J. Indian Inst. Sci., Sept.–Oct. 1995, 75, 517–535.

Shock dynamics

RENUKA RAVINDRAN
Department of Mathematics, Indian Institute of Science, Bangalore 560 012, India.

Received on December 21, 1994.

Abstract

This paper reviews the work of Prasad, Ravindran and collaborators in the area of shock dynamics. It focuses on why there was a need for a new theory of shock dynamics, and how this theory was developed. It justifies the development of the theory, gives some details of how this method can be carried over to a general hyperbolic system and fixes its attention on the equations of gas dynamics. It also mentions various applications and gives a complete bibliography of publications from this group.

Keywords: Shock dynamics, hyperbolic system, gas dynamics.

1. Introduction

The formation and propagation of shocks has been a challenging problem over the last two centuries. The list of investigators who have looked at this problem is most impressive—Poisson in the early 1800s, Stokes, Riemann, Rankine and Hugoniot in the late 1800s, to the more recent studies of Lighthill, Whitham, Thomas, Maslov, Hunter, Anile and Russo. The problem is one which involves both mathematical concepts—for example, weak solutions of partial differential equations, differential geometry—and physical laws—entropy increase across shocks, coordinate invariance, etc. The equations of gas dynamics have traditionally been used to study shock propagation. However, they are not the simplest. Though we will always keep gas dynamics as the final goal of our theory, the basic theory will first be illustrated using a simple model equation.

The shock in itself is an unknown boundary which is both influenced by and influences the flow behind it. Nonlinear effects are crucial, both geometric effects and the interaction with the flow behind. The position of the shock at any given time t is unknown and conditions have to be satisfied across this unknown boundary. In one space dimension and time, the problem has received a great deal of attention and the solutions of a number of these—traffic flow, flood waves in long rivers, glacier flow and piston with constant velocity—have been obtained. One of the most widely used approximations is the 'characteristic rule' of Whitham, where one merely transfers the relations that hold on a characteristic to the shock. It was found to give far more accurate results than one could imagine in a number of cases. However, a 'full analysis of the approximation' was never completed and 'no really satisfactory explanation of this was found'[1]. This approximation gave unbelievably good results in the case of geometrical acoustics for weak pulses, converging cylindrical or spherical shocks and for strong shocks propagating through a stratified layer.

The first development of an approximate geometrical theory for shocks in two- or three-dimensional space was with the help of 'rays'. The ray tube approximation involved considering the propagation of each element of the shock down each elementary ray tube as a problem of shock propagation in a tube with solid walls. It was also assumed that the local Mach number will be a function of the ray tube area. This lead to the utopian case, where one could calculate successive positions of a shock without calculating the flow behind the shock. This was too simple to be true! The rest of this article reports on the work of Prasad, Ravindran and collaborators, who took up the problem with the aim to propose a new theory of shock dynamics which would be both mathematically and physically realistic and correct.

2. Need for a new theory

The characteristic rule for shock propagation simplified the problem to such an extent that many researchers used it far beyond its region of validity. In this section, we consider a simple example where the characteristic rule fails. This shows beyond doubt that there is a need for a new theory of shock dynamics.

In a hyperbolic system, the equations can be expressed in characteristic form. This is an exact representation. The approximation involved is in replacing the characteristic velocity by the shock velocity in the relevant characteristic form of the equation and assuming that it is valid along the shock. The characteristic rule gives completely erroneous results when applied to the equation

$$u_t + uu_x + \frac{u^2 A'}{A} = 0 \quad \text{for} \quad -\eta \le x,\ t \ge 0, \tag{1}$$

where $\eta > -1$,

$$A \equiv A(x) = \left(\frac{\eta+1}{x+\eta+1}\right)^2, \quad -\eta \le x, \tag{2}$$

and the initial condition

$$u(x,0) = \begin{cases} \left(\frac{x+\eta}{\eta+1}\right)^2, & -\eta \le x \le 1, \\ 0, & \text{otherwise.} \end{cases} \tag{3}$$

The initial condition has a shock discontinuity at $x = 1$. $A(x)$ could be treated as an area of cross-section which varies with x only. The equation can be put in conservation form as

$$(uA^2)_t + \frac{1}{2}(u^2A^2)_x = 0, \tag{4}$$

giving the shock velocity

$$\dot{X}(t) \equiv U = (u_l + u_r)/2, \tag{5}$$

where u_l and u_r are the values of u to the left and right of the discontinuity $x = X(t)$.

Equation (1) can be solved with the help of the characteristic equation and the compatibility condition by the pair of ordinary differential equations:

$$\frac{du}{dt} = -\frac{u^2 A'}{A} \quad \text{on} \quad \frac{dx}{dt} = u. \tag{6}$$

The approximation according to the characteristic rule is that we treat the compatibility condition in (6) to be valid on the shock with u replaced by u_l:

$$\frac{du_l}{dt} = -\frac{u_l^2 A'}{A} \quad \text{on} \quad \frac{dX}{dt} = U.$$

Here, if we choose $u_r = 0$, *i.e.*, the shock propagates into an undisturbed medium, then we have $U = u_l/2$, which gives

$$\frac{du_l}{dt} = -\frac{u_l^2 A'}{A}, \quad \frac{dX}{dt} = \frac{u_l}{2}. \tag{7}$$

The system (7) can be solved easily using $A(x)$ as in (2) and $u_l(0) = 1$, $X(0) = 1$.

From (7) we can also deduce a general result: $u_l A^2 = \text{constant}$, which is the A–M relation for the problem. This gives the solution for the strength and position of the discontinuity as

$$u_l(t) = \frac{1}{\left(1 - \frac{3t}{2(\eta+2)}\right)^{4/3}}, \quad X(t) = -(\eta+1) + \frac{\eta+2}{\left(1 - \frac{3t}{2(\eta+2)}\right)^{1/3}}. \tag{8}$$

These expressions are valid only for a small range of values of t, $t \le (2/3)(\eta + 2)$. As $t \to (2/3)(\eta + 2)$, both u and X tend to infinity.

However, the situation is different if we consider the exact form of the equation for the shock. The restriction of (1) to the left subdomain can be written as

$$u_l + \frac{u}{2} u_x = -\frac{1}{2} u u_x - \frac{u^2 A'}{A}.$$

Taking the limit of this as we approach the shock from the left subdomain, we get

$$\frac{du_l}{dt} = -\frac{u_l (u_x)_l}{2} - \frac{u_l^2 A'}{A}, \quad \frac{dX}{dt} = \frac{u_l}{2}. \tag{9}$$

This differs from (7) in the pressure of the term $-(1/2)u_l(u_x)_l$. The system is not closed as $(u_x)_l$ is an unknown, which is not specified. The presence of the derivative $(u_x)_l$ in the equation for u_l is typical in equations determining shock front positions[2]. This term represents the effects on the shock of the waves which catch up with the shock from behind. In the case discussed here, $(u_x)_l$ can be evaluated by differentiating the implicit form of the solution of (1) and (2). However, the characteristic rule would amount to setting $(u_x)_l = 0$. We can solve (1)–(3) to get

$$u(x,t)=\left\{\frac{(\eta+1)(x+\eta+1)}{2t}\left[\sqrt{1+\frac{4t(x+\eta)}{(\eta+1)^2(x+\eta+1)}}-1\right]\right\}^2, \quad -\eta\le x\le X(t), \tag{10}$$

where $x = X(t)$ is the position of the discontinuity at time t. From this we deduce an expression of $u_x(x, t)$ in the form

$$u_x(x,t)=\frac{2u}{x+\eta+1}\left[1+\frac{1}{2(x+\eta)-\sqrt{u}(\eta+1)}\right]. \tag{11}$$

If this is used in (9), we get the position of the shock

$$X(t)=-(\eta+1)+1/\{1-(1/4t)(\eta+1)^2(s^2+2s)\}, \tag{12}$$

where s is the positive real solution of the quartic

$$s^4+\frac{4}{3}s^3-\frac{32}{3}\frac{t^3}{[(\eta+1)(\eta+2)]^3}=0. \tag{13}$$

The solution $u_l(t)$ satisfying $u_l(0) = 1$ when $X(0) = 1$ is obtained from the equation

$$u_l^{3/2}-3\left(\frac{X+\eta}{\eta+1}\right)u_l+2\left(\frac{X+\eta+1}{\eta+2}\right)^3=0. \tag{14}$$

It is easily verified that $u_l(t)$ given by (14) is the same as $u(X(t), t)$ as given by (10) when $x = X(t)$.

For $t \ll |\eta|$, the expressions for $X(t)$, $u_l(t)$ as given by (12) and (14) agree with $X(t)$, $u_l(t)$ given by (8) up to $O(t^2/\eta^2)$. If the flow behind were uniform (*i.e.*, $\eta \to \infty$) then the characteristic rule would give an accurate estimate of the shock position for $t = O(1)$. In all other cases, there is a large error involved in choosing the characteristic rule approximation on the shock. For $0 < 1 + \eta \ll 1$ and $t = O(1)$, the exact value of $u \sim (2(\eta + 1)/3t)^{1/2}$, whereas the characteristic rule gives $u \sim 1/(1-3t/2)^{4/3}$, which is completely in error and is defined only up to $t \sim 2/3$. (For details see Prasad *et al.*[3]).

For problems involving a hyperbolic system of two or more equations, such as gas dynamics equations[4], the normal derivative term (corresponding to the term $-(1/2)u_l(u_x)_l$ in (9)) may be evaluated by the short-wave approximation. The accuracy in the shock position will depend on the accuracy with which the normal derivative term has been evaluated.

The characteristic rule is a good approximation when the flow behind the shock is uniform at a given time t. When the flow behind the shock is nonuniform, the nonlinear effects which catch up with the shock are accounted for poorly by the characteristic rule. Although the characteristic rule allows for a simple elegant solution, it must be used with great caution as its use is justified only for a very restricted class of problems. Unfortunately, its simplicity so fascinates its users that they do not bother to check its validity.

3. A new theory of shock dynamics

The fact that the characteristic rule is not mathematically justified and could give erroneous results made it necessary to look for a new theory. For this, we turned to the work of Grinfel'd[5] and Maslov[6], who, for waves in elastic materials and nonviscous gases, respectively, derived an infinite system of compatibility conditions along certain curves in space–time, called shock rays. These theories had attracted little attention and there was no follow-up work at all. It is, however, these compatibility conditions which provide an extremely efficient system of equations to solve shock propagation problems. The efficacy of this method will be illustrated first with the help of a model equation:

$$u_t + \left(\frac{1}{2}u^2\right)_x = 0, \quad (x, t) \in \Re \times \Re_+ \tag{15}$$

with initial condition

$$u(x, 0) = \phi(x), \;\; t \in \Re \tag{16}$$

such that the solution is sufficiently smooth except for a single shock curve

$$x = X(t), \quad t \in \Re_+. \tag{17}$$

The solution behind the shock, *i.e.*, $x < X(t)$, is assumed to have a Taylor's series representation:

$$u(x, t) = \sum_{i=0}^{\infty} \frac{1}{i!} u_i(t)(x - X(t))^i \tag{18}$$

where u_i are spatial derivatives of u at the shock $x = X(t)$, $i = 0, 1, 2, \ldots$. Here $\lim_{x\to X(t)-0} u(x, t) = u_0(t)$. We assume that the state ahead of the shock (*i.e.*, for $x > X(t)$) is known, so that $\lim_{x\to X(t)+0} u(x, t) = u_r(t)$ is a known function of t. The shock velocity is given by

$$\frac{dx}{dt} = \frac{1}{2}(u_0 + u_r). \tag{19}$$

The solution behind the shock satisfies the partial differential equation $u_t + uu_x = 0$. Substituting (18) in the equation, setting $v_i(t) = u_i(t)/i!$, $i = 1, 2, \ldots$, and equating various powers of $x - X(t)$, we get

$$\frac{du_0}{dt} = -\frac{1}{2}(u_0 - u_r)v_1,$$

$$\frac{dv_i}{dt} = -\frac{i+1}{2}(u_0 - u_r)v_{i+1} - \frac{i+1}{2}\sum_{j=1}^{i} v_j v_{i-j+1}, \quad i = 1, 2, 3, \ldots \tag{20}$$

The infinite system of eqns (19) and (20) constitutes the required set of ordinary differential equations for the determination of the shock position $X(t)$, shock strength $u_0(t)$

and the spatial derivatives $u_i(t) \equiv i!\, v_i(t)$, $i \geq 1$. The initial values of X, u_0 and u_i are given by the initial data (16)

$$X(0) = X_0, \quad u_0(0) = \phi(X_0 - 0) \equiv u_{00},$$

$$v_i(0) = \frac{1}{i!}\left(\frac{\mathrm{d}^i \phi}{\mathrm{d}x^i}\right)_{x \to X_0 - 0} \equiv v_{i0}, \quad \text{say}, \tag{21}$$

where X_0 is the value of x at which $\phi(x)$ has the discontinuity.

If we set $v_{n+1} = 0$ in the $(n+1)$th equation in (20), then the first $(n+1)$ equations form a closed system. Let $\overline{X}, \overline{u}_0(t), \overline{v}_i(t)$, $i = 1, 2, 3, \dots, n$ be the solution of the truncated system of $(n+2)$ equations

$$\frac{\mathrm{d}\overline{X}}{\mathrm{d}t} = \frac{1}{2}(\overline{u}_0 + u_\mathrm{r}), \tag{22}$$

$$\frac{\mathrm{d}\overline{u}_0}{\mathrm{d}t} = -\frac{1}{2}(\overline{u}_0 - u_\mathrm{r})\overline{v}_1,$$

$$\frac{\mathrm{d}\overline{v}_i}{\mathrm{d}t} = -\frac{i+1}{2}(\overline{u}_0 - u_\mathrm{r})\overline{v}_{i+1} - \frac{i+1}{2}\sum_{j=1}^{i} \overline{v}_j v_{i-j+1}, \quad i = 1, 2, \cdots, n-1, \tag{23}$$

$$\frac{\mathrm{d}\overline{v}_n}{\mathrm{d}t} = -\frac{n+1}{2}\sum_{j=1}^{n} \overline{v}_j \overline{v}_{n-j+1}, \tag{24}$$

with initial conditions as in (21):

$$\overline{X}_0 = X_0, \quad \overline{u}_0(0) = u_{00}, \quad \overline{v}_i(0) = v_{i0}, \quad i = 1, 2, \dots, n. \tag{25}$$

Since the right-hand side of eqns (22)–(24) are Lipschitz-continuous at any point in $(\overline{X}_0, \overline{u}_0, \overline{v}_1, \dots, \overline{v}_n)$ space, a unique solution of (22)–(25) exists on a suitable interval $(0, T)$. Using this, the function $\overline{u}(x, t)$ can be constructed as follows:

$$\overline{u}(x,t) = \begin{cases} \overline{u}_0(t) + \displaystyle\sum_{i=1}^{n} \overline{v}_i (x - \overline{X}(t))^i, & x < \overline{X}(t), \\ \phi(x), & x > \overline{X}(t). \end{cases} \tag{26}$$

This is an approximate solution of the conservation law (15) with (16) in the following sense: on the approximate shock path $x = \overline{X}(t)$, the jump condition (22) is satisfied. In the region behind the approximate shock, *i.e.*, $x < \overline{X}(t)$, the substitution of $\overline{u}(x, t)$ in the conservation law (15) leaves a remainder containing a factor $(x - \overline{X}(t))^n$ on the left-hand side. This is small near $x = \overline{X}(t)$. The accuracy with which the conservation law is satisfied in a neighborhood of the shock increases as n increases.

For numerical computation, two special cases are considered.

Case 1.

$$\phi_1(x) = \begin{cases} \left(\frac{x+\eta}{\eta+1}\right)^2, & x \in (-\eta, 1), \\ 0, & \text{elsewhere.} \end{cases}$$

Case 2.

$$\phi_2(x) = \begin{cases} \alpha\, e^{\beta x}, & x < 0, \\ 0, & \text{elsewhere.} \end{cases}$$

For $\phi_1(x)$, $X_0 = 1$ and $\bar{v}_i(0) = 0$ for $i \geq 3$. Without loss of generality for $\phi_2(x)$, we can set $\alpha = 1$, $\beta = 1$ because by a change of variables

$$x = \beta t, \quad t = \frac{\beta t}{\alpha}, \quad u = \alpha u,$$

this can be obtained. Here $X_0 = 0$ and $\bar{u}_0(0) = \bar{v}_i(0) = 1$ for all $i \geq 1$.

The exact position of the shock in this simple case can be obtained from the equation

$$t\phi^2(\xi) + 2\int_{X_0}^{\xi} \phi(\mu)\, d\mu = 0, \tag{28}$$

where $\xi = X - u_0 t$. In Case 1 we have

$$t\left(\frac{\xi+\eta}{\eta+1}\right)^4 + \frac{2}{3}\frac{(\xi+\eta)^3 - (1+\eta)^3}{(1+\eta)^2} = 0 \tag{29}$$

and in Case 2

$$te^{2\xi} + 2(e^{\xi} - 1) = 0, \tag{30}$$

which give the value of ξ. This in turn helps to evaluate the other quantities on the shock:

$$u_0 = \phi(\xi), \quad v_1 = \frac{\phi'(\xi)}{1 + t\phi'(\xi)},$$

$$v_2 = \frac{\phi''(\xi)}{2[1 + t\phi'(\xi)]^3}, \quad v_3 = \frac{\phi'''(\xi)}{6[1 + t\phi'(\xi)]^4} - \frac{t[\phi''(\xi)]^2}{2[1 + t\phi'(\xi)]^5}, \tag{31}$$

and so on. In Case 1, where $\phi^i(\xi) = 0$ for $i > 3$, we have

$$v_i(t) = \frac{(-1)^i}{i!} \cdot \frac{1\cdot 3\cdot 5\cdot 7\cdots(2i-3)[\phi''(\xi)]^{i-1} t^{i-2}}{[1 + t\phi'(\xi)]^{2i-1}}, \quad i \geq 2. \tag{32}$$

$v_i(t) \to 0$ as $t \to \infty$ for all i and gives a value for comparison with the approximate results. The initial data in Case 1 are nonzero only in the interval $(-\eta, 1)$, with a shock at

Table I
$\eta = -0.5$.

	$t = 1.0$		$t = 5.0$		$t = 10.0$	
	u	*Error (%)*	u	*Error (%)*	u	*Error (%)*
Exact	0.47390445	–	0.24232081	–	0.17572092	–
$n = 1$	0.4421360	–5.6	0.21821789	–9.9	0.15617376	–11.0
$n = 2$	0.47171239	–0.46	0.23787367	–1.8	0.17140803	–2.5
$n = 3$	0.47366942	-0.50×10^{-1}	0.24120493	–0.46	0.17440411	–0.75
$n = 5$	0.47390183	-0.56×10^{-3}	0.24221465	-0.40×10^{-1}	0.17554484	–0.10
$n = 8$	0.47390560	0.24×10^{-3}	0.24230783	-0.12×10^{-2}	0.17570887	-0.68×10^{-2}
$n = 25$	0.47390561	0.24×10^{-3}	0.24231136	0.24×10^{-3}	0.17572129	0.22×10^{-3}

$x = 1$. As $\eta \to \infty$, the disturbance behind the shock approaches a constant state $u = 1$. For η close to -1, ϕ increases very rapidly from 0 to 1 over a very short distance, so that the spatial derivatives of u play an important role in this case. For numerical computation, we have chosen $\eta = -0.5$. The initial data in Case 2 are not of compact support.

Tables I and II give the values of u for initial values corresponding to Cases 1 and 2 at $t = 1.0$, 5.0 and 10, $n = k$ denotes that v_{k+1} is set equal to 0 and $k + 2$ eqns (22)–(24) with (25) are considered. For $n = 1$ the error in u is sizeable, but for $n = 2$ the error drops rapidly (< 1% in Case 2), while for $n = 3$ it is uniformly very small, as for $n = 5$, 8, 25 as well. Computation was done for Case 2 with $\alpha = 1$, $\beta = 1$.

For the equation $u_t + uu_x = 0$, with initial condition having a single jump discontinuity, the shock position $X(t)$, shock strength $u_0(t)$ and the spatial derivatives at the shock $u_i(t)$ are given by an infinite set of ordinary differential equations. If this set is truncated at any stage, involving five or more equations, this closed system of equations can be easily integrated numerically to give very accurate results. This is in sharp contrast to the characteristic rule, where only two equations are considered and the error involved could be extremely large.

For the accuracy of this theory for small t and for very large t, see Ravindran and Prasad[7] and Prasad and Ravindran[8].

Table II
$\phi = e^x, x < 0;\ \phi = 0, x \geq 0$

	$t = 1.0$		$t = 5.0$		$t = 10.0$	
	u	*Error (%)*	u	*Error (%)*	u	*Error (%)*
Exact	0.73205081	–	0.46332497	–	0.35825757	–
$n = 1$	0.70710678	–3.4	0.40824829	–12	0.30151134	–16
$n = 2$	0.73372900	0.23	0.46777169	0.96	0.36157950	0.93
$n = 3$	0.73200502	-0.63×10^{-2}	0.46355666	0.50×10^{-1}	0.35872978	–0.13
$n = 5$	0.73205096	0.26×10^{-4}	0.46331988	-0.11×10^{-2}	0.35825020	-0.21×10^{-2}
$n = 8$	0.73205081	0	0.46332497	0	0.35825765	0.22×10^{-4}
$n = 25$	0.73205081	0	0.46332496	0	0.35825757	0

4. The equations of gas dynamics

For a model conservation law, by truncating the infinite system of compatibility conditions, the new theory of shock dynamics is developed. The derivation is mathematically convincing and gives extremely good results. This theory gives not only the shock strength and shock position but also a few spatial derivatives behind the shock so that we can construct the unknown solution at any time by using a finite Taylor series.

One could now carry over the theory to the equations of gas dynamics in two space dimensions and time – (x, y, t). Derivation of the compatibility conditions is extremely complex and requires not only very careful mathematical calculations but also utmost care in the change of the order of certain differential operators, since they do not commute.

We consider the propagation of a shock front in a polytropic gas with γ as the constant ratio of the specific heats. For simplicity, we assume that the motion is two-dimensional, that there exists only one smooth shock manifold Ω in space–time ($\Re^3$) and that the fluid velocity $\boldsymbol{q} = (u, v)$, pressure p and density ρ are $C^\infty(\Re^3)$ functions except for a discontinuity of the first kind on Ω. We further assume that the shock front propagates into a gas in a uniform state and at rest ahead of the shock, *i.e.*, $\boldsymbol{q}_a = (0, 0)$, $p_a = p_0$ and $\rho_a = \rho_0$, where p_0 and ρ_0 are constants. The state behind the shock manifold Ω is denoted by $\boldsymbol{q}_b = (u_b, v_b)$, p_b and ρ_b. These functions, defined only in the domain behind the shock, are then extended as C^∞ functions on the whole of $\Re^3$. The extended functions $\boldsymbol{q}_b$, p_b and ρ_b are nonunique in the domain ahead of the shock.

Let $S(x, y, t) = 0$ denote the equation of the shock surface Ω in space–time. For a shock in an ideal gas with constant specific heats, it has been shown[9, 10] that the function S can be obtained by solving any one of a number of shock-manifold partial differential equations (SME), one of which is

$$S_t + C(S_x^2 + S_y^2)^{1/2} = 0, \tag{33}$$

with

$$C^2 = \frac{\rho_b}{\rho_0}\frac{p_b - p_0}{\rho_b - \rho_0}, \tag{34}$$

where p_b and ρ_b are in $C^\infty(\Re^3)$.

We define a function

$$\mu = \frac{\rho_b - \rho_0}{\rho_0} \in C^\infty(\Re^3) \tag{35}$$

whose value on Ω represents the shock strength. A shock front denoted by Ω_t at any time t is a curve in the (x, y) plane and is given by $S(x, y, t) = 0$, in which t appears as a parameter. The Rankine–Hugoniot conditions give the following relations on Ω:

$$C|_\Omega = a_0\left(\frac{2(1+\mu)}{2-(\gamma-1)\mu}\right)^{1/2}\Bigg|_\Omega, \tag{36}$$

$$p_b|_\Omega = p_0 \frac{2+(\gamma+1)\mu}{2-(\gamma-1)\mu}\bigg|_\Omega, \tag{37}$$

$$(u_b, v_b)|_\Omega = \frac{\mu C}{1+\mu}(N_1, N_2)\bigg|_\Omega, \tag{38}$$

where $a_0^2 = \gamma p_0/\rho_0$ and $(N_1, N_2) = (\cos\Theta, \sin\Theta)$ is the unit normal to the shock front Ω_t. The relations (36)–(38) are not valid in $\Re^3$ but only on Ω. The SME is valid in $\Re^3$ and hence it is possible to define Θ in $\Re^3$ with the help of the relation

$$(N_1, N_2) = \frac{(S_x, S_y)}{(S_x^2 + S_y^2)^{1/2}}. \tag{39}$$

The derivative

$$\frac{\mathrm{d}}{\mathrm{d}t} = \frac{\partial}{\partial t} + N_1 C\frac{\partial}{\partial x} + N_2 C\frac{\partial}{\partial y} \tag{40}$$

represents the time rate of change as we move along a characteristic curve of the eqn (33). When C is evaluated on Ω, then this is the time rate of change along a shock ray and is an interior derivative in Ω. We denote the normal and the tangential derivatives for the shock front by $\partial/\partial N$, $\partial/\partial T$, that is

$$\frac{\partial}{\partial N} = N_1\frac{\partial}{\partial x} + N_2\frac{\partial}{\partial y}, \quad \frac{\partial}{\partial T} = N_2\frac{\partial}{\partial x} - N_1\frac{\partial}{\partial y}. \tag{41}$$

We also denote the normal and tangential components of the fluid velocity by A and B, respectively; then

$$A|_\Omega = \frac{\mu C}{1+\mu}\bigg|_\Omega \tag{42}$$

and

$$B|_\Omega = 0. \tag{43}$$

We write the equations of conservation of mass, momentum and energy behind the shock as follows (after dropping the subscript b):

$$\frac{\mathrm{d}\rho}{\mathrm{d}t} + (A-C)\frac{\partial\rho}{\partial N} + \rho\left(\frac{\partial A}{\partial N} - A\frac{\partial\Theta}{\partial T}\right) + \left[B\frac{\partial\rho}{\partial T} + \rho\left(\frac{\partial B}{\partial T} + B\frac{\partial\Theta}{\partial N}\right)\right] = 0, \tag{44}$$

$$\frac{\mathrm{d}A}{\mathrm{d}t} + (A-C)\frac{\partial A}{\partial N} + \frac{1}{\rho}\frac{\partial p}{\partial N} + B\left[\frac{\mathrm{d}\Theta}{\mathrm{d}t} + (A-C)\frac{\partial\Theta}{\partial N} + \frac{\partial A}{\partial T} + B\frac{\partial\Theta}{\partial T}\right] = 0, \tag{45}$$

$$\frac{\mathrm{d}\Theta}{\mathrm{d}t} + (A-C)\frac{\partial\Theta}{\partial N} - \left(\frac{A-C}{A}\right)\frac{\partial B}{\partial N} - \frac{1}{\rho A}\frac{\partial p}{\partial T} - \frac{1}{A}\left[\frac{\mathrm{d}B}{\mathrm{d}t} + B\frac{\partial B}{\partial T} - AB\frac{\partial\Theta}{\partial T}\right] = 0, \tag{46}$$

$$\frac{dp}{dt}+(A-C)\frac{\partial p}{\partial N}-\frac{\gamma p}{\rho}\left(\frac{d\rho}{dt}+(A-C)\frac{\partial \rho}{\partial N}\right)+B\left[\frac{\partial p}{\partial T}-\frac{\gamma p}{\rho}\frac{\partial \rho}{\partial T}\right]=0. \tag{47}$$

In each of the above equations, either the quantity in the square bracket or its coefficient vanishes on Ω ($\partial B/\partial T=0$ since $B=0$ on Ω and $\partial/\partial T$ is an interior derivative). Equations (44), (45) and (47) on Ω form a system of equations in the unknowns ρ, A and p not involving B. From these equations, the quantities $\partial A/\partial N$ and $\partial p/\partial N$ can be eliminated to give on Ω

$$\frac{dp}{dt}-\rho(A-C)\frac{dA}{dt}+\left\{(A-C)^2-\frac{\gamma p}{\rho}\right\}\left\{\frac{d\rho}{dt}+(A-C)\frac{\partial \rho}{\partial N}\right\}-\rho A(A-C)^2\frac{\partial \Theta}{\partial T}=0. \tag{48}$$

In (48) C, p, A, ρ on Ω are known functions of μ (see (35)–(38)) and the derivative d/dt is an interior derivative in Ω, so that (35) to (38) can be used to determine dp/dt and dA/dt in terms of μ and $d\mu/dt$:

$$\left.\frac{dp}{dt}\right|_{\Omega}=p_0\frac{4\gamma}{\{2-(\gamma-1)\mu\}^2}\left.\frac{d\mu}{dt}\right|_{\Omega}, \tag{49}$$

$$\left.\frac{dA}{dt}\right|_{\Omega}=C\frac{4+3\mu-\gamma\mu}{2(1+\mu)^2(2-(\gamma-1)\mu)}\left.\frac{d\mu}{dt}\right|_{\Omega}; \tag{50}$$

where we have used the fact that $N_1^2+N_2^2=1$. This leads to the first compatibility condition on Ω along the shock rays:

$$\frac{1}{C}\frac{d\mu}{dt}=\mu\frac{Q}{S}\left[2\frac{\partial \Theta}{\partial T}-\frac{(\gamma+1)}{1+\mu}\frac{\partial \mu}{\partial N}\right] \quad \text{on } \Omega, \tag{51}$$

where

$$Q=2-\mu(\gamma-1), \quad S=8+5\mu-3\mu\gamma+\mu^2(\gamma^2-1). \tag{52}$$

The characteristic equations for the shock-manifold partial differential eqn (33) give[9] the shock ray equations:

$$\frac{dx}{dt}=N_1C, \ \frac{dy}{dt}=N_2C \tag{53}$$

and

$$\frac{d\Theta}{dt}=\frac{dC}{dT}. \tag{54}$$

On Ω, eqn (54) reduces to

$$\frac{1}{C}\frac{d\Theta}{dt}=\frac{\gamma+1}{2Q(1+\mu)}\frac{\partial \mu}{\partial T}. \tag{55}$$

Equations (53) (restricted to Ω), (55) and (51) form a coupled system of four equations for the position (x, y) of the shock, the inclination Θ of the normal to the shock and the shock strength μ. Since $\partial/\partial T$ is an interior derivative on Ω, $\partial\mu/\partial T$ and $\partial\Theta/\partial T$ are known on Ω if μ and Θ are known on Ω. The system, however, is not closed due to the presence of the normal derivative $\partial\mu/\partial N$ in (51). The presence of such a normal derivative rendering the system incomplete is typical of the compatibility conditions on Ω[7].

To obtain the second compatibility condition, the normal derivative of each of the equations of mass, momentum and energy is considered. The terms are then rearranged to give the following three equations on Ω (corresponding to eqns (44), (45), (47) in the first compatibility condition):

$$\frac{\mathrm{d}}{\mathrm{d}t}\left(\frac{\partial\rho}{\partial N}\right)+(A-C)\frac{\partial^2 A}{\partial N^2}+\rho\frac{\partial^2 A}{\partial N^2}+\rho\frac{\partial}{\partial T}\left(\frac{\partial B}{\partial N}\right)-\rho A\frac{\partial}{\partial T}\left(\frac{\partial\Theta}{\partial N}\right)$$

$$+\left(2\frac{\partial A}{\partial N}-A\frac{\partial\Theta}{\partial T}\right)\frac{\partial\rho}{\partial N}+\left(\frac{\mathrm{d}\Theta}{\mathrm{d}t}-C\frac{\partial\Theta}{\partial N}+\frac{\partial B}{\partial N}\right)\frac{\partial\rho}{\partial T}$$

$$+2\rho\frac{\partial B}{\partial N}\frac{\partial\Theta}{\partial N}-\rho A\left(\frac{\partial\Theta}{\partial N}\right)^2-\rho A\left(\frac{\partial\Theta}{\partial T}\right)^2-\rho\frac{\partial A}{\partial N}\frac{\partial\Theta}{\partial T}=0 \text{ on } \Omega, \tag{56}$$

$$\frac{\mathrm{d}}{\mathrm{d}t}\left(\frac{\partial A}{\partial N}\right)+(A-C)\frac{\partial^2 A}{\partial N^2}+\frac{1}{\rho}\frac{\partial^2 p}{\partial N^2}+\left(\frac{\mathrm{d}\Theta}{\mathrm{d}t}-C\frac{\partial\Theta}{\partial N}+\frac{\partial B}{\partial N}\right)\frac{\partial A}{\partial T}+\left(\frac{\partial A}{\partial N}\right)^2$$

$$+\frac{\partial B}{\partial N}\left(\frac{\mathrm{d}\Theta}{\mathrm{d}t}+(A-C)\frac{\partial\Theta}{\partial N}\right)+\frac{1}{\rho}\frac{\partial\rho}{\partial N}\left(\frac{\mathrm{d}A}{\mathrm{d}t}+(A-C)\frac{\partial A}{\partial N}\right)=0 \text{ on } \Omega, \tag{57}$$

$$\frac{\mathrm{d}}{\mathrm{d}t}\left(\frac{\partial p}{\partial N}\right)-\frac{\gamma p}{\rho}\frac{\mathrm{d}}{\mathrm{d}t}\left(\frac{\partial\rho}{\partial N}\right)+(A-C)\left(\frac{\partial^2 p}{\partial N^2}-\frac{\gamma p}{\rho}\frac{\partial^2\rho}{\partial N^2}\right)+\left(\frac{\mathrm{d}\Theta}{\mathrm{d}t}-C\frac{\partial\Theta}{\partial N}\right)$$

$$\times\left(\frac{\partial p}{\partial T}-\frac{\gamma p}{\rho}\frac{\partial\rho}{\partial T}\right)+\frac{\partial A}{\partial N}\left(\frac{\partial p}{\partial N}-\frac{\gamma p}{\rho}\frac{\partial\rho}{\partial N}\right)+\frac{\partial B}{\partial N}\left(\frac{\partial p}{\partial T}-\frac{\gamma p}{\rho}\frac{\partial\rho}{\partial T}\right)$$

$$-\gamma\left[\frac{1}{\rho}\frac{\partial p}{\partial N}-\frac{p}{\rho^2}\frac{\partial\rho}{\partial N}\right]\left[\frac{\mathrm{d}\rho}{\mathrm{d}t}+(A-C)\frac{\partial\rho}{\partial N}\right]=0 \text{ on } \Omega. \tag{58}$$

The fourth equation, corresponding to (46) in the first compatibility condition, is of interest only for evaluating the third compatibility condition. The expressions for $\partial A/\partial N$, $\partial B/\partial N$ and $\partial p/\partial N$ on Ω can be obtained in terms of μ, $\partial\mu/\partial T$, $\partial\mu/\partial N$, $\partial\Theta/\partial T$ and $\partial\Theta/\partial N$ from (45)–(47) and (51) and (55). These expressions are then substituted for all interior derivatives in eqns (56)–(58). The terms $\partial^2 A/\partial N^2$ and $\partial^2 p/\partial N^2$ are eliminated from the three eqns (56)–(58) to give the following equation:

$$\frac{1}{C}\frac{\mathrm{d}}{\mathrm{d}t}\left(\frac{\partial\mu}{\partial N}\right)=-\frac{1}{\zeta_1}\left[\frac{\mu(1+\gamma)}{2(1+\mu)^3}\frac{\partial^2\mu}{\partial N^2}+\zeta_2\left(\frac{\partial\mu}{\partial N}\right)^2+\zeta_3\frac{\partial\mu}{\partial N}\frac{\partial\Theta}{\partial T}\right.$$

$$+\zeta_4\left(\frac{\partial\Theta}{\partial T}\right)^2+\zeta_5\frac{\partial^2\mu}{\partial T^2}+\zeta_6\left(\frac{\partial\mu}{\partial T}\right)^2+\zeta_7\frac{\partial\mu}{\partial T}\frac{\partial\Theta}{\partial N}\Bigg], \tag{59}$$

where the coefficients $\zeta_1, \zeta_2, \ldots, \zeta_7$ are known functions of γ and μ. These coefficients are expressed in terms of $\delta_1, \delta_2, \ldots, \delta_6$ and $\beta_1, \beta_2, \ldots, \beta_6$, which appear in the equations

$$\frac{1}{C}\frac{\mathrm{d}}{\mathrm{d}t}\left(\frac{\partial p}{\partial N}\right)=\rho_0 C\Bigg[\beta_1\frac{\mathrm{d}}{\mathrm{d}t}\left(\frac{\partial\mu}{\partial N}\right)+C\beta_2\left(\frac{\partial\mu}{\partial N}\right)^2+C\beta_3\frac{\partial\mu}{\partial N}\frac{\partial\Theta}{\partial T}$$

$$+C\beta_4\left(\frac{\partial\Theta}{\partial T}\right)^2+C\beta_5\frac{\partial^2\mu}{\partial T^2}+C\beta_6\left(\frac{\partial\mu}{\partial T}\right)^2\Bigg] \text{ on } \Omega \tag{60}$$

and

$$\frac{1}{C}\frac{\mathrm{d}}{\mathrm{d}t}\left(\frac{\partial A}{\partial N}\right)=\delta_1\frac{\mathrm{d}}{\mathrm{d}t}\left(\frac{\partial\mu}{\partial N}\right)+C\delta_2\left(\frac{\partial\mu}{\partial N}\right)^2+C\delta_3\frac{\partial\mu}{\partial N}\frac{\partial\Theta}{\partial T}$$

$$+C\delta_4\left(\frac{\partial\Theta}{\partial T}\right)^2+C\delta_5\frac{\partial^2\mu}{\partial T^2}+C\delta_6\left(\frac{\partial\mu}{\partial T}\right)^2 \text{ on } \Omega. \tag{61}$$

The expressions for all these coefficients are known functions of γ and μ and are available in Ravindran and Prasad[11].

We define a quantity μ_i ($i = 1, 2, 3, \ldots$) by

$$\mu_i=\left\{\sum_{r+s=i}{}^{i}C_r N_1^r N_2^s\frac{\partial^i\mu}{\partial x^r\partial y^s}\right\}. \tag{62}$$

Obviously, $\mu_1 = \partial\mu/\partial N$ and it is easy to verify that

$$\frac{\partial^2\mu}{\partial N^2}+\frac{\partial\mu}{\partial T}\frac{\partial\Theta}{\partial N}=\mu_2.$$

The second compatibility condition (59) finally becomes

$$\frac{1}{C}\frac{\mathrm{d}\mu_1}{\mathrm{d}t}=-\frac{1}{\zeta_1}\Bigg[\frac{\mu(1+\gamma)}{2(1+\mu)^3}\mu_2+\zeta_2\mu_1^2+\zeta_3\mu_1\frac{\partial\Theta}{\partial T}$$

$$+\zeta_4\left(\frac{\partial\Theta}{\partial T}\right)^2+\zeta_5\frac{\partial^2\mu}{\partial T^2}+\zeta_6\left(\frac{\partial\mu}{\partial T}\right)^2\Bigg] \text{ on } \Omega. \tag{63}$$

The first compatibility condition (43) was originally incomplete due to the appearance of Θ and $\partial\mu/\partial N = \mu_1$ on the right-hand side. The shock ray equations partially removed the incompleteness by providing the equation (55) for Θ. The search for an equation for μ_1 led to the second compatibility condition (63), which is also incomplete due to the presence of μ_2. The search for an equation for μ_2 leads to the third compatibility condition[5] containing μ_3 and the process could continue, leading to an infinite system of compatibility conditions.

5. General approach

In this approach, we use the theory of 'singular surfaces' to study the propagation of discontinuities in general. The theory is based on a simple idea. From the governing system of equations, compatibility conditions are derived along singular surfaces. This involves a detailed study of

1. geometrical jump conditions associated with the surface,
2. kinetimatical conditions involving a specially defined time derivative, and
3. an infinite system of dynamical compatibility conditions.

This technique is based on the theory of moving singular surfaces and the theory of Grinfel'd[5]. Consider a moving surface $\Sigma(t)$ defined by equations $x^i = x^i(\xi^\alpha, t)$, where (x^i) is a fixed curvilinear coordinate system in Euclidean space and ξ^α are the surface coordinates (Latin indices take the rules 1, 2, 3 and Greek 1, 2). The term defined by

$$x^i_{.\alpha} = \frac{\partial x^i(\xi, t)}{\partial \xi^\alpha} \tag{64}$$

for a fixed α gives the components of a vector tangent to the surface $\Sigma(t)$ along the direction of ξ^α.

Given a function F, which is a function of spatial coordinates x^i and time t such that on either side of the moving surface $\Sigma(t)$, $F(x, t)$ and its derivatives are sufficiently smooth; then the jump in the covariant derivatives of F can be obtained in terms of the normal derivatives:

$$\left[F_{,i}\right]_-^+ = A_1 n_i + A_{0;\alpha} x_{i.}^{\alpha} , \tag{65}$$

$$\left[F_{,ij}\right]_-^+ = A_2 n_i n_j + A_\alpha (n_i x_{j.}^{\alpha} + n_j x_{i.}^{\alpha}) + A_{\alpha\beta} x_{i.}^{\alpha} x_{j.}^{\beta} , \tag{66}$$

where $[\ ]_-^+$ denotes the jump across $\Sigma(t)$: $[G]_-^+ = G_+ - G_-$, n_i are the components of a unit vector normal to the surface, $b_{\alpha\beta} = x^i_{.\alpha;\beta} n_i$ are the components of the symmetric second fundamental form of the surface $\Sigma(t)$, and

$$A_0 \equiv [F]_-^+,\ A_1 \equiv \left[F_{,i}\right]_-^+ n^i,\ A_2 \equiv \left[F_{,ij}\right]_-^+ n^i n^j,$$

$$A_\alpha \equiv A_{1;\alpha} + b_{\alpha.}^{.\beta} A_{0;\beta},\ A_{\alpha\beta} \equiv A_{0;\alpha\beta} - A_1 b_{\alpha\beta} .$$

Using Grinfel'd's formula, jumps in partial derivatives with respect to time can also be expressed in terms of special time derivatives of normal derivatives:

$$\left[\frac{\partial F}{\partial t}\right]_{-}^{+} = -A_1 C + \frac{\delta}{\delta t} A_0, \tag{67}$$

$$\left[\frac{\partial F_{,i}}{\partial t}\right]_{-}^{+} = n_i\left(-CA_2 + \frac{\delta}{\delta t} A_1 + A_{0;}^{\alpha} C_{;\alpha}\right) + x_{i.}^{;\alpha}\left(-CA_1 + \frac{\delta}{\delta t} A_0\right)_{;\alpha}, \tag{68}$$

where C is the normal surface velocity and

$$\frac{\delta F(x(\xi, t), t)}{\delta t} = \left.\frac{\partial F(x, t)}{\partial t}\right|_{\Sigma} + Cn^k F_{,k}|_{\Sigma} .$$

Using these expressions, if a smooth shock manifold $\Sigma(t)$ propagates in a polytropic gas, then jumps of the gas-dynamic equations across $\Sigma(t)$ are considered. This, on using the geometric and kinematical jump conditions, gives the first set of compatibility conditions. To obtain the $(N+1)$th set, $N = 1, 2, 3, \ldots$, we differentiate the gas-dynamic equations with respect to $x^{i_1}, \ldots, x^{i_N}$ and convolute with $n^{i_1}, \ldots, n^{i_N}$. An infinite system of compatibility conditions can thus be obtained. The first two equations of this system are precisely those mentioned in the previous section. For each $N \geq 1$ suitably defined Nth derivatives of density, velocity and pressure can be expressed as linear functions of a scalar $\Pi_N(\xi, t)$ and nonlinear functions of the scalars $\Pi_0(\xi, t), \ldots, \Pi_{N-1}(\xi, t)$ and their derivatives with respect to surface coordinates. Each of these compatibility conditions can be suitably combined to give a single equivalent equation in the scalars Π_i, $i = 0, 1, 2, \ldots$. The first two of these are

$$\frac{\delta}{\delta t} \Pi_0 = K_0 \Omega_1 - g\lambda_2 \Pi_1 , \tag{69}$$

$$\frac{\delta}{\delta t}\Pi_1 = K_1 \Pi_{0;\alpha} \Pi_{0;}^{\alpha} + K_2 \Omega_1^2 + K_3 \Pi_1 \Omega_1 + K_4 \Pi_1^2 + K_5 \Omega_2 + K_6 \Pi_{0;\alpha}{}^{\alpha} - \lambda_2 \Pi_2, \tag{70}$$

where K_i, $i = 0, 1, \ldots, 6$, and g, λ_2 are functions of Π_0 only and $\Omega_1 = b_{\alpha}^{\alpha}$, $\Omega_2 = b_{\alpha\beta} b^{\alpha\beta}$. The terms on the left-hand side, namely, $\delta\Pi_0/\delta t$ and $\delta\Pi_1/\delta t$ are the rate of change of Π_0 and Π_1, respectively, as we move along the normal to the surface $\Sigma(t)$. By a suitable choice of the surface coordinate system, for fixed ξ^α, $x^i = x^i(\xi, t)$ is a curve normal to the successive positions of the front, and for scalars Π_i

$$\frac{\delta}{\delta t} \Pi_i = \frac{\partial}{\partial t} \Pi_i, \quad i = 0, 1, 2, \ldots$$

As in the case of the new theory of shock dynamics, the infinite system of equations can be truncated and a closed system of partial differential equations obtained for the scalars Π_i, $i \leq N$. Details of this approach can be found in Lazarev *et al.*[12]

6. Applications of the theory

(i) Lazarev *et al.*[13] have used the new theory of shock dynamics to study the flow produced by a piston starting with a nonzero positive velocity and nonzero acceleration. The flow ahead of the piston consists of two regions: region I consisting of a constant state bounded by $t = 0$ and the shock path, and region II bounded by the shock path and the piston path. Comparing the results obtained by the new theory of shock dynamics in the general approach with a finite difference scheme, they observed that at the shock front there is very good agreement between the two, but there is some deviation of flow behind the shock. The finite difference scheme, however, gives good and stable results only when the perturbation from the uniform flow is small. It fails in the case of large acceleration of the piston and gives large errors for the decelerating piston. This method also requires complex grid refinement for large times.

On the other hand, the new theory of shock dynamics in the general approach is efficient and accurate in all cases up to an intermediate time range (t not too large) both for large accelerations and large decelerations of the piston. Besides, it is far more economical than the finite difference method with regard to computational time.

(ii) Kevlahan[14] studied the weak shock problem for a decaying N-wave and expanding cylindrical shock wave, using the new theory of shock dynamics. For the plane shock with an N-wave profile propagating into a fluid at rest, he showed that the theory predicts that the slope of the N-wave decreases like t^{-1} for large times, the shock strength decreases as $t^{-1/2}$ and the width of the N-wave increase as $t^{1/2}$. These results agree with those obtained by Courant and Friedrichs[15] and Whitham[1].

For the expanding cylindrical shock wave, Kevlahan[14] showed that in the weak-shock approximation, the strength of the shock decays as $t^{-3/4}$ while the width of the wave behind the shock increases as $t^{1/4}$, in agreement with the earlier findings of Landau[16].

(iii) Kevlahan[14] considered the challenging problem of an initially straight shock propagating into a steady sinusoidal velocity field, using the new theory of shock dynamics. In this case, the shock rays are no longer straight lines as in (ii). The nonuniformity of the upstream flow eventually causes the shock to focus and form two shock–shocks (discontinuities in shock strength) separated by a flat shock disk. The shock rays are curved and the weak-shock fails, indicating infinite shock strength at the focus.

In comparison with another purely numerical scheme, the new theory of shock dynamics up to second order gave remarkably good agreement even after the formation of shock–shocks. It thus stood the severe test of predicting correctly the shock–shock configuration. It can also be used to predict whether two shock–shocks will move towards each other or apart.

Germain and Guiraud[17] claimed that the presence of viscosity is a singular perturbation to the Euler equations and must always be included in the description of curved shocks. This is equivalent to saying that the thickness of curved shocks may never be neglected in deriving equations governing its propagation. Kevlahan has shown that the new theory of shock dynamics describes the shock–shock phenomenon accurately where the curvature is infinite at the kink, although it assumes that the shock strength is negligible.

7. Conclusions

The new theory of shock dynamics provides an efficient, simple and, above all, reliable method for studying the propagation of shocks in fluid media. With the help of a model equation, the theory was tested in a number of cases where the exact solutions are available. The results are incredibly good. In the case of fluid-dynamic equations, an infinite system of compatibility conditions which are valid on the shock front can be derived. By suitably truncating the system, one obtains a closed system involving the flow variables and their derivatives up to any desired order. Using the general approach, one obtains at each order a single equation in a scalar quantity, from which the flow variables and their derivatives behind the shock can be calculated.

The shock equations cannot be solved exactly, except in a few simple cases. In the case of a plane N-wave weak shock and an expanding cylindrical weak shock, the analytical solution of the shock equations compare exceedingly well with the known solutions. The numerical solution of the shock equations was checked against a powerful numerical flow solver for the case of an initially plane shock propagating into a sinusoidal shear flow. The agreement was 'excellent', even at the time of focus when shock-shocks develop. The numerical flow solver actually resolves the shock structure, while the shock equations assume that the shock is discontinuous. The good agreement between the two indicates that one may neglect shock thickness, even in the case of curved shocks.

The new theory of shock dynamics holds a lot of promise for further theoretical and numerical work. For the reader interested in general reading and/or further details of the topics discussed here, a number of references[18–32] are given at the end of the reference list.

References

1. Whitham, G. B. — *Linear and nonlinear waves*, 1974, Wiley.
2. Srinivasan, R. and Prasad, P. — On the propagation of a multi-dimensional shock of arbitrary strength, *Proc. Indian Acad. Sci.: Math. Sci.*, 1985, 94, 27–42.
3. Prasad, P., Ravindran, R. and Sau, A. — On the characteristic rule for shocks, *Appl. Math. Lett.*, 1991, 4, 5–8.
4. Ravindran, R. and Prasad, P. — Kinematics of a shock front and resolution of a hyperbolic caustic, *Advances in nonlinear* waves, Vol. II (L. Debnath, ed.), 1985, Pitman.
5. Grinfel'd, I. M. — Ray method for calculating the wavefront intensity in nonlinear elastic material, *PMM J. Appl. Math. Mech.*, 1978, 42, 958–977.
6. Maslov, V. P. — Propagation of shock waves in an isentropic non-viscous gas, *J. Sov. Math.*, 1978, **13**, 119–163.
7. Ravindran, R. and Prasad, P. — A new theory of shock dynamics, Part-I: Analytical consideration, *Appl. Math. Lett.*, 1990, 3, 77–81.
8. Prasad, P. and Ravindran, R. — A new theory of shock dynamics, Part-II: Numerical results, *Appl. Math. Lett.*, 1990, 3, 107–109.

9. PRASAD, P. Kinematics of a multi-dimensional shock of arbitrary strength in an ideal gas, *Acta Mech.*, 1982, **45**, 168–176.

10. ROY, R. AND RAVINDRAN, R. A note on the equivalence of shock manifold equations, *Acta. Mech.*, 1988, **73**, 239–244.

11. RAVINDRAN, R. AND PRASAD, P. On an infinite system of compatibility conditions along a shock ray, *Q. J. Mech. Appl. Math.*, 1993, **46**, 131–140.

12. LAZAREV, M. P., RAVINDRAN, R. AND PRASAD, P. Shock propagation in gas dynamics: explicit form of higher order compatibility conditions, Communicated for publication.

13. LAZAREV, M. P., PRASAD, P. AND SINGH, S. K. An approximate solution of one-dimensional piston problem, *ZAMP*, 1995, **46**, 752–771.

14. KEVLAHAN, N. K. -R. *Structure and shocks in turbulence*, Ph.D. Thesis, 1994, University of Cambridge.

15. COURANT, R. AND FRIEDRICHS, K. O. *Supersonic flow and shock waves*, 1948, Interscience.

16. LANDAU, L. D. On shock waves at large distances from their place of origin, *Sov. J. Phys.*, 1945, **9**, 496–500.

17. GERMAIN, P. AND GUIRAUD, J.-P. Conditions de choc et structure des ondes de choc dans unécoule ment non stationnaire de fluid dissipatif, *J. Math. Pures Appl.*, 1966, **45**, 311–358.

18. RAMANATHAN, T. M., PRASAD, P. AND RAVINDRAN, R. On the propagation of a weak shock front: theory and application, *Acta Mech.*, 1984, **51**, 167–177.

19. RAMANATHAN, T. M. *Huyghen's method of construction of weakly nonlinear wavefronts and shock fronts with application to hyperbolic caustic*, Ph.D. Thesis, Indian Institute of Science, Bangalore, 1985.

20. PRASAD, P. Extension of Huyghen's construction of a wavefront to a nonlinear wavefront and a shock front, *Curr. Sci.*, 1987, **56**, 50–54.

21. SRINIVASAN, R. *A mathematical theory on the propagation of multidimensional shock and nonlinear waves*, Ph.D. Thesis, Indian Institute of Science, Bangalore, 1987.

22. PRASAD, P. On shock dynamics, *Proc. Indian Acad. Sci.: Math. Sci.*, 1990, **100**, 87–92.

23. RAVINDRAN, R. Shock propagation in fluids. *Theory of differential equations and applications to oceanography* (Deo, S. G. and Prahalad, Y. S., eds), 1992, Affiliated East-West Press.

24. SUNDAR, S., PRASAD, P. AND RAVINDRAN, R. A case study with the new theory of shock dynamics, *Appl. Math. Lett.*, 1992, **5**, 89–92.

25. PRASAD, P. *Propagation of a curved shock and nonliear ray theory*, Pitman Research Notes in Mathematics, 292, 1993, Longman.

26. RAVINDRAN, R., SUNDAR, S. AND PRASAD, P. Long time behaviour of the solution of a system of equations from the new theory of shock dynamics, *Comp. Math. Applic.*, 1994, **27**, 91–104.

27. PRASAD, P. A nonlinear ray theory, *Wave Motion*, 1994, **20**, 21–31.

28. RAVINDRAN, R. AND SUNDAR, S. Two shock interaction using new theory of shock dynamics, *Comp. Math. Applic.*, 1994, **28**, 37–47.

29. Lax, P. D. — Hyperbolic system of conservation laws II, *Commun. Pure Appl. Math.*, 1957, **10**, 537–566.

30. Sturtevant, B. and Kulkarny, V. A. — The focusing of weak shock waves, *J. Fluid Mech.*, 1976, **73**, 651–671.

31. Anile, A. M. and Russo, G. — Generalised wavefront expansion I: Higher order corrections for the propagation of a weak shock wave, *Wave Motion*, 1986, **8**, 243–258.

32. Anile, A. M. and Russo, G. — Generalised wavefront expansion II: The propagation of stop shocks, *Wave Motion*, 1988, **10**, 3–18.