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# BOOK REVIEWS

## Differentiable manifolds. A first course by Herbert Amann *et al.*, Birkauser Verlag AG, Klosterberg 23. CH-4010 Basel, Switzerland, 1993, pp. 395, SFr. 88.

The book under review is a leisurely introduction of differentiable manifolds. Mastery of this material will indeed prepare a student for advanced topics courses and seminars in differential topology and geometry.

The first chapter on topological manifolds is concisely written and the core concepts are dealt with. However, in the section on Quotient constructions or elsewhere in the book, I would have preferred quotient constructions coming from group actions on manifolds. Proper discontinuous and free actions of groups on manifolds could have been introduced with examples to show that the quotient space is an orbifold and not necessarily a manifold when the action is not free. The theory of orbifolds initiated by Satake and Thurston forms an interesting area of modern mathematics and an introduction to orbifolds would have been a welcome addition.

Chapters 2 and 3 cover the standard topics on local and global calculus on manifolds. The section on Degree theory and the proof of 'half' of the fundamental theorem of algebra is illuminating. Chapter 4 is on Frobenius integrability. The next chapter is on Lie groups and Lie algebras. The author did a wise thing by assuming all manifolds. bundles, Lie groups, etc., to be smooth of class $C^\infty$. This results in conceptual simplicity of the treatment of Lie groups. Usually the study of Lie groups is carried out in the real analytic ($C^\omega$) category and in this category some results are cumbersome to prove. Since $C^\infty$ groups can be real analytic, no generality is lost by the $C^\infty$ approach.

The author has chosen a pedagogically sound method of first introducing covectors and 1-forms together with first cohomology and then introducing multilinear algebra in the next chapter. Usually multilinear algebra is first introduced and then the topic on the space of forms is developed. But this method could sometimes tax a beginner with more than necessary abstractness at first go. The author choosing the other way round does not have this advantage and moreover prepares the students to see the higher degree forms and cohomology as a natural generalisation.

The important chapter on Integration and cohomology is also well written (like the rest of the book). This chapter culminating in the Poincare duality and the de Rham Theorem is well laid out for the student. The next chapter on Forms and folliations is small and compact. Inclusion of the Godbillon–Vey class in the exercises and the treatment on Foliations is notable. The last chapter on Riemannian geometry succinctly covers the core topics. The appendices in the book form a collection of some basic results used often in mathematics.

The exercises in the book complement very well the topics treated. It is very clear that the author has addressed the book to a fresh student qualifying for Ph. D. and hence is strongly recommended for such students. Even the teachers will find it book stimulating.

K. GURUPRASAD

Department of Mathematics
Indian Institute of Science
Bangalore 560 012.

## Floquet theory for partial differential equations, Vol. 60, by Peter Kuchment, Birkhauser Verlag AG, P. O. 133, CH-4010 Basel, Switzerland, 1993, pp. 368, SFr. 138.

Differential equations with periodic coefficients appear both in ordinary (ode) and partial (pde) differential equations. There are a number of physical applications like inverse scattering, resonance theory, homogenization, elasticity theory, etc., where such equations arise.

An ode $dy/dt = Ay$ where $A$ is an $n \times n$ matrix with periodic coefficients can be handled as follows: with a suitable substitution $z = P(t)y$, the above system can be transformed to a system with constant coefficients. Then any solution of such a system is given by the form (Euler's theorem) $z(t) = e^{i\lambda t} \sum_{k=0}^{n} a_k t^k$ which in turn will give a solution $y(t) = e^{i\lambda t} \sum_{k=0}^{n} b_k(t)t^k$ (Floquet solution) where $b_k$s are periodic in $t$. This is the Floquet theory for odes.

This simple approach does not work for pdes as there are many difficulties. One of them, of course, is the question of transforming the given pde to an equation with constant coefficients which, in general, is not possible (see the introduction of the book under review). Before going further, let me define the Floquet solution for an elliptic equation $L(x, D)u = 0$ of order 2 with smooth periodic coefficients. A solution $u$ is called a Floquet solution if it is given by $u(x) = e^{i\lambda x} \sum_k x^k g_k(x)$, where $k$ runs over all integer $n$-tuples and $g_k$s are period functions. The complex vector $\lambda \in \mathbb{C}^n$ is known as the *quasimomentum* and $z = e^{i\lambda} = \left(e^{i\lambda_1}, \ldots, e^{i\lambda_n}\right)$ is the *Floquet exponent*. If the sum has only the term $k = 0$, then $u(x)$ is called a *Bloch solution*. Understanding the distribution of Floquet exponents and obtaining the expansion of any solution in terms of Floquet solutions are very important in applications. These are all well studied in Chapter 3 for elliptic pdes and we come to this later.

The present book under review begins with a long basic chapter on holomorphic Fredholm operator functions. In this chapter, the author describes operators (in particular, Fredholm operators), Banach vector bundles, Fredholm morphisms, etc. Many results with proofs from complex analysis both in one and several dimensions are also recalled in this chapter. Moreover, holomorphic dependence of Fredholm operators is also worked out here.

Test function spaces and operators required for the study of $Lu = 0$ are described in Chapter 2. These function spaces with suitable modifications can be used for hypoelliptic equations, elliptic systems or pseudo differential equations. A transform that plays the role of Fourier transform, its effect on the function spaces and operators are also investigated here. More precisely, this transform enables us to obtain an isomorphism between spaces of test functions and certain spaces of holomorphic functions introduced in Chapter 1. Moreover, the operators become multiplication operators by some holomorphic morphisms. This is in coincidence with the standard Fourier transform which converts the differential operators to multiplication operators and *vice versa*.

As mentioned earlier, in Chapter 3 the author studies elliptic equation $Lu = 0$ using the results of previous chapters. Here, he shows that any solution of exponential growth, *i.e.*, $| u(x) | \leq C \exp(a | x |)$ can be represented as finite summation of integrals of Floquet solutions (p. 111, Theorem 3.2.1.). The completeness of Floquet solutions in a class of solutions of faster growth, *i.e.*, the linear hull of Floquet solutions of $Lu = 0$ is dense in the topology of $C^\infty_{loc}(\mathbb{R}^n)$ in the set of all solutions $u$ with $| u(x) | \leq C \exp(a | x |^p)$, $p < 1 + \frac{1}{n-1}$, is also obtained in this chapter (p. 112, Theorem 3.3.1). Among the other class of equations, the elliptic systems can be handled as above. Hypoelliptic operators are not, in general, Fredholm in the standard Sobolev spaces and involve a lot of technical considerations which are not presented in this book and only the references are cited. Some open problems regarding the pseudo differential equations are also mentioned.

Many interesting results on the properties of solutions of periodic equations, particularly on the spectral theory, are discussed in Chapter 4. This is one of the most interesting chapters of this book. Distribution of Floquet exponents, the emptiness of the discrete spectrum of the Shrödinger operators $-\Delta + q$, etc., are available here. The solvability of the non-homogeneous equation ($Lu = f$ is solvable in $L_2(\mathbb{R}^n)$ if and only if there are no Floquet exponents in $\{z \in \mathbb{C}^n : |z_j| = 1, j = 1, \ldots n\}$) is studied in Section 4.2. and the Bloch property (existence of nonzero solution with exponential growth implies the existence of Bloch solution of the same type) is given in Section 4.3. The remaining sections, 4.4 and 4.5, are, respectively, devoted for the study of quasimomentum dispersion relation and problems from spectral theory. Many results regarding the positive Bloch solutions and the set of all quasimomentum for positive Bloch solutions are worked out in Section 4.6.

The study of evolution equations of the form $du/dt + A(t)u = 0$, $A(t)$ being a periodic unbounded operator with coefficients in a Hilbert space is carried out in Chapter 5. Section 5.1 is devoted to the study of abstract hypoelliptic evolution equations. The hypoellipticity is a concept more general than the parabolicity. A crucial result in this section is that the set $\Lambda$ of all Floquet exponents is either discrete or all of $\mathbb{C}\setminus\{0\}$. The author goes on to describe the exponentially increasing solutions in terms of Floquet solutions (p. 200, Theorem 5.1.7). In Section 5.2, the author investigates certain degenerate cases. The completeness of Floquet solutions in the class of all solutions of the Cauchy problem is investigated in Section 5.3. Here he considers the equation $du/dt + Au = B(t)A^{\alpha}u$ with $\alpha < 1$, where $A$ is a positive self-adjoint operator and $B(t)$ is a bounded operator function. The elliptic and parabolic problems posed on general cylindrical domain $\mathbf{R}^n \times G$, $G$ being a bounded domain in $\mathbf{R}^m$, are considered in Section 5.4. Here the author obtains similar results as in Section 3.2.

Floquet solutions for equations with deviating arguments (equations of the form $dy/dt = a(t)y(t-1)$) need not be complete. Results in this direction for pdes have been studied in the first section of Chapter 6. Problems related to the equations with coefficients that do not depend on some variables are studied in the second section, while Section 6.3. is devoted to the study of invariant differential equations on Riemannian symmetric spaces of noncompact type.

Now, let us look at some salient features of the book. The clarity of presentation is to be appreciated. It clearly indicates the thorough understanding of the subject by the author. It contains many beautiful results as one can see from the earlier paragraphs. Each chapter ends with a section on comments and references. In these sections, the author quotes [illegible] references required for the results presented and for related problems as well. This is extremely helpful to locate the right reference, particularly for those who want to pursue research in this direction. But it may not be an ideal book for beginners as a first course. I am sure the main results from Chapters 3 to 6, the core material, can be understood by people with reasonable background in pde, but the proofs heavily depend on the first chapter. Here the student needs a thorough background of operator theory, complex analysis, liftings, bundles, morphisms, etc. Once one is familiar with these notions, the book makes enjoyable reading and will be an asset to any library, all the more, when only a few books are available on Floquet theory for pdes.

I am looking forward to the promised joint publication with A. Miloslavski incorporating more results on hypoelliptic evolution equations.

TIFR Centre, IISc Campus
Bangalore 560 012, India

A. K. Nandakumaran

## Periodic solutions of singular Lagrangian systems by A. Ambrosetti and V. Coti Zalati, Birkhauser Verlag AG, P.O. Box 133, CH-4010, Basel, Switzerland, 1993, pp. 157, SFr. 78.

This is a book on dynamical systems; in fact, the main examples treated here come from celestial mechanics, namely, Kepler's problem and the $N$-body problem. Newton in his book *Principia mathematica philosophiae naturalis* published in 1687 has recorded the periodic solutions of Kepler's problem and laid the foundations of western science. Since then, the study of periodic solutions of Kepler's problem (more generally, Lagrangian and Hamiltonian systems) has occupied a prestigious place in the development of mathematics. Many great minds have worked on such problems. One of the famous treatises to appear on the subject was Poincaré's *Méthodes nouvelles de la mécanique céleste*. He organized, for the first time, inherent complexities in Hamiltonian systems and showed that most of them would contain trajectories with widely irregular behaviour. Before him, attention was paid only to the so-called completely integrable systems which exhibit only stable and regular motions.

Periodic solutions lie at the interface between these two types of motion: on the one hand, they represent regular motions, and on the other, a slight perturbation of them may result in other kinds of motion as illustrated by today's computer pictures (see [illegible]). Thus, the knowledge of periodic solutions (and quasiperiodic solutions) helps us understand the structure of the phase space of the dynamical system and our own solar system. That is why Poincaré termed the problem of finding periodic solution as the fundamental problem and considered it as the only opening through which we can force our way into an otherwise impenetrable citadel.

For planar dynamical systems, we have the classical method of Poincaré–Benedixon to find periodic solutions. In the case of integrable systems, they are obtained by clever and explicit manipulations. For other systems, they are obtained by clever and explicit manipulations. For other systems lying near integrable ones, perturbation arguments have been the main tool to prove that periodic solutions persist. Of course, this attempt does not always succeed. In particular, there is the famous problem of small divisors to be overcome. Such results rely on, more or less, refined versions of inverse function theorem. The main results in this direction are the celebrated KAM theorem and Liapunov Centre theorem (see Arnold[2] and Moser[3]).

All the above results are local. Poincaré also initiated some global methods. He reduced the problem of finding periodic solution to the one of finding fixed points (or periodic points) for the so-called Poincaré map. His last geometric functional theorem concerns with latter problem (see Arnold[2]).

The method followed in the present work is variational and so different from the above ones. This is based on the least action principle which states that periodic solutions are critical points of a suitable action principle which states that periodic solutions are critical points of a suitable action $f$ defined on a class of closed curves. The most obvious candidate of the critical point is the point at which the minimum of the functional is attained. But, unfortunately, the functional is not bounded below and this was a fundamental difficulty. The idea to overcome this is to look for critical points lying on higher levels of energy. (Recall minimum corresponds to ground state). This idea has already been implemented to find eigenvectors of self-adjoint matrices which are nothing but critical points of Rayleigh quotient $R(u)$. This is the so-called mini-max characterization given by Courant–Fischer.

$$\lambda_k = \min_{\substack{V_k \subset H \\ \dim V_{k=k}}} \max_{u \in V_k} R(u).$$

To handle nonlinear situations, the above principle is generalized as follows:

$$c = \min_{A \in \mathcal{A}} \max_{u \in A} f(u),$$

where $\mathcal{A}$ is a suitable class of subsets not linear of the state space. The first such example was provided by Lusternik and Schnirelman (L.S.) in 1929, where $\mathcal{A}$ is defined in terms of an interesting topological invariant called category. Of course, the aim is to show that $c$ is a critical value, *i.e.*, there exists $u$ such that $f'(u) = 0$ and $f'(u) = c$. Some of the ingredients used to achieve this are a compactness condition called Palais–Smale Condition (P.S.) and the notion of pseudo-gradient flow or Ekeland's variational principle. The necessity of a compactness criterion is evident even for minimization problems. These results are simply recalled without proof in Chapter 1 of the book. For proofs, we refer the reader to Mawhin and Willem[4] and Struwe[5].

Efforts were made to obtain critical points without using category. A remarkable result in that direction is the so-called Mountain Pass Theorem (MPT) due to Ambrosetti and Rebinowitz. The curious name is due to the following interpretation of the theorem. Imagine two villages in two separate valleys surrounded by mountains. To go from one to another, the canonical route is the path on which the highest altitude is minimized. In doing so, we cross a critical point situated at the top of the mountain. Obviously, this is a deep extension of the familiar Rolle's Theorem in Calculus. In addition, if the Hamiltonian is convex then duality techniques can also be used. The power of these and other fascinating methods can be seen in the study of smooth Hamiltonian systems (see Ekeland[6], Mawhin and Willem[4] and Struwe[5]).The problems considered are of following two types:

*Type A*: The Hamiltonian depends on time and is periodic with a given period $T$. In this case, one looks for $T$-periodic solution.

*Type B*: The Hamiltonian is independent of time. It is then well known that the energy is conserved. It therefore makes sense for periodic solutions lying on a given energy surface.

The present book attempts to carry out the programme indicated above to the case of Hamiltonian involving potentials which are singular at the origin. The singularity introduces extra complications and so the application of mini-max theory is not straightforward. The associated functional is defined only on an open subset instead of the whole space and thus we need to control its behaviour as we approach the boundary. In order to do this, the authors introduce Strong Force Assumption which enables them to treat first strong potentials and then *via* an approximation, they could handle weak potentials as well. They introduce a set of assumptions which imply (PS) condition which is not automatic otherwise. This is done in Chapters III and IV.

In Type B problems, the main difficulty is that the energy surface is not compact any more. To overcome this, a nice variant of the least action principle which is quite ingenius is used (see page 22). Once again, the problem is reduced to finding critical points of a certain functional for which they use the mini-max theory (Chapter V). The study of the $N$-body problem occupies Chapter VI. There are many other results which are presented in Chapter VI without proofs.

This book has taken nontrivial strides into some realistic problems. The research is still in evolution and therefore the results are not to be taken as the final ones. Even though the LS theory and MPT are heavily used, the book simply takes the results of these theories for granted. At many places, the description is very brief and one has a feeling that there are not enough comments, interpretations of the results and hypotheses and historical remarks. Definition and notations introduced earlier are freely used later without any reference. Further, no index is provided. It simply presents a section of results taken from the published articles and of course this does not give an amateur a feeling for the subject. This book, written by the recognized leaders in the critical point theory, is therefore recommended along with the companion books cited earlier.

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TIFR Centre
Bangalore 560 012.

M. Vanninathan

## An introduction to *G*-convergence by Gianni Dal Maso, Birkhauser Verlag AG, P.O. Box 133, CH-4010 Basel, Switzerland, 1993, pp. 340 SFr. 118.

The Γ-convergence and its techniques were introduced in the early seventies by De Giorgi, an Italian mathematician, to study the variational problems associated with integral functionals. Because of the same reason, Γ-convergence is also referred to as 'variational convergence'. This convergence has close relationship with the G-convergence of operators which was also studied, in depth, by De Giorgi and Franzoni.

Let $F_h$ be a sequence of real-valued functionals defined on a topological space $X$. We say $F_h \to F$ as $h \to \infty$ pointwise in $X$ if $F_h(x) \to F(x)$ for every $x \in X$. This is equivalent to

(i) $F(x) \leq \liminf_{h\to\infty} F_h(x)$ and (ii) $F(x) \geq \limsup_{h\to\infty} F_h(x)$, $\forall\, x \in X$.

Assume that $X$ satisfies the axiom of countability. Then the Γ-convergence is equivalent to the following definition.

We say that $(F_h)$ Γ-converges to $F$ if and only if

(a) For every $x \in X$ and for every sequence $(x_n) \in X$ converging to $x$ *in* $X$, we have $F(x) \leq \liminf_{h\to\infty} F_h(x_h)$;

(b) For every $x \in X$, there exists a sequence $(x_n)$ converging to $x$ *in* $X$ such that $F(x) \geq \limsup_{h\to\infty} F_h(x_h)$.

Now, if we observe the conditions (i), (ii) and (a), (b), it is easy to see that (a) is stronger than (i). On the other hand, (b) is weaker than (ii). Thus, the definition of sequential Γ-convergence looks as a very simple modification of the pointwise convergence. But the beauty of Γ-convergence lies in the fact that, under suitable conditions like equicoercivity, this allows us to pass to the limit in the minimization problems. Hence, it is an important tool in practical problems like homogenization, various relaxation problems related to optimal design of structures, equilibrium of crystals, martensitic phase transitions, etc.

The main advantage of Γ-convergence is its good compactness properties. Further, if all the functions $F_h$ are represented as integrals, say, of the form $F_h(u) = \int f_h(x, Du(x))\,dx$, then, under suitable assumptions, the Γ-limit can also be represented as an integral.

Though giving more properties is beyond the scope of this review, as a last point, I would like to mention that, in general, Γ-limit differs from the pointwise limit.

Before giving a brief sketch of its contents let us leaf through the beautiful book.

This book is written by an expert and it is a great pleasure to read it. I am afraid that I may not be able to project its complete beauty, but would like to point out its various features. Lengthy chapters most often put a reader off. The author has taken a lot of pain to prepare small chapters but without sacrificing continuity. Even a novice reader would not find it difficult to locate a particular item. It is enough to glance through the contents. Another good aspect is that each theorem is followed by one or more examples which makes the theorem more transparent and hence provides a good understanding of the results. The later part of the book contains two sections, namely, guide to literature and bibliography. The former is a good help, especially, to nonexperts. The latter contains a vast amount of references not only on the subjects discussed in the book, but also on topics related directly or indirectly. Perhaps, a major omission in this section is a paper by G.Nguetseng (A general convergence result for a functional related to the theory of homogenization, *SIAM J. Math. Anal.*, 1989, 20(3)) 608-623).

The book is divided into 25 small chapters together with a 'guide to literature' and a 'bibliography'. It begins with a small introduction explaining the importance of Γ-convergence. The first three chapters are devoted for the review of direct methods of calculus of variations, minimum problems for integral functionals and relaxation methods. After introducing the notion of Γ- and $K$-convergences (a notion of convergence of sets in the sense of Kuratowski) in Chapter 4, the author studies the various properties of Γ-limits and comparison with pointwise convergence (Chapters 5 and 6). In Chapter 4, he proves that $(F_h)$ Γ-converges to $F$, then the corresponding epigraphs converge in the sense of Kuratowski.

Chapter 7 is devoted to the study of convergence of minimization problems. He proves that under certain conditions like equicoercivity, the Γ-convergence implies the convergence of minima and minimizers.

Sequential characterization, as mentioned earlier when $X$ satisfies the first axiom of countability, is introduced in Chapter 8 with additional results when $X$ is a Banach space endowed with its weak topology. When $X$ is a metric space, the author examines the relationship between the Γ-convergence and Moreau–Yosida approximations (Ch. 9). A topology $\tau$ is introduced on $S(x)$ of all lower semi-continuous functionals on $X$ (Ch.10) and in $S(x)$, the Γ-convergence implies the convergence in the topology, but the converse holds if $X$ is Hausdorff space with local compactness or the first axiom of countability. The relationship between the Γ-convergence of quadratic forms and the $G$-convergence of its operators is studied in Chapters 12 and 13. In Chapter 11, the author proves that the Γ-limit of a sequence convex functions is still a convex function with a similar result for quadratic forms.

The remaining chapters up to 20 are developed with applications in mind. The crucial question is whether the Γ-limit of a sequence of integral functionals is an integral functional? The author mainly restricts his attention to the space $X = L^p(\Omega)$, $1 \leq p < \infty$, $\Omega$ a bounded open set in $\mathbf{R}^n$. He develops the 'localization method' for the study of such functions. He introduces a weaker notion called $\overline{\Gamma}$-convergence and proves that under certain conditions $\overline{\Gamma}$-convergence is also induced by a topology (Ch.14–17). Under suitable conditions, like $F_h$ satisfies uniform fundamental estimates, the $\overline{\Gamma}$ limit of a sequence of measures is also a measure is proved in Chapters 18 and 19 and the integral representation theorem is proved in Chapter 20. The estimates used in the localization methods can also be useful to deal with Dirichlet or mixed boundary conditions (Ch. 21).

$G$-convergence of elliptic operators is studied in Chapter 22. The remaining three chapters are devoted to the study of homogenization problems using the techniques developed earlier. The author concludes the book with a guide to literature.

I strongly recommend the book to all beginners on nonlinear problems in PDE, asymptotic analysis and many other research topics mentioned earlier. It is a good reference for the other researchers as well.

A. K. NANDAKUMARAN

TIFR Centre
Bangalore 560 012.

**Non-classical equations of mixed type** by A.G. Kuz'min, Birkhauser Verlag AG, P.O. Box 133, CH–4010 Basel, Switzerland, 1993, pp. 298, SFr. 158.

The mathematical study of mixed-type equations (elliptic hyperbolic) and its applications to the difficult problems of transonic gas dynamics goes back to the days of Lord Raleigh, Tricomi and Chaplygin. With the advent of powerful serial and parallel computers, computational methods have been developed to a high degree of sophistication by physicists and engineers. The fundamental problem of unicity, uniqueness and stability of the transonic boundary-value problem still remains unclear.

This book is primarily devoted to this aspect of the problem which would add a certain amount of rigour to the cookery which engineers resort to derive practical solutions to problems. The topic of the book (non-classical type of mixed-type equations) refers to the mixed type of second-order partial differential equations which is irreducible by change of variables to one of the two well-known equations having a single-type degeneracy line that is not tangent to the characteristic direction at any point.

The book is divided into five main chapters, covering 290 pages. Chapter I deals with the geometrical interpretation of the characteristic lines for the two-dimensional problems of partial differential equations of the second order. The qualitative aspects of the mixed type of equations are discussed based on the characteristic theory. Chapter II which is the most important one for the understanding of the subsequent chapters dealing with model gas dynamic equations deals mainly with the applications of functional analysis to mixed type of equations and discusses in detail the solvability of the boundary-value problems in Sobolev space. The theorems quoted are all based on the author's own contributions and should be of interest to mathematicians. Chapters III to V deal rigorously with the classical problems of continuous flow in a de Laval nozzle, non-isentropic flows and acoustics field in non-uniform flow. Frankl, a Russian mathematician had first proved the problem in the hodograph plane (godograph, a translation error!). But the solution could not be interpreted in the physical plane. The author treats the linear problem in the stream function formulation. Hence, flows with shocks could not be treated. Although the author treats the direct problem of the nozzle, the most important problem of relevance to the design engineer, the shockfree aerofoil, is rather sketchy. The significant development made in this field by Paul Garabedian with his colleagues Bauer and Korn who were the first to design shockfree aerofoil has not even been mentioned.

Non-existence of continuous transonic irrational flow over an aerofoil was shown by Cathleen Morawetz. However, the breakthrough in transonic flow computation by Murman and Cole has not been discussed.

Considering the enormous difficulties involved in proving the transonic boundary-value problem (in fact, multiple solutions for potential and Euler equations have been obtained by CFD workers), the author needs to be commended in bringing out the fundamental mathematical issues involved in solving the mixed type of boundary-value problem. As pointed out in the preface by M. Prottere it seems likely that the bits and pieces of the results will be put together eventually to form a single comprehensive theory for a mixed-type equation in $R^2$ space similar to the classical theory for the standard elliptic, hyperbolic and parabolic equations. At present, there still are more questions to be answered.

Apart from these comments, the book would be of greater use to physicists and mathematicians than to aircraft and rocket design engineers as mentioned in the introduction.

CSIR Centre for Mathematical Modelling and Computer Simulation
National Aerospace Laboratory
Bangalore 560 037.

N. R. SUBRAMANIAN

**Operator extensions, interpolation of functions and related topics** by A. Gheondea *et al.*, Birkhauser Verlag AG, P.O. Box 133, CH-4010 Basel, Switzerland, 1993, pp. 224, SFr. 92.

In the past few decades Romania has nurtured one of the strongest schools in operator theory. The conferences organised by this school have attracted participants from all over the world. It is heartening to note that this tradition has survived the upheavals of the last few years. The volume under review contains 12 papers presented at the 14th conference held in this series in June 1992. The papers are:

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| 1. D. ALPAY, V. BOLOTNIKOV, A. DIJKSMA AND H. DE SNOO | On some operator colligations and associated reproducing kernel spaces. |
| 2. J. BRASCHE, H. NEIDHARDT AND J. WEIDMANN | On the spectra of selfadjoint extensions |
| 3. P. COJUHARI | On the spectrum of singular nonselfadjoint differential operators |
| 4. A, DIJKSMA, M. A. DRITSCHEL, S. MARCANTOGNINI AND H. de SNOO | The commutant lifting theorem for contractions on Krein spaces |
| 5. M. A. DRITSCHEL | A method for constructing invariant subspaces for some operators on Krein spaces |
| 6. T. FURUTA | Applications of the Furuta inequality to operator inequalities and norm inequalities preserving some orders |
| 7. A. GHEONDEA | Quasi-contractions on Krein spaces |
| 8. S. HASSI AND K. NORDSTROEM | Antitonicity of the inverse and $J$-contractivity |
| 9. M. D. MORAN | Unitary extensions of a system of commuting isometric operators |
| 10. A. A. NUDELMAN | Some generalisations of classical interpolation problems |
| 11. I. SUCIU | The Kobayashi distance between two contractions |
| 12. L. WAELBROEK | The category of quotient bornological spaces |

Let $K$ be a linear space with a Hermitian sesquilinear (but not positive) inner product $< .,. >_K$. Suppose $K$ is the algebraic direct sum of two subspaces $K_+$ and $K_-$ which are orthogonal with respect to $< .,. >_K$ and which are Hilbert spaces with the inner products $< .,. >_K$ and $- < .,. >_K$, respectively. Then $K$ is called a Krein space. Such spaces with an indefinite inner product, and linear operators on them, have been studied extensively in the last 20 years. Papers 4, 5, 7, 8 as their titles indicate are devoted to this subject. (A quasi-contraction is a linear operator which is contractive when restricted to some subspace of finite codimension).

Another area associated with the name of Krein is the spectral theory of selfadjoint extensions of semi-bounded operators on a Hilbert space. Krein studied this problem mainly for the case of finite deficiency indices. In paper 2 the authors extend some of these results to operators with infinite deficiency indices.

Operator inequalities (in the positive semidefinite ordering) are studied in papers 6 and 8. In the latter paper the authors derive some conditions under which the inverse operator function is antitone on selfadjoint operators.

In paper 10 some matrix versions of the Nevanlinna–Pick, the Caratheodory and the Schur interpolation problems are studied. Such problems arise in the design of optimal linear systems.

Unitary and isometric realisations for a class of operator-valued functions are studied in the first paper in the collection.

Most of the papers in this collection are meant for the expert well into the respective area. Normally they would appear as articles in journals to which libraries may already be subscribing. The book budget is always smaller and this makes the task of recommending such books to libraries harder. I am glad I received this book as a reviewer and not as a member of a library committee.

Indian Statistical Institute
New Delhi 110 016.

RAJENDRA BHATIA

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