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A modified area–product method for the design of inductors and transformers

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Abstract

A design method which is an improvement over the area–product method (Ramanarayanan, V., *Power electronics class notes*, 1996) for the design of inductors and transformers working at 50 Hz is proposed in this work. An iterative method is developed over the existing area–product method to achieve an optimal core for a given set of input specifications. A large database of cores and user-interactive software is developed. The contributions made in this work are: (i) a design method for inductors which achieves substantial (30%) savings in the material when compared to the conventional method, (ii) a similar design method for transformers that add more features to the conventional method, and (iii) a method to estimate the parasitic leakage inductance (error < 40%) in transformers based on stored energy principles.

1. Introduction

Magnetic components such as transformers and inductors are electromagnetic circuit elements. Transformers transform electrical energy through an intermediate magnetic link and inductors store energy in a magnetic field. In power electronics systems (PES), these elements constitute a major part. In majority of the systems, inductors and transformers contribute to as much as 30% of the overall cost. The design of these elements has been a challenging and demanding task with the increasing complexity of the PES. The designer of modern magnetics must possess a wealth of information in the form of an extensive database. Both inductors and transformers are not available in wide range of ratings but are usually custom-designed and constructed. In this situation, the equipment designer or the user must be knowledgeable about the design.

There are two methods employed for the design of magnetic elements, namely, area–product method[1] and core-geometry method.[2] The area–product method is more popular in its simplicity. This method gives a feasible design but does not guarantee an optimal design. Most industries maintain a catalogue of past designs which are successfully made and new designs evolved based on the past experience. Such designs use the VA rating and Volt per turn for every core being used. Such a procedure also provides a feasible design but does not guarantee an optimal design. The limitations of the previous methods[1, 2] being the motivating factor, a new *iterative* method is proposed as an improvement over the conventional area–product method.

The objectives of this work are:

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(1) To improve the area–product method for the design of transformers and inductors at power frequency,

(2) To incorporate in the new method an empirical correction for fringing effect in the case of inductors and parasitic leakage inductance in the case of transformers, respectively,

(3) To develop a user-interactive software for the conventional and modified area–product methods and verify experimentally.

The iterative method developed has several advantages. They are: (a) selection of the right magnetics for inductors and transformers, (b) multiple design options for a given specification and (c) estimating leakage inductance of transformers with reasonable accuracies.

2.1. *Area–product method*

The area–product is defined as the product of core area (A_c) and the window area (A_w) for a core. The basic design requirements involved in magnetics are the selection of an appropriate core which can carry the flux without saturating and winding, and the required amount of current. The various parameters involved in the design of magnetics are divided into three categories: design requirement, material constraints and manufacturing constraints. The design requirements for inductors are the inductance (L), peak current (I_p) and RMS current (I_{rms}). Similarly, the design requirements for transformers are the Volt Ampere (VA) and the frequency of operation (f). The material constraints include current density (J) for winding conductors and flux density (B_m) for the core to be used. The manufacturing constraints are the window utilisation factor (K_w) and core-stacking factor (K_s). The relationships between these design parameters and the area–product (A_cA_w) for both inductors and transformers are given below, respectively.

$$A_c A_w = LI_pI_{rms}/K_w\, K_s\, B_m\, J, \tag{1}$$

$$A_c A_w = \mathrm{VA}/2.22 f B_m\, K_w\, K_s\, J. \tag{2}$$

The above equations can be interpreted as the relationship between the energy-handling capacity (LI_pI_{rms}/VA) to the size of the core (A_cA_w), the material constraints (B_m, J) and the utilisation factors (K_w, K_s).

In this method, given the design requirement, one can select the appropriate material and manufacturing constraints using the past experience and can calculate the area–product to choose the core and winding to complete a feasible design.

2.2. *Modified area–product (iterative) method*

This method is a modification proposed over the area–product method. In this section, we discuss the key parameters involved in the proposed method and the need for the proposed method. A design algorithm for the same is proposed and the results are presented subsequently.

Considering the design equations (1) and (2), apart from the design requirements, one needs to initialize the parameters B_m, J, K_w, K_s to start the design process. The maximum unsaturated flux density (B_m) is around 1 to 1.6 web/m^2 for iron cores and its exact value for a particular design depends on the maximum allowable temperature in the core. Also, the maxi-

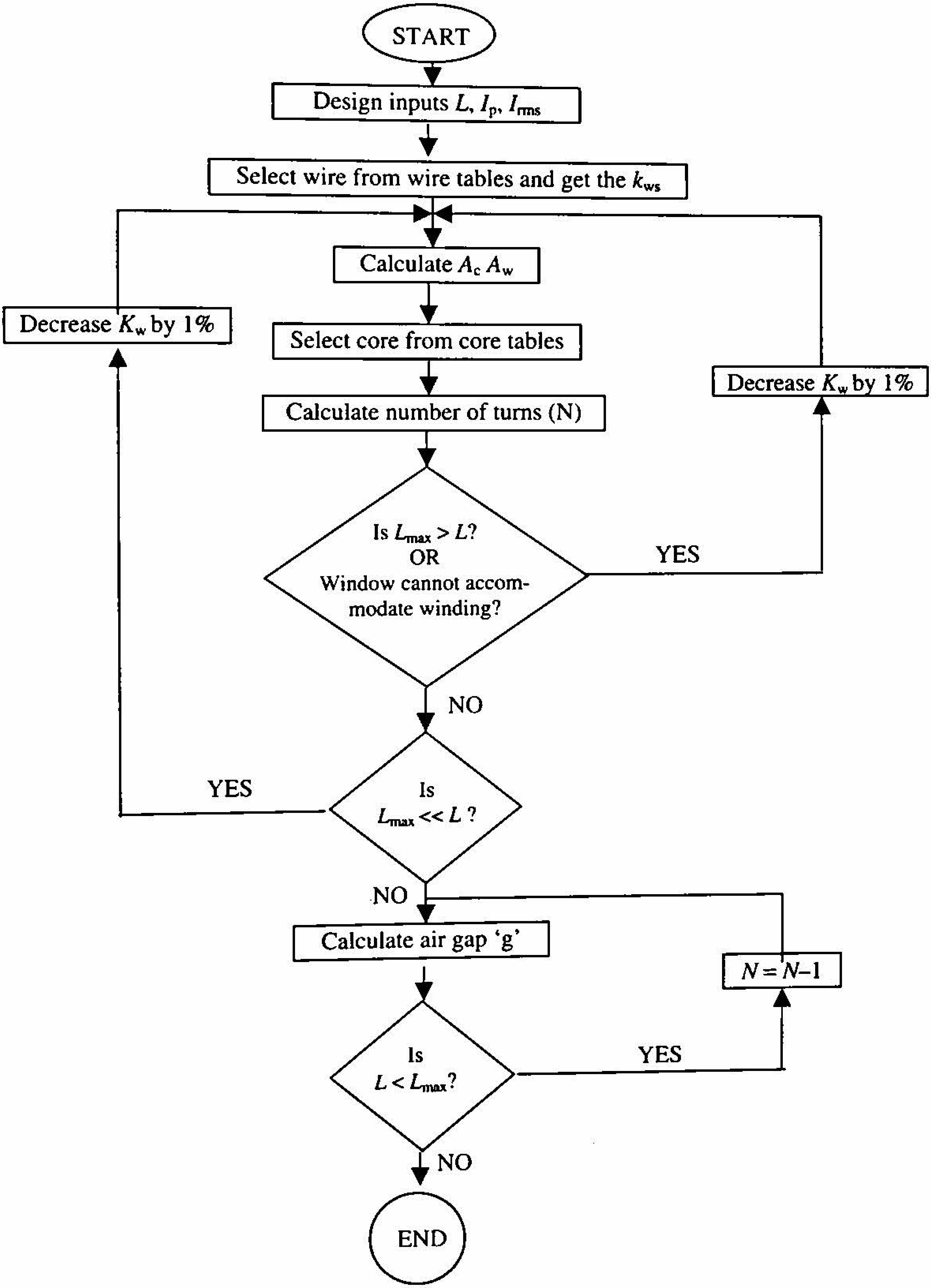

FIG. 1. Iterative algorithm for the design of inductors.

Table I
Results for (a) 2.5 mH inductor with $I_{rms} = 6$ A and $I_p = 12$ A and (b) 5 mH inductor with $I_{rms} = 20$ A and $I_p = 50$ A

Parameters	Area–product method		Iterative method	
	(a)	(b)	(a)	(b)
Inductance (mH)	3.4	7.06	2.59	5.22
Copper (kg)	0.17	1.492	0.15	1.36
Iron (kg)	0.54	6.92	0.49	4.7

mum allowable current density (J) for copper conductors is in the range of 2 to 5 A/mm^2, which again depends on the allowable temperature in the winding. The value of core stacking factor (K_s) depends on the type of the core being used and it is typically in the range of 0.9 to 0.95 for iron cores. The only parameter which is not in the control of the user and lies within a wide range of 0.3 to 0.6 is K_w. The optimal core can be obtained by selecting the right value of K_w, which in normal cases is difficult to predict for a set of given design inputs. In order to find an exact value of K_w for a particular design, one should consider several designs and decide on the one that is optimal. To get the exact value of K_w, an iterative method is proposed for the design procedure.

To carry out this method, one should have a complete database of cores and bobbins. A user-interactive program is developed to carry out this method iteratively and efficiently by creating a database. The method is different for inductors and transformers because of the difference in their number of windings.

In the case of inductors, an estimated value of K_w (K_{ws}) is taken from the wire table, which picks an initial core, and iterations are carried out to obtain an exact core ultimately. The estimated value of K_{ws} for each wire is given by

$$K_{ws} = a_w / D_o^2, \tag{3}$$

where a_w is the cross-sectional area of the bare wire and D_o the overall diameter of the wire.

After selecting the value of K_{ws}, using (1) we can calculate the area–product required for the design and also select a core from the core tables. For a selected core, the number of turns required is calculated using (4) and subsequently the maximum inductance achievable is calculated using (5):

$$N = k_w A_w / a_w, \tag{4}$$

$$L_{max} = N A_c B_m / I_p. \tag{5}$$

From the values of N and L_{max}, iterations are carried out to adjust K_w that, in turn, selects a final core to be used for the specifications given. After selecting the right core, more iterations are carried out to adjust air gap and the number of turns to achieve the right value of inductance. The algorithm based on this method which is self-explanatory is illustrated in Fig. 1. This design method is more advantageous since (i) it reduces both the iron and copper weights, (ii) calculates the air gap more exactly by considering the fringing effect[3] and also (iii) adjusts the number of turns of the inductor to get the exact value of inductance. The results for the inductors based on area–product and iterative methods are presented in Table I.

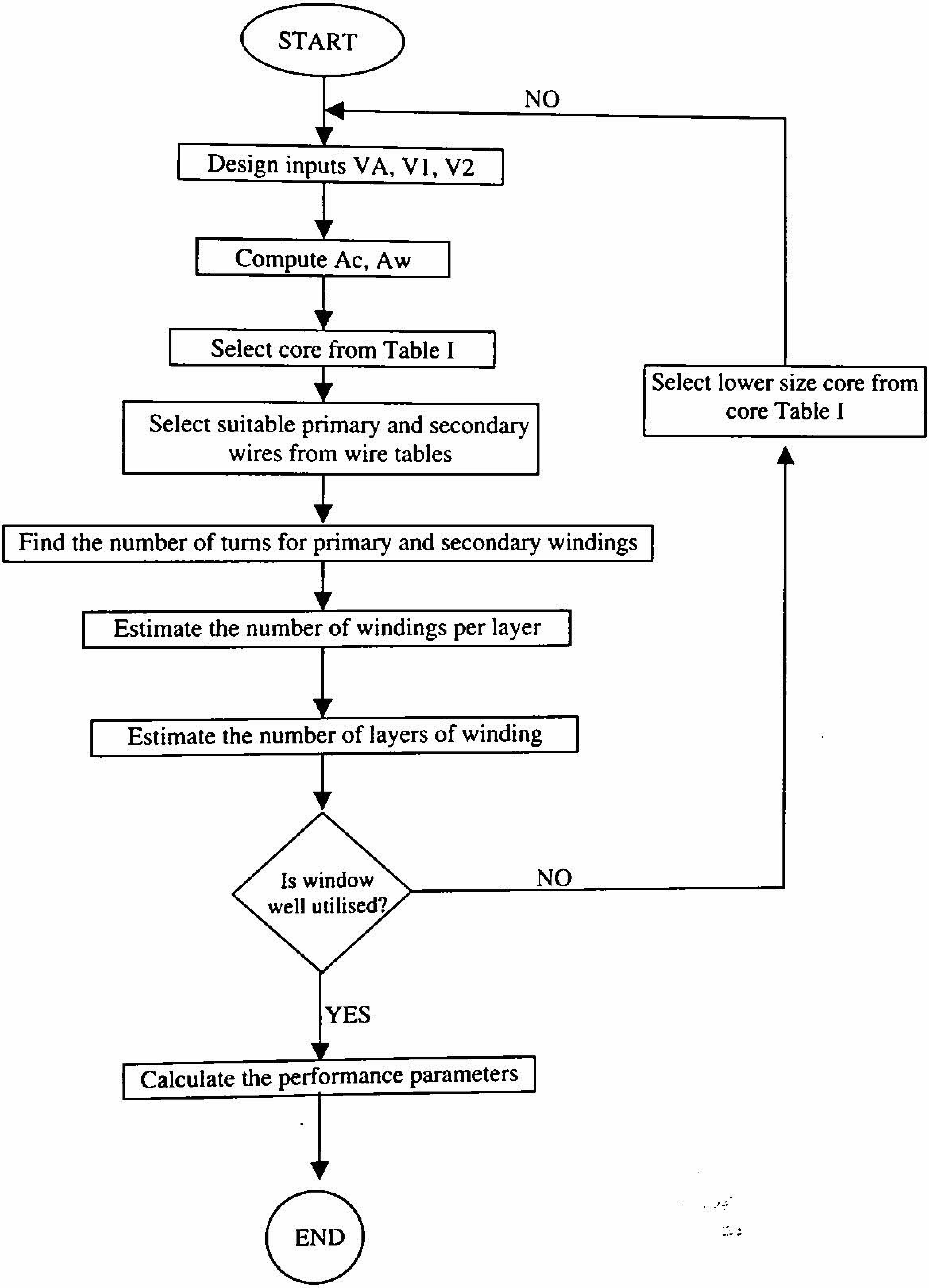

FIG. 2. Iterative algorithm for the design of transformers.

Table II
Results for (a) 240 VA single secondary transformer, (b) 342 VA multiple secondary transformer and (c) 500 VA auto transformer

Parameters	Area product method			Iterative method		
	(a)	(b)	(c)	(a)	(b)	(c)
Copper (kg)	0.72	1.18	0.75	1.02	1.74	0.96
Iron (kg)	4.65	4.7	4.9	2.75	2.81	3.1
Total weight (kg)	5.37	5.88	5.65	3.75	4.43	4.4

In the case of transformers, two tables of cores are made, one each in the ascending order of area–products and core sizes. To start the algorithm, a minimum value of K_w is chosen and $A_c A_w$ is calculated using (2). Based on this area–product, a core is selected from the first table and by making use of the second table a more exact core is selected for the given specifications. The algorithm based on this method which is self-explanatory is given in Fig. 2 and the practical results are presented in Table II. The design method achieves a considerable percentage decrease in iron weight but has a small percentage increase in copper weight. Ultimately, an overall weight reduction in the design process is obtained. This method is advantageous in cases where the iron cost is comparable to the copper cost (which is the case with higher-grade iron materials like cold-rolled grain-oriented (CRGO) steel.

2.3. *Estimation of leakage inductance in transformers*

A modelling technique for E-I cores is derived for the estimation of the leakage inductance of transformers. This method is based on the stored energy principles.[4] The leakage field distribution is calculated by deriving the equations in the winding space of the transformer. The field equation which is used to estimate the leakage inductance is given below:

$$L_l = \frac{\mu_0}{I^2} \iiint H(\rho)^2 \mathrm{d}v, \tag{6}$$

where $H(\rho)$ is the field intensity inside the winding volume at a distance ρ from the core, dv the volume element and I the current in the reference winding.

This method is applied to the transformers built with E-I type cores. On evaluating the integral (6) for a two-winding case, (7) is obtained.

$$L_1 = 2\,\mu_0 N_1^2/3b\,(h_1(\mathrm{CW} + \mathrm{CB} + 3h_1) + h_2(\mathrm{CW} + \mathrm{CB} + 4\,h_1 - 8\,h_2)), \tag{7}$$

where N_1 is the number of turns in the primary of the transformer, b the height of the windings, h_1 and h_2 are the widths of the primary and secondary windings, respectively, and CW and CB the core width and core build dimensions of the core.

This leakage calculation feature is incorporated in the design software and applied on the transformers designed (Table III). From the results, we can observe that the values of leakage inductance calculated by the designed algorithm (Fig. 2) are nearer to the actual values obtained by the short-circuit test on the transformers. The errors are within 50%, which is a reasonable prediction, since the leakage inductance in a transformer is a second-order effect (order of magnitude estimates are acceptable).

Table III

A comparison of calculated and actual values of leakage inductance in transformers

Type	Transformer rating (VA)	Calculated value (mH)	Actual value (mH)	% Error
Single secondary transformers	10	44.83	60.0	–28
	240 (Sample 1)	1.67	2.71	–38.4
	240 (Sample 2)	5.8	8.17	–29.0
	1000	1.02	1.25	–18.4
Multiple secondary transformers	18.9	32.29	52.0	–37.9
	342 (Sample 1)	4.84	6.54	–26.0
	342 (Sample 2)	13.92	14.23	–2.2
	2560	0.9	1.88	–52.1
Auto transformers	50	10.98	15.3	–28.3
	500 (Sample 1)	1.75	2.34	–25.4
	500 (Sample 2)	4.61	5.57	–17.3
	900	1.41	1.86	–24.2

3. Conclusions

This work presents an iterative algorithm for the design of transformers and inductors. This iterative algorithm is an improvement over the conventional area–product method in achieving an optimal design. The method also includes the estimation of leakage inductance of transformers. User-interactive software is developed for both the conventional and iterative methods. The design specifications obtained from the software are tested on practical inductors and transformers. These results indicate an overall achievement of nearly 20% savings in weight and commensurate savings in cost. The design methods also cover important variations such as multiple winding transformers.

Another important aspect in the design methodology proposed is an attempt to calculate parasitic inductance associated with transformers. Appropriate modelling technique based on the leakage energy has been effectively used to predict the leakage inductance of various types of transformers. The accuracy in this prediction (approx. 30%) is a reasonable prediction considering the leakage as parasitic.

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