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BOOK REVIEWS

Algorithms for random generation and counting by A. Sinclair, Birkhauser Verlag AG, Klosterberg 23, CH-4010 Basel, Switzerland, 1993, pp. 146, SFr. 78.

During my last year as a graduate student at Berkeley, I was once sitting in a campus eatery with some cronies from Statistics when Prof. David Aldous came by. He was then teaching us a course on continuous time, continuous (*i.e.*, real-) valued stochastic processes. One of us asked him what he was currently working on and his response was: "Believe it or not, finite Markov chains". Those were heady days for continuous time/state stochastic processes. Martingale theory was at its peak. 'Large deviations' *a la* Donsker and Varadhan had recently burst forth on the scene and was going strong. 'Malliavin calculus' had just arrived and was on its way to becoming the centre of hyperactivity that it soon did. On this backdrop, finite state, discrete time Markov chains were passe', so to say, left essentially to the philistines of probability theory, such as operations researchers and electrical engineers. It was certainly not something one expected a young star like Prof. Aldous, recent winner of the Rollo Davidson award and already well known for his work on the subsequent principle and prediction processes, to be after. This explained the 'Believe it or not', but still left me, for one, wondering as to what exactly he was up to in finite Markov chains. In retrospect, one knows what was afoot. Aldous, along with a few others, was about to initiate what has come to be known as the 'finite Markov chain renaissance'—a whole stream of new ideas in Markov chains that were to have a major impact on other disciplines, ranging from theoretical computer science to computational Beyesian statistics. At the beginning of the nineties, it is perhaps the highest profile activity in probability theory, already accounting for a special year on 'Emerging applications of probability' organized by the Institute of Mathematics and its Applications in 1993-94. A flavor of the extent and pitch of the activity can be gleaned from the report of the Committee on Applied and Theoretical Statistics of the National Research Council of USA titled 'Probability and algorithms', which now forms an entire issue of *Statistical Science* (Vol. 8, Feb. 1993).

With hindsight, this may not seem all that surprising. As observed by Prof. Gian Carlo Rota in an interview some years back, trends in mathematics have always been dictated largely by which natural or applied science is concurrently undergoing an explosive growth. Early this century, quantum mechanics spurred a lot of activity in operator theory and such like. In the post-World War boom for 'heavy' engineering, hydrodynamics, partial differential equations, etc. found their feet. The revolution in the past few decades has been in computer science and it was natural that discrete mathematics should get a new lease to life. Probability theory, with a thumb in every pie, has faithfully reflected these trends. After serving the cause of physics (Brownian motion, functional integration), communication engineering (prediction theory of stationary processes) and so on, it is now building a deep relationship with theoretical computer science.

The book under review, part of a series titled 'Progress in Theoretical Computer Science', is a landmark event in this mini-revolution-in-the-making. It is a slightly revised version of the author's Ph. D. thesis submitted to the University of Edinburgh in 1988. This fact itself says much about the nature of the book—it is a topical contribution to a fast-moving field. It addresses two related issues: that of approximately counting the number of elements in a set and of generating a random element thereof with uniform distribution. (That these two problems are related is not a trivial fact, but a consequence of earlier work by Jerrum, Valiant and Vazirani. Jerrum, incidentally, supervised this Ph.D. thesis). The book is quite slim (about a hundred and forty-odd pages) and consists of four chapters.

The first chapter is intended to serve as a quick resume of the background material. The most important concept discussed here is that of self-reducibility, again due to Jerrum, Valiant and Vazirani. Self-reducibility of a set implies the possibility of constructing the set from a smaller set of similar nature through an inductive process. It is such sets that are the objects to be 'approximately counted' or 'randomly generated'. The second chapter forms the core of the book. It presents the relevant Markov chain theory. In problems like these (or those of Monte Carlo simulations, computational Bayesian statistics, etc.), one needs to produce a prescribed probability distribution on a given set. The way Markov chains come into the picture is that one achieves this aim by constructing an ergodic Markov chain on this set with a relatively simple transition mechanism and having the desired distribution as its equilibrium distribution. The question then arises as to how fast the Markov chain approaches its equilibrium behaviour. This leads to the concept of a 'rapidly mixing' Markov chain. These are ergodic Markov chains for whom the distance between the distribution at time n and the equilibrium distribution (with respect to a suitable norm) is within a prescribed error bound $\varepsilon > 0$ for n bounded by a polynomially bounded function of $\ln(1/\varepsilon)$ and a parameter depending on the 'size' of the problem.

Now recall that the equilibrium probability vector is simply the unique left eigenvector of the Markov chain transition matrix corresponding to the eigenvalue 1, which is also its maximum (in modulus) eigenvalue. The distribution at time n is obtained by left multiplied by the initial probability (row) vector of the nth power of the transition matrix and approaches the equilibrium distribution at an exponential rate governed by the absolute value of the second largest eigenvalue. Thus rapid mixing has crucial links with the second eigenvalue of the transition matrix. One of the author's major contributions here is the introduction of the notion of 'conductance' of a Markov chain in terms of which he estimates its second eigenvalue and gives conditions for rapid mixing.

The last two chapters give applications of this theory to specific instances, such as the problem of approximating the permanent of a matrix and the partition function of a monomer–dimer systems, to mention just two. These are followed by an 'appendix' that gives a rapid overview of the developments that followed the thesis while it was evolving into a book.

As already mentioned, this is a topical contribution to a fast-moving field. It will no doubt be subsumed eventually by texts and treatises that put these and other results in a broader perspective made possible by hindsight, but that is not the point. The value of this book is in its timely appearance. It is likely to influence the course of developments in this area in 'real-time', the afore-mentioned hindsight well come only later. Also, it has the immediacy and freshness of new discoveries which no text or treatise can ever hope to convey.

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Mathematical control theory : An introduction by Z. Zabczyk, Birkhauser Verlag AG, P.O Box 133, CH–4010 Basel, Switzerland, 1992, pp. 272, SFr. 108.

In general, books can be divided into two groups. One class of books chooses a certain subject and a topic in it and dwells on it exclusively and deeply. The other class touches upon various topics in the subject, introducing the various concepts involved, scattering some examples and presenting the landmark theorems in the overall development of the subject. The present book falls into the second category.

The subject dealt with by this book is mathematical aspects of control theory. The motivation for this subject stems from the numerous applications it has. Some examples of applications are cited in this book. However, I am tempted to cite the following paper: R. Narasimha and K.R. Sreenivasan, Relaminarization of fluid flows, *Advances in Applied Mechanics*, Vol.19, (1979). In this article, one can see a variety of situations in fluid flows which are turbulent and a number of ways to make them laminar. These ways amount to fixing suitable control which act on the system (*e.g.*, through the boundary or in the interior). Thus, we have the feeling that all unwanted perturbations in an unstable system can be killed through the devise of suitable controls. Nothing can be more satisfying than this ! This example illustrates beyond any doubt the usefulness and the need of control theory in infinite dimensions. In this set-up, the state equation, in many cases, is a set of partial differential equations (PDE). Part IV of this book is devoted to some elementary aspect, control of PDEs. However, the recent developments in this area are not included here.

The other three parts of the book concentrate on finite dimensional controllability problems wherein the equation of state is described by a set of ordinary differential equations (ODEs). These state equations have parameters called controls. The study of their solutions, when the parameters vary, with regard to stability, bifurcations, etc., is one of the hot topics in dynamical systems. However, the nature of the questions posed in control theory is quite different. Roughly, one looks for controls so that the solution of the corresponding state equations has desired properties. In other words, in the context for control theory, we do not have closed systems and we interact with the open systems through the choice of control parameters. Let us now briefly indicate the various questions discussed in the course of this book.

Given a system, the question is to take the system from a given state to another desired state. This property of the system is called (exact) Controllability. Linear controllable systems are characterized in terms of algebraic condition called Rank Condition. Thus analytic problem of controllability is reduced to some algebraic problem.

The second question to be discussed is that of observability. In practice, sometimes, it is not possible to observe the system fully: it is possible to know only a part of it. The aim is to go back in time and determine the initial state of the system with this partial information of the system over an interval of time. If we have the full information on the system at any instant then obviously we can go back in time and determine uniquely the initial condition because of the reversibility of the system. The situation at hand is different: we have only partial information on the state of the system but over a range of a time interval.

Another important issue discussed is that of stabilizability of the system around an equilibrium point of the system. The new idea here is the introduction of feedback controls which depend on the state itself. In the theory of differential equations, there exist several methods to determine whether a given equilibrium state is a stable one. However, the question of choosing one feedback control for which equilibrium point is stable is a new one. Similar questions can be raised in the case of practically observable systems as described earlier.

In control theory, one is also interested in the transformation which associates the solution of the state equations to the controls. The problem at hand is to understand the structure of this transformation by looking for the 'simplest' (or) 'canonical' forms of it. This is called Realization Theory.

Apart from the above-mentioned topics, the book also considers the problem of optimal control. Here the choice of the control is dictated by an optimal strategy defined by the so-called cost functional. In this part, the book touches upon, among others, some classical topics like Bellman's principle in dynamic programming and Pontriagin's maximum principle.

The above questions are addressed in the case of linear and nonlinear state equations. Parts dealing with Realization theory are not classical. Separate books/monographs/survey articles exist on most of the individual topics discussed here. This is especially so in the case of PDEs which have seen enormous progress since the appearance of the seminal survey paper of Russell. (See J-L. Lions, *Controllabilite exact.* Tomes 1, 2 Masson, Paris, 1988). In the infinite dimensional control systems, there are different classes of controls (*e.g.*, boundary controls) apart from the interior controls which already exist in the finite dimensional systems. Because of these possibilities the richness of the subject has increased. The methods of solution naturally become much more complicated. The book, of course, does not report on the recent activities in this area. May be this is not possible given the aim of this volume which is to present a bird's eye view of the entire subject of controllability. In the limited space available, the author has made his best by touching upon a wide variety of topics without going too much in depth. The presentation is clear and simple. Prerequisites are kept to a basic knowledge of linear algebra, calculus and differential equations. The book may not be of much use to researchers in the field but definitely will help them and other teachers to design a course on Control theory.

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Barcelona seminar on stochastic analysis 1991 edited by D. Nualart and M. Sanz Sole, Birkhauser Verlag, Klosterberg 23, CH-4010 Basel, Switzerland, 1993, pp. 234, SFr. 88.

The theory of continuous time stochastic processes since the early days of its development, has had a close interrelationship with several branches of analysis, most notably harmonic analysis and the theory of partial differential equations. An early instance of this is Kolmogorov's forward and backward equations to describe the timeevolution of transition probabilities of a Markov process, which served as the seed for the semigroup-theoretic approach to Markov processes by Feller and Dynkin and still later, for the 'martingale formulation' of Stroock and Varadhan. The traffic of ideas in the reverse direction was also not negligible—probability theory has contributed much to potential theory and through 'functional integration' to mathematical physics as well, the high point of which is the Feynman–Kac formula. A natural outgrowth of this interaction in recent years has been the emergence of a body of ideas constituting 'stochastic analysis' which goes one step further and derives stochastic analogs of certain concepts in analysis. This includes analytic aspects of stochastic differential equations and stochastic partial differential equations, calculus on Wiener space (the 'Malliavin' calculus') and the theory of large deviations, which may be viewed as 'stochastic asymptotic analysis'. The volume under review is a timely contribution to the field, taking stock of the current trends in the field through the material presented at a seminar held in Barcelona in 1991.

The theory of stochastic differential equations depends on the Ito calculus which involves the integration of a stochastic process with respect to a Brownian motion (more generally, local martingale). This requires a severe condition on the process: it should not anticipate (*i.e.*, should be statistically independent of) the future increments of the Brownian motion. This condition not only ensures a nice natural 'definition' of the integral, but makes the resultant process a 'local martingale', putting the vast machinery of martingale theory at your service. Nonetheless, it can be a handicap at times, which prompted the search for an integration theory without the nonanticipativity condition. One such integral is the Skorohod integral which surprisingly re-emerged as the 'dual' object of a certain operator at the heart of Malliavin calculus. This led to a resurgence of interest in 'anticipative' stochastic calculus and associated 'Skorohod' stochastic differential equations, led by Paradoux, Nualart, Ocone, among others. The success of the programme has been limited to specific classes of problems and the area remains active. Not surprisingly, this topic dominates the volume under review. The first paper, by Baldi and 'Sanz–Sole', considers the (stochastic) flow of diffeomorphisms generated by a stochastic differential equation viewed as a map from initial condition to its state at time t. They find its modulus of continuity in the space variable, with an eye on application to an anticipative s. d. e.. The proof uses an estimate due to Azencott of the probability that the solution lies outside a prescribed 'tube' while the driving Brownian motion lies inside another prescribed 'tube' in a given interval, the two 'tubes' being in a natural relationship with one another. A short proof of this estimate along with some consequences is given in a paper by Leandre in this volume. Buchdahn in his article gives conditions on the coefficients of a Skorohod s. d. e. that ensure a local existence result. Millet and Sanz–Sole characterize the support of the law of a process satisfying a Skorohod s.d.e. in the spirit of the celebrated Stroock–Varadhan support theorem for diffusions. Oskendal and Zhang study a Volterra-type Skorohod equation and establish the existence of a unique solution in a suitable 'generalized' sense.

The remaining contributions are on diverse topics. Cruzeiro and Zambrini introduce a class of stochastic processes called Bernstein processes, motivated by quantum mechanical problems, and show that the classical Ornstein–Uhlenbeck process can be viewed as one. Gorostiza and Lopez–Mimbela give a test for convergence in law of measure-valued processes and an application thereof to 'superprocesses' recently introduced by Dynkin. Malliavan' s paper on 'universal Wiener space' addresses certain abstract issues in Gaussian probability spaces. Nualart and Zakai present results on the 'positivity index' of Wiener functionals. Ocone shows that a local martingale with 'cadlag' paths has conditionally independent increments and symmetric jumps if and only if its law is invariant under integral transformations which preserve quadratic variation. Shepp and Zeitouni concern themselves with certain conditional expectations of exponentials of Wiener functionals and apply them to computation of 'Onsager–Machlup functionals' which estimate the probability of a diffusion process belonging to a prescribed 'tube' in an asymptotic sense. Williams explores the conditions on the domain of a reflected Brownian motion that ensure certain regularity properties for its representation as a semimartingale. Finally, Zabczyk discusses 'stochastic factorization', an analytic tool introduced by him and coworkers, and sketches its applications to stochastic partial differential equations and stochastic convolutions.

To summarize, this volume contains contributions of a very high quality, giving a timely indication of where stochastic analysis is currently headed. It is a valuable source for researchers in the field, though a bit overspecialized for the stochastic process community as a whole.

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Computational algebraic number theory by Michael E. Pohst, Birkhauser Verlag AG, P.O. Box 133, CH-4010 Basel, Switzerland, 1993, pp. 88, SFr. 34.

From a practical viewpoint any mathematical theory must ultimately be subjected to the question "What can be computed?". Hence it is natural to suppose that the more a theory develops, the more computable various objects within this theory become. This is evident in the case of Algebraic Number Theory, where many of the latest theorems and techniques can be used to find 'faster' ways of computing and sometimes even show that computations are indeed possible. In many cases better algorithms for applications outside the realm of Number Theory have also been found using the new results from this area. Conversely, the need to compute things more 'effectively' has stimulated research in Algebraic Number Theory.

Computational Algebraic Number Theory is thus a lively and interesting topic on which the author has chosen to write a book. It is somewhat unfortunate therefore that (perhaps due to constraints of space) this slim volume largely takes a cookbook approach to the problem of computations in Algebraic Number Theory; the proofs and details are often relegated to the references—either current research papers or the book by the author and H. Zassenhaus[1]. Perhaps this is in the nature of the seminar series of the Deutsche Mathematiker Vereinigung (DMV). The aim seems to be to present the latest material in greater detail leaving the parts known to experts to be filled in by the references. Given this approach the book should make very interesting reading to anyone with an interest in Algebraic Number Theory. One who wishes to see the mathematics behind the computations could of course follow up the references–especially Pohst and Zossenhaus[1].

The subject matter of Algebraic Number Theory is finite field extensions of the field $\mathbf{Q}$ of rationals; such fields are called algebraic number fields. By the primitive element theorem any algebraic number field K takes the form $K = \mathbf{Q}\ [T]/(f(T))$, where $f(T)$ is an irreducible polynomial. The first invariant of such a polynomial and its associated field is its degree d. Within K we find the subring of algebraic integers $\mathcal{O}_K$ which consists of elements of K that satisfy a monic polynomial equation with integer coefficients. The primitive element theorem is not true in general for such rings and thus the best we can do is to write $\mathcal{O}_K$ as the quotient of the polynomial ring $\mathbf{Z}[T_1,...,T_r]$ by a suitable ideal. We have invariants naturally associated with the field K which are in fact invariants constructed using commutative algebra for such (integral) $\mathbf{Z}$-algebras. We have the discriminant which is the norm of the different or the relative dualising module. From a more modern standpoint we must also keep track of the component of the discriminant at 'infinity' which can be described as follows. Let r_1 denote the number of embeddings of K into the field $\mathbf{R}$ of real numbers and let r_2 denote the number of pairs of conjugate complex embeddings of K in the field $\mathbf{C}$ of complex numbers. Then we have the identity $r_1 + 2r_2 = d$ and we think of r_2 as the component of the discriminant at 'infinity' This is in any case an additional invariant associated with the field K. The ring $\mathcal{O}_K$ is a Dedekind domain and hence has an associated Class group C_K, *i.e.*, the group of all fractional ideals modulo the subgroup of principal fractional ideals. This group measures the extent to which $\mathcal{O}_K$ deviates from being a unique factorization domain. The subgroup $\mathcal{O}_K \otimes 1 \subset K \otimes_{\mathbf{Q}} \mathbf{R}$ is a lattice. The unit U_K of $\mathcal{O}_K$ is similarly contained in the group of units $(K \otimes_{\mathbf{Q}} \mathbf{R})^*$. The class group and the group of units thus give us two additional invariants of the field. Last but by no means the least we have the Galois group of the Galois closure of K over $\mathbf{Q}$ and its action on the various objects described above.

In essence we have described above the fundamental classical invariants associated with an algebraic number field K. We should hasten to point out that algebraic K-theory has thrown up a number of other invariants which are of arithmetic interest. There is also the Dedekind zeta-function $\zeta_K(s)$ of K which is a meromorphic function of one complex variable and contains within it a wealth of arithmetical data of interest to algebraic number theorists (conjecturally it contains a lot more).

The book gives us various algorithms to compute some of these fundamental classical invariants associated with an algebraic number field. After a brief chapter on motivation the book begins in Chapter II with the study of polynomials in one variable over finite fields. The methods of Berlekamp and Cantor–Zassenhaus are presented for the complete factorization of such polynomials. This factorization is not only essential to factoring polynomials over $\mathbf{Q}$ but is also required in the computations of the Galois group. In fact, computations over finite fields form the basis of most computations in Algebraic Number Theory.

Chapter III is devoted to the study of lattices in $\mathbb{R}^n$ or equivalently matrices. The fundamental reduction for the latter is the Hermite Normal Form and the fundamental result about the former is the convex body theorem due to Minkowski. These are stated and used as motivation for the Lenstra–Lenstra–Lovász reduction. This reduced form can be conveniently computed by a polynomial time algorithm also due to Lenstra–Lenstra–Lovász. While this reduction is not as strong as Minkowski reduction it is good enough for many computational purposes. Two applications are provided in Chapter III. First of all the powerful algorithm also due to Lenstra–Lenstra–Lovász which factors a polynomial over $\mathbb{Q}$ into irreducible factors in polynomial time. A modified version of this algorithm that avoids some needless computations is also presented. The second application is to compute all points in a lattice that lie within a given ellipsoid (or in modern language to compute points of bounded height).

The stage is now set for computations regarding algebraic number fields since we can recognize irreducible polynomials over $\mathbb{Q}$. Chapter IV introduces algebraic number fields and their rings of integers. The reader is introduced to the idea that to compute in an algebraic number field we need to specify its ring of integers *via* a basis $\{\omega 1 ..., \omega d\}$ for O_K as a free abelian group and the multiplication table given by

$$\omega_i \cdot \omega_j = \sum_{k=1}^{d} \Gamma_{ij}^{k} \omega^k .$$

To compute such a basis we begin with the ring $\mathbb{Z}\,[T]/(f(T))$ (by elementary reductions we can assume that $f(T)$ is monic with integer coefficients). The discriminant of this ring divides the discriminant of $\mathcal{O}_K$. Thus the problem can be reduced to the computation of the completion of $\mathcal{O}_K$ at each prime p dividing the discriminant. The rest of the computation is then an application of an algorithmic version of the Chinese Remainder Theorem. In Chapter V we find two methods (both due to Zassenhaus) to compute the completion of $\mathcal{O}_K$ at a prime p both base of an algorithmic version of Hensel's lemma. The first method (the Round-Two method) is simpler but works badly for polynomials of large degree. The second method (the Round-Four method) improves on the performance of the Round-Two method.

Chapters VI and VII which show how the groups C_K and U_K can be computed. For the group of units one uses an algorithmic version of the Dirichlet unit theorem replacing the Minkowski reduction thereby the Lenstra–Lenstra–Lovász reduction. Another approach is to generalize to arbitrary number fields the method of continued fractions for totally real number fields (where $r_1 = d$ or equivalently $r_2 = 0$). To compute the Class group C_K one again uses a bound obtained *via* Minkowski's convex bopy theorem for the 'size' of ideals that are required to generate C_K and the Lenstra–Lenstra–Lovász algorithm which makes the reduction to this bound explicit then gives us the relations in this ideal.

The first appendix deals with the Number Field Sieve approach to factorising large numbers—a procedure that is often required but computationally time-consuming. The second appendix describes the implementation KANT of the algorithms described in the book. Unfortunately, the package KANT is not available at the site which is mentioned in the book. An erratum to this effect should perhaps be included in the book.

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Representations of finite groups by C.Musili, Hindustan Book Agency (India), 17-0-B, Jawahar Nagar, Delhi 110 007, 1993, pp. 237, Rs 185.

Representation theory of finite groups now has a history of more than a hundred years, starting with the work of Frobenius over the fields of characteristic zero. Though this theory was greeted with initial scepticism from Burnside, its famous application to the proof of the solvability of groups of order divisible by only 2 primes (Theorem

3.9.6 in this book) convinced Burnside of its utility to the study of the structure of finite groups. Since then, the theory has been developed (and used) extensively: modular representation theory and block theory (Brauer, Green, Alperin, Broue, ...); representation theory of groups of Lie type (Green, Deligne–Lustig, Lustig, Curtis, Springer, ...) and Coxeter groups (Schur, Lustig, ...); representation theory of solvable groups (Dade, Isacs, Huppert, ...); and more recently, the representation theory of Artin algebras (Gabriel, Auslander–Reiten, ...). It has been a major tool in the study of finite groups (significantly in the recent, monumental, classification of finite simple groups) and the inspiration to initiate the study of the representations of topological and Lie groups at the beginning of this century.

This theory has several excellent expositions of various kinds: short and introductory[1-3]; encyclopedic[4]; state-of-the art 'now' kind of works on a particular topic[5]; works devoted to specific topics[6-8]; works with a definite slant (Benson[9], for example, with emphasis on cohomology) and some concise modern introductions[10-11] etc.

The book under review emerged out of a set of notes of the author used to teach M. Phil. students at the University of Hyderabad, India.

The author starts with a detailed study of the theory of semi-simple rings (Chapter 2); proves some basic facts about ordinary representations and characters (Chapter 3); discusses induced representations, elements of Clifford theory, Mackey irreducibility criteria, Frobenius reciprocity, Wigner–Mackey method of little groups (Chapter 4); and the ordinary representation theory of specific groups S_n, A_n and D_n (Chapters 5–8).

The merit of this book is that it has plenty of details, exercises and quizzes ('true or false statements') and so could be used for self-study. As the author says, the material on B_n and D_n is not easily available in book form.

Now, for some quibbles: the list of definitions and theorems with occasional (mostly, trivial) examples in Chapter 1 and the theory of semi-simple rings is in greater generality and detail than necessary for the representation theory developed in the book. In consequence, one has to go through a third of the book to come across the definition of a representation, the topic of the book. This is not very satisfactory also because the utility of the section on the simple algebras and the Brauer group is limited by the lack of any reference to the proofs of the theorems stated.

While the first three of the aims the author sets himself (in the preface) are adequately met, given the intended audience, a few pages spent on indicating the main themes in the theory not discussed in the text (rather than just a list of references) would perhaps have better met the fourth stated aim of helping the student 'look ahead and around'.

The book is well brought out and proof-read (an occasional 'parallels', p.79, does not do much harm). The affordability and the careful job of the author should commend this book to a wide audience among (at least Indian) students. The trim series in which this book appears so far seems to have maintained these features, thus meeting a need, however, partially, felt long since among the students of mathematics in India.

On the whole, I recommend this book for self-study and/or a first course on representations.

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Infinite dimensional Morse theory and multiple solution problems by Kung Ching Chang, Birkhauser Verlag AG AG, P.O. Box 133, CH-4010 Basel, Switzerland, 1993, pp.312, SFr. 118.

In dealing with the existence of solutions of nonlinear differential equations some of the main tools available are: 1. Theory of fixed points, 2. Method of sub and super solutions, 3. Bifurcation theory, and 4. Variational methods.

However, in many situations one is able to find at least one solution just by inspection and in most cases the solution thus found is not the relevant solution one is looking for. For example, in the case of autonomous ODE one is interested in finding nontrivial periodic solutions. In dealing with nonlinear scalar field equations that lead to semilinear elliptic boundary-value problems (solitons in nonlinear Klein–Gordan or Schrödinger equations, etc.), the nonlinear elliptic equations take the form

$$-\Delta u = F(u) \text{ in } \mathbb{R}^n$$

with conditions at infinity. Usually $F(0) = 0$ in these situations and $u \equiv 0$ will always be a solution. But one is interested in ground state solution which is different from $u \equiv 0$.

In dealing with Hamiltonian systems, given $T > 0$, one can ask the question if there exists a solution with period T. Also one can prescribe an energy and ask if one can find a periodic solution with this energy.

The above-mentioned situations clearly indicate to the need to find methods which are useful to find multiple solutions for a wide variety of problems.

Use of Degree Theory is quite well known in finding multiple solutions. However, even if the problem under consideration has a variational structure, to use 'Degree' one needs a priori bounds for solutions and these are not always obtainable. Hence there is definite need to find other methods if one wants to look the question of multiple solutions.

Morse Theory is a very appropriate tool to use when the problem has a variational structure. The book under review covers in a very elaborate and interesting way a wide range of applications of Morse Theory in finding multiple solutions of differential equations. The main advantages in using Morse Theory are, there is no need to have any a priori bounds, but one needs only a certain compactness condition which can be verified even if there is no a priori bound. Also, the theory gives information about each critical point.

The basis object of Morse Theory is to relate the topological type of critical points of a functional and the topological structure of the manifold on which the functional is defined.

The topological type of a critical point is described by the critical groups of Morse. The topological structures of the manifold is described by its Betti numbers. As already said Morse Theory relates these.

Chapter 1 of the book by K.C Chang starts with a basis review of algebraic topology required and goes on to develop infinite dimension Morse Theory. The main results are various deformation lemmas (the crux of infinite dimensional theory), Morse inequalities and Morse handle body theory. Also some extensions and generalization are discussed. It must be mentioned that, it is to prove the deformation results that one needs the compactness (the so-called Palais–Smale condition), mentioned earlier. Also the deformation results are proved using the negative gradient flow.

In Chapter II various abstract critical point theorems, specially those of linking type are looked at, from the point of view of the Morse Index of the critical points one obtains using these critical point theorems. Also these critical point theorems are proved using More Theory.

Chapter III deals with application of Morse theory to study multiple solutions of different types of semilinear elliptic problems. The classification of these elliptic problems depends on the kind of nonlinearity involved.

Chapter IV deals with periodic solutions for Hamiltonian systems. The results here complement those of J. Mawhin and M. Willem, *Critical point theory and Hamiltonian systems* (Springer-Verlag). Some of the main results in the book under review deal with second-order systems with singular potentials, the double pendulum equation, Arnold conjectures on symplectic fixed points and on Lagrangian intersections.

Chapter V deals with the application to problems of harmonic maps and minimal surfaces. The problem of harmonic maps presents very serious difficulties due to lack of compactness. In the context of what has been said earlier (while commenting on the first chapter), the Palais–Smale condition does not hold. Hence there is need to prove the deformation results in these situations using different approach. Recall that the deformation results in Chapter I are proved using negative gradient flow; however, in this chapter the deformation results are obtained using heat flow. This point is one of the highlights of the book.

Concluding comments. The book deals with a very active and highly interesting research topic. The book gives very elaborate details, and can be of use to experts and researchers only.

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