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Mathematical programming in electric power capacity investment planning

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Abstract

Mathematical programming has long been an important tool for electric utility planners. This paper presents a survey of state-of-the-art mathematical programming methods as applied to electric power capacity expansion planning. The focus is on modelling features which make it possible to investigate important power system issues such as reliability, uncertainty and environmental impacts which linear programming models cannot address without considerable simplification. Solution methodologies are also described.

Keywords: Electricity utilities, mathematical programming, linear programming, power systems, power generation.

1. Introduction

The use of mathematical programming for planning capacity expansion for electric utilities has a long history[1,2], and has remained one of the most important and active applications of management science/operations research. Since the earliest days, the basic problem has remained fundamentally the same: How to select optimally power plant capacities and investment times so that total capital and operating costs are minimized, while meeting customer demand and physical constraints over a given planning horizon. What has changed over the intervening years are the issues which have been considered and the methods used to solve the problem. Linear programming models have given way to models requiring state-of-the-art techniques in formulation and solution. Algorithmic and computational advances in solving complex, large-scale mathematical programs have been paralleled by increasing awareness of important issues in power system planning beyond minimizing cost. Principal among these issues, in order of their historical appearance in the literature, are reliability of supply, uncertainty in demand and most recently, consideration of environmental consequences. Up to the early 1970s, with demand growing rapidly and taxing the ability of electric power to meet it, reliability was a major concern of power systems planners. As price shocks hit and demand growth became more unpredictable planners needed ways of dealing with uncertainty in the planning process. With growing awareness of the role of power generation in environmental issues such as acid rain, global warming and resource depletion, attention has turned to incorporating environmental costs and demand-side management programs into planning models.

It is important to emphasize that the mathematical program is not the planning process itself, but rather is one of the many inputs that are used by decision makers to develop rational plans which meet the objectives of the stakeholders. In the words of Anderson, 'the search for an investment program that satisfies engineering and economic criteria is an iterative, multidisciplinary process.' While capturing many of the important economic and physical processes involved in electric power systems, mathematical programming models fail to consider many important aspects of the planning process, such as environmental impact reporting, pricing and other regulatory and financial characteristics. The methodology also cannot address one of the most important aspects of electric power systems planning: politics.

Math programming models have been widely used for a number of years by just about every major player in electric power investment. Researchers at the de Electricité France, realizing that 'electrical investment serves to create not a unique product, the kilowatt hour, but a group of related products'[1], discovered that linear programming techniques would be ideal to balance the cost of investment in electric power with the benefits that electricity provides. While researchers envisioned solving large, realistic power systems models, their size was limited by the ability to solve them. During the 1970s, as computers became faster and more accessible, expansion planning models grew larger and more sophisticated. Large-scale linear programming models were developed by Brookhaven National Laboratory which integrated power planning into the larger context of planning national energy strategies, and considered all phases of energy consumption[3]. The World Bank has also developed power system planning models for newly industrializing and developing countries[4]. An early model developed for the International Energy Agency, WASP, is still widely used in energy planning in Europe and many countries in the developing world as well[5].

This paper will attempt to trace the evolution of the state-of-the-art in mathematical programming approaches to electric power generation investment planning. This evolution is based on investigation of issues which have had the widest treatment in the literature; thus, approaches which address very important (but somewhat system-specific) considerations such as fuel transportation, nonutility generation, demand-side management, privatization and interconnection of decentralized systems are not considered in detail. The important areas of transmission and distribution planning and power dispatching are also not considered. The scope of this paper is confined to the electric power sector; studies which incorporate power sector planning as part of a broader economic framework are not considered in any detail. While not meant to be a comprehensive survey of the literature in this field, papers have been selected based on the diversity of the approaches taken. The paper is organized in the following manner. Section 2 describes the basic principles involved in capacity expansion and discusses the linear formulation and its limitations. Section 3 discusses some recent nonlinear programming approaches to the problem, including integer, stochastic and multiobjective programming, to incorporate reliability, uncertainty and environmental concerns into capacity planning. We present some concluding remarks in Section 4.

2. Electric capacity expansion modelling

A simple statement of the electric power capacity expansion problem is to find the type, size and introduction times of electric power generation resources so that demand for electricity is met over a specified planning horizon at the least cost. As first noted by Masse and Gibrat[1], this problem lends itself quite nicely to solution via techniques of mathematical programming. Before a model is constructed, a number of important questions need to be addressed, such as what the length of the planning horizon is, how demand is to be specified and what type of generation technology is available. Anderson[2] provides an excellent survey of the state-of-the-art in modelling up to the early 1970s. A brief description of the basics of capacity expansion models is now provided.

Two types of decision variables are present: *investment* variables, which determine the capacity of a particular generation type to be installed in each time period, and *operational* variables, which specify the level of utilization of each technology. In linear models investment variables will almost always refer to the total capacity required of a particular technology type, rather than the size or number of a particular plant. Generation type is thus classified according to the type of fuel used. More detailed specification of power plants is considered in nonlinear programming models.

Demand for electricity varies over time, and can be described graphically as a curve which provides the instantaneous power required at each point over the given time period. Figure 1a depicts a typical daily variation; a typical yearly curve might resemble 365 such curves strung together. The load duration curve (Fig. 1b) is obtained by rearranging the instantaneous demands in decreasing order. For use in mathematical programming models, it is convenient to approximate the load curve into a number of discrete segments, or blocks, each with a corresponding power requirement and duration. This discretization can be either with respect to the horizontal or vertical axis; Figures 2a and b show both types of discretization.

At the very minimum, the system must satisfy the following constraints in each time period in the planning horizon:

- generation must be less than the available capacity for each technology,

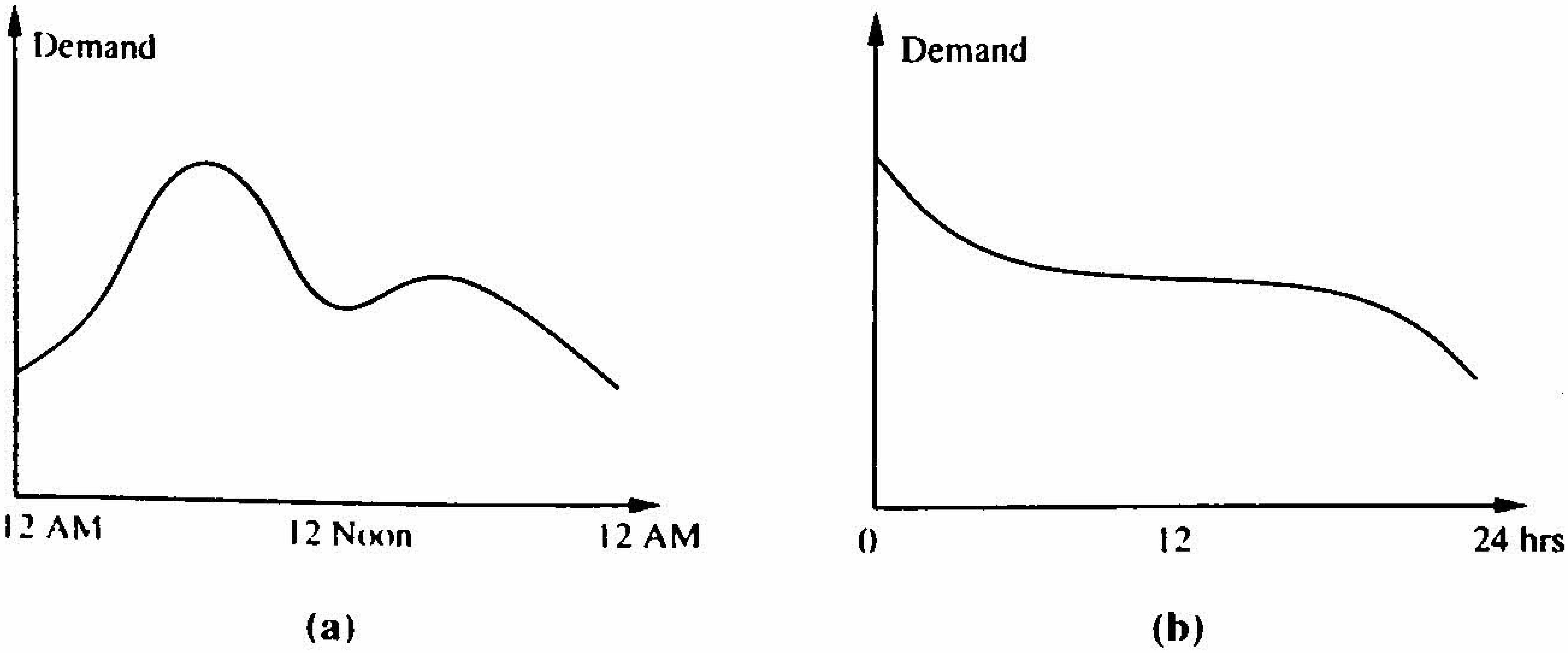

FIG. 1. Daily (a) and cumulative (b) load duration curves.

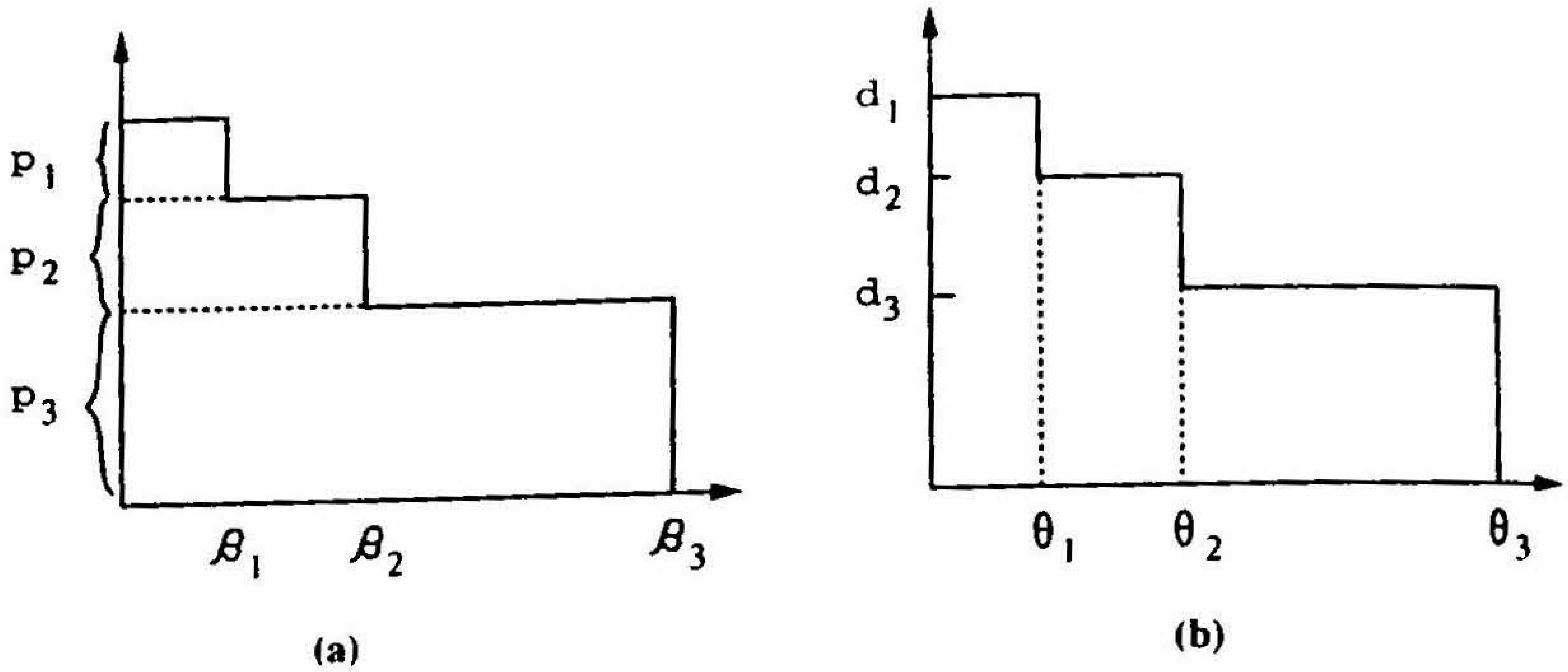

FIG. 2. Horizontally (a) and vertically (b) discretized load duration curves.

- generation must be adequate to meet projected demand in each of the N demand blocks.

Other constraints that are normally considered in order to reflect the reality of the system under study are:

- upper and lower bounds on capacity additions,
- requiring sufficient reserve capacity above peak demand,
- budgetary restrictions within each time period,
- seasonal hydro energy limitations.

A typical linear programming model for determining the optimal capacity expansion minimizing the net present value of capital and operating costs can now be presented*:

[LP]
$$\underset{x,y}{\text{Minimize}} \quad \sum_{t=1}^{T} \left(\sum_{j\in J} c_{jt} x_{jt} + \sum_{j\in J} \sum_{n=1}^{N} f_{jt} \theta_n y_{jnt} \right) \tag{1}$$

subject to

$$\bar{a}_j \bar{x}_j + \sum_{\tau=1}^{t} a_{j\tau} x_{j\tau} - y_{jnt} \geq 0 \quad \text{for all } j, n, t, \tag{2}$$

$$\sum_{j\in J} y_{jnt} \geq d_{nt} \quad \text{for all } n, t, \tag{3}$$

*Whenever possible, notation from the original literature cited has been maintained. In some cases the notation differs slightly from the original citation in order to make our notation consistent.

$$\sum_{j\in J}\bar{a}_j\bar{x}_j + \sum_{\tau=1}^{t}\sum_{j\in J}a_{j\tau}x_{j\tau} \geq (1+m)d_{1t} \qquad \text{for all } t, \tag{4}$$

$$x_{jt},\ y_{jnt} \geq 0, \tag{5}$$

where x are the investment variables and y, the operating variables. Constraint (2) requires generation to be no greater than existing capacity, (3) requires generation to be at least equal to demand and (4) requires a sufficient reserve capacity, m, to be on line in every period. Constraint (5) requires generation and expansion to be nonnegative. The set J indexes generation resources while $\bar{x}_j$ denotes capacity of type j which is currently online and N is the set of blocks the load duration curve has been approximated by. a_{jt} is an availability factor which accounts for scheduled maintenance and derating of performance over time. d_{1t} is the peak demand required in period t. If the load duration curve is discretized in a horizontal rather than a vertical fashion, constraint (2) is expressed as

$$\bar{a}_j\bar{x}_j + \sum_{\tau=1}^{t}a_{j\tau}x_{j\tau} - \sum_{n=1}^{N}y_{jnt} \geq 0. \tag{2a}$$

In addition, the vertical load duration parameter θ_n is replaced by horizontal load duration parameter β_n in the objective function and vertical segment power demands d_{nt} are replaced by horizontal segment demands p_{nt} in constraint (4). With this realization, the number of constraints (2) is reduced by a factor of N.

Note that while the investment variables are continuous, generation technologies are commercially available only in certain sizes. It may not be possible to break down the aggregate power requirement into a set of commercially available units optimally. Thus, the linear programming formulation is not very useful for actual project selection. What is of greatest importance here is the generation mix; that is, the total capacity of coal, nuclear, hydro, etc., power that is required in each time period.

Anderson[2] extends his treatment of the basic linear programming model, discussing marginal analysis (starting with a reference solution, then seeking to improve it by making marginal substitutions), simulation models (finding the least-cost operating schedule for a given system/demand configuration) and global models (selecting the optimal investment plan from all the available options). Anderson also discusses several possible extensions to the basic linear model. These extensions include finding optimal replacement times for old power stations, rudimentary inclusion of interregional transmission and treatment of pumped storage hydro plants. The method by which the model is extended is to introduce new decision variables and constraints necessary to depict the system accurately. The extensions do not alter the LP structure of the model.

A large-scale linear-programming-based electricity supply model for India was developed by Sengupta[6]. This model is a part of a larger study on commercial energy policy. The model follows closely that described by Anderson[2].

3. Recent modelling approaches

3.1. *Nonlinear programming*

At the time Anderson did his survey, the growing size of capacity expansion models was rapidly outstripping the ability to solve them. Early attempts at nonlinear programming approaches were motivated in part by a desire to increase computational efficiency by reducing the number of constraints. The introduction of *merit order operation* facilitated this approach. In merit order operation, the order in which plants are brought on line to meet the required load is predetermined by sequencing the plants in order of their operating costs. This implicitly allows the operating variables to be dropped from the formulation, although now the load curve must be integrated directly to obtain the energy produced, and thus the cost, from operating each plant. The problem of finding the system operating cost given the operating costs and reliabilities of the component plants, known as *probabilistic production costing*, is a rich field in itself. The reader is referred to Lin *et al.*[7] for a review of methods to solve this problem.

While linear programming models have been useful in long-term power capacity expansion planning, it was not possible to incorporate directly many important aspects of real-world power systems into a linear framework. Many methods have been proposed which go beyond linear programming in their efforts to address problems faced by power systems planners. Some of these, such as the multiobjective models described below, retain a linear programming framework. Nonlinear programming approaches, however, can allow for the explicit inclusion of realibility of electricity supply. In models using linear programming, the reserve margin is chosen without regard to size, number or reliability of individual plants, and consider only reliability implicitly by derating the nominal capacity of plants by an annual availability factor. In addition, the lumpy, discontinuous nature of capacity additions cannot accurately be reflected with linear programming models. One way that these modelling issues are addressed is through the assignment of integer, or binary, values to the investment variables. Instead of installed capacity taking continuous values (which may not correspond to realistic or commercially available unit sizes), project-specific capacities are specified, such as a 1000 MW coal-fired plant, and a binary variable is associated with it such that a value one is assigned if the project is to be built in a given time period and zero if it is not. A typical model can thus be formulated as:

[MIP]

$$\underset{x,y}{\text{Minimize}} \sum_{t=1}^{T} \left(\sum_{j \in J} c_{jt} x_{jt} + \sum_{k \in (I,J)} \sum_{n=1}^{N} f_{kt}\, \theta_n\, y_{knt} \right) \tag{6}$$

subject to

$$\bar{a}_i \bar{x}_i - y_{int} \geq 0, \quad \text{for all } i, n, t, \tag{7}$$

$$a_{jt} \hat{x}_{jt} x_{jt} - y_{int} \geq 0 \quad \text{for all } j, n, t, \tag{8}$$

$$\sum_{i \in I} y_{int} + \sum_{j \in I} y_{int} \geq d_{nt} \quad \text{for all } n, t, \tag{9}$$

$$y \geq 0, \tag{10}$$

where the decision variables x are binary-valued. As in [LP], eqns (8) and (9) are the capacity and demand constraints. I is the set of plants currently on line, with capacities $\bar{x}_i$ and availabilities a_i. $\hat{x}_j$ is the capacity associated with project j and c_{jt} is the present-value cost of building project j in period t. This use of integer variables is essentially the approach followed by Noonan and Giglio[8]. Other features which can be modelled using this framework are constraints prohibiting more than one project of a certain predefined set from being built (allowing for consideration of multiple projects on the same site, for instance.) Also, cost functions exhibiting economies of scale or other general characteristics can be introduced by describing the capital cost in terms of piecewise linear functions. Integer variables can be used in other contexts as well, as in the model by Scherer and Joe[9], where they are used to model system states where the reliability requirement is met. Mixed-integer formulations also have been used in applications to power sector planning in India[10,11].

Another method for handling nonlinear elements in the capacity expansion problem is the use of dynamic programming. Dynamic programming has been used extensively in solving both systems simulation and capacity expansion problems. See, for example, Booth[12], Brookhaven National Laboratory's DESOM model[13], the WASP model[5], the EGEAS model[14] and Dapkus and Bowe[15]. In dynamic programming, a multistage optimization problem is broken down into a series of simpler problems. These subproblems often involve only the evaluation of an objective function, rather than solving an optimization problem, and in general the objective function need not be linear. At each stage, there exist a number of possible configurations which satisfy the requirements of the system, given the initial system and the expansion options available at each stage. These configurations are the states of the system, and the goal of dynamic programming approaches is to find the sequence of decisions that lead from the initial state to the least-cost state in the final stage. For example, the EGEAS and WASP models simulate the production cost for each feasible state of the system, and identify the least-cost transitions to each state in the next stage. Once the state in the final stage with the least total cost is found, a backward path from the final state along the least-cost transitions specifies the optimal sequence. A major drawback of dynamic programming is the *curse of dimensionality*. This is the potential for an enormous number of problems which need to be solved as the process moves along.

From the 1970s onward, more sophisticated approaches to modelling power systems were taken to describe them more accurately. What follows are brief summaries of papers that have made contributions to modelling and methodology in capacity expansion planning.

3.2. *System reliability*

The goal of the utility is to provide customers the electricity desired, ideally without failure. However, shortages may result from failure of equipment (supply uncertainty) or from demand exceeding available supply (demand uncertainty). Improvement in supply

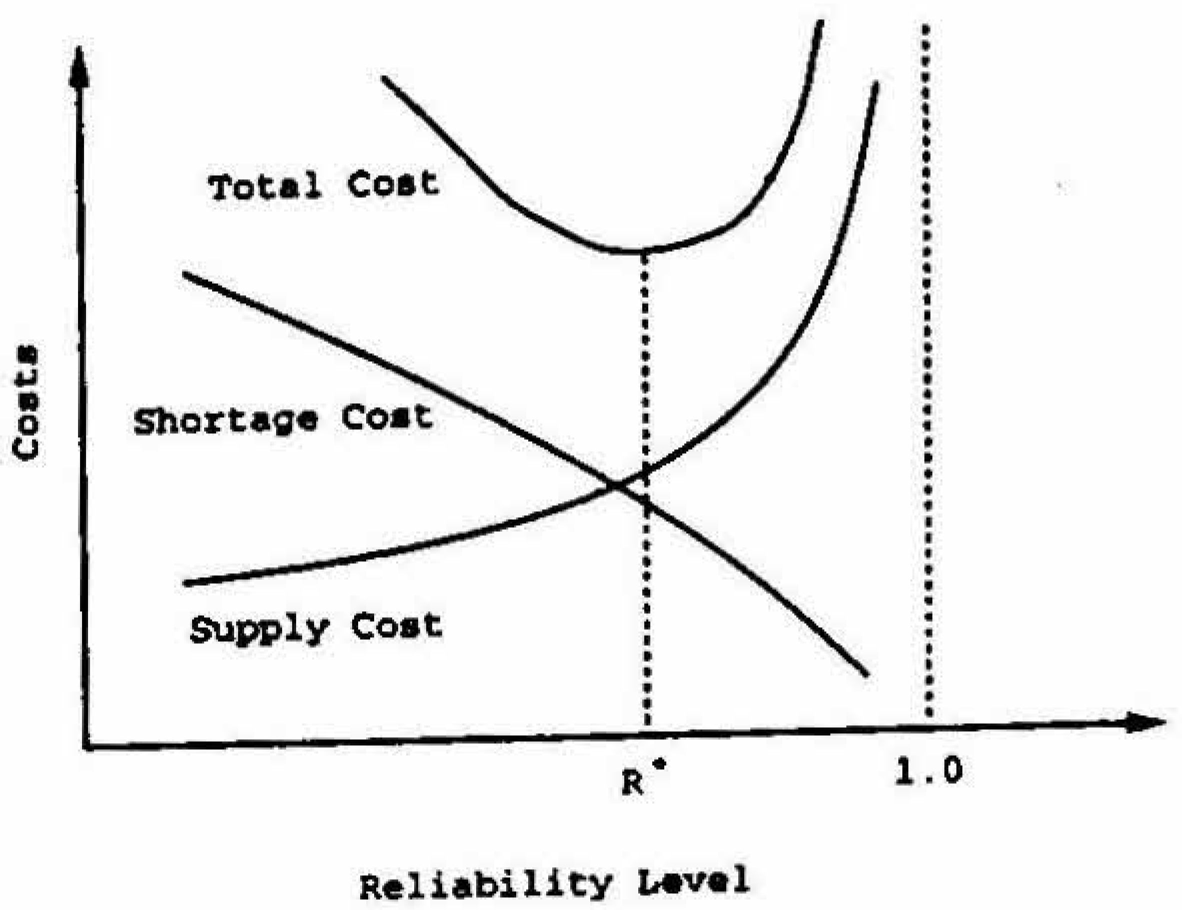

FIG. 3. Trade off between cost and reliability level for electric power system.

reliability can be made by either reducing the outage probability of individual plants or by using an increased number of plants of smaller size (for a given load requirement). There is, however, a tradeoff between the cost of increased reliability and reducing the level of electricity provided due to power shortages. The economic cost associated with providing different levels of reliability is shown in Fig. 3. Munasinghe[16] provides a summary of the issues involved in optimal reliability, pricing and planning in an economic framework. The typical approach in system planning, however, is to fix a reliability level, and optimize the system around this value. Few math-programming-based capacity planning models specifically optimize the reliability level, although a family of solutions parameterized over different levels can be obtained if computation times allow.

Noonan and Giglio[8] formulate a mixed-integer program to determine the type and size of electric generation facilities. Specifically, the model considers the addition of thermal, conventional hydro and pumped storage hydro units to the existing system under a systems reliability requirement. This model is more detailed than other models in that demand and operating conditions are decomposed into typical weeks, primarily to reflect changing hydro conditions.

The objective of the model is to minimize total discounted cost. The systems reliability requirement is modelled by including constraints of the following form for each planning period t:

$$\mathrm{Prob}\,\{\beta_t C_t \le d_t^{\mathrm{pk}}\} \le R^*, \tag{11}$$

with C_t a random variable representing total available capacity at peak demand, β_t, a parameter to adjust total capacity for seasonal inefficiencies and scheduled maintenance and peak demand given by d_t^{pk}. R^* is the maximum allowable loss of load probability (LOLP). These chance constraints are converted to nearly equivalent deterministic constraints for use in the model. Defining decision variable y^c as the demand requirement for restoring reservoir levels for pumped-storage units, the model can be described as follows:

[NG]
$$\text{Minimize}_{x,y} \sum_{t=1}^{T} \sum_{j\in J} \sum_{m=1}^{M(t,j)} c_{mjt}\, x_{mjt} + \sum_{t=1}^{T} \sum_{i=1}^{I} \sum_{j\in J} \sum_{n=1}^{N} q_i \theta_n f_{jt} y_{it} \tag{12}$$

subject to

$$\sum_{m=1}^{M(t,j)} x_{mjt} \le 1 \quad \text{for all } j, t, \tag{13}$$

$$\sum_{j\in J} y_{jnit} - \sum_{p\in P} y^{c}_{pnit} \ge d_{nit} \quad \text{for all } n, i, t, \tag{14}$$

$$\overline{a}_{ij}\overline{x}_j + \sum_{\tau=1}^{t} \sum_{m=1}^{M(\tau,j)} a_{ij}\overline{x}_{j\tau im} x_{j\tau im} - y_{jnit} \ge 0 \quad \text{for all } j, n, i, t, \tag{15}$$

along with the deterministic equivalents to eqn (11) and nonnegativity restrictions. Equation (13) states that at most one of the $M(t, j)$ possible projects of generation type j available for construction in time t can be built in time t. Equation (14) requires total generation less the generation required to restore pumped storage facilities P to meet system demand in each demand block and time period. Additional constraints on the operational variables include constraints specifying the maximum and/or minimum energy levels allowed and a constraint which requires reservoir levels for pumped storage units to equate at the beginning and end of each week.

A methodology which has been found to be successful in solving complex formulations of the power capacity expansion problem is the use of *decomposition methods*. The most popular decomposition technique which has been applied to the solution of the present problem is the generalized Benders' decomposition. Benders' decomposition is a technique for solving programs which have two (or more) distinct classes of decision variables, one of which complicates the problem considerably. By fixing the values of the complicating variables, a mathematical program in terms of the noncomplicating variables results, referred to as the subproblem. Ideally, this problem should be significantly easier to solve than the original program. Information from the solution of this program is incorporated into what is called the master problem. This program is used to determine improved solutions for the complicating variables. The process continues until an optimal (or satisfactory) solution is reached. For a complete discussion of the generalized Bender's decomposition, see Lasdon[17]. In the power capacity expansion problem, the investment options, such as plant size, type and number can be thought of as the complicating variables. Once the investment plan is fixed, the least-cost operating strategy for the system can be obtained.

Scherer and Joe[9] develop a mixed-integer program which uses binary-valued decision variables to incorporate probabilistic reliability requirements. Recognizing the increasing adoption of probabilistic approaches to reliability planning, constraints incorporating a probabilistic measure are introduced, with loss of load probability used as the measure of reliability. In contrast to other planning models, this considers a single time period. No discussion of issues involved in extending the model to the multiperiod case is made. In addition, only the ability of the system to meet peak capacity requirements is of concern; no load duration curve or relative operating costs are considered.

The decision variables are y_j, which is set to one if plant j is to be built and zero if it is not, and g_j, the size of the plant. The other decision variable is the system state. Every possible combination of running and failed plants is assigned a binary value, x_i, which takes the value one whenever the plants which are operating in state i can meet demand and zero otherwise. For a system with n candidate plants, this results is 2^n possible system states to be considered. The model is given as:

[SJ]
$$\text{Minimize} \sum_{j=1}^{n} (c_j g_j + f_j y_j) \tag{16}$$

subject to

$$\sum_{i=1}^{2^n} P_i x_i \geq P_s \,, \tag{17}$$

$$\sum_{j \in S(i)} g_j \geq \mathrm{d}x_i \quad \text{for all } i \,, \tag{18}$$

$$g_j \leq \bar{g}_j y_j \quad \text{for all } j \,, \tag{19}$$

$$g_j \geq 0 \quad \text{for all } j, \tag{20}$$

where f_i is the fixed charge associated with plant j and c_i is the slope of the capacity cost function. $S(j)$ is the set of plants that are operating in state j and P_j is the probability that the system is in state j. Constraint (17) guarantees that the probability the system is in a state that can meet demand exceeds or meets the required reliability level, P_s, defined as $1 - \text{LOLP}$. Constraint (18) says that each state in which the capacities of plants are sufficient to meet system peak demand d will assign the value one to its state variable x_i. Constraint (19) restricts the maximum capacity of required plants.

Since the calculation of the P_j's is made with the *a priori* assumption that all plants will be built, the question arises as to whether a solution having fewer than n plants will be feasible. A proposition is provided demonstrating that a solution to the n-plant problem, having $n - m$ plants at zero capacity, is also a solution to the m-plant problem (formed by removing from consideration the plants at zero capacity in the n-plant problem, and recalculating the P_j's), assuming independence of plant failures.

The number of integer variables in the model grows exponentially with the number of plants considered. The number of variables can be reduced, however, by setting the values corresponding to some of the states beforehand. For instance, it is unlikely that a state with relatively few plants operating will be able to meet the demand and those variables will be fixed at zero whereas the state with all plants up will definitely meet the demand (for well-formulated problems, at least) and its variable will be fixed at one.

Bloom[18] presents a formulation with the goal of integrating math programming planning models with probabilistic methods for measuring the reliability of the system, rather than using *a priori* estimates of reserve margins. In this case, the reliability standard is expressed as the maximum demand which is allowed to be not served by the system. The problem is defined as follows:

[B1]
$$\underset{x,y}{\text{Minimize}}\ c'x+\sum_{t=1}^{T} EF_t(y_t) \tag{21}$$

subject to

$$EG_t(y_t) \le \varepsilon_t \quad \text{for all } t, \tag{22}$$

$$y_t \le \delta_t x \quad \text{for all } t, \tag{23}$$

$$x, y_t \ge 0, \tag{24}$$

where the decision variables are x, the vector of plant capacities, and y_t, the vector of plant utilization levels in period t, with c the discounted capacity costs, $EF_t(y_t)$, the present value of the expected operating cost in period t, $EG_t(y_t)$, the expected unserved energy, and ε_t the desired reliability level. The matrix δ_t allows for temporal changes in the merit order caused by the addition of new plants and changing operating costs. While there are no integer variables, Bloom notices that the planning problem naturally decomposes into two parts, namely, determining investments in new capacity and determining the operating cost and reliability of the system, and proposes to solve it using generalized Benders' decomposition. A master problem is defined which is used to generate a set of trial plant capacities. The subproblems calculate the minimum expected cost of operation and the reliability in each time period for the given capacity vector. For each period t, the subproblems are formulated:

[B1-S]
$$\underset{y}{\text{Minimize}}\ EF_t(y)=\sum_{i=1}^{I} f_i p_i \int_{U^{i-1}}^{U^i} G_i(Q)\,\mathrm{d}Q \tag{25}$$

subject to

$$EG_t(y_t)=\int_{U^I}^{\infty} G_{I+1}(Q)\,\mathrm{d}Q \le \varepsilon, \tag{26}$$

$$0 \le y_i \le x_i \quad \text{for all } i, \tag{27}$$

with I plants in merit-ordered index, i, plant utilization level y_i, operating cost f_i and availability p_i. G_i is the equivalent load duration function seen by the ith plant at level of demand Q and U^i is the cumulative capacity of the first i plants (also called the loading point), defined recursively by $U^i = U^{i-1} + x^i$, $U^0 = 0$. The plant capacities x_i are fixed, provided by the master problem. The equivalent load duration curve is obtained by a recursive relationship known as probabilistic simulation: $G_{i+1}(Q) = p_i G_i(Q) + (1-p_i)\, G_i(Q-x^i)$. Concisely, $G_i(Q)$ is the expected load duration function seen by plant i, considering the outage probability of the previous plant in the merit order. After solution of the T subproblems, cuts constructed from their dual multipliers are added to the master problem.

The subproblems are written in the form of nonlinear optimization problems. However, an approach is developed which does not require the use of optimization algorithms to solve the subproblems. Since merit order minimizes operating costs, plants are loaded according to merit order until the reliability constraint (26) is satisfied or the subproblem is found to be infeasible. A method for calculating the dual multipliers is described requiring computation and manipulation of the load duration functions. Using a similar approach, Côté and Laughton[19] developed a mixed-integer program which uses stochastic production costing to solve the operational subproblems. Li and Billinton[20] deal with reliability in a linear programming framework. They incorporate customer damage functions to represent the cost of unserved energy into the cost minimization objective, rather than a system aggregate value. Monte Carlo sampling is used simulate these damage functions over different load levels.

3.3. *Uncertainty in demand and model parameters*

In the previous studies cited the issue of primary concern, along with cost minimization, is system reliability. Another issue which has received considerable attention has been resolving the uncertainty inherent in model parameters, most notably uncertainty in demand. Uncertainty influences many aspects of power system planning. This includes factors normally considered in planning models, such as fuel prices and plant availability as well as factors not normally considered such as climate and regulatory requirements. Uncertainty manifests itself in many ways in power planning, but the most important of these is simply the unpredictability of the future. Models require data representing demand, cost, availability, and so on, for every period in the planning horizon. Several methods have been proposed to incorporate uncertainty into capacity planning problems[21–23]. In the *deterministic equivalent* method, the best available forecast, or the expected values from forecast distributions of parameters are used. New information is incorporated as it becomes available. However, optimality for a particular period is dependent on the future unfolding as predicted. The *scenario analysis* method handles uncertainty by identifying a range of values for the uncertain variable of interest (*e.g.*, *low*, *base* and *high*). Often the *base* case is recommended without further analysis. More sophisticated approaches to scenario analysis subject the optimal solutions of each scenario to other scenarios to assess their sensitivity and performance, and to determine plants which appear in all, or many, solutions (so-called 'robust' plants). However, a solution which is best, in some sense, over all the scenarios is difficult to determine. The method also may require solving a large numbers of problems, depending on the number of scenarios considered. *Stochastic programming* has also been proposed as a tool to address this challenging problem facing system planners. In stochastic programming, parameters are allowed to take on a range of values corresponding to particular scenarios that reflect possible futures which the system may have to operate under. As each scenario has a certain probability of occurring, optimization of the objective function is made with respect to the expected value of the capital and operating costs. This provides the solution that is 'optimal' in this sense over all scenarios. Other methods are also used, such as *Monte Carlo simulation* and *options valuation*, but are not of direct concern in mathematical programming approaches.

In the paper by Murphy *et al.*[24], two alternative models for incorporating uncertainty in demand are compared. The first is a two-stage stochastic program with recourse while the second is a linear program using an expected load duration curve constructed from the individual scenario curves. If there are S possible realizations of demand, each with a corresponding load duration curve and a probability of occurrence p_s, the two-stage program can be written as (only over one period, for clarity):

[M]
$$\underset{x, y_s}{\text{Minimize}} \quad \sum_{j \in J} c_j x_j + \sum_{j \in J} \sum_{n=1}^{N} \sum_{s=1}^{S} p_s f_j \theta_n y_{jns} \tag{28}$$

subject to

$$x_j - \sum_{n=1}^{N} y_{jns} \geq 0 \quad \text{for all } j, s, \tag{29}$$

$$\theta_n \sum_{j \in J} y_{jns} \geq d_{ns} \quad \text{for all } n, s, \tag{30}$$

$$x_j, y_{jns} \geq 0, \tag{31}$$

where the notation is the same as in Section 2. Constraints (29) and (30) require that generation restrictions and demand are met for each scenario. The xs are the first-stage decision; when a particular scenario is revealed, say $\bar{s}$, then second-stage recourse operating strategy $y_{jn\bar{s}}$ is followed.

A method for constructing the parameters for the expected load curve from the break points and probabilities of the individual scenario curves is given and a deterministic equivalent program identical to [LP] is derived, with demand and load duration parameters d_n and θ_n replaced by the corresponding parameters of the expected load duration curve. It is then shown that under fairly general conditions the optimal first-stage solution to the program using the expected load curve is equivalent to the stochastic programming solution. This equivalence does not hold if the operating costs of the generating technologies are load-dependent; that is, if the costs vary according to which segment of the load curve is being served. An important implication is that a stochastic program having S possible demand curve realizations (scenarios) can be transformed into a linear problem having a factor of S fewer capacity constraints, although the number of demand constraints remains the same.

A more general approach to solving the capacity expansion problem under uncertainty is developed by Borison *et al.*[25]. The method, called the state-of-the-world (SOW) decomposition, solves the dynamic probabilistic problem by breaking it down into a set of static deterministic problems which are linked dynamically. The linkage is enforced by Lagrange multipliers. The paper first gives the SOW decomposition accounting only for the dynamic interaction, and follows with an equivalent decomposition for dealing with uncertainty. Each state-of-the-world is identified as a time-period/outcome pair. The decomposition for accounting for both dynamics and uncertainty is then developed. In the joint formulation, each technology is indexed by type, installation date and information of past outcomes. This allows for the possibility of contingent decisions and is a main feature of the model: an ability to identify optimal purchase strategies as future uncertainties are revealed with time. The problem is formulated as:

$$\text{[SOW]} \qquad \underset{x}{\text{Minimize}} \sum_{j\in J} c_j x_j + \sum_{t=1}^{T} \sum_{u=1}^{U} W_{tu}(y_j(t,u)) \tag{32}$$

subject to

$$y_j(t,u) \le \bar{x}_j \quad \text{for all } j, (t,u) \in L_j, \tag{33}$$

$$y_j(t,u) \le x_j \quad \text{for all } j, (t,u) \in L_j, \tag{34}$$

$$y_j(t,u) = 0 \quad \text{for all } j, (t,u) \notin L_j, \tag{35}$$

where $W_{tu}(y_j(t,u))$ is the probability-weighted discounted operating cost incurred under state-of-the-world (t, u), and L_j, the set of states-of-the-world in which technology j is an option. Constraints (33) and (34) together restrict generation to installed capacity and keep installed capacity below the allowable upper bound. If technology option j is not allowed in a particular state of the world, its generation is fixed at zero (constraint (35)). Demand is implicitly satisfied via the operating cost function. From this problem, the Lagrangian dual problem is formed and simplified. An algorithm is proposed to solve the problem which, in summary, iteratively solves the Lagrangian dual problem for each state-of-the-world using dynamic programming and updates estimates of the Lagrange multipliers until the projected gradient at the solution is zero. A nondegenerate solution will result only if the operating cost function W is convex.

An example is provided which illustrates the procedure and shows that it produces reasonable contingency plans. There are two time periods with uncertainty in nuclear fuel costs. Reproducing the results in Table I shows that the uncertainty in nuclear fuel cost discourages the purchase of nuclear capacity in the first period, but different plans are recommended in the subsequent period, depending on the outcome of the uncertain event.

Another decomposition method which has been used in investment planning under uncertainty is the Dantzig–Wolfe decomposition. This approach is used by Sanghvi and Shavel[26] to solve a very-large-scale model for hydrothermal system expansion for the US Pacific Northwest. A unique feature of the model is that conservation of energy through demand-side management is explicitly included as a supply option. Uncertainty in hydro energy availability, along with load growth uncertainty, are considered.

Table I

Purchase decisions from state-of-the-world decomposition

1990	*2000 Nuclear fuel cost*	
	Low	*High*
5000 MW Coal 3000 MW Oil	4000 MW Nuclear	4000 MW coal

Other models using techniques of stochastic programming have been proposed as well. Louveaux[27] presents a general method for solving multistage stochastic programs which is applied to the energy investment problem. Dapkus and Bowe[15] use a stochastic dynamic programming approach incorporating uncertainty demand, availability of new technology, and possible loss of service from failure, regulatory action or lack of fuel. A stochastic program with recourse to handle uncertain demand and fuel costs is developed by Janssens de Bisthoven *et al.*[28]; a technique based on nested decomposition and cutting plane methods is proposed for its solution. Bienstock and Shapiro[29] developed a two-stage mixed-integer, stochastic model for resource acquisition that includes the ability to model complex features such as supply contracts contingent on future outcomes. Gorenstin *et al.*[23] minimize the maximum regret associated with each scenario, rather than the expected cost. The *regret* of a scenario is the difference between the actual cost (as determined by the model) and the cost optimal for a given scenario. Malcolm and Zenios[30] present a stochastic formulation which produces solutions that are robust to uncertain outcomes by weighting deviations from the expected value.

3.4. *Environmental considerations*

Environmental impacts from power generation range from local (*e.g.*, particulates) to global (*e.g.*, climate change), and have clearly been recognized as an important part of power system planning. While many models have acknowledged this, few actually include such considerations nor do they discuss the implications of such considerations on the model. There are several methods that have been used to integrate environmental considerations with power systems planning models[31]. One is to include in the constraint set equations limiting emissions of pollutants. In many models which use this method, these constraints are nonbinding; they have no effect in determining the optimal plan and simply aggregate total emissions. Also, in order for this method to be compatible with the existing models, such constraints must be linear. Linearization is a crude approximation to pollution production and transport phenomena, and thus will fail to capture impacts accurately. Alternatively, a (weighted) term reflecting environmental costs can be added to the objective function. This method requires estimation of mitigation cost on technology-by-technology basis. A survey of these methods and their application in many widely used models is provided by Markandya[31].

A few models have been developed which explicitly include environmental factors in their formulations. Remmers *et al.*[32] implemented the first method described above by including constraints representing emissions restrictions. The system is modelled as a network, with primary energy sources as inputs to conversion technologies and end users, with a set of emission reduction technologies assigned to each energy conversion technology. Decision variables y_{jt} thus denote the flow of energy through various branches in the system. The model is fundamentally the same as LP, with the following additional constraints:

$$\sum_{j \in EET} e_{jkt}\, y_{jt} \leq L_{kt} \qquad \text{for all } k, t, \tag{36}$$

$$\phi_{lk} \sum_{j \in FUL_i} e_{jkt}\, y_{jt} + \sum_{j \in ERT_{ik}} e_{jkt}\, y_{jt} \geq 0 \quad \text{for all } k, l, t, \tag{37}$$

with ϕ_{lk} the reduction efficiency of emission reduction technology l for pollutant k, and e_{jkt}, the emission factor. The summation in eqn (36) is made over all emissions and emissions-reducing technologies, EET. This constraint limits the aggregate emission of pollutant k over time t to L_{kt}. In eqn (37) the first summation is made over all primary energy sources feeding energy conversion technology i, while the second is over the set of emissions-reducing technologies. The model was run under scenarios requiring different emissions reduction targets to forecast long-term primary energy consumption.

Environmental concerns such as air pollution, acid rain and global climate change have caused regulatory agencies in the US to develop an interest in incorporating environmental externalities into the planning and regulatory process[33,34]. Nearly half of all the US states require incorporation of environmental externality costs in either the planning and acquisition or rate-making process. The goal of these methods is to increase the likelihood of less environmentally harmful technologies appearing in the generation plan, even if their economic costs are higher than least-cost alternatives. On the rate-making side, a greater rate of return is often allowed for construction and use of environmentally sound resources. Several methods of incorporating environmental externalities are being used by state Public Utilities Commissions in the planning/acquisition process. These include[34]:

Qualitative treatment of environmental concerns, such as giving preference to more environmentally sound technologies.

Direct quantification of costs as a part of integrated resource planning.

Percentage adder/subtractor applied to the cost of supply/demand-side resources.

However, to our knowledge, no math programming models which directly incorporate any of the above methods have been published.

In the past, the planning process was dominated by utilities, or central planning authorities, with economic or social equity goals being the main concern. Today, however, there has been an increasing involvement of many diverse interests with different and often conflicting objectives. The objective nature of optimality also changes under a multicriteria regime; the values and biases of the decision makers affect the solution. The advent of multiobjective programming has led to many model formulations which include planning goals in addition to cost minimization and explicitly account for the tradeoffs between conflicting goals.

A representative multiobjective model, developed by Kavrakoglu and Kiziltan[35], considers the following three criteria: economic cost (F_1), environmental impact (F_2) and risk (F_3). These are described mathematically as :

$$F_1 = \sum_{j \in J} \sum_{t \in T} c_{jt}\, x_{jt} + f_{jt}\, y_{jt}\,, \tag{38}$$

$$F_2 = \sum_{j \in J} \sum_{t \in T} I_{jt}\, y_{jt}\,, \tag{39}$$

$$F_3 = \sum_{j \in J} \sum_{t \in T} R_{jt} \sum_{\tau=0}^{T} x_{j\tau}\,, \tag{40}$$

subject to the constraints of [LP]. The quantities I_{jt} and R_{jt} are values which represent the environmental impact and risk associated with technology j. To obtain solutions to the multiobjective model weights w_i, with $\sum_i w_i = 1$, are assigned to each of the objective criteria functions and the reulsting linear program solved for various combinations of w_i. The object is to find the efficient frontier of solutions, *i.e.*, a solution $z^* = (x^*, y^*)$ is said to be efficient if, for all objectives i, $F_i(z^*) \leq F_i(z)$ for all solutions z. Techniques exist for varying the weights w_i so that efficient solutions are obtained. From this set of efficient, or noninferior, solutions the 'best compromise' solution can be found, or some subset of these solutions chosen for ultimate consideration by the decision makers. The model was applied to the Turkish electric power system. Amagai and Leung[36] use a similar compromise programming approach on the objectives of minimizing cost, emissions and fuel supply risk for the Japanese power sector.

Many other approaches which have integrated multiple objectives with traditional linear and dynamic programming tools (such as WASP) have been developed. Evans *et al.*[37] use multiattribute utility theory to measure the utility's objectives and dynamic programming to choose the expansion plan. Yang and Chen[38] integrate a multicriteria decision procedure based on the analytical hierarchy process with a dynamic programming method. Kim and Ahn[39] integrate a preference-order ranking scheme with WASP to determine the relative desirability of expansion plans. A two-stage solution procedure is developed by Climaco *et al.*[40]. In the first stage, solutions to the multicriteria problem are generated, with some being retained for further consideration. In the second stage, the coarse solutions of the first stage are fine-tuned by evaluating them against a second set of criteria. A review of a variety of multiobjective programming techniques and their application to power systems modelling is presented by Psarras *et al.*[41]

4. Conclusion

Over the past three decades, the electric power industry has been a rich source of application for the state-of-the-art mathematical programming approaches, particularly in the area of capacity expansion. We hope that the reader has acquired an appreciation of the variety of mathematical programming applications to the electric power sector. This paper has described how mathematical programming methods and models have been used to address some of the issues critical to planning electric power capacity expansion. Primary among these issues have been reliability of supply, uncertainty in demand and environmental externality costs in planning models. However, as the regulatory, financial and technical structure of the utility industry changes, more issues have arisen which complicate the planning process. Several of these issues stand out[42]. Demand-side management has become an important concern of utilities, extending beyond traditional policies such as encouraging conservation. Nonutility generation (independent power production) significantly increased in the US following the Public Utility Regulatory Policies Act of 1978, and is sure to affect future generation plans of utilities in many ways. As transmission networks become more extensive and interconnections between systems become stronger, the possibility of inter-regional trade and the formation of power pools are viable alternatives for most utilities. Not least importantly, deregulation and privatization of utilities is taking place throughout the world; state-owned, centrally planned institutions are being replaced by enterprises governed by market forces. Each of these issues has features which will pose challenges to modellers who wish to investigate their effects in a mathematical programming environment.

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