $\lambda_0=4.00$ cms. $\epsilon=2.4$

around the wire within which 90% of the power is propagated as a function of the dielectric constant of the coating for different values of wavelengths and

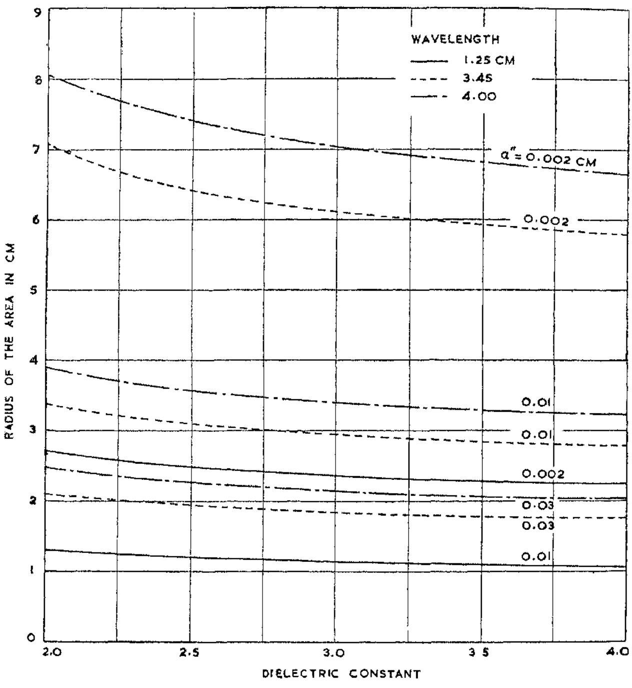

FIG. V

Radius of the area around the wire, with in which 90% of the power is propagated as a function of the dielectric constant, for different wavelengths and coating thicknesses. Wire radius $a = 0.10$ cm.

coating thicknesses is plotted in Fig. V. From these two sets of graphs, the superior influence of the dielectric coating thickness in reducing the radial field spread than the dielectric constant is brought out more clearly.

PHASE VELOCITY

eld line :—The phase velocity v_p of the Sommerfeld line is ession[2]

$$v_p = C\,[1 - 0.63\,|\xi|\,\mathrm{Cos}\,\theta/(k\,a)^2] \qquad [5]$$

locity of light

$$\theta = -(\pi/4)[1 - 1/(ln\,|\xi| + 1)]$$

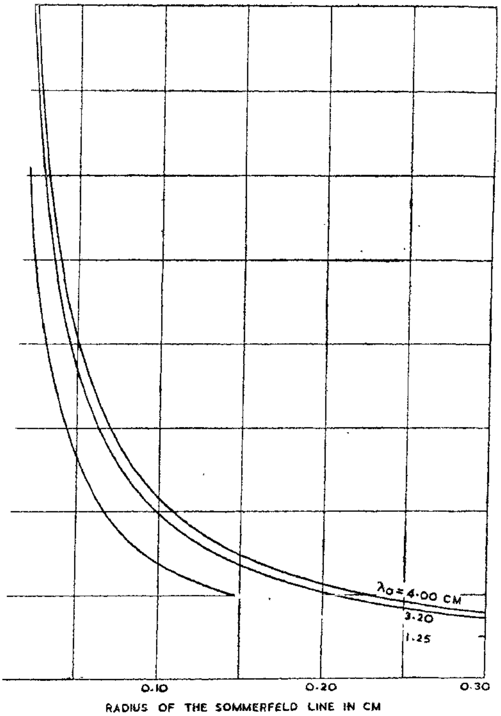

FIG. VI

ge reduction in phase velocity of the wave as a function of wire radius, for different wavelengths

The percentage reduction in phase velocity as a function of the wire radius a for different values of wavelength λ_0 is plotted in Fig. VI. It is seen that the percentage reduction in the phase velocity of the wave is very small for increasing values of wire radius; and as the conductivity of the conductor increases with decreasing values of wavelength, the reduction in the phase velocity of the wave decreases with increasing frequency.

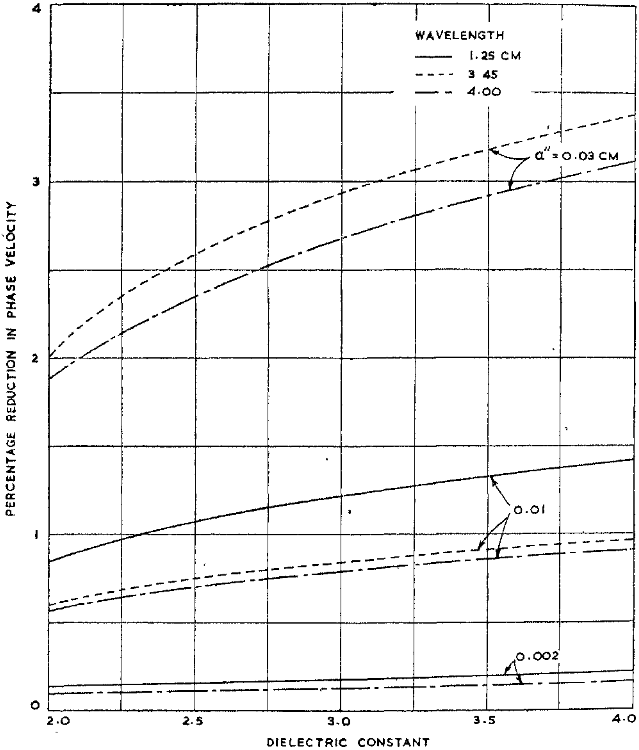

FIG. VII

Percentage reduction in phase velocity of the Harms-Goubau line, as a function of the dielectric constant for different wavelengths and coating thicknesses. Wire radius $a=0.10$ cm.

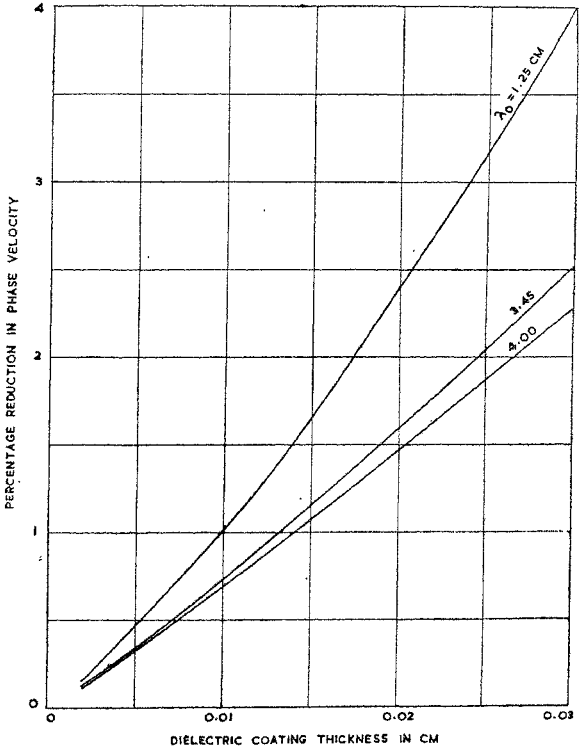

FIG. VIII

Percentage reduction in phase velocity of the Harms-Goubau line, as a function of the dielectric coating thickness, for different wavelength. $a=0.10$ cm. $\epsilon=2.4$

(ii) *Harms-Goubau Line* :—With a knowledge of the radial decay factor γ', the propagation constant h of the guided wave can be derived. It is given by

$$h^2 = -[(\gamma')^2 + k^2]. \qquad [6]$$

If the line is assumed to be lossless, $h^2 = -\beta^2$, where β is the phase constant of the guided wave. For the wire radius and coating thickness considered earlier, $[\gamma'^2/k^2] << 1$. Hence we get

$$\beta = k\,[1 + 1/2 \cdot \gamma'^2/k^2] \qquad [7]$$

and the phase velocity v_p of the wave is given by

$$v_p = \omega/\beta = C\,[1 - \gamma'^2/2k^2]. \qquad [8]$$

The percentage of reduction in v_p as a function of the dielectric constant ϵ for different values of wavelength λ_0 and dielectric coating thickness a'' is shown in Fig. VII. Fig. VIII shows the variation of $\delta\, v_p/v_p\,\%$ with respect to the thickness a'' at $\epsilon = 2.40$ for different wavelengths. In both these graphs, the wire radius a is taken to be 0.10 cm.

For sake of comparing the relative values of the reduction in v_p between the Sommerfeld line and Harms Goubau line, Fig. IX is plotted for variations in wire radius a, for $\lambda_0 = 4.00$ and 1.25 cms. It is seen clearly that $\delta v_p/v_p\%$ is appreciably large in the case of the Harms-Goubau line than the Sommerfeld line.

Distribution of Power in the Harms-Goubau Line : – The power transmitted around the line is given by[4]

$$P_0 = -\frac{\beta I_0^2}{4\pi\omega\epsilon_0}[\ln 0{\cdot}89\, \gamma'\, a' + 0.5] \qquad [9]$$

where I_0 is the peak value of the current and ϵ_0 is the free space dielectric constant.

The power flowing in the dielectric coating is given by

$$P_i = \frac{\beta\, I_0^2}{4\,\pi\,\omega\,\epsilon_d} \ln\,(a'/a) \qquad [10]$$

where ϵ_d is the absolute dielectric constant of the coating. The above two equations have been derived with the assumption that the arguments of the cylindrical functions are very small and hence the small argument approximations for the Hankel and Bessel functions are valid.

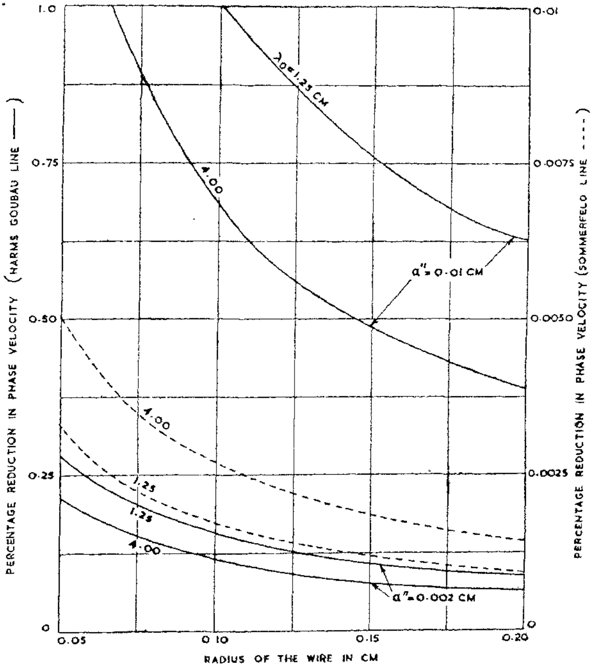

FIG. IX

Percentage reduction in phase velocity of the wave as a function of the wire radius for different wavelengths. $\epsilon=2.4$ for the Harms-Goubau line.

The percentage of the total power flowing in the dielectric coating is then given by

$$[P_i/(P_i+P_0)]\% = (P_i/P_t)\% = ln(a'/a)/\{ln(a'/a) - \epsilon[ln(0.89\,\gamma' a') + 0.5]\} \qquad [11]$$

where $\epsilon = \epsilon_d/\epsilon_0$, is the relative dielectric constant of the coating.

This percentage of the power decreases as the dielectric constant of the coating increases *i.e.*, a part of the energy goes into the air medium from the dielectric medium. The radial decay factor γ' increases at a lower rate for

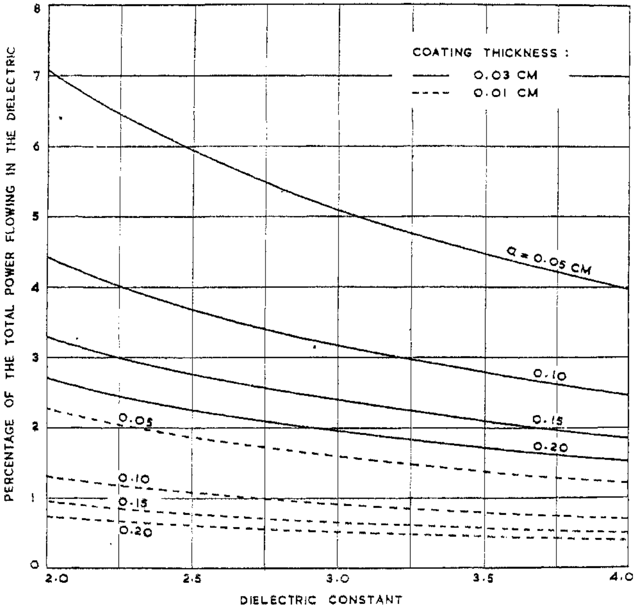

FIG. X

Percentage of the total power flowing in the dielectric coating as a function of the dielectric constant, for different wire radii $\lambda_0 = 4.00$ cms.

increasing values of ϵ and so P_i/P_t % decreases. Figures X and XI show the variation of P_i/P_t % as a function of ϵ, for wavelengths $\lambda_0 = 4.00$ cms and 3.45 cms respectively. Curves are drawn for two different coating thicknesses $a'' = 0.01$ cm and 0.03 cm.

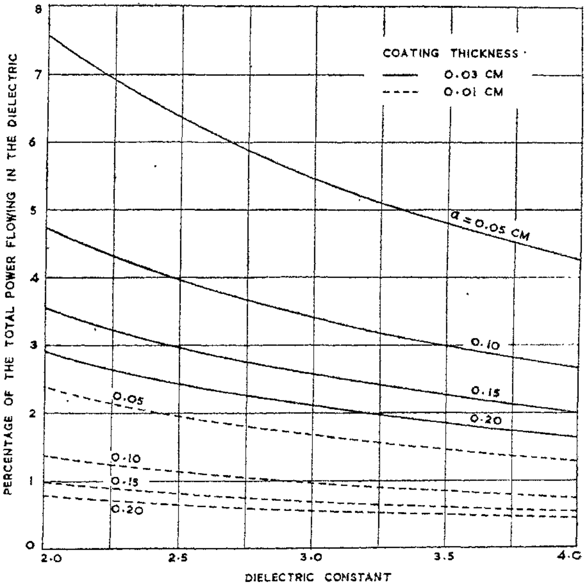

FIG. XI

Percentage of the total power flowing in the dielectric coating as a function of the dielectric constant for different wire radii. $\lambda_0 = 3.45$ cms.

The effect of the dielectric coating thickness a'' on the percentage of P_i/P_t is brought out in Figure XII and XIII for two different wire radii $a = 0.05$ cm and 0.20 cm. The curves are drawn for $\lambda_0 = 4.00$ cms and 3.45 cms. respectively. The slow rate of variation of this with the dielectric constant is also seen in the intermediate curves drawn for $\epsilon = 2.0$ to 4.0. To bring out the relation between the wavelength and the percentage of the power more explicitly, Figure XIV has been presented for three different wire radii $a = 0.05$ cm, 0.10 cm, and 0.20 cm.

It is observed from this family of curves that P_i/P_t % increases as (i) the thickness of dielectric coating increases (ii) the wire radius is made smaller and (iii) the wavelength is decreased. However it decreases with increasing value of the dielectric constant.

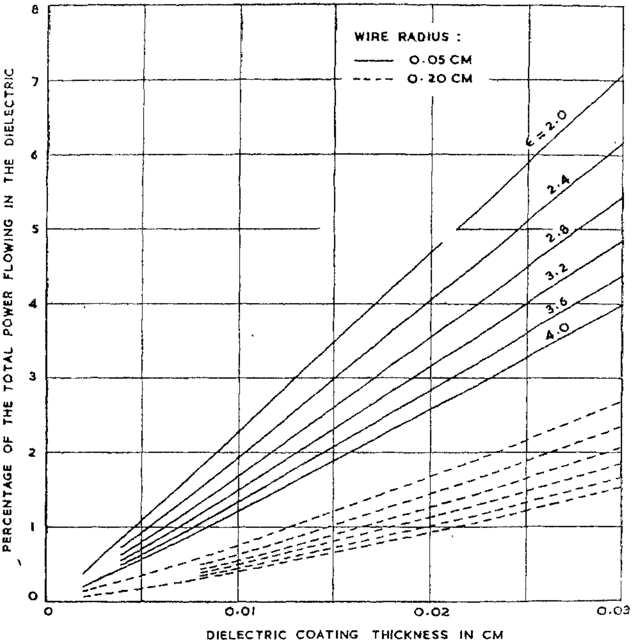

FIG. XII

Percentage of the total power flowing in the dielectric coating as a function of the coating thickness, for different dielectric constants. $\lambda_0 = 4.00$ cms.

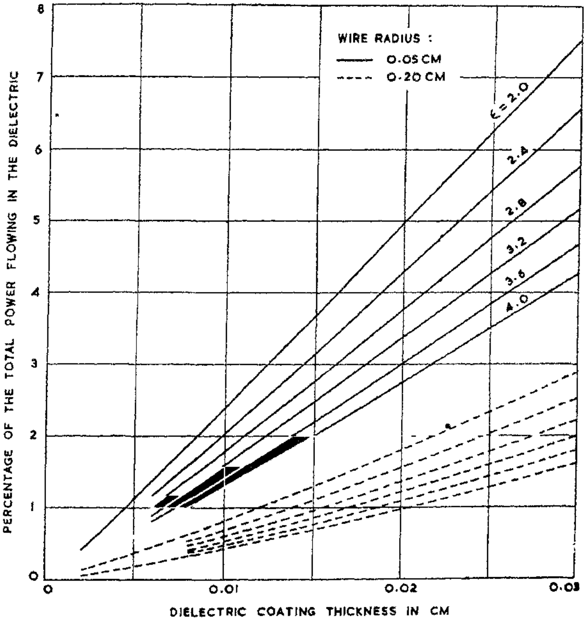

FIG. XIII

Percentage of the total power flowing in the dielectric coating as a function of the coating thickness, for different dielectric constants. $\lambda_0 = 3.45$ cms.

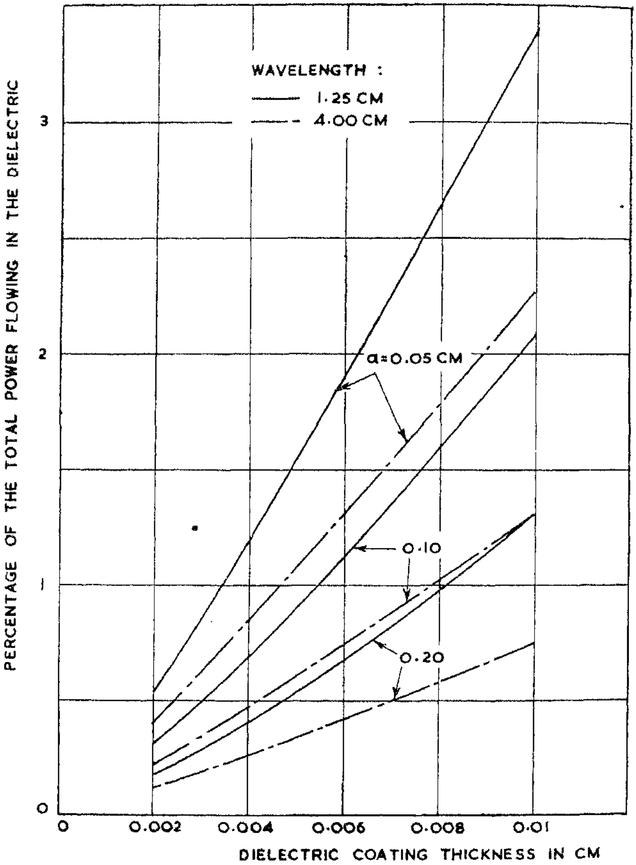

FIG. XIV

Percentage of the total power flowing in the dielectric coating as a function of the coating thickness, for different wire radii. $\epsilon=2.0$.

COMPARATIVE STUDY

To enable a comparative study of the two lines, namely Sommerfeld and Harms-Goubau line, with particular reference to the reduction in phase velocity of the guided wave and the radial field spread, the following two tables are presented. It is seen from the respective figures, the superiority of the Harms-Goubau line, over that of Sommerfeld line, as a surface wave guide for single wire transmission.

TABLE I

Radial field spread for different percentages of power flow.
Wire radius for both the lines, $a=0.10$ cm. Dielectric constant $\epsilon=2.40$ and coating thickness $a''=0.01$ cm. for the Harms-Goubau line.

Percentage of Power flow	Sommerfeld line		Harms-Goubau Line	
	$\lambda_0=4$ cms.	$\lambda_0=1.25$ cms.	$\lambda_0=4$ cms.	$\lambda_0=1.25$ cms.
90%	28.44 cms.	12.19 cms.	3.61 cms.	1.21 cms.
75%	11.10 ,,	5.42 ,,	1.74 ,,	0.68 ,,
50%	2.30 ,,	1.44 ,,	0.68 ,,	0.34 ,,
25%	0.43 ,,	0.38 ,,	0.27 ,,	0.20 ,,
10%	0.18 ,,	0.17 ,,	0.15 ,,	0.13 ,,

TABLE II

Percentage reduction in phase velocity of the surface wave.
Wire radius for both the lines, $a=0.10$ cm.

Dielectric coating thickness in cm	$\lambda_0=4,00$ cms.			3.45 cms			1.25 cms.		
	Summerfeld line	Harms-Goubau line		Summerfeld line	Harms-Goubau line		Sommerfeld line	Harms-Goubau line	
		$\epsilon=2.0$	$\epsilon=4.0$		$\epsilon=2.0$	$\epsilon=4.0$		$\epsilon=2.0$	$\epsilon=4.0$
0	.00269			.00256			.00170		
0.002		.0970	.1526		.1008	.1589		.1315	.2108
0.004		.2087	.3299		.2174	.3437		.2911	.4728
0.006		.3268	.5187		.3423	.5430		.4683	.7666
0.008		.4486	.7153		.4717	.7503		.6554	1.0830
0.01		.5732	.9167		.6060	.9672		.8536	1.4230
0.02		1.2213	1.9852		1.3198	2.1266			
0.03		1.8875	3.1138		2.0861	3.3869			

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