SOME INVESTIGATIONS ON DIELECTRIC ROD AERIALS
Part VI

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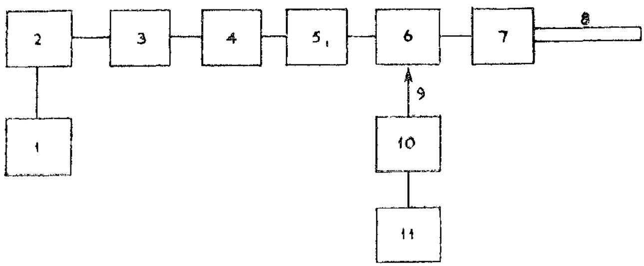
ABSTRACT

The applicability of the concept of wave impedance for a dielectric rod supporting the HE_{11} mode has been discussed. The input impedance for dielectric rods of varying length, diameter, and dielectric constant has been experimentally determined in both the cases when the free end is fully exposed and covered with tin foil.

INTRODUCTION

In a previous paper[1], the authors have derived an expression for the electric field intensity as a function of the diameter, length and dielectric constant of a dielectric rod aerial excited in HE_{11} mode, by considering that radiation takes place from the cylindrical surface as well as the end of the rod.



1. Universal Klystron Power Supply. P.R.D. Type 801-A, No. 795
2. Microwave Oscillator. 723 A/B Klystron, Western Electric
3. Buffer Attenuator 0-20 db. P.R.D. Type 160-A, No. 106
4. Frequency Meter 8.2—10 KMc/s. P.R D. Type 558-A, No. 128
5. Precision Attenuator. P.R.D. Type 185-B, No. 337
6. Slotted Section. P.R.D. Type 202-A, No 223
7. Mode Transformer (Constructed in the Laboratory)
8. Dielectric rod Aerial
9. Probe. P.R.D. Type 250-A, No. 2105
10. Bolometer. P.R.D. Type 610-A
11. Universal Power Bridge. P.R.D. Type 650-B, No. 740

FIG. I

Block schematic of the experimental Setup for the measurement of impedance

In order to determine the part played by the free end in radiation, different metal discs and rings were used to suppress radiation from the free end. The length of the rod was adjusted by pushing it into or pulling it out of the mode transformer. This may give rise to mismatch inside the mode transformer. It was therefore thought worthwhile to study the input impedance of the dielectric rod aerial, when the free end was fully exposed and when different discs and rings were used to cover the free end as well as when the length was varied. The report presents an experimental study of the input impedance of a dielectric rod aerial as a function of length, diameter and dielectric constant of the rod. It is believed that the present paper along with the previous one[1] will be helpful in understanding the proper mechanism of radiation from a dielectric rod aerial.

THE CONCEPT OF IMPEDANCE

At low frequencies, the definition of impedance is unique, but at high frequencies curl **E** differs increasingly from zero and the voltage difference between any two points defined as the line integral of the electric field between the points

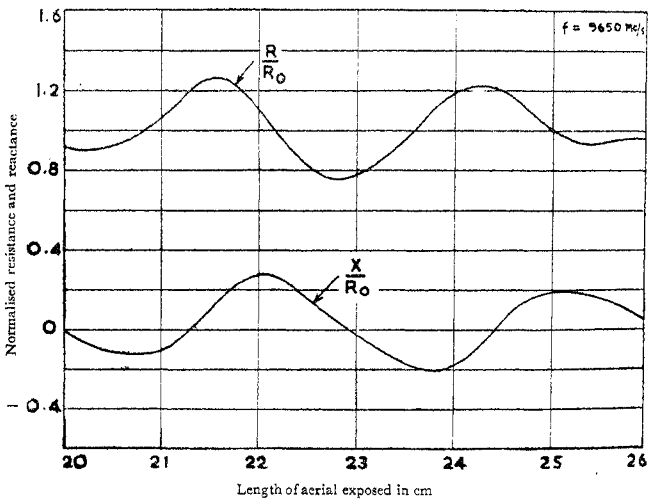


FIG. III
Input impedance for Rod B_2 with free end exposed

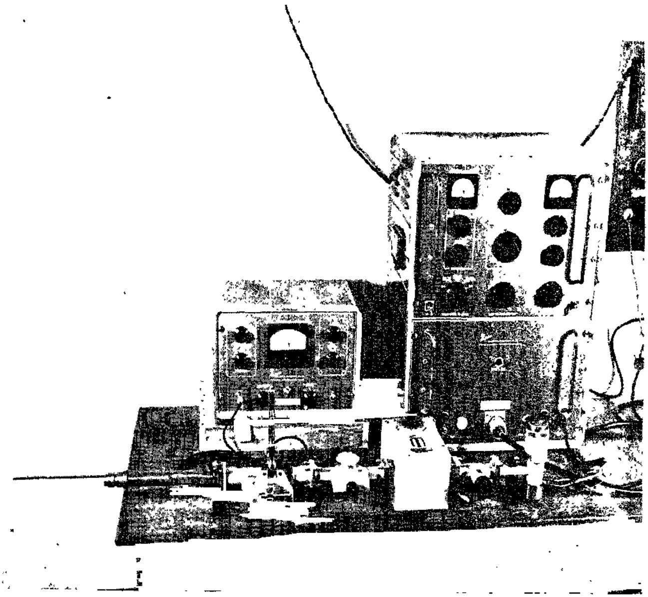


FIG. II

Experimental setup for determining the input impedance of dielectric aerials ceases to be unique. Consequently, the definition of a unique impedance is not possible in such cases. Schelkunoff's[2] impedance concept leads to the definition of the impedance matrix in cartesian coordinate system as follows:

$$[\eta] = \begin{vmatrix} E_x/H_x & E_x/H_y & -E_x/H_z \\ -E_y/H_x & E_y/H_y & E_y/H_z \\ E_z/H_x & -E_z/H_y & E_z/H_z \end{vmatrix} \qquad [1]$$

But this impedance is not a dyadic, as it does not transform to another coordinate system in the manner prescribed for the transformation of a dyadic. However, in many cases of guided transmission, the ratio of the electric field to the magnetic field which are perpendicular to each other and to the direction of transmission of power is unique and is called the wave impedance. This definition has enabled a better understanding of some of the electromagnetic phenomena such as the reflection of electromagnetic waves at a discontinuity in a waveguide, representation of complicated waveguide junctions by equivalent circuits, matching of waveguides, etc. and has helped to improve considerably

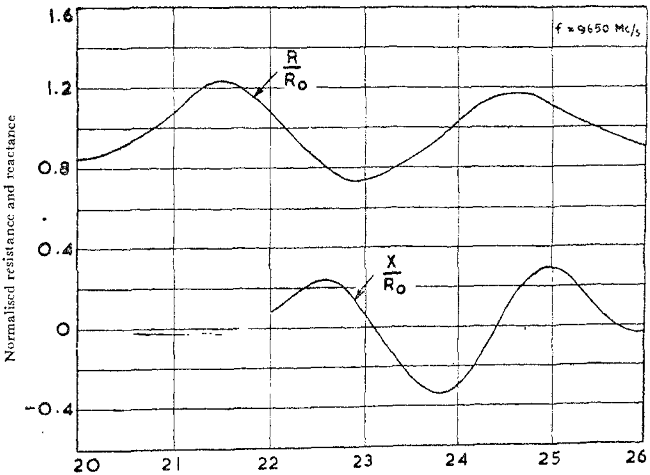


FIG. IV

Input impedance for Rod B_2 with free end covered with Disc No. 1

the measurement technique at microwave frequeneies. Impedance of a composite waveguide, such as cylindrical waveguide containing dielectric has been studied theoreticelly by Chatterjee, *et al.*[3-5]. A variational expression for the terminal admittance of a semi-infinite dielectric rod has been derived by Angulo and Chang.[6]

WAVE IMPEDANCE AND ORTHOGONALITY OF FIELDS

The wave impedance of a plane wave travelling along the z-direction in an unbounded medium, defined by the relation :

$$E_x/H_y = -E_y/H_x = \eta \qquad [2]$$

yields the following orthogonality relation,

$$E_x H_x + E_y H_y = 0 \qquad [3]$$

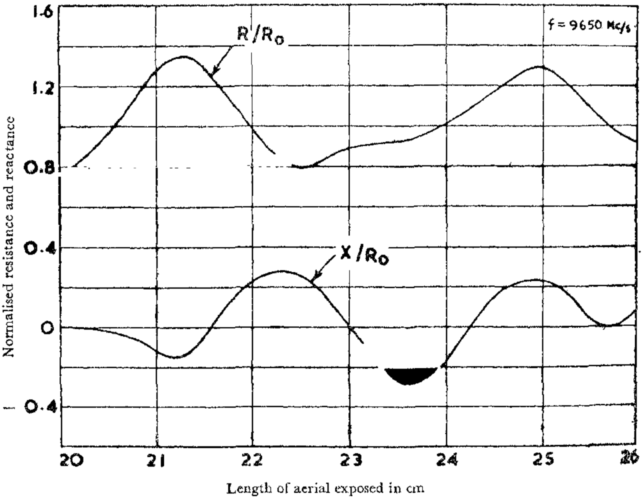


FIG. V

Input impedance for rod B_2 with free end covered with disc No. 2

between E and H. It is obvious that the existence of a unique wave impedance along a direction implies the orthogonality of electric and magnetic fields in the plane perpendicular to the direction and vice versa.

Wave Impedance of Guided Waves

For H and E waves, the impedances are respectively

$$\eta_H = j\omega\mu / \gamma \qquad [4]$$

and

$$\eta_E = \gamma / j\omega\epsilon \qquad [5]$$

where the electric and magnetic fields satisfy the orthogonality relation. But such a wave impedance does not necessarily exist in the case of multimode transmission. Consider a combination of two pure H waves denoted by subscripts 1 and 2 as follows. η_1 and η_2 represent the wave impedances for the two modes

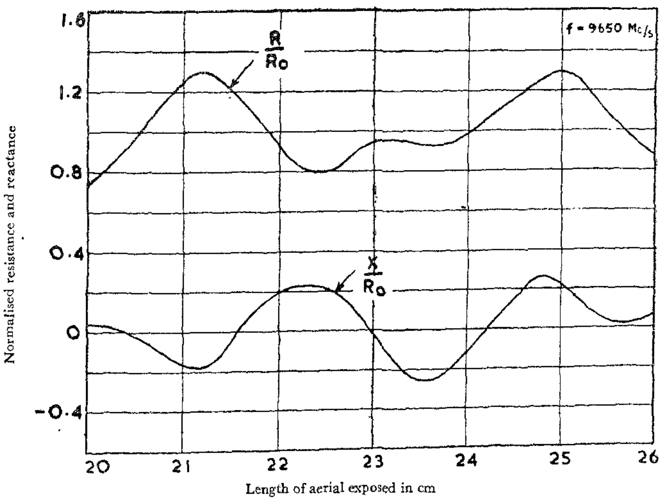


Fig VI

Input impedance for rod B_2 with free end covered with disc No. 3

$$\mathbf{H}_1 = H_{1x}\,\mathbf{i} + H_{1y}\,\mathbf{j} + H_{1z}\,\mathbf{k} \qquad \mathbf{E}_1 = \eta_1\, H_{1y}\,\mathbf{i} - \eta_1\, H_{1x}\,\mathbf{j}$$

$$\mathbf{H}_2 = H_{2x}\,\mathbf{i} + H_{2y}\,\mathbf{j} + \mathrm{H}_{2z}\,\mathbf{k} \qquad \mathbf{E}_2 = \eta_2\, H_{2y}\,\mathbf{i} - \eta_2\, H_{2x}\,\mathbf{j} \qquad [6]$$

For the combination,

$$\mathbf{H} = \mathbf{H}_1 + \mathbf{H}_2 \quad \text{and} \quad \mathbf{E} = \mathbf{E} + \mathbf{E}_2 \qquad [7]$$

The condition for the existence of a unique wave impedance is

$$(\eta_2 - \eta_1)\,(H_{2x} H_{1y} - H_{1x} H_{2y}) = 0 \qquad [8]$$

which is, in general, not satisfied. Hence, a unique wave impedance cannot be defined in the case of a combination of H waves. But if $\eta_1 = \eta_2$, the above relation is satisfied. A similar conclusion holds in the case of combination of pure E waves.

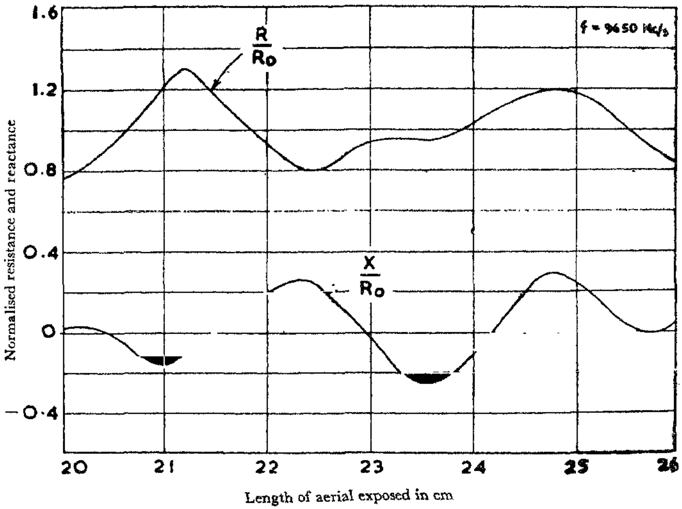


FIG. VII
Input impedance for rod B_2 with free end covered with disc No. 4

In the case of combination of a single H wave and a single E wave, the following expressions can be written

$$\mathbf{H}_h = H_{hx}\,\mathbf{i} + H_{hy}\,\mathbf{j} + H_{hz}\,\mathbf{k} \qquad \mathbf{E}_h = \eta_H H_{hy}\,\mathbf{i} - \eta_H H_{hx}\,\mathbf{j}$$

$$\mathbf{H}_e = H_{ex}\,\mathbf{i} + H_{ey}\,\mathbf{j} \qquad \mathbf{E}_e = \eta_e H_{ey}\,\mathbf{i} - \eta_e H_{ex}\,\mathbf{j} \qquad [9]$$

$$\mathbf{H} = \mathbf{H}_h + \mathbf{H}_e \quad \text{and} \quad \mathbf{E} = \mathbf{E}_h + \mathbf{E}_e \qquad [10]$$

The condition for the existence of a unique wave impedence is

$$(\eta_H - \eta_e)\,(H_{hx} H_{ey} - H_{ex} H_{hy}) = 0 \qquad [11]$$

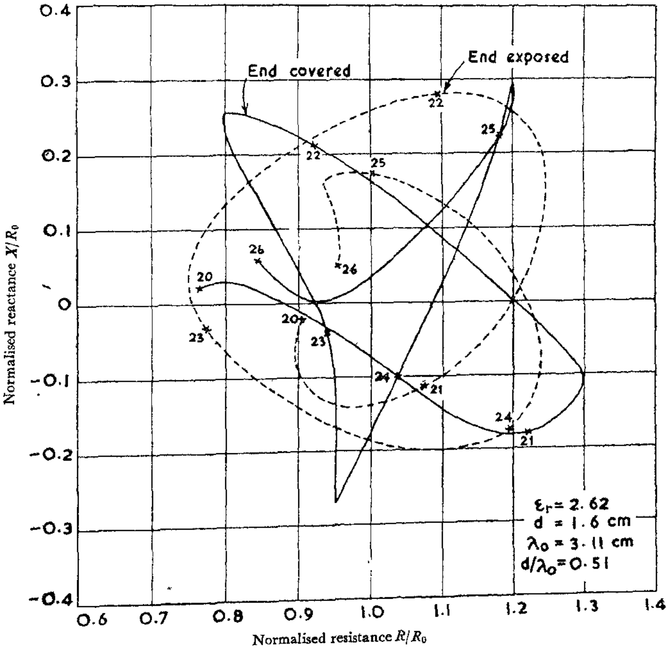


FIG. VIII

Impedance diagram for rod B_1 (numbers on curves indicate lengths of aerial exposed) which is not satisfied in general. Hence a unique wave impedance cannot be defined in general for a combination of H and E modes.

WAVE IMPEDANCE OF HE_{11} MODE OF PROPAGATION ALONG A DIELECTRIC ROD

For a unique wave impedance to exist, equation [11] must be satisfied *i.e.* $\eta_e = \eta_H$ or, $\beta = \omega\sqrt{(\mu\epsilon)}$. For the impedance to be unique everywhere, inside the dielectric rod we should have

$$\beta = \omega\sqrt{(\mu_1 \epsilon_1)} \qquad [12]$$

and for the impedance to be unique everywhere, outside the dielectric rod we should have

$$\beta = \omega\sqrt{(\mu_2 \epsilon_2)} \qquad [13]$$

As the wave travels in both the dielectric rod and the surrounding space, both the equations [12] and [13] should be satisfied. This is not possible for the

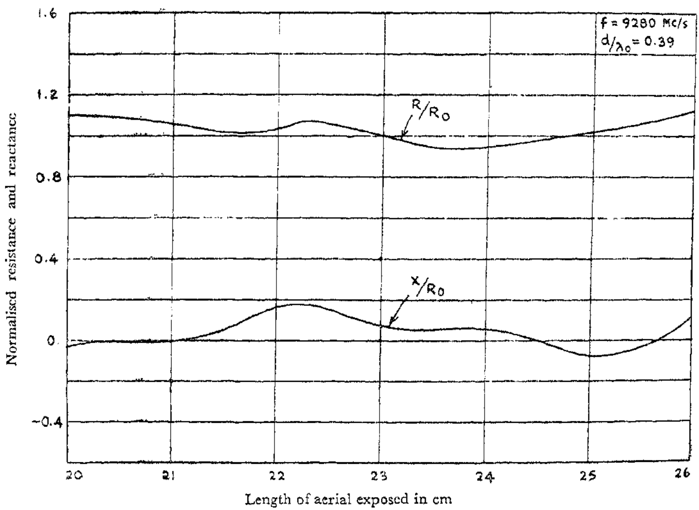


FIG. IX
Input impedance for rod B_1

case of a dielectric rod having different dielectric constant from the surrounding medium as $\mu_1 = \mu_2$. As the propagation constant β for the HE_{11} mode lies in between that for plane waves in the two media, neither of the two equations [12] and [13] is satisfied except in the limiting case of a rod of infinite diameter and of vanishingly small diameter respectively. Hence a unique wave impedance does not exist for the HE_{11} mode supported by a dielectric rod in the free space.

However, if (Appendix I) we make,

$$\omega^2 \mu_1 \epsilon_1 \cong \beta^2 \cong \omega^2 \mu_2 \epsilon_2$$

then for the two media, the following expressions hold

$$E_{\rho 1}/H_{\phi' 1} = -E_{\phi' 1}/H_{\rho 1} = j\omega\mu_1/\gamma = \eta_1 \qquad [14]$$

and

$$E_{\rho 2}/H_{\phi' 2} = -E_{\phi' 2}/H_{\rho 2} = j\omega\mu_2/\gamma = \eta_2 \qquad [15]$$

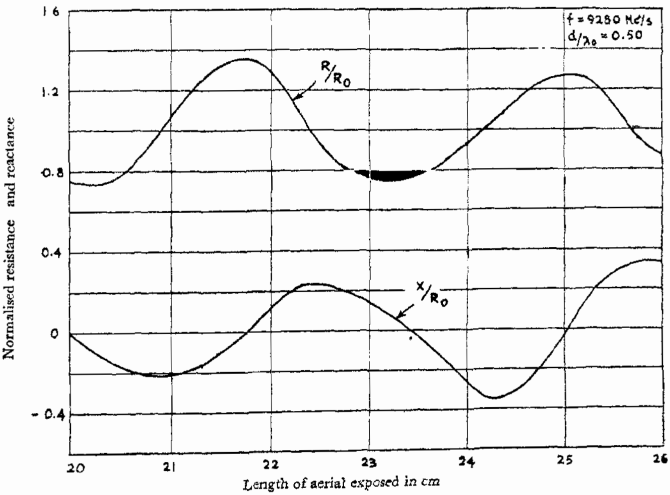


FIG. X

Input impedance for rod B_1

For $\mu_1 = \mu_2$, η_1 becomes equal to η_2 and a unique wave impedance can be defined as

$$\eta = (k/\beta)\, \eta_0 \qquad [16]$$

where, $\eta_0 = \sqrt{(\mu_2/\epsilon_2)}$, the free space impedance. This equation is valid only for rods of low ϵ.

EXPERIMENTAL

The experimental set up for determining the input impedance of the aerial is shown by the block schematic (Fig. I) and the photograph (Fig. II). The impedance is determined as usual from the measurement of v.s.w.r. and the position of first minimum from the load end and the use of Smith Calculator. It is to be noted that this input impedance refers to the input impedance of the system as a whole of the aerial fitted with the mode transformer. It has not been possible to evaluate the input impedance of the aerial without the mode transformer.

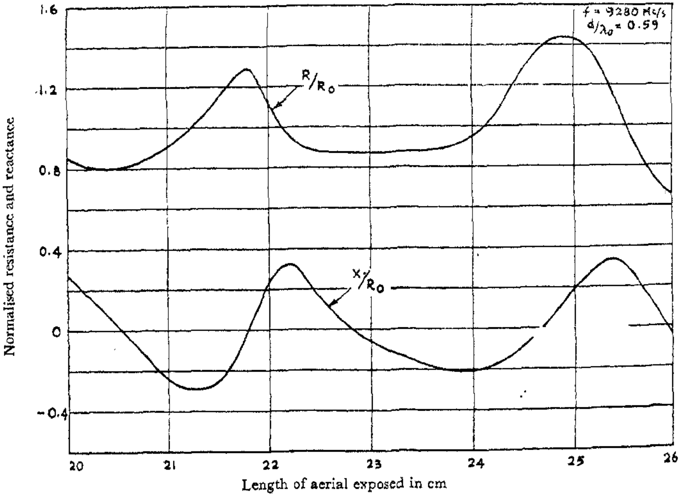


FIG. XI
Input impedance for rod B_3

The input impedance in the case of rod B_2 ($\epsilon = 2.62$, $2a = 1.60$ cm.) used as an aerial has been determined for different lengths (l) of the rod exposed and for different terminations (discs) of the free end. The variation of the normalised resistive and reactive components of the input impedance with length, when the free end of the rod is terminated with different discs is shown in Figures III to VII.

(i) *Variation with the exposed length of the rod*: The v s.w.r. is found to vary with the length of the rod and the termination of the free end. This implies that the power input to the aerial varies with the termination of the free end of the rod as well as the length exposed. However, the v.s.w.r. is small, the maximum observed being only 1.45 with different combinations of discs and lengths. This corresponds to a power reflection of 3.4% of the input power.

The v.s.w.r. input resistance and input reactance are found to vary periodically with l. The interval of length $\lambda_x/2$ at which $x = 0$ and the interval of length $\lambda_R/2$ at which $R/R_0 = 1$ obtained from Figs. III to VII are given in Table I.

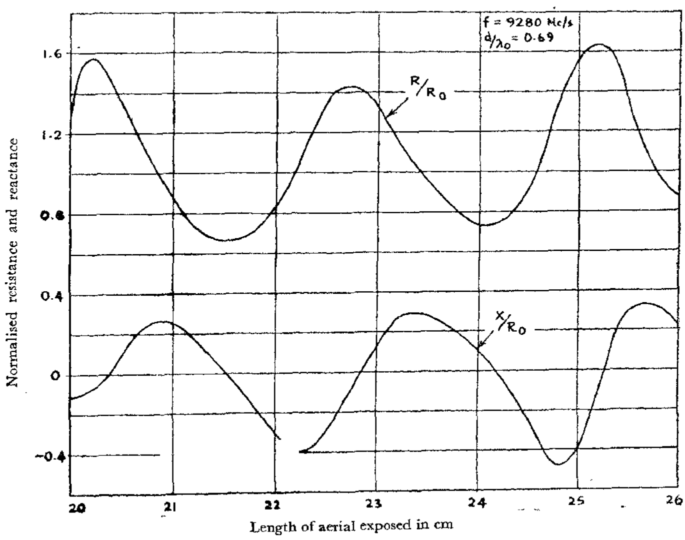


FIG XII
Input impedance for rod B_4

TABLE I

Effect of covering the free end of the rod on periodicity of impedance : Rod B_2 ; Frequency 9650 Mc/s

Termination	l for $X/R_0=0$ in cms.	$\lambda_x/2$ cms.	Mean $\lambda_x/2$ cms.	l (cms.) for $R/R_0=1$	$\lambda_R/2$ cms.	Mean $\lambda_R/2$ cms.	$\lambda_x/2+\lambda_R/2$ cms.
Exposed	21.30			20.80			
	22.92	1.62		22.14	1.34		
	24.39	1.47	1.55	23.58	1.44		
				25.02	1.44	1.41	2.96
Disc No. 1	21.10			20.78			
	23.11	1.41		22.15	1.37		
	24.44	1.33		23.90	1.75		
	25.70	1.26	1.33	25.42	1.52	1.55	2.88
Disc No. 2	21.55			20.60			
	23.00	1.45		21.98	1.38		
	24.25	1.25		23.96	1.98		
	25.70	1.45	1.38	25.70	1.74	1.70	3.08
Disc No. 3	20.40			20.56			
	21.56	1.16		21.86	1.30		
	22.96	1.40		24.05	2.19		
	24.24	1.38	1.31	25.66	1.61	1.70	3 01
Disc No. 4	20.36			20.62			
	21.46	1.10		21.85	1.23		
	22.92	1.46		23.90	2.05		
	24.18	1.26	1.27	25.50	1.60	1.63	2.90

Mean=2.97 cms.

TABLE II

Variation of periodicity of impedance with rod diameter

Material : Perspex ; Frequency : 9280 Mc/s

Rod No.	$2a$ cms	l (cms) for $X/R_0=0$	$\lambda_x/2$ (cms)	Mean $\lambda_x/2$ (*cms*)	l (cms) for $R/R_0=1$	$\lambda_R/2$ cms	Mean $\lambda_R/2$ *cms*	$\lambda_x/2+\lambda_R/2$ cms
B_1	1.27			Irregular				
B_2	1.60	20.00			20.90			
		21.74	1.74		22.40	1.50		
		23.46	1.72		24.18	1.78		
		25.04	1.58	1.68	25.68	1.50	1.59	3.27
B_3	1.90	20.54			21.26			
		21.80	1.26		22.16	0.90		
		22.81	1.01		24.14	1.98		
		24.66	1.85		25.52	1.38		
		25.96	1.30	1.35			1.42	2.77
B_4	2.22	20.20			20.89			
		21.52	1.32		22.16	1.27		
		22.85	1.33		23.43	1.27		
		24.17	1.32		24.55	1.12		
		25.25	1.08	1.26	25.76	1.21	1.22	2.48
B_5	2.54	22.33			22.69			
		23.49	1.16		23.88	1.19		
		24.62	1.13		25.12	1.24		
		25.72	1.10		26.11	0.99		
		26.93	1.21		27.20	1.09		
		27.97	1.13				1.13	2.26

For a simple transmission line with constant load, both these intervals would be equal to one-quarter of a wavelength. In the present case, the two are nearly equal when the free end of the rod is exposed. As the area of the free end increases $\lambda_x/2$ decreases and $\lambda_R/2$ increases but the sum nearly remains constant. The variation of the input impedance is presented in the complex plane in Fig. VIII.

(*ii*) *Variation with different diameters*: The variation of input impedance with length for rods of different diameters (Rode B_1 to B_5) has also been studied and the results are shown in Figs. IX to XIII. The free end of the rod was fully exposed in each case.

Higher values of v.s.w.r. were observed for thicker rods which resulted in greater mismatch. But the v.s.w.r. obtained in the case of rod B_5 was smaller than that for the rod B_4. The input resistance curves are periodic with respect to length but the curves are irregular for the rod B_1.

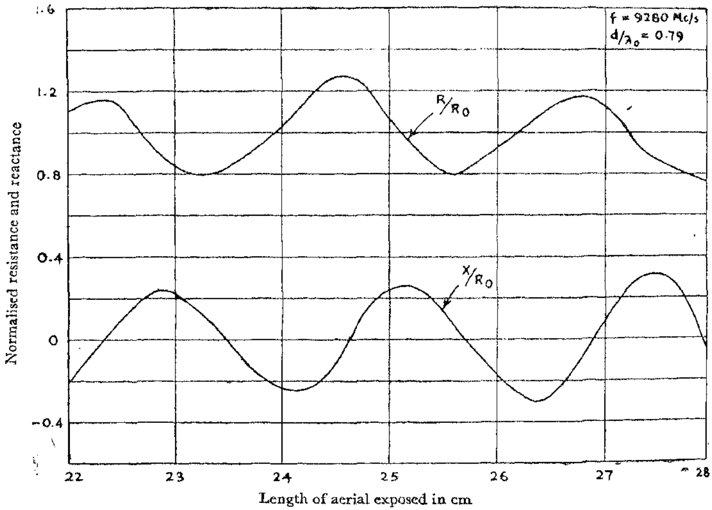


FIG. XIII
Input impedance for rod B_5

An analysis of the periodicity of impedance is shown in Table II. It is seen that $\lambda_X \simeq \lambda_R$ in each case. The interval of length $(\lambda_X + \lambda_R)/2$ at which the impedance repeats corresponds to a half wavelength in the case of an ideal transmission line with constant load. This value is found to decrease with the increase in the rod diameter.

(*iii*) *Variation of input impedance with length for rods of different dielectric constant*: The variation of input impedance with the length of the rod exposed for rods of teak wood ($\epsilon = 2.5$), perspex ($\epsilon = 2.62$) and bakelite ($\epsilon = 3.8$) has also been studied and the analysis of the periodicity of impedance with dielectric constant of the rod is shown in Table III. It is seen that $\lambda_X = \lambda_R$ for each rod. $(\lambda_X + \lambda_R)$ does not vary progressively with the dielectric constant of the rod.

TABLE III

Variation of periodicity of impedance with electric constant of the rod

Diameter : 1.60 cms. Frequeney : 9280 Mc/s

Rod No.	ϵ	l (cms.) for $X/R_0 = 0$	$\lambda_x/2$ cms.	Mean $\lambda_x/2$ cms.	l cms. for $R/R_0 = 1$	$\lambda_R/2$ cms.	Mean $\lambda_R/2$ cms.	$\lambda_x/2 + \lambda_R/2$ cms.
A*	2.05	21.74			21.30			
		22.96	1.22		22.45	1.15		
		23.93	0.97		23.33	0.88		
		24.97	1.04		24.47	1.14		
		25.92	0.95	1.05	25.50	1.03	1.05	2.10
B_2	2.29	20.00			20.90			
		21.74	1.74		22.40	1.50		
		23.46	1.72		24.18	1.78		
		25.04	1.58	1.68	25.68	1.50	1.59	3.27
C	3.80	21.00			20.50			
		22.06	1.06		21.78	1.28		
		23.38	1.32		22.90	1.12		
		24.40	1.02		23.82	0.92		
		25.42	1.02	1.10	24.70	0.88	1.05	2.15

*Tapered end was outside the mode transformer.

Comparison between perspex and bakelite rod shows that the interval of length at which the impedance repeats decreases with increase in the dielectric constant of the rod. The maximum v.s.w.r. observed with the teak wood, perspex and the bakelite rod were 1.08, 1.47 and 1.62 respectively showing that the mismatch between the antenna and the feed system increaces with the dielectric constant of the rod.

DISCUSSION

It may be pointed out that the experimental investigations reported in the present paper relates to the determination of impedance what the slotted section 'saw' looking into the mode transformer. The entire assembly of the dielectric rod, the mode transformer and the associated fittings has been regarded as the aerial. The mode transformer was present in all the experiments and the observed changes in the impedance were essentially due to change in the parameters of the dielectric rod. Impedance was specified at the mode transformer. The experiments show that the input impedance is a function of the length, diameter and dielectric constant of the rod as well as the termination of the free end.

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