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Magnetohydrodynamic modelling of some aspects of the solar cycle

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Abstract

After summarizing the relevant observational data and the basic theoretical framework, this article discusses the contributions made by the IISc group in the following three general problems: (i) properties of the solar dynamo operating in a thin layer at the base of the solar convection zone; (ii) formation of sunspots due to the buoyant rise of the toroidal flux created by the dynamo; (iii) evolution of the poloidal magnetic field produced by the dynamo.

Keywords: Magnetohydrodynamics, dynamo theory, solar cycle, sunspots.

1. Introduction

It was noticed in the middle of the 19th century that the number of sunspots seen on the solar surface increases and decreases with time in a cyclic fashion, the average period of each cycle being about 11 years. The sunspots were also seen to migrate towards the lower solar latitudes with the progress of the solar cycle. In other words, most of the sunspots in the early phase of a solar cycle are seen between 30° and 40°. As the cycle advances, new sunspots are found at lower and lower latitudes. Afterwards a fresh cycle begins again with sunspots appearing at high latitudes. Individual sunspots live from a few days to a few weeks.

A first clue to the physical nature of the sunspots came in 1908 when Hale[1] used the newly discovered Zeeman effect to establish that sunspots have magnetic fields. The typical magnetic field of a large sunspot is about 3000 G. For comparison, the Earth's magnetic field is only about 0.3 G. Hale[2] also noted that often two large sunspots are seen side by side and they are invariably found to have opposite magnetic polarities. The line joining the centres of such a bipolar sunspot pair is usually found to be nearly parallel to the solar equator. Joy[3], however, noticed that there is a very small systematic tilt of this line with respect to the equator which increased with higher latitude. The relation between the tilts of bipolar sunspot pairs and the latitude is often called 'Joy's law'.

The magnetic field on the solar surface outside sunspots is usually quite weak (we shall not discuss about the fibril flux tubes in this paper). With the development of the magnetograph by Babcock and Babcock[4], it became possible to study this weak magnetic field. This weak diffuse field is the strongest near the poles (having values of the order of 10 G), where the polarity of the field reverses at the time of the maximum of the solar cycle[5]. It was also found that this weak diffuse magnetic field migrates to higher latitudes with the solar cycle in contrast to the sunspots, which migrate towards lower latitudes[6,7].

A proper theory of the solar cycle would involve an understanding of the origin of the solar magnetic fields as well as of the behaviours of sunspots and weak magnetic fields outside them. Only fragments of a full theory are available at present. There are several other manifestations of the solar activity cycle (such as the shape of the corona, occurrence of solar flares, etc.), which will not be discussed here. It is to be noted that the amplitude of the solar cycle has a period of 11 years, as seen from the sunspot number counts. However, if we consider both the amplitude and the direction of the magnetic field, then the actual period of the cycle is seen to be 22 years.

The aim of the present paper is to put together the contributions of the IISc group over the last few years towards understanding various aspects of the solar cycle. This is not a review of the complete field. After summarizing the theoretical background in the next section, we highlight only those problems which have been studied by the IISc group.

2. Some theoretical considerations

Theoretical studies of the solar cycle are based on the equations of magnetohydrodynamics (MHD), which combines fluid mechanics with electromagnetic theory. MHD was systematically developed in the 1930s and 1940s by scientists like Alfvén, Cowling, Elsasser, and finally was firmly established as an important new discipline with the publication of Alfvén's classic monograph in 1950[8]. For an introduction to MHD, the readers are referred to the books of Cowling[9], Moffatt[10], Parker[11] and Priest[12].

The evolution of the magnetic field in a conducting fluid is governed by the induction equation, which is the central equation of MHD:

$$\frac{\partial \boldsymbol{B}}{\partial t} = \nabla \times (\boldsymbol{v} \times \boldsymbol{B}) + \eta \nabla^2 \boldsymbol{B}, \tag{1}$$

where $\eta = c^2/4\pi\sigma$ is called the magnetic diffusivity (σ is electrical conductivity). The induction equation is exactly analogous to the equation satisfied by vorticity in an ordinary incompressible fluid. The term $\eta \nabla^2 \boldsymbol{B}$ implies that the magnetic field diffuses away due to the electrical resistance of the medium. The other term $\nabla \times (\boldsymbol{v} \times \boldsymbol{B})$ implies that the magnetic field is 'frozen' in the conducting fluid and is carried with the fluid motions – a result similar to the Kelvin–Helmholtz theorem for vorticity.

The subdiscipline of MHD, in which one studies whether motions of conducting fluids can sustain magnetic fields, is called the dynamo theory. The first important result in dynamo theory is a negative theorem due to Cowling[13]. Cowling in 1934 showed that the induction equation does not allow the sustenance of axisymmetric magnetic fields by axisymmetric fluid motions. Hence, one must need more complicated fluid motions to solve the dynamo problem. The first clear positive breakthrough came in 1955 when Parker[14] realized that turbulence may play an important role in the dynamo problem.

Most astrophysical bodies have some angular momentum and are subject to convective instability in some parts. Convective turbulence in the presence of rotation gives rise to helical fluid motions, as we see in cyclones and anticyclones in the Earth's atmosphere. Parker[14] showed that such helical fluid motions in a turbulent conducting fluid can maintain a magnetic field.

The turbulent dynamo theory is based on the equations satisfied by the mean magnetic field in the turbulent fluid. The mean field theory for the turbulent magnetic fields was developed systematically by Steenbeck *et al.*[15] This theoretical framework gave a rigorous justification to Parker's original heuristic ideas. The equation satisfied by the mean magnetic field in the presence of helical turbulence is the dynamo equation:

$$\frac{\partial \boldsymbol{B}}{\partial t} = \nabla \times (\alpha \boldsymbol{B}) + \nabla \times (\boldsymbol{u} \times \boldsymbol{B}) + \eta \nabla^2 \boldsymbol{B}, \tag{2}$$

where $\boldsymbol{u}$ is the mean fluid velocity. Although (2) resembles the induction equation superficially, there are important differences. Firstly, $\boldsymbol{B}$ here is the mean magnetic field and not the total magnetic field as in (1). Secondly, η now is the turbulent diffusivity rather than the molecular diffusivity as in (1). The coefficient α is the measure of the average helical motion $\langle v \cdot \nabla \times v \rangle$ in the turbulence and is given in the so-called 'first-order smoothing approximation' by

$$\alpha \approx -\frac{1}{3} \langle v \cdot \nabla \times v \rangle \tau, \tag{3}$$

where τ is the correlation time of the turbulence. The term $\nabla \times (\alpha \boldsymbol{B})$ is generally nonzero in the interiors of rotating fluid bodies and is the crucial source term responsible for the dynamo maintenance of magnetic fields.

One often has to solve the dynamo equation in axisymmetric astrophysical systems with spherical geometry. The fluid velocity in such systems can be written in spherical coordinates as

$$\boldsymbol{u} = \boldsymbol{u}_{\mathrm{p}} + r \sin\theta \, \omega(r, \theta) \hat{\boldsymbol{e}}_{\varphi}, \tag{4}$$

where $\omega(r, \theta)$ is the angular velocity and $\boldsymbol{u}_{\mathrm{p}}$ is the meridional flow. Similarly, the magnetic field can be expressed as

$$\boldsymbol{B} = \nabla \times [A(r, \theta) \hat{\boldsymbol{e}}_{\varphi}] + B(r, \theta) \hat{\boldsymbol{e}}_{\varphi}, \tag{5}$$

where $B\hat{e}_{\varphi}$ is the toroidal component and $\nabla \times [A\hat{e}_{\varphi}]$ the poloidal component – often written as $\boldsymbol{B}_{\mathrm{p}}$. Substituting (4) and (5) in (2), we find the evolution equations for the toroidal and the poloidal components:

$$\frac{\partial B}{\partial t} + r \sin\theta \, (\boldsymbol{u}_{\mathrm{p}} \cdot \nabla) \left(\frac{B}{r \sin\theta} \right) = r \sin\theta (\boldsymbol{B}_{\mathrm{p}} \cdot \nabla) \omega + \nabla \times (\alpha \boldsymbol{B}_{\mathrm{p}}) + \eta \left(\nabla^2 - \frac{1}{r^2 \sin^2\theta} \right) B, \tag{6}$$

$$\frac{\partial A}{\partial t} + \frac{1}{r \sin\theta} (\boldsymbol{u}_{\mathrm{p}} \cdot \nabla)(r \sin\theta A) = \alpha B + \eta \left(\nabla^2 - \frac{1}{r^2 \sin^2\theta} \right) A. \tag{7}$$

It is clear from (7) that the source term for the poloidal field is αB, whereas (6) shows that the toroidal field has two source terms $r \sin\theta (\mathbf{B}_p \cdot \nabla)\omega$ (let us call it T_1) and $\nabla \times (\alpha \mathbf{B}_p)$ (let us call it T_2). The term T_1 is nonzero only if there is differential rotation which stretches out the poloidal field $\mathbf{B}_p$ in the toroidal direction. If $|T_1| \ll |T_2|$ (as in the case of nearly solid body rotation), then the dynamo is said to be an α^2 dynamo. On the other hand, if $|T_1| \gg |T_2|$ (as in the presence of strong differential rotation), then we call it an $\alpha\omega$ dynamo. If neither of the terms is negligible, then the dynamo is said to be of the $\alpha^2\omega$ type. It is found that α^2 dynamos can sustain nonoscillatory fields with comparable toroidal and poloidal components, whereas $\alpha\omega$ dynamos are responsible for propagating wavelike oscillatory solutions with a predominant toroidal component (resulting from the stretching of field lines due to the strong differential rotation). The planetary dynamos are supposed to be of α^2 type, whereas stellar dynamos are probably more like $\alpha\omega$ type.

Where does the dynamo action take place in the interior of the Sun? Theoretical models suggest that the inner core of the Sun, up to about seven-tenths of its radius (the solar radius is 7×10^{10} cm), is stable against convection. This is called the radiative core, where heat is transported by radiative transfer. The outer envelope of the Sun, called the convection zone, is convectively unstable, and heat is transported there by convective motions. The sunspots appear cooler compared to surroundings because the magnetic stresses inside the sunspots inhibit convective heat transport there. Since the dynamo has to be fed by convective turbulence, it used to be assumed that the dynamo action takes place in the convection zone of the Sun. In recent years, however, it has been realized that there are difficulties in making the dynamo work in the convection zone.

To see the nature of these difficulties, we first have to understand the very important idea of magnetic buoyancy suggested by Parker[16] in 1955 to explain the origins of the bipolar sunspot pairs. Let us consider a horizontal cylindrical region of the conducting fluid having a strong magnetic field B with very little magnetic field outside. Since a pressure balance has to be maintained across the surface of this cylindrical region, keeping in mind that the magnetic field would have a pressure $B^2/8\pi$, we write down

$$p_{\text{out}} = p_{\text{in}} + \frac{B^2}{8\pi}, \tag{8}$$

so that

$$p_{\text{in}} < p_{\text{out}},$$

which very often, though not always, implies that material inside the cylindrical flux tube must be lighter than the surroundings. Hence, such a magnetic flux tube is expected to be buoyant. If the differential rotation inside the Sun stretches out the nearly 'frozen' field lines to produce a strong toroidal magnetic field, it is expected that parts of that strong field would become buoyant and rise in the form of a flux tube. Figure 1 shows the initial and final configurations of such a buoyant flux tube. Once the top of the flux tube has passed through the solar surface, the flux tube intersects the solar surface in two regions, which become the two sunspots. This scenario explains why the two sunspots have opposite polarities and why the line joining their centres is nearly parallel to the equator.

Magnetic buoyancy is particularly destabilizing in the interior of the convection zone, where convective instability and magnetic buoyancy reinforce each other. Calculations of buoyant rise by Parker[17] showed that any magnetic field would be removed from the convection zone quickly. Hence, it is difficult to make the dynamo work in the convection zone, since the magnetic field has to be stored in the dynamo region for a sufficient time to allow dynamo amplification.

It is expected that there is a thin overshoot layer (probably with a thickness of the order of 10^4 km) at the top of the radiative core just below the convection zone. This is a layer which is convectively stable according to a local stability analysis, but convective motions are induced there due to convective plumes from the overlying unstable layers overshooting and penetrating there. Several authors (Spiegel and Weiss[18], van Ballegooijen[19]) pointed out that this layer is a suitable location for the operation of the dynamo. Though there will be enough turbulent motions in this layer to drive the dynamo, magnetic buoyancy will be suppressed by the stable temperature gradient there. There have also been other suggestions for suppressing magnetic buoyancy at the base of the convection zone. Parker[20] proposed 'thermal shadows', whereas van Ballegooijen and Choudhuri[21] showed that an equatorward meridional circulation at the base of the convection zone can help suppressing the magnetic buoyancy there. In summary, the current belief is that the solar dynamo operates in a thin layer at the bottom of the convection zone, where magnetic buoyancy is at least partially suppressed. A support to this hypothesis comes from the recent helioseismology observations[22], which indicate the presence of strong differential rotation at the bottom of the convection zone. The IISc group

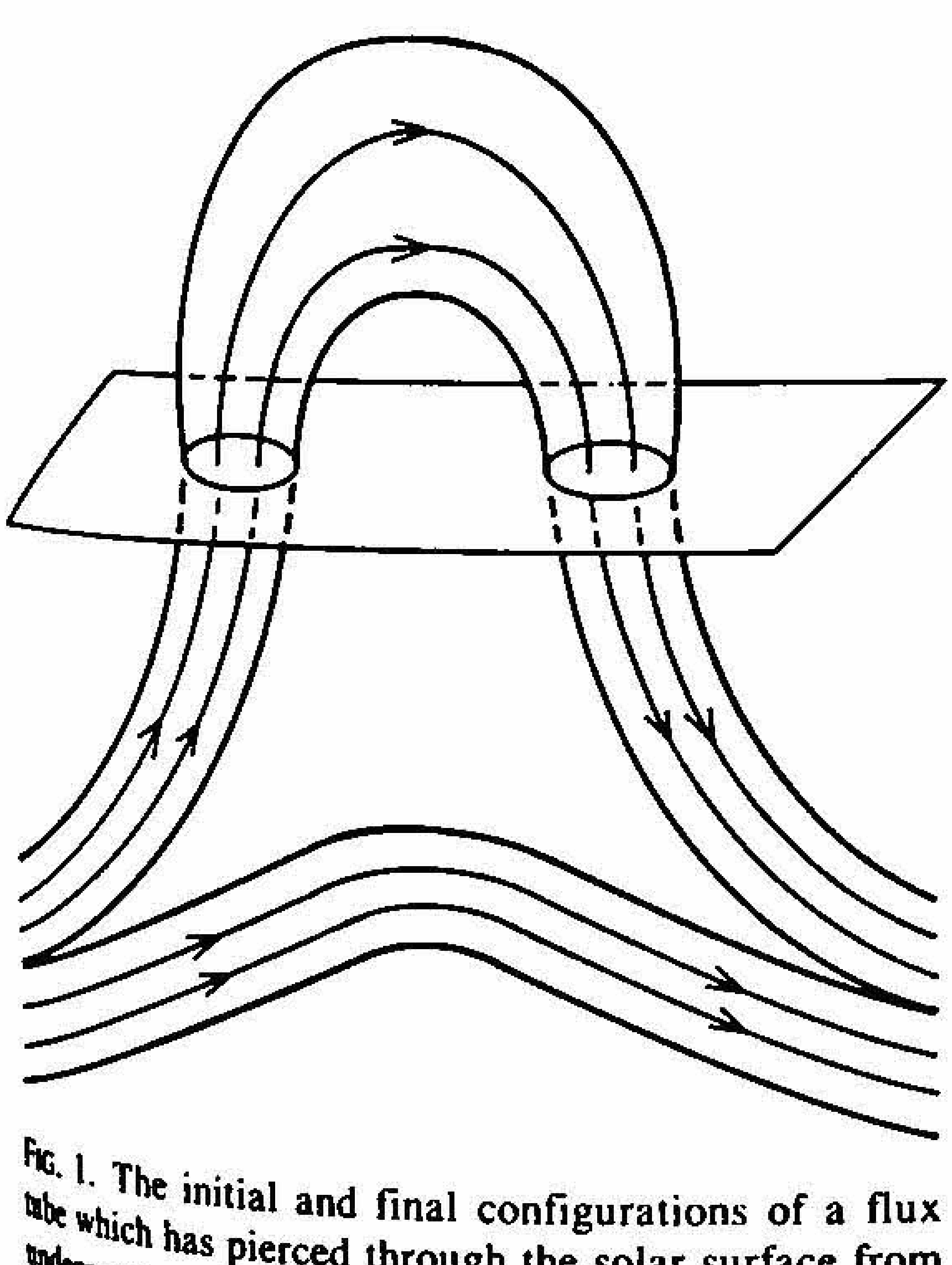


FIG. 1. The initial and final configurations of a flux tube which has pierced through the solar surface from underneath.

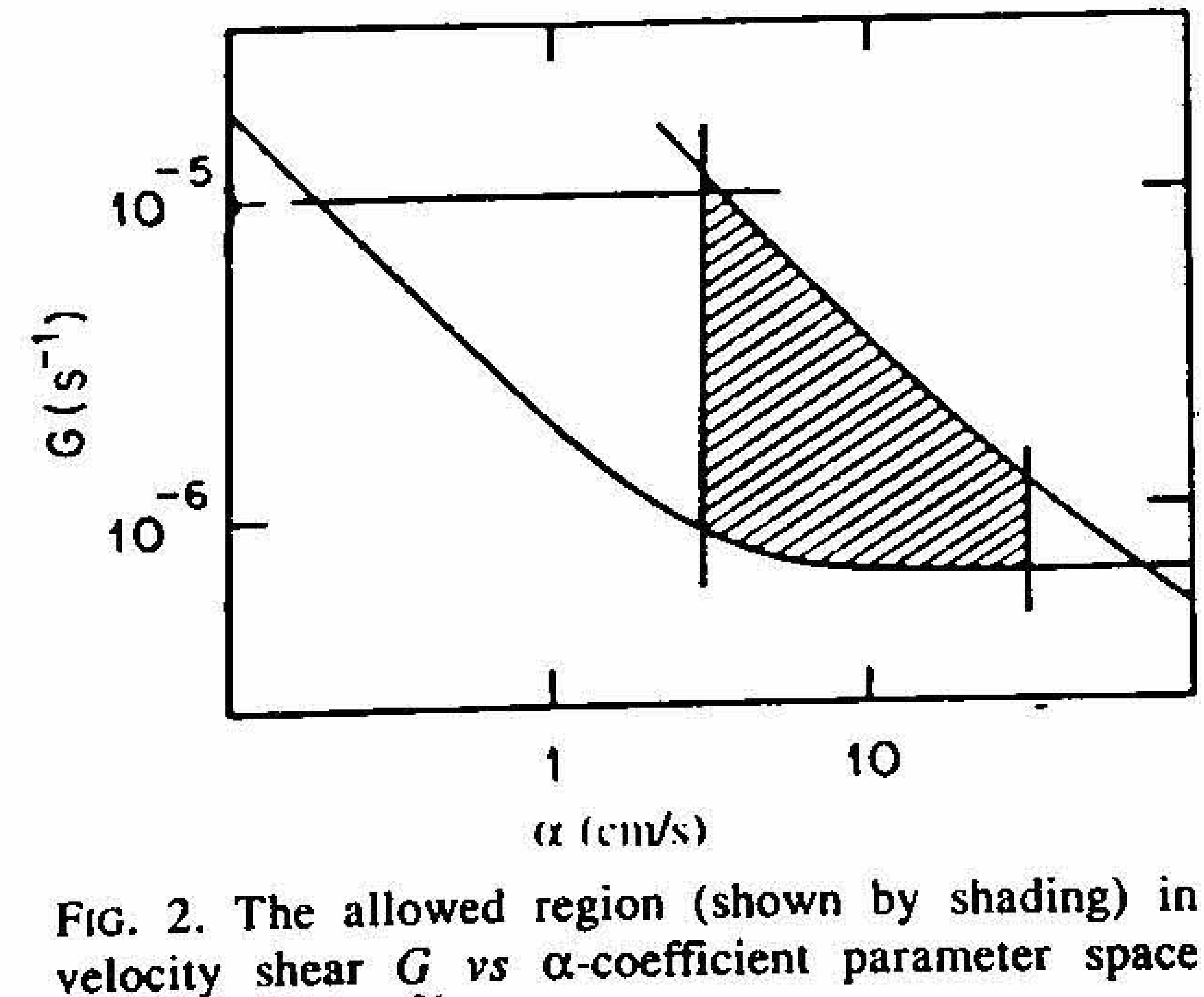


FIG. 2. The allowed region (shown by shading) in velocity shear G vs α-coefficient parameter space (from Choudhuri[26]).

has studied some of the implications of this hypothesis of a boundary layer dynamo below the convection zone. The next few sections describe the problems studied by this group.

3. The dynamo generation of magnetic fields

In order to solve the dynamo equations (6) and (7), one has to specify α, η, and the velocity field. In what is called the kinematic approach, all these quantities are specified on the basis of some plausibility arguments and then the ensuing solutions of the dynamo equations are studied. In principle, one can try to solve the problem self-consistently by solving as well for the velocity field, which satisfies Navier–Stokes equation with one additional Lorenz force term due to magnetic stresses. Although there have been some attempts at numerical simulations of velocity and magnetic fields simultaneously (Gilman[23], Glatzmaier[24], Brandenburg *et al.*[25]), it is extremely difficult to carry out detailed simulations for sufficiently realistic models of astrophysical systems and the subject of numerical dynamo simulations is still in its infancy.

Choudhuri[26] carried out kinematic calculations for the dynamo in a thin layer. The local properties of the dynamo in a region can be found by considering a local Cartesian frame. The dynamo equations in a local Cartesian frame without any meridional flow (*i.e.*, $u_p = 0$) are slightly simpler than the equations (6) and (7) in spherical coordinates. Still one ends up with a formidable dispersion relation for a Fourier mode of the solution. Determining the appropriate boundary conditions to be applied is also a tricky question. The readers are referred to the original paper for a discussion of the boundary conditions. By solving the dispersion relation numerically, it is eventually possible to find how the period and wavelength of the dynamo are related to each other.

We know that the dynamo must have a period of 22 years. Looking at the distribution of sunspots in latitude, one concludes that the half-wavelength of the dynamo should be of the order of about 40° in latitude. If we demand that the dynamo operating in the thin layer at the bottom of the convection zone has the appropriate period and wavelength, then Choudhuri[26] showed that the various parameters specified in the kinematic calculations (such as α, η, the velocity shear) are restricted to have values within certain ranges. Figure 2 shows the range of α and the velocity shear G which Choudhuri[26] found. On the basis of crude mixing length arguments, the ranges of various quantities suggest a rather small length scale of turbulence (a few hundred km) – much smaller than all the relevant lengths and scale heights in the dynamo layer[26, 27]. Parker[28] worked out a model of a layer dynamo with the regions of shear and α-effect separated and found similar values for various dynamo parameters.

The permissible ranges of various dynamo parameters seem to indicate that the dynamo is more probably of the $\alpha^2\omega$ type rather than of $\alpha\omega$ type[26]. Since an $\alpha\omega$ dynamo produces a much weaker poloidal field compared to the toroidal field and the polar field of the Sun is much weaker than the sunspot fields (a manifestation of the toroidal component), it used to be assumed in the early years of dynamo research that the Sun has an $\alpha\omega$ dynamo operating in the convection zone. However, if the dynamo operates in the overshoot layer, then we can allow for an $\alpha^2\omega$ type dynamo which produces more substantial poloidal field, since that field may not be able to leak to the solar surface easily. The physics of the poloidal field will be discussed more in Section 5. We will now discuss another argument for the dynamo being of $\alpha^2\omega$ type.

Although the solar magnetic cycle is roughly periodic, one finds noticeable irregularities superposed on it. The most famous irregularity is a prolonged stretch of time during the 17th century (often called the Maunder minimum), when sunspots were not seen at all. If α, η and the velocity field are assumed to be given, then dynamo equations (6) and (7) are linear and lead to regular periodic solutions. Actually, however, the Lorenz force of the magnetic field is expected to back-react on the fluid flows and inhibit the turbulence. Hence, the full dynamo problem is an intrinsically nonlinear problem as can be seen in the simultaneous simulations of magnetic and velocity fields. Within the framework of kinematic approach, Stix[29] studied the quenching of the α-effect by the magnetic field by taking

$$\alpha = \frac{\alpha_0}{1+|B|^2/B_0^2}. \tag{9}$$

Several other nonlinear calculations[30–33] have been carried out by incorporating the feedback of the magnetic field in the kinematic equations in similar fashions. Although limited evidence of chaos has been found in some toy models[34], the nonlinearities seem to make the regular solutions more stable rather than producing sustained irregularities. This is because a sudden increase (decrease) in the magnetic field would inhibit (enhance) the dynamo source term and make the field decrease (increase) again. Thus, it has not been possible to model the irregularities of the solar cycle satisfactorily by looking for deterministic chaos due to the nonlinear terms.

Since the dynamo equation is obtained by averaging over turbulence, the stochastic fluctuations of different quantities around the mean may be another source of irregularities (see Hoyng[35]). Choudhuri[36] carried out numerical studies of a one-dimensional dynamo model in which the α-coefficient was taken to be stochastically varying around a mean value. Although the correlation time for such fluctuations is expected to be a few months, the irregular patterns in the solar cycle often persist for a few decades. Choudhuri[36] found that this type of qualitative behaviour can be obtained for $\alpha^2\omega$ dynamos if the nonlinearities are not very strong. $\alpha\omega$ dynamos are not much affected by the stochastic fluctuations, whereas α^2 dynamos become completely irregular. Figure 3 shows the solutions for stochastic fluctuations in α having an amplitude of 10% around the mean. The three panels correspond to $\alpha\omega$, $\alpha^2\omega$, and α^2 dynamos. The middle panel qualitatively resembles the solar cycle. It is to be noted that the stochastic fluctuations get suppressed if the nonlinearities are made sufficiently strong.

Hoyng[37] christened this as 'Choudhuri's problem' and showed that many of Choudhuri's numerical results can be understood from the theory of stochastic equations. On the other hand, Moss *et al.*[38] carried out simulations in more complex three-dimensional models and found the results similar to Choudhuri's.

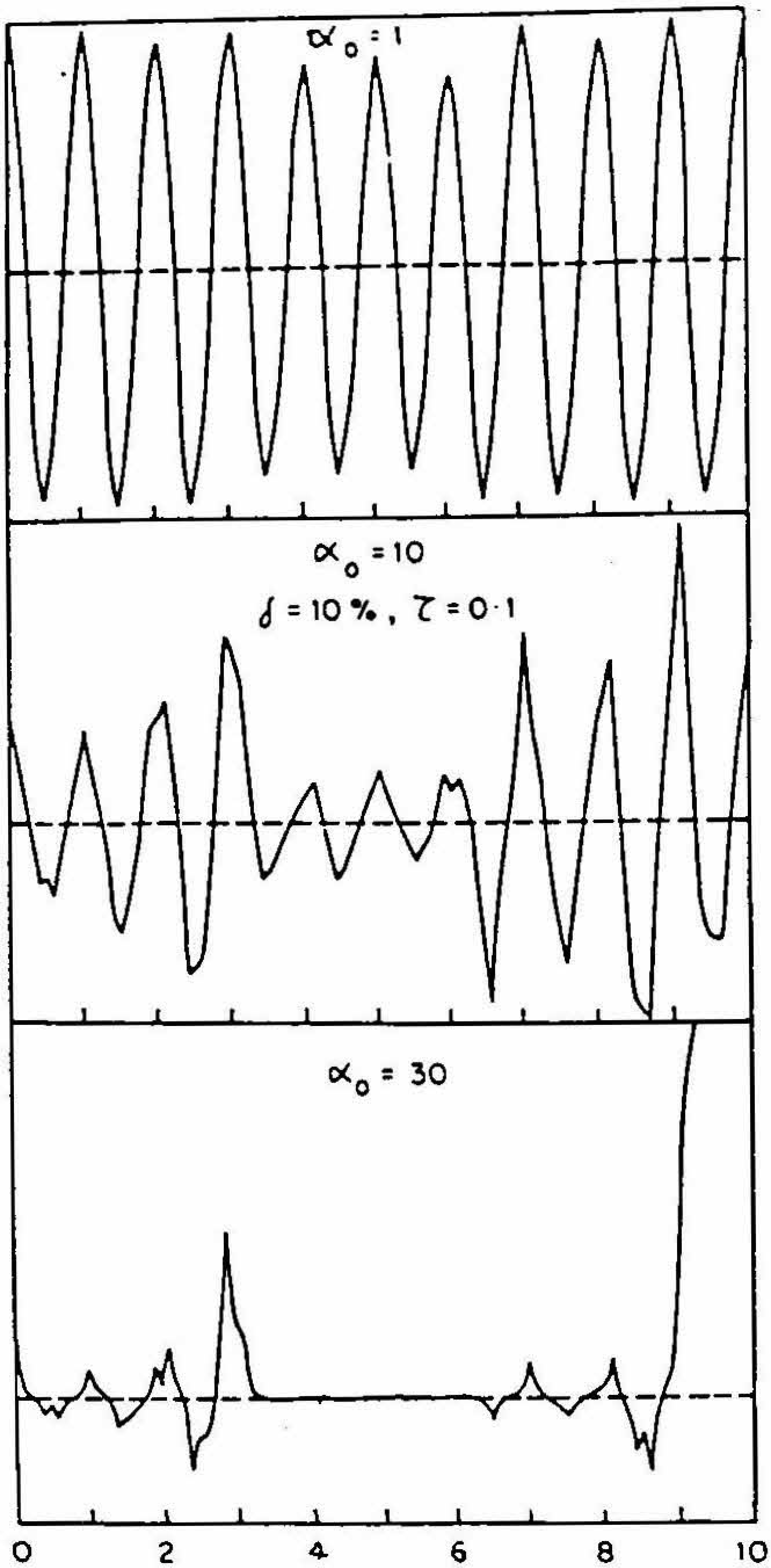


FIG. 3. The variation in magnetic field due to a dynamo with stochastic fluctuations in the α-coefficient. The three panels correspond to $\alpha\omega$, $\alpha^2\omega$, and α^2 dynamos (from Choudhuri[36]).

The readers are referred to the recent reviews on the solar dynamo by Brandenburg[39] or Schmitt[40] for discussions of other aspects of the problem.

4. Flux tube dynamics in the convection zone

If the dynamo operates in a stable layer at the bottom of the convection zone, then one expects that parts of the toroidal field created there would occasionally come out of the stable layer due to turbulent fluctuations. Such strands of magnetic field coming out of the stable layer would be subjected to magnetic buoyancy and would rise in the form of flux tubes as indicated in Fig. 1. Hence, to understand the formation of bipolar sunspot pairs, one has to study the dynamics of flux tubes rising from the bottom of the convection zone due to magnetic buoyancy. Starting from the full MHD equations, Spruit[41] derived an equation for the dynamics of 'thin flux tubes' embedded in an ambient medium. Here 'thin' means that the cross-sectional radius of the flux tube has to be small compared to various scale heights. For such a flux tube of interior density ρ (with external density ρ_e), the acceleration at a point is given by

$$\frac{dv}{dt} = \left[-\frac{1}{\rho} \hat{l} \cdot \nabla p + \hat{l} \cdot g \right] l + \frac{\rho v_A^2}{\rho + \rho_e} k + \frac{\rho - \rho_e}{\rho + \rho_e} (\hat{l} \times g) \times \hat{l}, \tag{10}$$

where $\hat{l}$ is the unit tangent vector, k, the curvature vector and $v_A = B/(4\pi\rho)^{1/2}$ is the Alfvén speed inside the flux tube. The last term in (10) corresponds to magnetic buoyancy and the last but one term to magnetic tension. Choudhuri[42] noted a slight inconsistency in Spruit's derivation of (10) and hence a correction has to be applied to this equation. This correction, however, is based on slightly subtle arguments and does not change the results of calculations qualitatively. So we shall not discuss the correction here.

Since (10) is nonlinear, one usually has to study the dynamics of flux tubes by numerically integrating it. Moreno-Insertis[43] developed the first code to integrate Spruit's equation in two dimensions and showed that the intuitive scenario sketched in Fig. 1 indeed follows from detailed calculations. The magnetic buoyancy factor $(\rho - \rho_e)/(\rho + \rho_e)$ in (10) is related to the magnetic field inside the flux tube and hence one has to specify an initial value of the magnetic field to start the simulations. It is expected that the dynamo will produce a magnetic field roughly in equipartition with fluid turbulence, *i.e.*,

$$\frac{B^2}{8\pi} \approx \frac{1}{2} \rho v^2. \tag{11}$$

The typical equipartition field is not more than about 10^4 G[20]. We shall see below reasons to expect the starting magnetic field to be as strong as 10^5 G. The calculations of the IISc group described below always were done for a range of values of the initial magnetic field.

The buoyant rise time of 10^4 G flux tubes is of the order of several weeks. Since the rotation period of the Sun is about 27 days, Choudhuri and Gilman[44] pointed out that the Coriolis force due to the Sun's rotation would play a very important role in the dynamics of these flux tubes. To illustrate the effect of the Coriolis force, Choudhuri and Gilman considered flux rings symmetric about the rotation axis which are released at the bottom of the convection zone at different latitudes. Figure 4 shows the trajectories of such flux rings in the solar convection zone for three values of the initial magnetic field. It is found that the dynamics of flux rings with equipartition magnetic fields is completely dominated by the Coriolis force. They move parallel to the rotation axis to emerge at

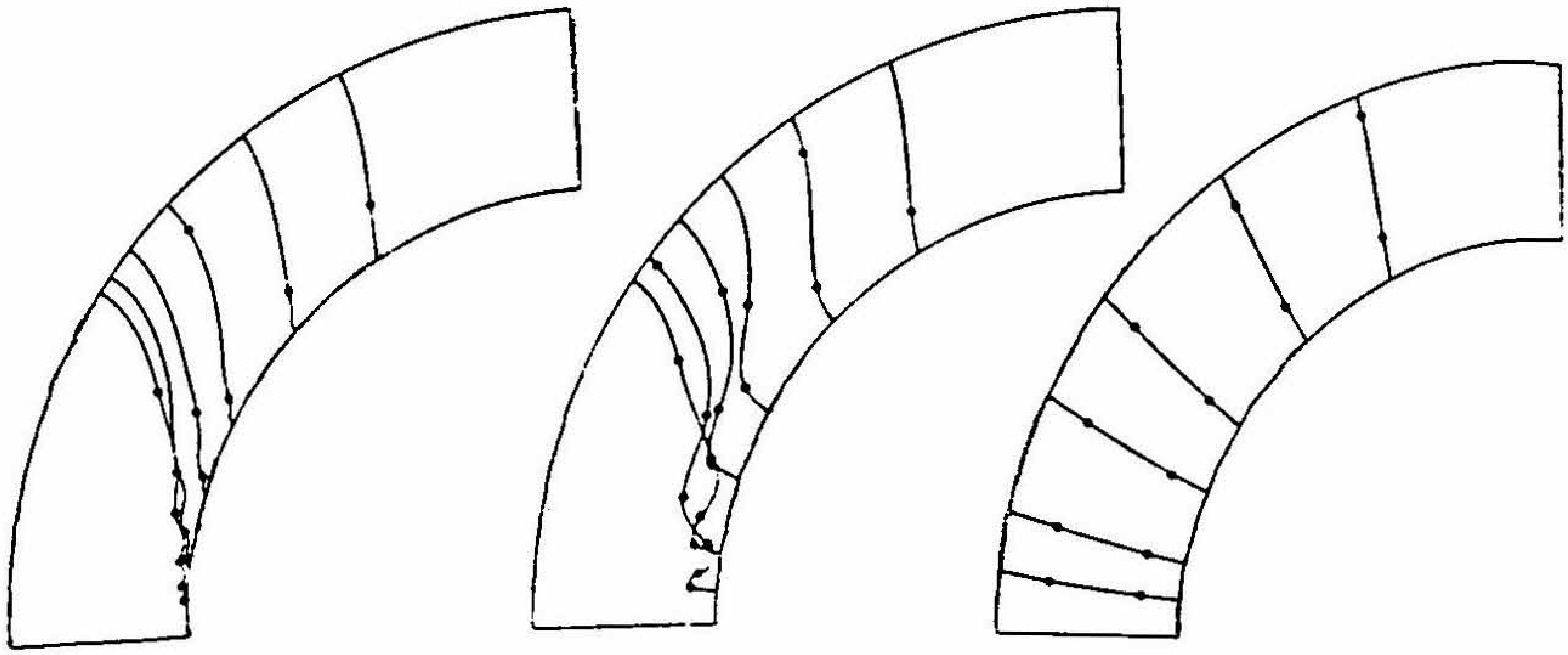
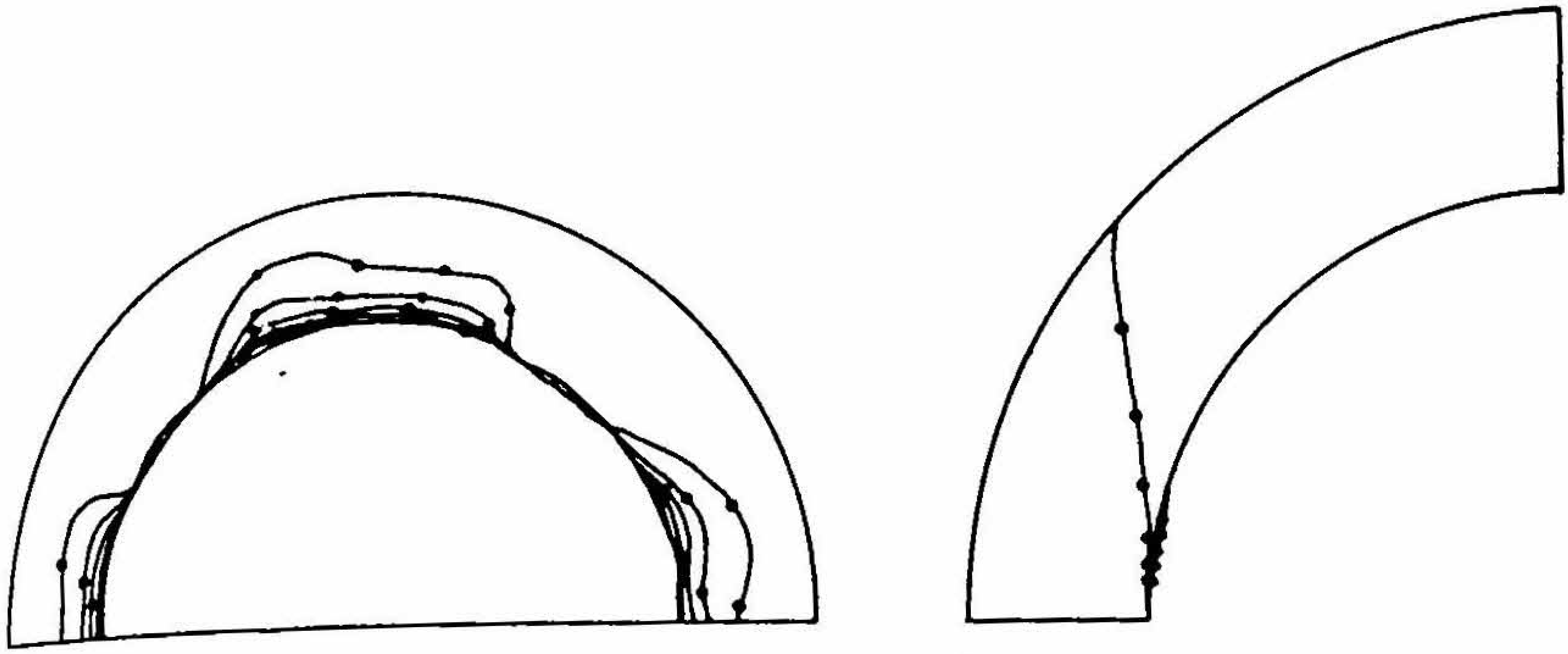


FIG. 4. Trajectories of flux rings starting from different latitudes at the base of the convection zone. The three cases are for the initial magnetic fields of 1.7×10^4 G (left), 5.4×10^4 G (middle) and 1.7×10^5 G (right) (from Choudhuri and Gilman[44]).

latitudes much higher than the typical sunspot latitudes. Only when the starting magnetic field is made of the order of 10^5 G is the magnetic buoyancy sufficiently overpowering and the flux rings starting from low latitudes rise radially to emerge at low latitudes.

Choudhuri[45] developed the first three-dimensional code for solving Spruit's equation in spherical geometry with the Coriolis force. This code was used to study the evolution of a nonaxisymmetric undulated flux ring of which the lower parts remained anchored in the stable layer under the convection zone and the upper parts rose due to magnetic buoyancy. Figure 5 shows the results for such a nonaxisymmetric flux ring starting from 5° latitude with an initial magnetic field of 1.7×10^4 G. Figure 5a is a polar plot of the successive configurations in the (r, ϕ) plane, whereas Figure 5b shows the trajectories in the (r, θ) plane of the highest and the lowest points of the flux ring with the growing loops. We still find that we are unable to get the flux out at typical sunspot latitudes, unless we take the initial magnetic field to be much larger than the equipartition value of 10^4 G.

D'Silva and Choudhuri[46] looked at the tilts of the bipolar sunspots which would result from the rising upper parts of the flux tubes. Figure 6 gives a plot of the observational dependence of tilt on latitude and the theoretical plots which one gets for different values of the initial magnetic fields. The observational plot is based on the analysis of Wang and Sheeley[47]. It is seen that an initial magnetic field of 10^5 G (for which the upper part of flux tube will rise more or less radially) gives a good fit with observations. This constitutes the first quantitative theoretical model of Joy's law nearly three-quarters of a century after its discovery. For weaker magnetic fields, the Coriolis force makes the flux to emerge only at high latitudes with tilts at variance with Joy's law. On the other hand, the tilt becomes negligible for stronger fields, which rise so fast that the Coriolis force does not have enough time to produce an appreciable tilt. We get the correct tilts



a. b.

FIG. 5. Evolution of a partially anchored flux ring with an initial magnetic field 1.7×10^4 G. The successive configurations in (a) or the dots in (b) are at intervals of 14 days (from Choudhuri[45]).

only when the magnetic field is of such value that the magnetic buoyancy force and the Coriolis force become comparable. It was also noted that a bipolar pair with smaller separation will be subject to stronger magnetic tension and hence will have less tilt. Howard[48] was almost immediately able to verify this prediction of D'Silva and Choudhuri[46] using the data already in his possession. Fan *et al.*[49] developed a code similar to Choudhuri's to model the asymmetries of the bipolar sunspot pairs.

The theoretical models seem to fit the observational data well if the magnetic field at the bottom of the convection zone is taken to have a value of 10^5 G. But is such a high value reasonable? It is not clear if the dynamo can produce such a strong field or if the dynamo can even operate in the presence of such a strong field which would inhibit the α-effect completely. Hence, the IISc group has also considered several special mechanisms for suppressing the Coriolis force in the case of more moderate fields of 10^4 G

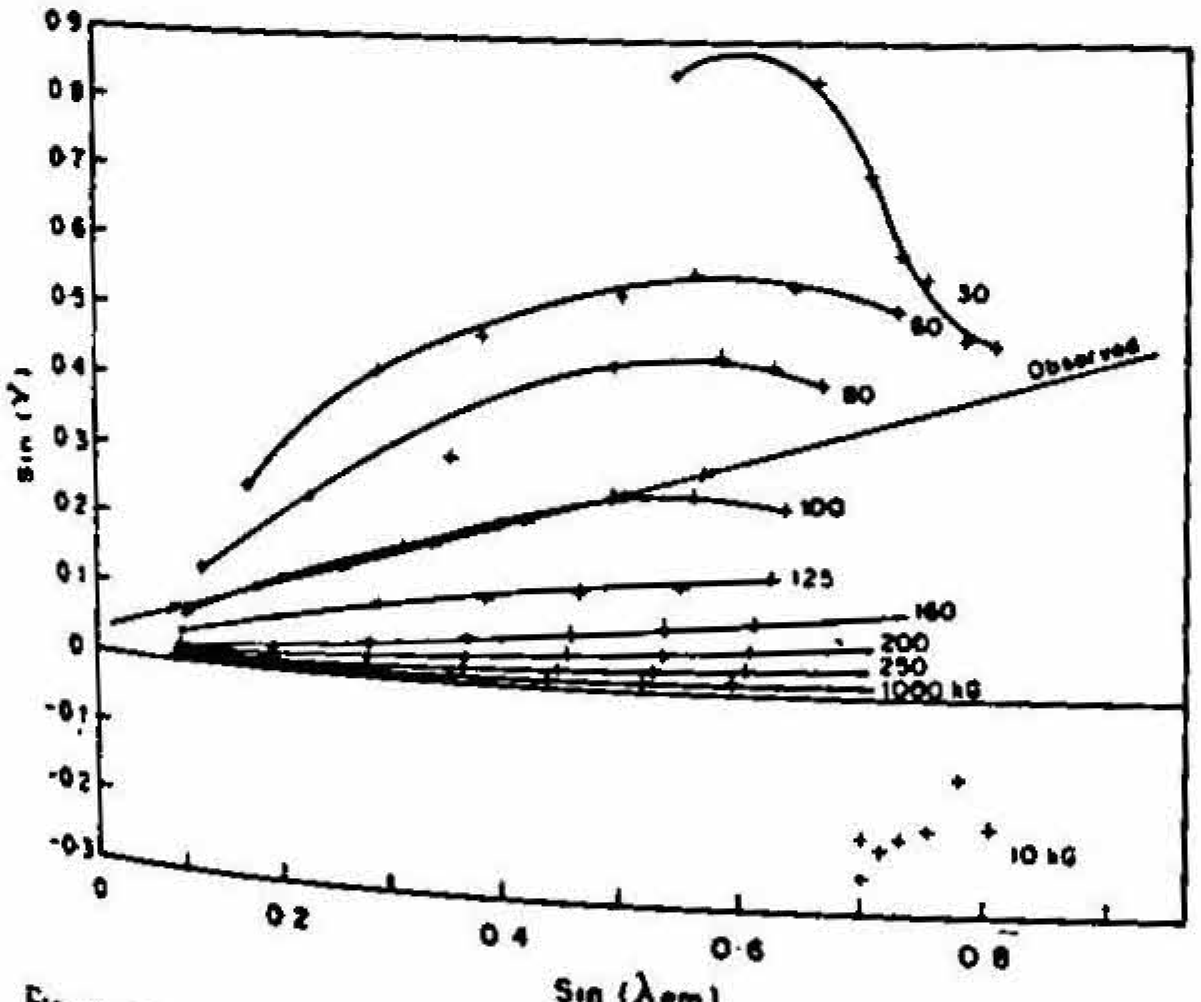


FIG. 6. Plots of sin(tilt) against sin(latitude) obtained for different initial values of the magnetic field indicated in kG (from D'Silva and Choudhuri[46]).

such that the flux tubes with such fields may be made to rise radially. Choudhuri and D'Silva[50] found that if the cross-sectional radius of the flux tube is sufficiently small (a few hundred km), then there may be sufficiently rapid exchange of angular momentum with the surroundings to suppress the Coriolis force. For flux tubes of such small cross-sectional radius, D'Silva and Choudhuri[51] later found another way of suppressing the Coriolis force: Kelvin–Helmholtz instability coupled with the drag of giant convection cells. D'Silva[52] showed that some of these mechanisms can make the bipolar sunspots appear with the correct tilt. However, all these special mechanisms for suppressing the Coriolis force assume some special characteristics of convective turbulence and it is not completely clear if these mechanisms are really operative. The small cross-sectional radius demanded for these mechanisms to be operative may be a result of the fragmentation of a large flux tube during rise[53,54]. Such a fragmentation can account for many aspects of the emergence of sunspots[55].

The arguments for and against both 10^4 and 10^5 G magnetic fields are discussed by Moreno–Insertis[56] in the fist invited review of flux tube dynamics in an important conference. With review articles being written and several groups around the world (Hawaii, Berkeley, Kitt Peak, Freiburg, Tenerife, Budapest) working on it, the subject of flux tube dynamics has suddenly gained prominence. This gives us great pleasure and company as the IISc group was the only one in the world working on this subject for several years. Fan *et al.*[49] recognized our contribution by providing a long summary of our work before presenting the results obtained with a code similar to ours.

5. Evolution of the weak diffuse field

It has already been mentioned that the weak diffuse fields outside sunspots migrate polewards. Even when averaged over latitude, one finds predominantly one polarity in a belt of latitudes which drifts towards the pole[7,57]. There is some observational evidence[58] for a meridional circulation which is poleward at the surface and hence is expected to be equatorward at the bottom of the convection zone. Presumably this circulation plays an important role in the poleward drift of weak diffuse fields.

It is natural to interpret the weak diffuse field as the poloidal component produced by the dynamo. However, the early researchers[59–63], who modelled the solar dynamo to be of $\alpha\omega$ type and to operate in the convection zone, found that the toroidal and poloidal components are strongly coupled together and drift in the same direction. Hence, these classical models could not account for the sunspots (resulting from toroidal component) and diffuse fields (resulting from poloidal component) migrating in opposite directions. In view of this failure, a flux transport model has been developed in which the weak diffuse field is thought to be due to the decay of bipolar sunspot regions. Leighton[64] suggested that turbulent diffusion would make the magnetic flux to disperse from sunspots and the inclinations of bipolar sunspot pairs could account for one polarity predominantly diffusing to higher latitudes. A detailed model of flux transport incorporating the meridional circulation has been developed by the NRL group (see Wang *et al.*[57] and the references therein). Though this model had remarkable success in matching magnetogram data, a conceptually unsatisfactory aspect of the model is that it treats the magnetic field as a scalar residing on the solar surface and satisfying a two-dimensional advection–diffusion equation. It is not clear if this model can be extended to three dimensions to take note of the vector nature of the magnetic field.

Since the dynamo is now believed to operate at the bottom of the convection zone, Dikpati and Choudhuri[65] pointed out that the toroidal and poloidal components should be coupled only there and not inside the convection zone. This again opens up the possibility of regarding the weak diffuse field as the poloidal component of the dynamo. Dikpati and Choudhuri[65] have developed a detailed model to study the evolution of the poloidal field in the convection zone by solving (7) with $\alpha = 0$ (because there is no dynamo action in the convection zone). A running wave corresponding to the dynamo is taken as the bottom boundary condition and acts as the source of the poloidal field. The upper boundary condition is that the field lines smoothly match a potential field outside the Sun. Figure 7 shows the successive configurations of poloidal field lines during a half-cycle obtained by solving (7) with a suitable u_p which is equatorward at the bottom of the convection zone and poleward at the top. The strong toroidal field (which produces bipolar sunspots due to magnetic buoyancy and is coupled to the poloidal field in the dynamo layer) must be propagating equatorward at the base of the convection zone to account for the equatorward drift of sunspots. The weaker poloidal field, for which magnetic buoyancy can be shown to be much less important (because it goes as B^2), is seen in Figure 7 to drift poleward at the top of the convection zone, although the lower parts of the poloidal field lines move equatorward with the dynamo. After demonstrating that the model works, we are now trying to find the best combination of various parameters which will fit the observational data properly[66].

It may be noted that the magnetic fields outside sunspot regions are responsible for transporting energy to the corona *via* MHD waves which eventually dissipate and raise the coronal temperature to values much higher than the Sun's surface temperature. Choudhuri *et al.*[67] developed a model for this energy transport (see also Choudhuri *et al.*[68]). Muller *et al.*[69] recently used this model to analyse their observational data and found that there will be enough energy transported to the corona to account for the high temperature there.

6. Conclusion

Although the aim of this paper has been to present the work done by the IISc group, we trust that it will give the reader some general idea of the current status of theoretical modelling of the solar cycle. We can observe only the sunspots and the diffuse fields on the solar surface. Any inference about the interior processes has to be purely theoretical. It is quite remarkable that the hypothesis of a layer dynamo at the interface between the radiative core and the convection zone can provide a unified scenario for so many aspects of the solar cycle. The sunspots are supposed to result from the toroidal component produced by the dynamo, whereas the weak diffuse field probably results from the poloidal component. It should be emphasized that many pieces of the jigsaw puzzle are still missing. A cloud of uncertainty hangs over the value of the magnetic field produced by

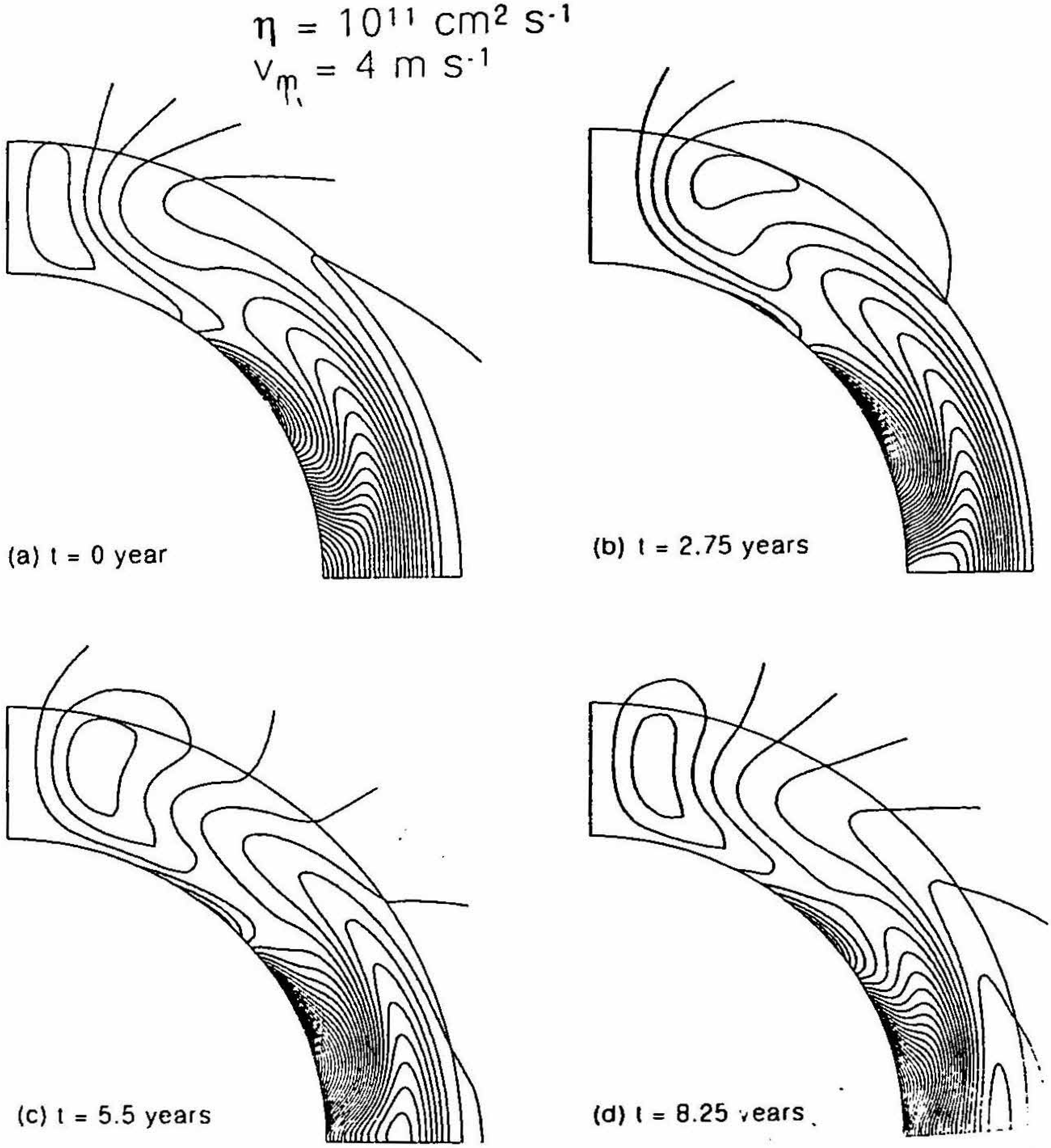


FIG. 7. Four successive configurations of poloidal field lines during a half-cycle (from Dikpati and Choudhuri[65]).

the dynamo. In spite of such shortcomings, the progress in the last few years towards understanding the solar cycle has been noteworthy.

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