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Laminar–turbulent transition modelling at IISc

J. DEY, O. N. RAMESH, M. JAHANMIRI, S. V. S. PHANI KUMAR AND A. PRABHU
Department of Aerospace Engineering, Indian Institute of Science, Bangalore 560 012, India.

Abstract

An overview of laminar–turbulent transition zone modelling research at IISc is presented. The linear-combination model, based on the principle of combining mean laminar and turbulent profiles, in proportions determined by the intermittency, has been found to provide an excellent representation of the transition zone in two-dimensional incompressible boundary-layer flows. Ongoing investigations in a three-dimensional constant pressure flow with lateral streamline divergence indicate that many transitional characteristics are similar to those in two-dimensional constant-pressure flows for modelling purposes.

Keywords: Boundary layer, incompressible flow, laminar–turbulent transition, turbulent spot, spot propagation characteristics, spot structure, intermittency, transition zone modelling, streamline divergence, pipe, channel, axisymmetric body.

1. Introduction

The need for an appropriate laminar–turbulent transition zone model in technological applications has, if anything, become more important in recent years, leading to extensive research work addressing this aspect of viscous flow computation around a body, such as aircraft wing or turbine blade. In fact, Cebeci *et al.*[1], have called the representation of transition 'perhaps the most important immediate modelling problem'. Although a variety of transition zone models are now available, the principal means of transition-zone modelling since the late 1950s is through Narasimha's[2] intermittency distribution and the Dhawan–Narasimha[3] transition zone model originating from the Aeronautical (now Aerospace) Engineering Department here at IISc. This earliest transition zone model for constant-pressure flows is capable of predicting all parameters (including mean velocity profiles) in the transition zone provided the onset of transition is given.

The transition zone of concern here is the region of flow that begins with the appearance of turbulent spots and ends through an asymptotic approach to a fully turbulent state far downstream with the flow in this region having intermittent turbulent fluctuations due to the passage of spots[4]. Generally speaking, the events leading to such type of transition in a large class of boundary-layer flows are as follows. As shown in Fig. 1, the boundary layer on any surface is steady and laminar for some distance from the leading edge. As the Reynolds number for instability is exceeded, two-dimensional (2D) Tollmien–Schlichting waves set in, followed by three-dimensional (3D) secondary instability waves further downstream. With subsequent growth of these waves, the flow

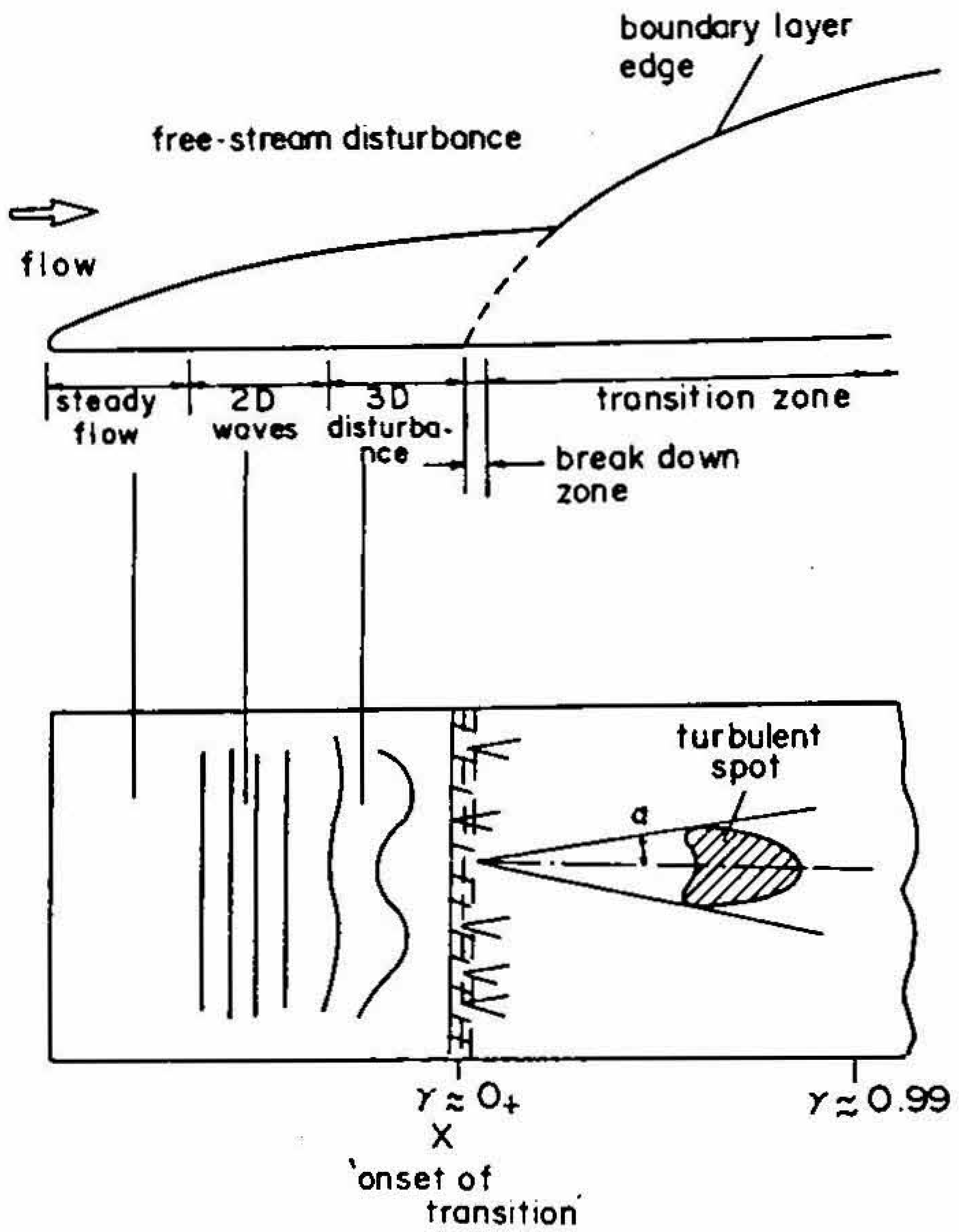

FIG. 1. Schematic representation of various stages of laminar–turbulent transition[17]. α is the spot spread angle; γ is the intermittency.

eventually 'breaks down' with the appearance of turbulent spots leading to intermittent turbulent fluctuations beginning at a streamwise location called the 'onset' of transition; this onset location is denoted by x_t in Fig. 1. Finally, the flow attains a fully turbulent state further downstream. It may be noted that some of these events can even be completely bypassed in highly disturbed environment[5]. Unfortunately, the basic fluid-dynamical problems associated with many of the above-mentioned events remain poorly understood, as the physical processes preceding, during and following the onset of transition are highly complex, as reviewed in the past[5–8]; the reader may also refer to the recent workshop[9] on 'end-stage transition' in this regard. A somewhat oversimplified view of a turbulent spot is that it has a sharp reference boundary within which the flow is fully turbulent, the flow elsewhere being laminar. In terms of the intermittency factor (γ), which is defined as the fraction of time the flow is turbulent, the transition zone may be conveniently defined as beginning where γ has just departed from zero and extending up to where it is nearly unity ($\gamma \approx 0.99$).

The emphasis in the development of transition zone model here at IISc has been, following Dhawan and Narasimha[3], on linear-combination models rather than algebraic, differential or higher-order models, as classified by Narasimha[7]. A linear-combination model considers the mean flow during transition as a linear combination, in the proportion $(1-\gamma):\gamma$, of the mean flow in the laminar and turbulent boundary layers, respectively. In algebraic models, one tackles the time-averaged equations of motion with appropriate model for the Reynolds stress that is gradually turned on in the transition zone in proportion to intermittency (Cebeci and Smith[10], for example). Differential models also use the Reynolds-averaged equations of motion with either one- or two-equation turbulent closure models like the popular K–ε (turbulent kinetic energy–dissipation rate) model (*e.g.*, Vancoillie[11], Wang *et al.*[12]). Higher-order models, on the other hand, involve direct Navier–Stokes simulations (*e.g.*, Wray and Hussaini[13], Orszag and Patera[14], Laurien and Kleiser[15]). With the advent of high-speed computers such simulations may become ubiquitous in future, but they usually require very large computing time. (Reshotko[16] mentions a simulation requiring 8000 hours of CRAY–YMP time.) Also, simulations have not yet covered the transition zone[17], as outlined above, and have to do better before they can be used as 'validation experiments for simpler methods'[16]. The reader is referred to a review of the available transition zone models for 2D incompressible boundary layer flows by Narasimha and Dey[17]. Investigations in the past[18–20] have clearly demonstrated that the Dhawan–Narasimha-type linear-combination model performs as well as other models; in certain cases it is even superior[18]. In this paper, we make a brief presentation of the development in transition-zone modelling research at IISc. The plan of the paper is as follows. Section 2 deals with 2D incompressible flows; the results of investigation on 3D flows are reported in Section 3. In Section 4 investigations in ducts and axisymmetric bodies are discussed.

2. Two-dimensional flows

In this section a brief commentary on the linear–combination-type transition zone model developed for 2D incompressible flows is presented.

2.1. *Earlier work: constant-pressure flows*

For constant-pressure flows, considering the laminar and turbulent boundary layers to originate from the stagnation point and the onset location further downstream, respectively, Dhawan and Narasimha[3] take the mean velocity, u, and skin friction coefficient, c_f, respectively, as

$$u = (1-\gamma)\,u_{\mathrm{L}} + \gamma u_{\mathrm{T}}, \tag{1}$$

$$c_{\mathrm{f}} = (1-\gamma)\,c_{\mathrm{fL}} + \gamma c_{\mathrm{fT}}. \tag{2}$$

Here suffixes L and T refer, respectively, to laminar and turbulent boundary layers, each starting from its respective origin, and the velocity is normalized with the free-stream velocity $U(x)$, x denoting the streamwise distance. The intermittency distribution in eqns (1) and (2) is the universal intermittency distribution of Narasimha[2] based on the hypothesis of concentrated breakdown, *i.e.*, all spots appear at the onset location (x_t) over the entire span of the flow. The intermittency distribution is given by

$$\gamma = 1-\exp(-0.41\xi^2), \quad \xi = (x-x_t)/\lambda, \tag{3}$$

$$\lambda = x(\gamma = 0.75) - x(\gamma = 0.25) \tag{4}$$

is a measure of the transition zone length. This γ-distribution is now well established by various measurements in constant-pressure flows[7, 21, 22].

Dhawan and Narasimha[3] also correlate the transition zone length λ to x_t by the relation

$$Re_\lambda = 5Re_{xt}^{0.8}, \tag{5}$$

where Re denotes the Reynolds number. Further, they draw the interesting conclusion that the extent of the transition zone varies as the inverse square root of the breakdown rate n (defined by the number of spots born/time and spanwise length at the point of breakdown), so that

$$\tilde{n} = n\sigma\nu^2/U^3 = 0.41Re_\lambda^{-2} = 0.016Re_{xt}^{-1.6}. \tag{6}$$

Here σ is a nondimensional spot propagation parameter and ν is the kinematic viscosity. There was no attempt by Dhawan and Narasimha to predict x_t. Using the measured intermittency data, x_t was inferred by extrapolating the linear variation of $F(\gamma)$ $(=[-\ln(1-\gamma)]^{1/2})$ with x, a consequence of eqn (3), to the point where $F(\gamma)=0$. This process is particularly desirable as low and high values of γ are difficult to measure[7].

Dhawan and Narasimha[3] have shown by extensive comparison with experiments that eqns (1) and (2) do provide an excellent description of the mean velocity profiles, skin friction, and all integral parameters in the transition zone in constant-pressure flows.

2.2. *Recent studies: pressure gradient flows*

As a follow-up of the transition zone model mentioned above, subsequent investigations were aimed at developing a transition zone model that is simple and useful at the design level. Toward this end, the effect of pressure gradients on the γ-distribution has been studied[23] and the zone length is prescribed[7] from an appropriate new nondimensional spot formation rate, N, rather than using the relations (5) and (6).

Regarding γ-distribution in pressure gradient flows, Narasimha *et al.*[23] observe that the γ-distribution (3) is also valid in mild pressure gradients. In stronger pressure gradients, an important parameter in the γ-distribution, however, is the location of the pressure gradient relative to the onset: lengthening of the transition zone is observed for the case of a favourable pressure gradient applied near the onset, and a 'kink' is seen in the $F(\gamma)$ *vs* x plot (see Fig. 2) due to what has been called a 'subtransition'[7]. In such cases, nevertheless, the γ-distribution (3) holds in segments, and hence was used by Dey and Narasimha[18] in their estimation of transitional boundary-layer parameters in nonzero pressure gradient flows.

The prescription of the zone length from the appropriate nondimensional spot formation rate is based on the observation[7] that the breakdown rate scales with the local boundary-layer thickness, δ, and ν. Thus, the new nondimensional parameter N is

$$N \equiv n\sigma\theta_t^3/\nu, \tag{7}$$

where θ_t is the momentum thickness at x_t and is preferred to δ as it is more precisely defined. This scaling follows from the basic γ-distribution

$$\gamma = 1 - \exp(-n\sigma(x - x_t)^2/U), \tag{8}$$

the basis for eqn (3), and the correlation

$$Re_\lambda = C\, Re_{xt}^{0.75}, \tag{9}$$

where C is a constant. The correlation (9), though just an improvement upon eqn (5) (slight change in the index from 0.8 to 0.75), is remarkable, for it provides a powerful scaling in eqn (7). It is easy to show from eqns (7) and (8) that

$$N = 0.41\, Re_{\theta t}^3 / Re_\lambda^2. \tag{10}$$

This parameter is then used to study the effect of free-stream turbulence[7, 24] and pressure gradient[7, 25–27] on the transition zone length in a more meaningful way than before.

It may be noted that the prediction of onset being an open problem, correlations like that by Govindarajan and Narasimha[28] to predict x_t are being currently used; elsewhere, many other correlations have also been proposed[20, 27].

With the zone length from the nondimensional spot formation rate, Dey and Narasimha[18] successfully extended the Dhawan–Narasimha model to 2D incompressible pressure gradient flows, taking appropriate care of subtransitions in the intermittency distribution whenever they arise. For the computational domains shown in Fig. 3, the integral method of Dey and Narasimha[18] uses the estimation of the laminar parameters using a modified Thwaites method[29] and turbulent parameters using the lag-entrainment method of Green *et al.*[30] Some typical examples elucidating the successful prediction of

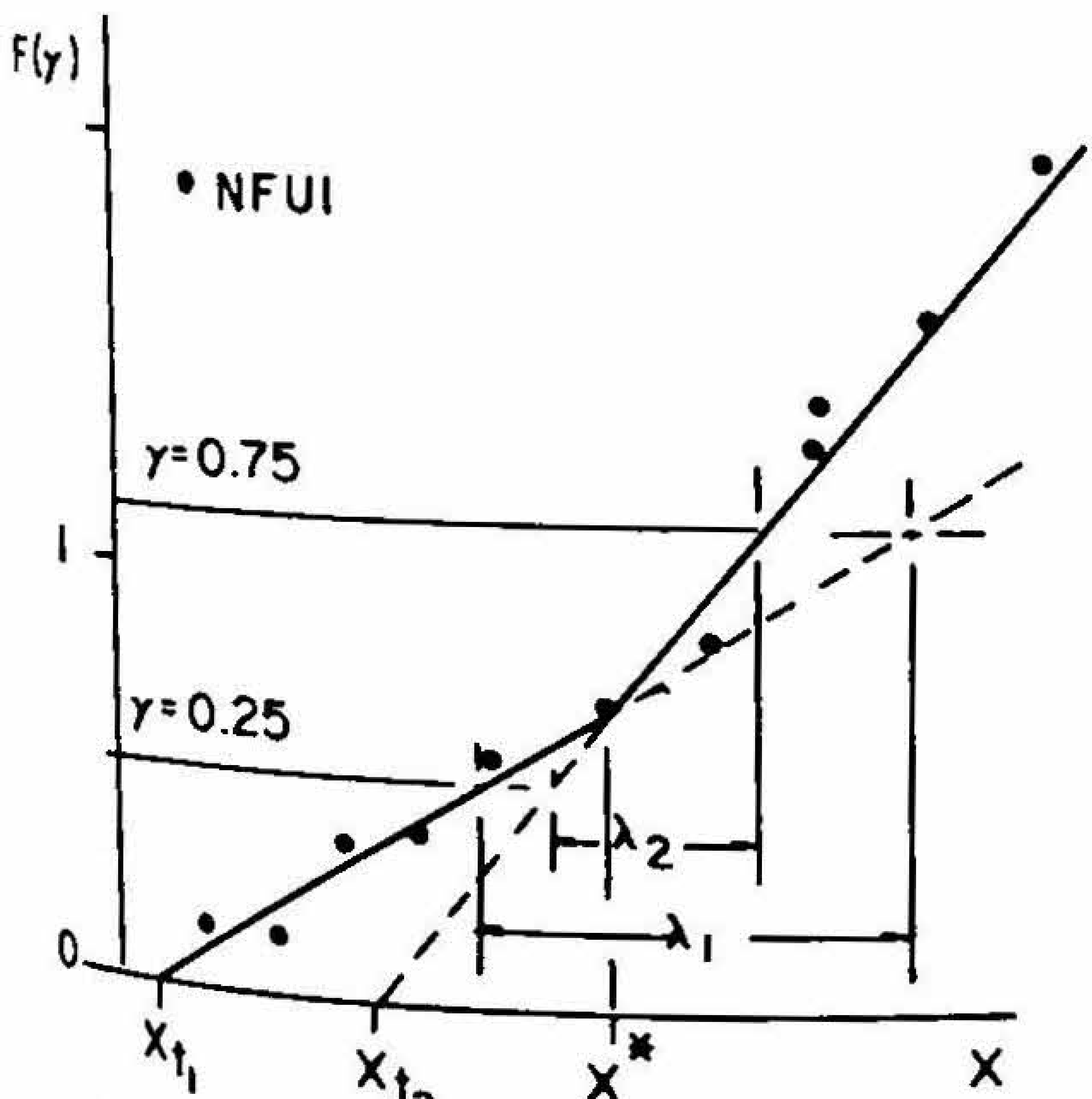

FIG. 2. Segmented linear variation of $F(\gamma)$ for the favourable pressure gradient flow (NFU1) of Narasimha *et al.*[2] x^* is the location of the kink.

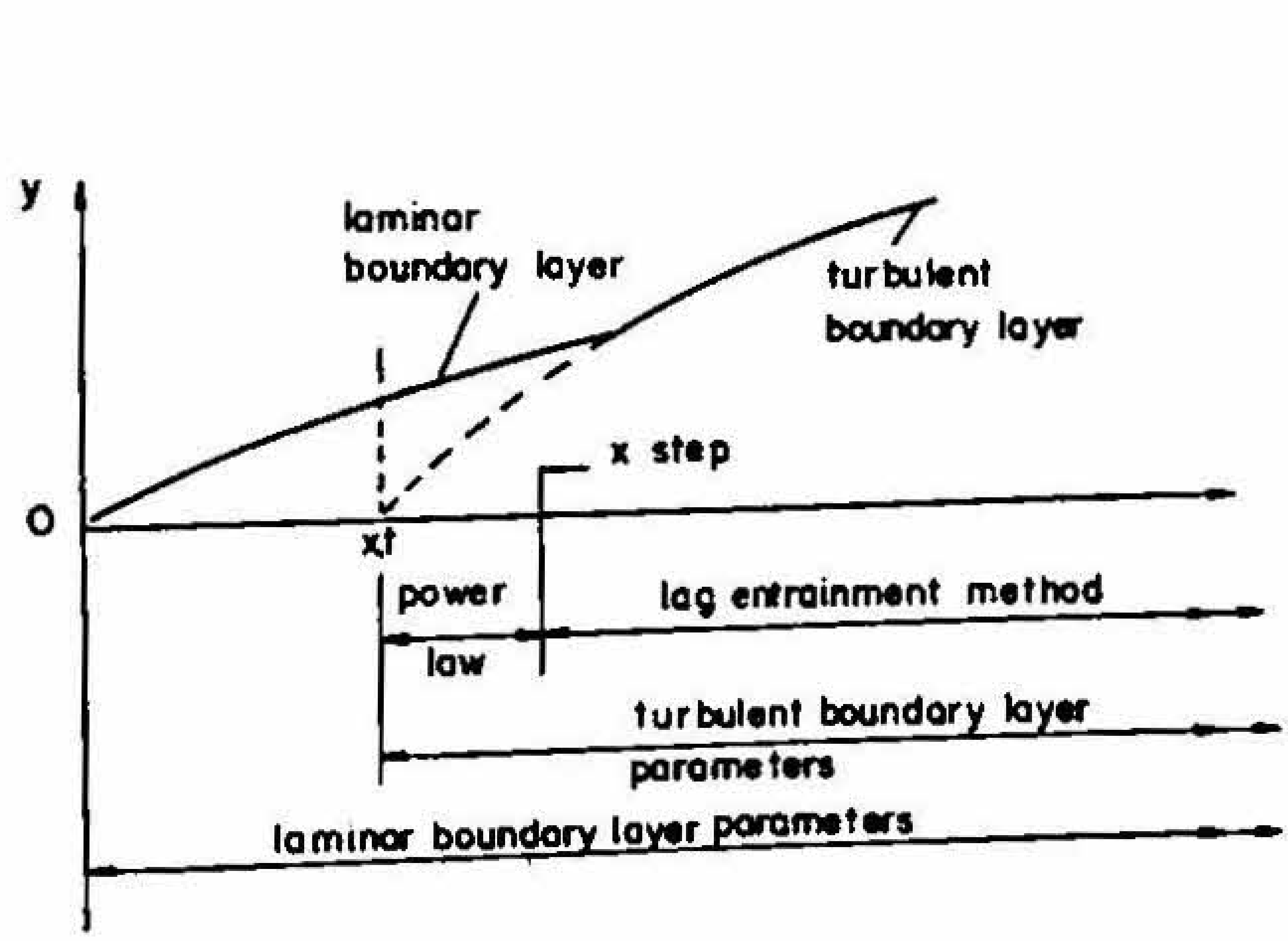

FIG. 3. Various computational regimes as adopted by Dey and Narasimha[18].

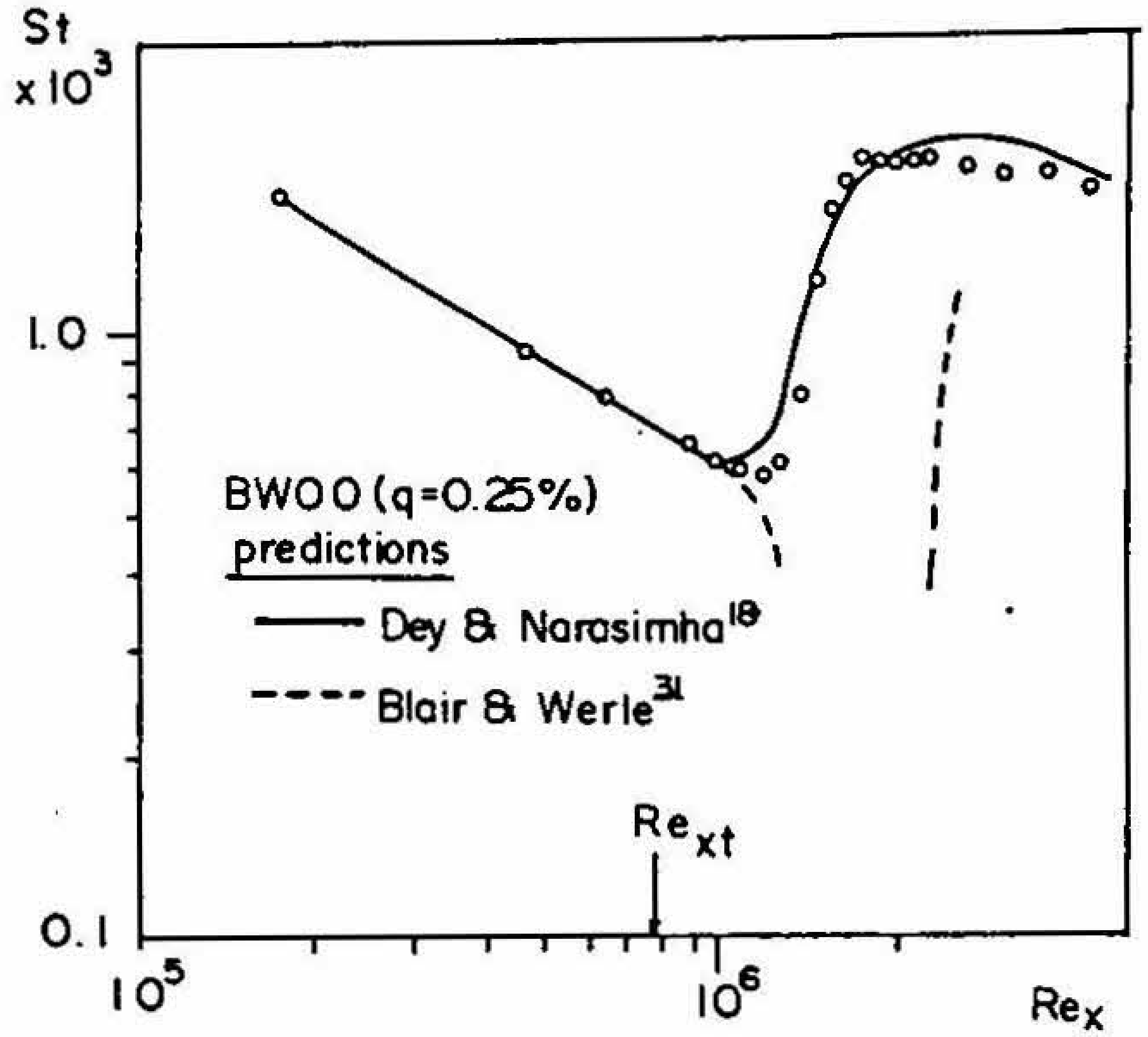

FIG. 4. An example of Dey and Narasimha's comparison of their prediction with measurements (open symbol) and with the prediction of the Stanton number by Blair and Werle; BW00 is a constant-pressure flow[31] at a free-stream turbulence level $q = 0.25\%$.

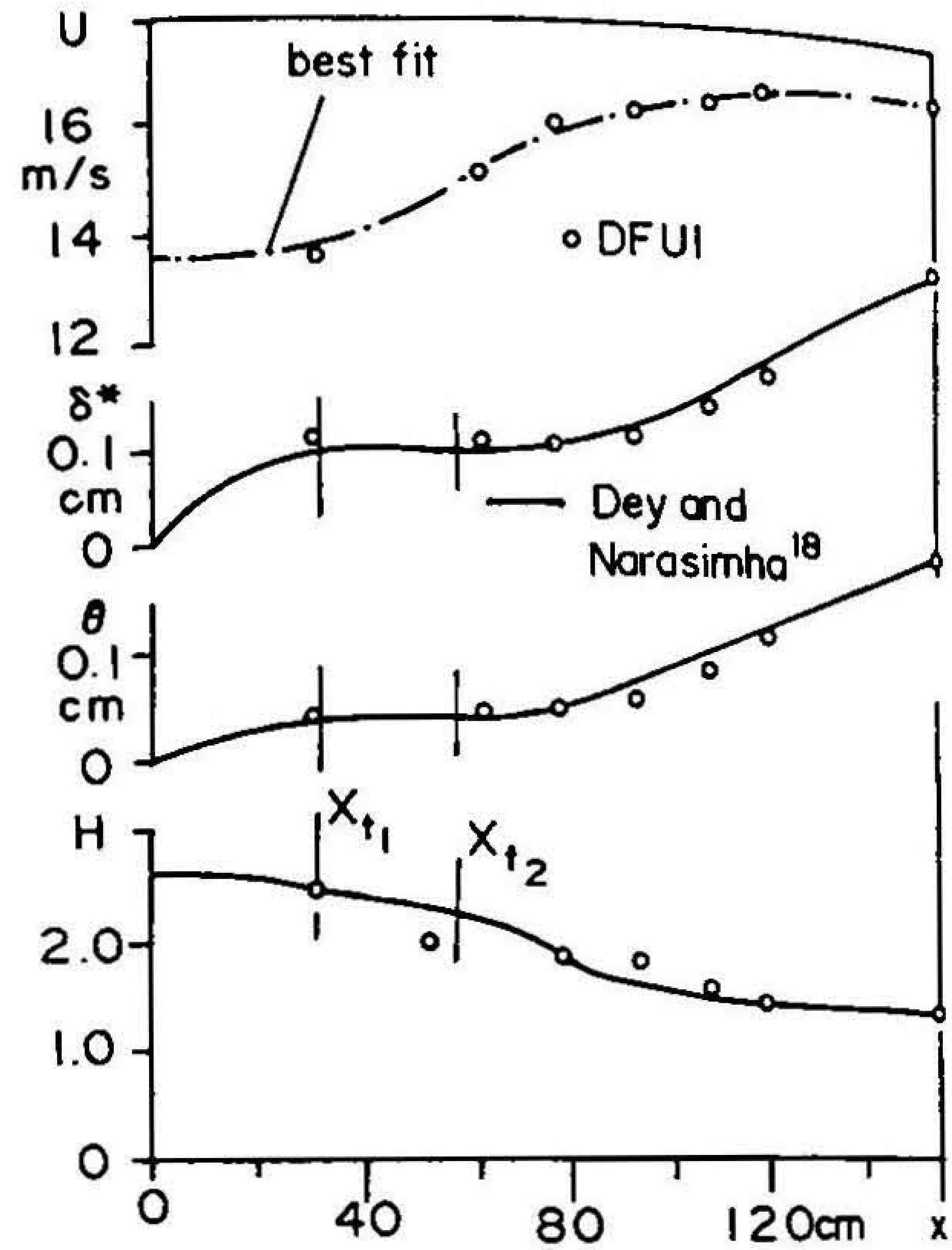

FIG. 5. An example of Dey and Narasimha's prediction of δ^*, θ and H for the favourable pressure gradient flow (DFU1) of Narasimha et al[23].

the transitional boundary-layer parameters by Dey and Narasimha[18] are shown in Fig. 4 and 5. Figure 4 shows Dey and Narasimha's[18] comparison of their prediction with Blair and Werle's[31] measurement of the Stanton number, St, and prediction based on a differential model for a constant-pressure flow (BW00) at a free-stream turbulence level $q = 0.25\%$. A similar comparison is shown in Fig. 5 for the measurements of displacement thickness (δ^*), momentum thickness (θ) and shape parameter ($H = \delta^*/\theta$) by Narasimha et al[23] for their favourable pressure gradient flow (DFU1). It can be seen in Figs 4 and 5 that this linear-combination integral model provides an adequate representation of the transition zone in 2D incompressible flows.

Recent investigations[19, 20, 27] (with modified prediction schemes for the onset or zone length) have further demonstrated the effectiveness of the Dhawan–Narasimha model in predicting the transitional boundary-layer parameters in 2D incompressible flows.

3. Three-dimensional flows

In this section we present some results of the ongoing investigations aimed at modelling the transition zone in 3D flows. Transition zone models for 3D flows are rare mainly because such flows are usually associated with many parameters like pressure gradient, streamline divergence/convergence, streamline curvature, etc., rendering them difficult for a meaningful study. The present investigation has been initiated to study the effect of streamline divergence alone on transition, as streamline divergence and convergence are known to produce lateral straining effects on turbulent boundary-layer development[32, 33], and may lead to large effects on transition characteristics as well. A 3D constant-pressure diverging flow is created for this purpose. The propagation characteristics of artificially generated turbulent spots and the intermittency distribution in such a flow have been studied, as reported below. Potti[34] has studied the effect of divergence on transition in a flow field created by an axisymmetric body on a flat plate. This flow, however, is not free from longitudinal pressure gradient, as in the present study, which pays more attention to the structure and propagation of a turbulent spot. Potti[34] also finds that the linear-combination model is adequate for describing the transition zone.

3.1. *Brief details of experimental set-up*

Details of the experimental set-up and signal analysis have been reported elsewhere[35-37]. Briefly, measurements are made on a flat plate placed in a distorted duct (see Fig. 6) having one diverging side wall at 10° and a converging tunnel roof at 5.9° maintaining a nearly constant flow area with diverging streamlines; the pressure in the test section is constant within ±2% of the mean dynamic pressure. Loudspeaker-driven artificial spot is

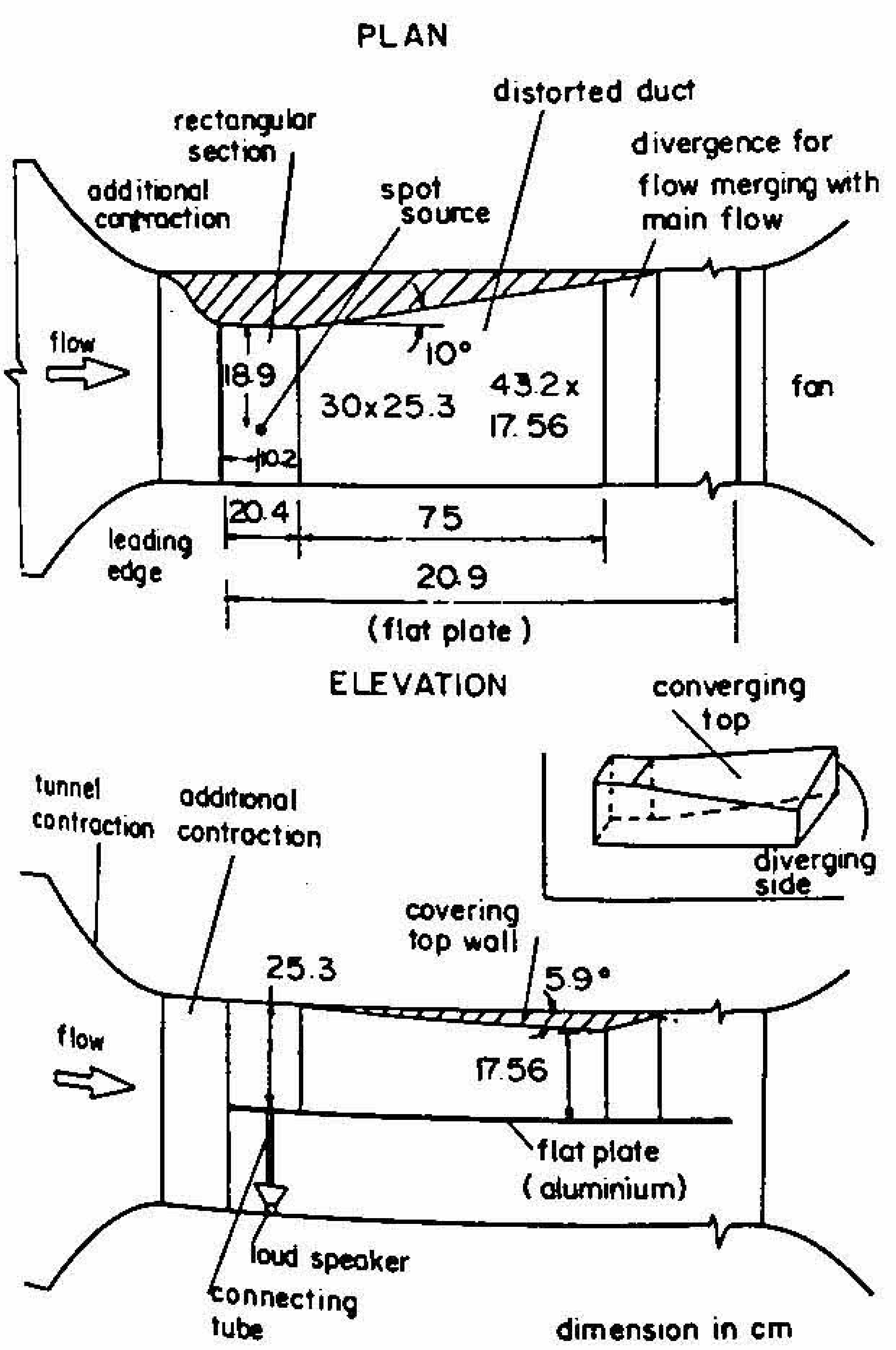

FIG. 6. Schematic view of the experimental set-up for 3D constant-pressure diverging flow.

created through a 1 mm hole on the flat plate at a distance of 10.2 cm downstream (see Fig. 6) of the plate leading edge. A constant-temperature hot wire anemometer has been used for velocity measurements. Signal processing involves double differentiation ($D \equiv d^2e'/dt^2$) of the digitized time series of the voltage fluctuation (e') and squaring (D^2) of the double differentiated signal. The threshold for the turbulent/nonturbulent decision is obtained from the probability density distribution (D^2 *vs* the frequency of occurrence (f) of that magnitude). For filtering out spurious data, a hold time of 5 × sampling time, based on the Kolmogoroff scale for small eddies, has been used.

Spot shape measurements are made at a constant phase inferred from various measurements of leading- and trailing-edge velocities at different stream- and spanwise locations. The mean structure of the spot and its substructures has been investigated by Jahanmiri[35, 38] and Phani Kumar[37]. Natural transition is investigated in the same set-up by blocking the hole used for creating the turbulent spot; the intermittency distribution and the mean velocity profiles during transition have been measured[36]. The free-stream velocity in these experiments is in the range of 6–12 m/s.

3.2. *Results*

A very interesting feature of the spot propagation characteristics studied by Jahanmiri *et al.*[39] is that streamline divergence does not alter the overall spot propagation characteristics from those in 2D constant-pressure flows. As in 2D flows[40–42], the wedge angle of the spot envelope is about 20° (see Fig. 7); similarly, the celerities of the leading and

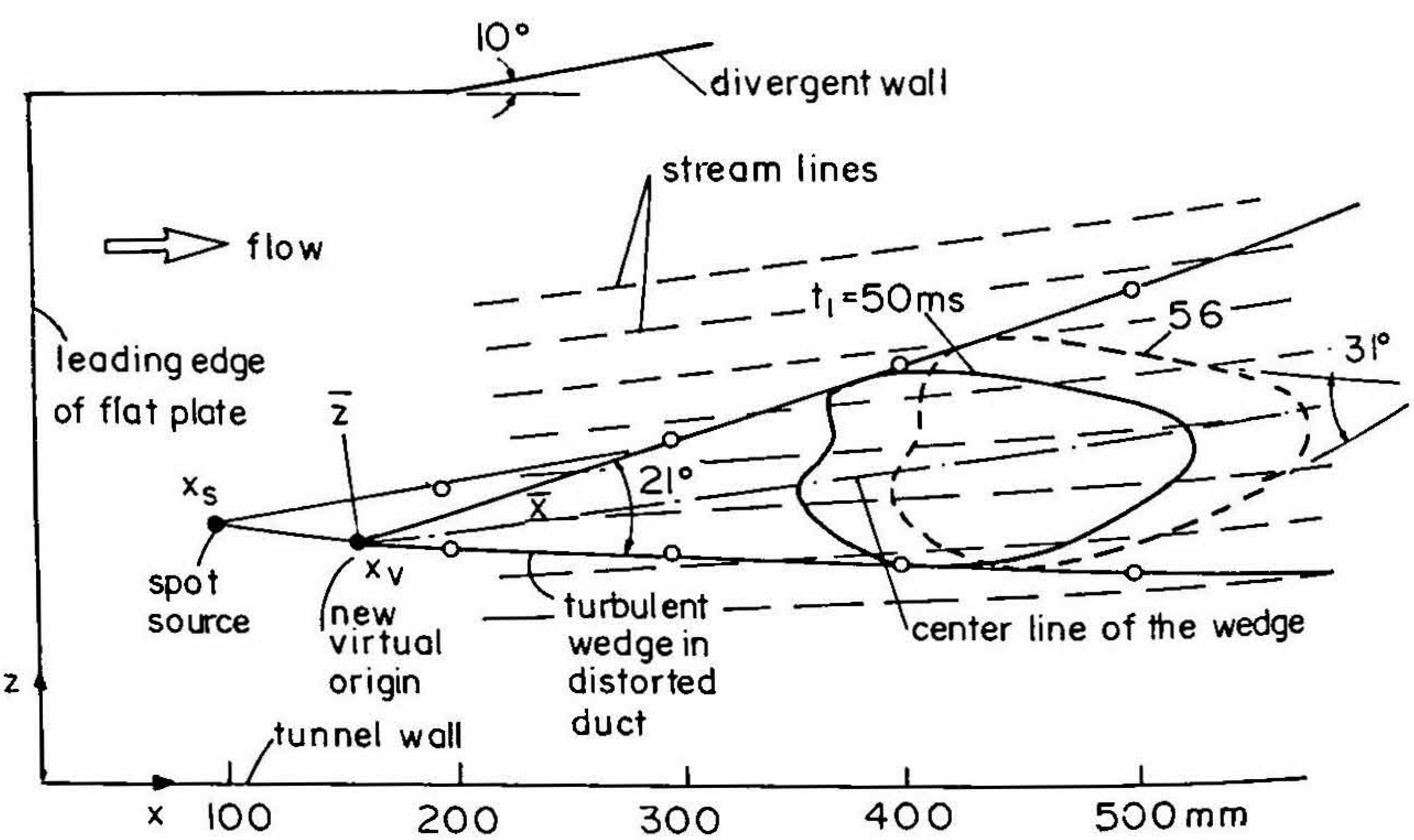

FIG. 7. Spanwise spot envelope in a diverging flow; spot shapes are at constant time[35].

trailing edge of the spot are respectively $0.85U$ and $0.56U$. As shown in Fig. 7, the spot shape, however, is distorted, and it is for the first time that such a distorted spot is seen[35, 39] in contrast to the usual symmetrical spots studied extensively by many investigators in the past[40–44]. This distortion in spot shape is attributed to the flow asymmetry; in addition, the effect of centrifugal force the flow encounters while entering the diverging section also contributes to the spot distortion. It is also found that the spot propagation wedge does not make a constant angle with the local streamlines (see Fig. 7), as in 2D flows, in contrast to the assumptions made by Chen and Tyson[45] in the development of their intermittency model. As a consequence of the distortion of spot shape, the spot overhang and the spot height variation in the spanwise direction are nonsymmetrical[35].

The study on the internal structure of the distorted spot[37] using variable interval time averaging (VITA) technique reveals that the structures within the spot increase in number in the streamwise direction at a rate similar to that of 2D flows[46, 47]. The asymmetry of the spot is confirmed by the variation in the number of events in the spot in the spanwise direction. The event frequency—the number of structures per unit time of activity in a spot—of these structures does not vary much in the streamwise direction along the centreline of the spot, whereas the mean intensity of the events decreases along this direction.

In Fig. 8 the measured γ-distributions in the diverging section are compared with Narasimha's model[2] [eqn (3)]; the intermittency is measured along the streamlines ST1, ST2 and ST3 shown in the inset. The streamwise distance x in eqn (3) is measured from reference locations, which are at a distance of 300 mm from the flat plate leading edge (see Fig. 8). It can be seen that the 2D intermittency model [eqn (3)] shows remarkable agreement with the measured data even in the diverging flows studied here.

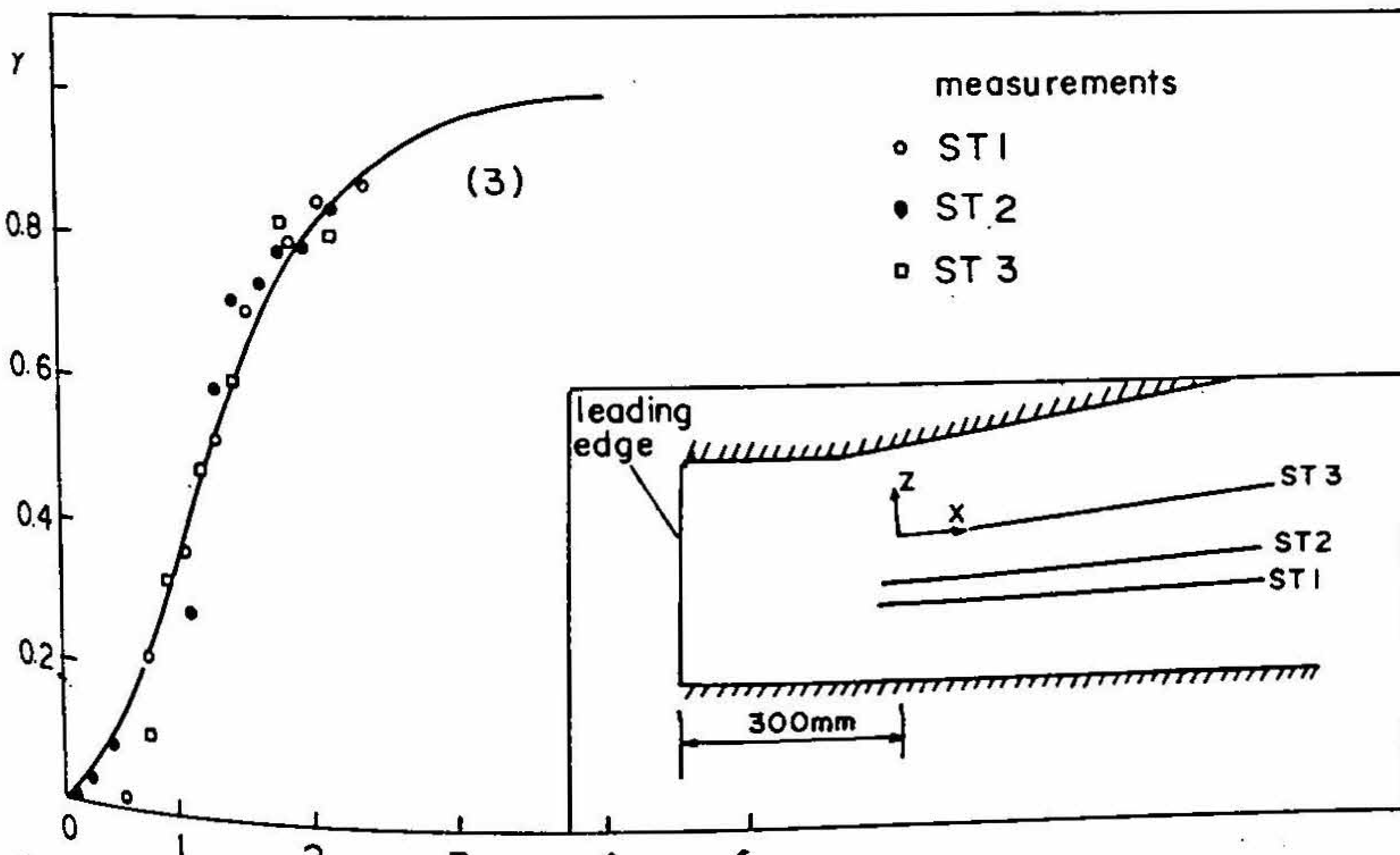

FIG. 8. Comparison of the measured γ-distribution in a diverging flow with Narasimha's intermittency model[2]. (Inset: streamlines along which intermittency is measured.)

As the intermittency distribution and many gross features of the spot propagation characteristics like spot wedge angle and celerities have a lot in common with 2D flows, an interesting indication here is that the Dhawan–Narasimha model may also be as effective in 3D constant-pressure diverging flow studied here as in 2D flows. In fact, the similarity between the Blasius flow and the laminar flow in the present geometry noted by Ramesh *et al.*[48] further indicates that this 3D constant-pressure diverging flow may be similar to a 2D constant pressure flow as far as the transition-zone modelling is concerned.

4. Transition zone in other flows

Apart from the transition-zone modelling work for 2D flows mentioned in the previous sections, transition in pipe[49], channel[50] and axisymmetric bodies[51] has also been investigated, as reviewed briefly in this section.

Pantulu[49] finds that the γ-distribution in a pipe when it is filled by turbulent slugs follows the one-dimensional law,

$$\gamma = 1 - \exp(-1.1\xi). \tag{11}$$

This γ-distribution is also found to be valid in a channel[50].

On an axisymmetric body of circular cross-section, Rao's[51] investigation reveals some interesting results. Rao finds that a spot generated on the cylinder surface initially propagates as in plane 2D flows, mentioned in Section 2, up to a certain distance. It then wraps around the body like a 'sleeve', which then propagates in one dimension. As a result, the γ-distribution is found to follow the 2D law [eqn (3)] over some distance and the 1D law [eqn (11)] thereafter.

5. Conclusions

In this paper we have presented a survey of transition-zone modelling research undertaken at IISc till date. The focus has been on the past and present work with respect to 2D and 3D incompressible flows. The linear–combination model developed for 2D flows provides an adequate representation of the transition zone. For 3D flows with lateral divergence alone, many transitional boundary layer characteristics are found to be similar to 2D constant-pressure flows.

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References

1. Cebeci, T., Stewartson, K. and Whitelaw, J. H. Calculation of two-dimensional flows past airfoils, *Proc. Numerical and Physical Aspects of Aerodynamic Flows II* (T. Cebecci, ed.), 1983, pp. 1–4, Springer.
2. Narasimha, R. On the distribution of intermittency in the transition region of a boundary layer, *J. Aeronaut. Sci.*, 1957, **24**, 711–712.

3. DHAWAN, S. AND NARASIMHA, R. — Some properties of boundary layer flow during the transition from laminar to turbulent motion, *J. Fluid Mech.*, 1958, **3**, 418–436.

4. EMMONS, H. W. — The laminar–turbulent transition in a boundary layer, Part I, *J. Aeronaut. Sci.*, 1951, **18**, 490–498.

5. MORKOVIN, M. — *On the many faces of transition, Viscous drag reduction* (C. S. Wells, ed.), 1969, pp. 1–31, Plenum.

6. TANI, I. — Boundary layer transition, *A. Rev. Fluid Mech.*, 1969, **1**, 169–196.

7. NARASIMHA, R. — The laminar–turbulent transition zone in the boundary layer, *Prog. Aero. Sci.*, 1985, **22**, 29–80.

8. HERBERT, T. — Secondary instabilities of boundary layers, *A. Rev. Fluid Mech.*, 1988, **20**, 487–526.

9. LAGRAFF, J. E. — *Proc. Workshop on End-stage Boundary Layer Transition*, 15–18 August 1993, Syracuse, NY.

10. CEBECI, T. AND SMITH, A. M. O. — *Analysis of turbulent boundary layers*, 1974, Academic Press.

11. VANCOILLIE, G. — A turbulence model for the numerical simulation of transitional boundary layers, *Proc. 2nd IUTAM Symp. on Laminar–Turbulent Transition* (V. V. Kozlov, ed.), 1984, pp. 87–92. Springer-Verlag.

12. WANG, J. H., JEN, H. F. AND HARTEL, E. O. — Airfoil heat transfer calculation using a low Reynolds number version of a two-equation turbulence model, *J. Engng Gas Turbine Power*, 1985, **107**, 60–67.

13. WRAY, A. AND HUSSAINI, M. Y. — Numerical experiments in boundary-layer stability, *Proc. R. Soc. Lond. A*, 1984, **392**, 373–389.

14. ORSZAG, S. AND PATERA, A. T. — Secondary instability of wall-bounded shear flows, *J. Fluid Mech.*, 1983, **128**, 347–385.

15. LAURIEN, E. AND KLEISER, L. — Numerical simulation of boundary-layer transition and transition control, *J. Fluid Mech.*, 1989, **199**, 403–440.

16. RESHOTKO, E. — Transition zone modelling, *Proc. Workshop on End-stage Boundary Layer Transition*, 15–18 August, 1993, Syracuse, NY.

17. NARASIMHA, R. AND DEY, J. — Transition zone models for 2D boundary layers: A review, *Sadhana*, 1989, **14**, 93–120.

18. DEY, J. AND NARASIMHA, R. — Integral method for the calculation of incompressible 2D boundary layers, *J. Aircraft*, 1990, **27**, 859–865.

19. SINGER, B. A., DINAVAHI, S. P. G. AND IYER, V. — *Testing of transition-region models: Test cases and data*, 1991, NASA CR 4371.

20. GOSTELOW, J. P., HONG, G., WALKER, G. J. AND DEY, J. — Modelling of boundary layer transition in turbulent flows by linear combination integral method, ASME paper No. 94-GT-358, *Conf. on Gas Turbine & Aerospace Cong. & Exposition*, The Hague, June 1994.

21. GOSTELOW, J. P., BLUNDEN, A. R. AND BLUNDEN, W. R. — Measurements and stochastic analysis of boundary layer transition for a range of free-stream turbulence levels, *2nd Int. Symp. on Transport Phenomena*, Honolulu, April 1988 (preprint).

22. HAZARIKA, B. K. AND HIRSCH, CH. — Boundary layer transition on a sharp leading edge plate, *5th Int. Symp. on Refined Flow Modelling & Turbulence Measurements*, 1993, Paris (preprint).

23. NARASIMHA, R., DEVASIA, K. J., GURURANI, G. AND BADRI NARAYANAN, M. A. Transitional intermittency in boundary layers subjected to pressure gradient, *Exp. Fluids*, 1984, 2, 171–176.

24. NARASIMHA, R. AND DEY, J. Transitional spot formation rate in 2-dimensional boundary layers, *Proc. 3rd Symp. on Numerical and Physical Aspects of Aerodynamic Flows* (T. Cebeci, ed.), 1985, pp. 57–74, Springer-Verlag.

25. DEY, J. AND NARASIMHA, R. Effect of favourable pressure gradient on transitional spot formation rate, *Expl Thermal Fluid Sci.*, 1991, 4, 192–197.

26. GOSTELOW, J. P., BLUNDEN, A. R. AND WALKER, G. J. Effects of free-stream turbulence and adverse pressure gradients on boundary layer transition, ASME paper No. 92-GT-380, *Int. Gas Turbine and Aeroengine Cong. Exposition*, Cologne, July 1992.

27. FRASER, C. J., HIGAZY, M. G. AND MILNE, J. S. End-stage boundary layer transition models for engineering calculations, *Proc. Inst. Mech. Engrs*, 1994, **208**, 47–58.

28. GOVINDARAJAN, R. AND NARASIMHA, R. *The role of residual non-turbulent disturbances on transition onset in two-dimensional boundary layers*, NAL TM DU 8901, 1989, National Aerospace Laboratory, Bangalore.

29. DEY, J. AND NARASIMHA, R. An extension of the Thwaites method for calculation of incompressible laminar boundary layers, *J. Indian Inst. Sci.*, 1990, **70**, 1–17.

30. GREEN, J. E., WEEKS, D. J. AND BROOMAN, J. W. F. *Prediction of turbulent boundary layers and wakes in compressible flow by a lag-entrainment method*, RAE Tech. Rep. 72231, 1973, Farnborough, UK.

31. BLAIR, M. F. AND WERLE, M. J. *The influence of free-stream turbulence on the zero pressure gradient fully turbulent boundary layers*, Report R 80-914388-12, United Tech. Res. Center, Connecticut, 1980.

32. SADDOUGHI, S. G. AND JOUBERT, P. N. Lateral straining of turbulent boundary layers, Part 1. Streamline divergence, *J. Fluid Mech.*, 1991, **229**, 173–204.

33. HAFEZ, S. AND JOUBERT, P. N. Some aspects of the effects of streamline convergence on a fully developed turbulent boundary layer, *11th Australian Fluid Mech. Conf.*, Hobart, 1992, pp. 551–554.

34. POTTI, M. G. S. *Transition in laterally convergent and divergent flow*, Ph.D Thesis, Indian Institute of Science, Bangalore, 1991.

35. JAHANMIRI, M. *Experimental studies of a distorted turbulent spot*, Ph.D Thesis, Indian Institute of Science, Bangalore, 1994.

36. RAMESH, O. N., DEY, J. AND PRABHU, A. Transitional intermittency distribution in a three-dimensional constant pressure diverging flow, Fluid Mech. Rep 95FM5, Department of Aerospace Engng, Indian Institute of Science, Bangalore, 1995.

37. PHANI KUMAR, S. V. S. *Analysis of turbulent spot using VITA technique*, M.Sc. (Engng) Thesis, Indian Institute of Science, Bangalore, 1994.

38. JAHANMIRI, M., PRABHU, A. AND NARASIMHA, R. Turbulent spot in 3-D constant pressure flow, *IUTAM Symp. on Laminar–Turbulent Transition*, Sendai (Japan), 1994.

39. JAHANMIRI, M., DEY, J., NARASIMHA, R. AND PRABHU, A. Spot characteristics in a 3-D flow with streamline divergence under zero pressure gradient, *Proc. Asian Fluid Mech. Cong.* (K. S. Chong and D. H. Choi, eds), Korea, 1992.

40. SCHUBAUER, G. B. AND KLEBANOFF, P. S. *Contributions on the mechanics of boundary layer transition*, NACA TM 3489, 1955.

41. GUTMARK, E. AND BLACKWELDER, R. F. On the structure of a turbulent spot in a heated boundary layer, *Expl Fluids*, 1987, 5, 217–229.

42. WYGNANSKI, I., SOKOLOV, M. AND FRIEDMAN, D. On a turbulent spot in a laminar boundary layer, *J. Fluid Mech.*, 1976, 78, 785–819.

43. GOSTELOW, J. P., HONG, G., LEE, J. AND MELWANI, D. The evolution of a turbulent spot under an adverse pressure gradient, *Proc. 11th Australasian Fluid Mech. Conf.*, Hobart, Dec. 1992 (preprint).

44. VAR HEST, B. F. A., PASSCHIER, D. M. AND VAN INGEN, J. L. Development of a turbulent spot in a boundary layer with adverse pressure gradient, *IUTAM Symp. on Laminar-Turbulent Transition*, Sendai, Japan, Sept. 1994 (preprint).

45. CHEN, K. K. AND THYSON, N. A. Extension of Emmons' spot theory to flows on blunt bodies, *AIAA J.*, 1971, 9, 821–825.

46. SANKARAN, R., SOKOLOV, M. AND ANTONIA, R. A. Substructures in a turbulent spot, *J. Fluid Mech.*, 1988, 197, 389–414.

47. SANKARAN, R. AND ANTONIA, R. A. Flow patterns and organisations within a turbulent spot, *Phys. Fluid A*, 1991, 3, 1560–1571.

48. RAMESH, O. N., DEY, J. AND PRABHU, A. Three-dimensional laminar boundary layer in a constant pressure diverging flow–Blasius equivalent, *AIAA J.*, 1994, 32, 209–210.

49. PANTULU, P. V. Studies on the transition from laminar to turbulent flow in a pipe, M.Sc. (Engng) Thesis, Indian Institute of Science, Bangalore, 1962.

50. BADRI NARAYANAN, M. A. AND NARAYANA, T. Some studies on transition from laminar to turbulent flow in a two-dimensional channel, *ZAMP*, 1967, 18, 642–650.

51. RAO, G. N. V. Mechanics of transition in axisymmetric boundary layer on a circular cylinder, *ZAMP*, 1974, 25, 63–75.