Book Review

Symmetry in mechanics

by S. F. Singer, Eirkhauser Verlag AG, Klosterberg 23, CH-4010 Easel, Switzerland, 2001, pp. 193, sFr. 58.

This book provides an overview of symmetry, symplectic concepts and symplectic reduction in mechanics. These concepts have gained much importance in recent years. Therefore, a book devoted t9 such topics is very timely. Along the way, the author also provides an introduction to differential geometry and Lie groups. As promised by the subtitle of the book, the treatment is gentle and treats mechanics using modern concepts. However, this approach is not unique to this book. The reader may wish to consult Rasband (N. Rasband, Dynamics, Wiley, 1984) for a similar approach to mechanics though Rasband does not treat topics like symplectic reduction.

The book starts with a brief preliminary chapter on notations and basic mathematical concepts. In the first chapter, there is a detailed treatment of the two-body problem from celestial mechanics. The primary aim of this chapter is to derive Kepler's laws from Newton's laws of motion and his universal law of gravitation. This is achieved in a simple manner using centre of mass coordinate system and conservation of angular momentum. The author is careful to use mathematically correct notation throughout. The concept of duality between quantities such as velocity and momentum, force and displacement is explained along with the mathematical concept of dual vector spaces.

The second chapter introduces the important concept of symplectic manifold. The approach is yery intuitive in that the author starts with phase spaces of simple well-known mechanical systems and shows how the symplectic 2-form comes into picture in specific cases. Therefore the reader is introduced to the idea of symplectic 2-form through concrete examples which one can work through before encountering the abstract definition. Along the way, other basic building blocks of symplectic theory of mechanics-covectors, vector fields, differential forms, etc.- are introduced. The chapter ends with a simple example of pulling back forms. Throughout this chapter and other chapters, there are numerous exercises which invite the reader to work out additional examples and complete derivations. Solutions to selected exercises are also given at the end of the book.

Chapter 3 is devoted to a brief but standard treatment of differential geometry. The author gives both informal and formal definitions of key concepts like manifolds, differentiable functions, etc. Again there are many worked-out examples. Chapter 4 intr-oduces the concepts of Hamiltonian function and Hamiltonian vector field. The inner product of the Hamiltonian vector field and the symplectic 2-form is shown to be equivalent to Hamilton's equations of motion. Using this formalism, simple examples such as particles on the line and magnetism in three spaces are studied. Next it is shown that Hamiltonian flows preserve both the Hamiltonian function and the symplectic form. Using level sets of the Hamiltonian function, dynamics of different systems are predicted.

Chapt~rs 5, 6 and 7 form the core of the book. Chapter 5 starts with definition of groups. Next, matrix Lie groups are introduced. Standard examples like SO(2) and SO(3) are considered. Group actions, orbits of group actions and quotient spaces are other important concepts introduced in this chapter. Equally important, the connection between group actions and symmetry of physical systems is explained. Chapter 6 is devoted to Lie algebras, in particular, matrix Lie algebras. One-parameter subgroups are introduced and explained through examples from 50(3), SI x SI, etc. This is then used to define the vector field associated with the Lie algebra. In both the chapters, only key concepts are described in detail, which is fine for an introductory book. Chapter 7 explains in detail the momentum map and its properties. Further, it is shown that linear and angular momentum, and conserved quantities, which play an important role in physics, are examples of momentum maps. The chapter ends with an advanced (optional) topic wherein the momentum maps associated with group actions on mechanical phase spaces are derived using more sophisticated differential geometric tools.

The final chapter returns to the analysis of the two-body problem fIrst considered in Chapter I. The treatment in this chapter is from a modern perspective. In particular, the important technique of symplectic reduction plays a key role in the analysis. Using the momentum map associated with the action of the 3-d group of translations, the first symplectic reduction is performed yielding a reduced Hamiltonian. This reduction is nothing but the passage to centre of mass coordinates. The second and final symplectic reduction is carried out using the momentum map associated with the rotation action of S0(3). This reduction is mathematically equivalent to using angular momentum conservation.

In summary, the book fulfills more than amply its objective of introducing symmetry in mechanics using modem mathematical language and notation. It will be useful to both mathematics and physics students.

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