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Aeroacoustic analysis of the two-duct reverse-flow perforated elements

M. L. MUNJAL, K. NARAYANA RAO* AND A. D. SAHASRABUDHE
Department of Mechanical Engineering, Indian Institute of Science, Bangalore 560 012.

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Abstract

This paper deals with the derivation of the transfer matrices of the two-duct reverse-flow (expansion and contraction) elements, using both the segmentation and distributed parameter methods. These have been used to evaluate transmission loss (TL) of the expansion element. TL predictions over the entire plane wave frequencies range have been compared. A near complete agreement between the two curves at different mean flow Mach numbers indicates the correctness of the transfer matrices derived by the two methods.

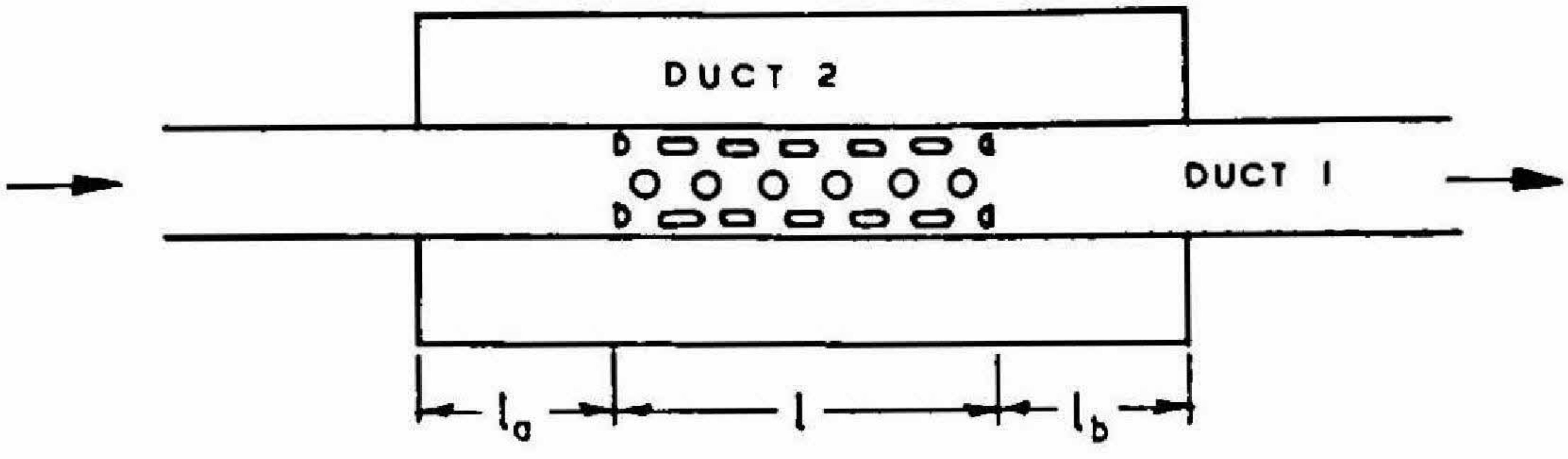
Key words: Aeroacoustic analysis, mufflers, perforated elements, noise control, transfer matrix method.

1. Introduction

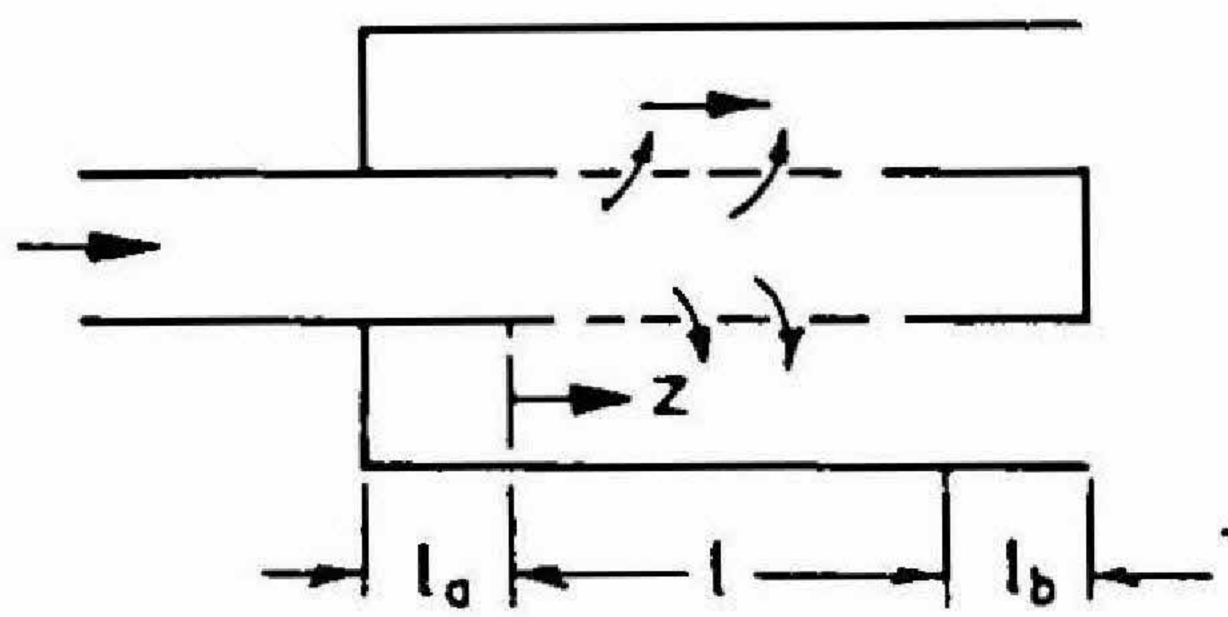
Commercial automotive mufflers (or silencers) make extensive use of perforated elements. A uniformly perforated tube opens into an expansion chamber, leading the mean flow in and out. Of late, two substantially different methods have been developed for the aeroacoustic (or flow-acoustic) analysis of these perforated elements. The first of these methods is the Segmentation method due to Sullivan[1,2] and the second is the Distributed parameter method due to Rao and Munjal[3,4]. In the Segmentation method, the perforations are notionally lumped at the junctions of an arbitrary number of segments, whereas in the distributed parameter method, a uniformly perforated tube is treated as such. While the former involves simultaneous solution of a number of algebraic equations (for the usual assumption of the sinusoidal time dependence, characteristic of the frequency-domain analysis), the latter involves simultaneous solution of two or three ordinary differential equations, coupled through the impedance of the perforate.

Both these methods have been made use of to derive transfer matrices of concentric tube resonators, two-duct cross-flow expansion, two-duct cross-flow contraction, three-duct cross-flow chamber, and three-duct reverse-flow chamber (fig. 1). These matrices have been verified indirectly by comparing the computed values of the performance (in terms of transmission loss (TL) of the mufflers made from various

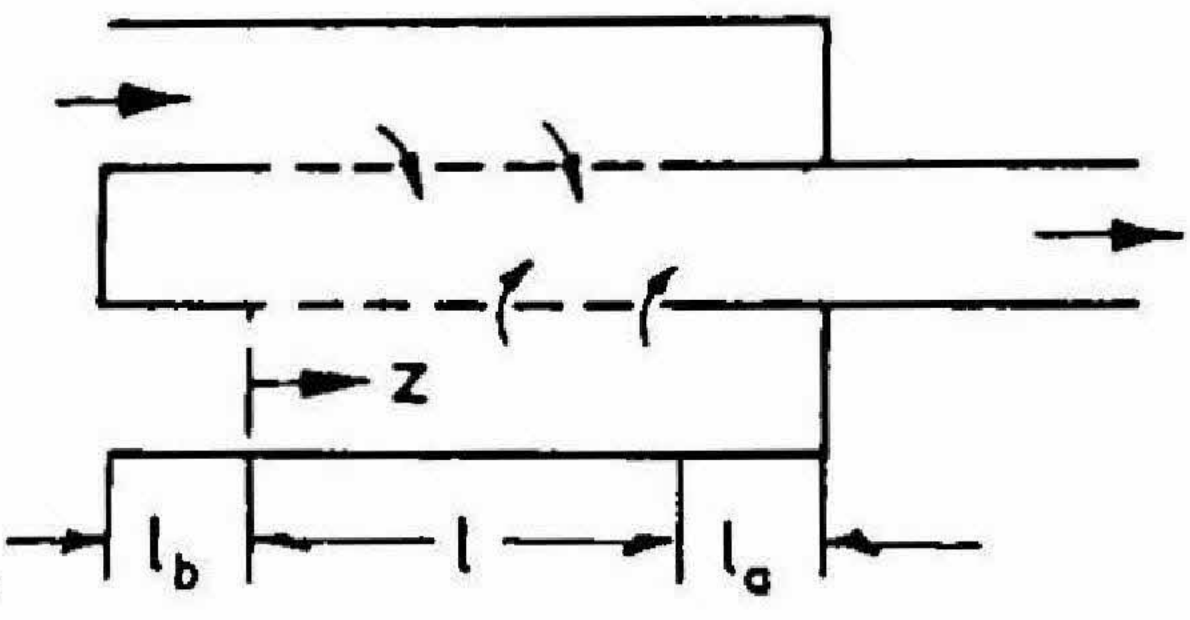
*Permanent address: Department of Mechanical Engineering, University College of Engineering, Osmania University, Hyderabad 500 007.



a. Concentric-tube resonator.



b. Cross-flow expansion element.



c. Cross-flow contraction element.

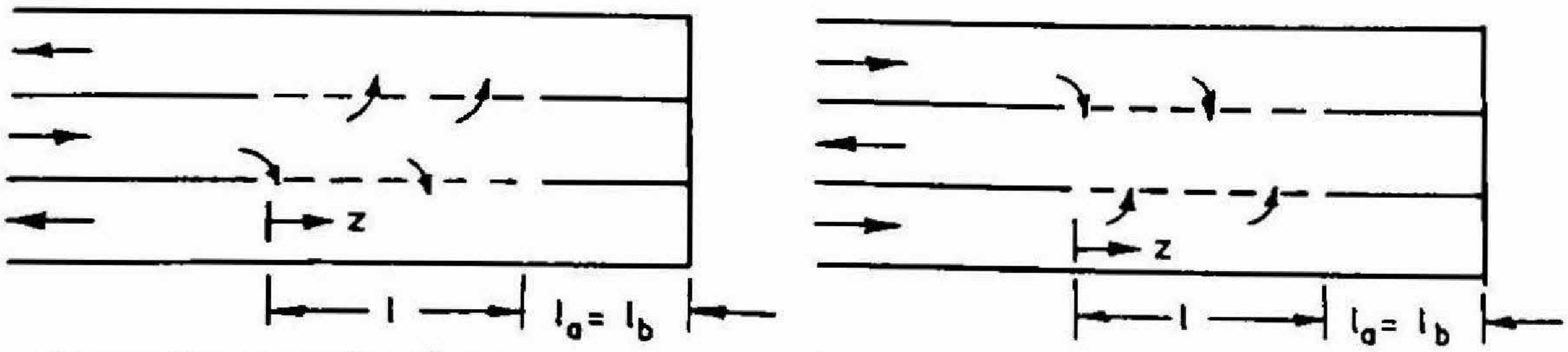
FIG. 1. Two-duct perforated forward-flow elements.

perforated elements, with those observed experimentally over the entire frequency range relevant to pure plane wave propagation[1-4].

In this paper attention is directed to the yet unexplored elements, the two-duct reverse-flow perforated elements (fig. 2). The two elements differ only in the direction of mean flow, and as will be clear later, their transfer matrices would be inverse of each other. So, attention is restricted to the first configuration (fig. 2a) for most part of this article. The transfer matrix of this element is derived by means of both the methods. These two substantially different forms of the transfer matrix are used to estimate values of TL at all frequencies (relevant to pure plane wave propagation). These are then compared to check the correctness of the transfer matrices derived by the two methods.

2. The Segmentation method

Two flow ducts are joined together by a perforated section of length L and specific acoustic admittance $a(x)$. Duct 1 has cross section S_1 and termination acoustic impedances $Z_1(0)$ and $Z_1(\mathrm{L})$. These and the corresponding parameters for duct 2 are shown in fig. 3. Mean flow mach numbers M_1 and M_2 vary all along the duct due to mean flow interchange between ducts. Temperature gradients along the duct length are neglected.



a. Reverse-flow expansion element. b. Reverse-flow concentration element.

FIG. 2. Two-duct perforated reverse-flow elements.

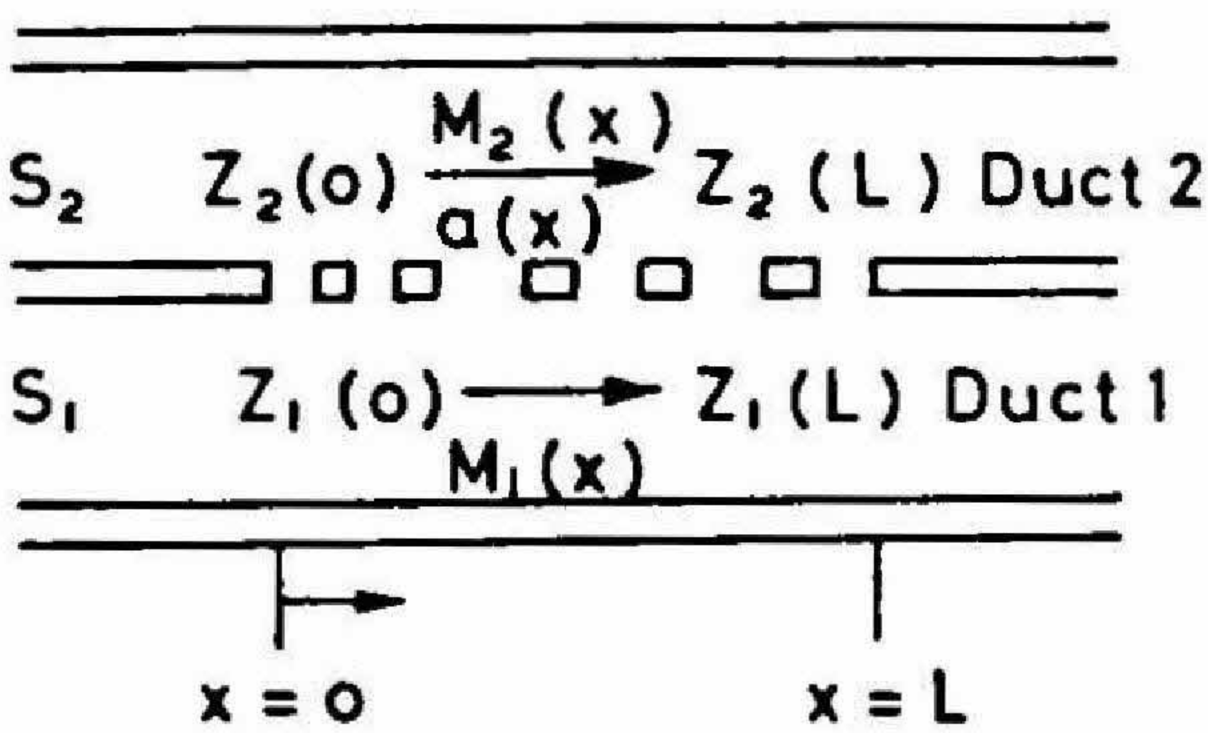


FIG. 3. Basic two-duct element.

In an approximation, the admittance is lumped at discrete intervals or branches. Between branches, the admittance is zero (rigid wall) and convected plane wave motion is assumed. Thus each duct is reduced to a finite number of parallel branch points, each of which can be assigned a spacing, an admittance and a mean flow mach number to suit the configuration to be modelled (fig. 4).

The equations necessary for determining the acoustic pressures and velocities at the branch are derived from consideration of momentum balance, and continuity of energy and mass flow.

Assuming that the spatial variations in both mean and fluctuating components over each face of the control volume, shown in fig. 5, are negligible, the energy and mass balance equations for the jth branch may be solved simultaneously to obtain[1,5]

$$\begin{bmatrix} P_{1,2j} \\ U_{1,2j} \\ P_{2,2j} \\ U_{2,2j} \end{bmatrix} = [\, G_j \,] \begin{bmatrix} P_{1,2j-1} \\ U_{1,2j-1} \\ P_{2,2j-1} \\ U_{2,2j-1} \end{bmatrix} \tag{1}$$

where $[\, G_j \,]$ is the jth branch 4×4 transmission matrix, the elements of which are as follows[1,5]:

$$G_{11} = (1 - B_3(B_4 + B_5))/E_1;$$

$$G_{12} = (B_1 - B_3(1 + B_1(B_4 + B_5 - B_2)))/E_1;$$

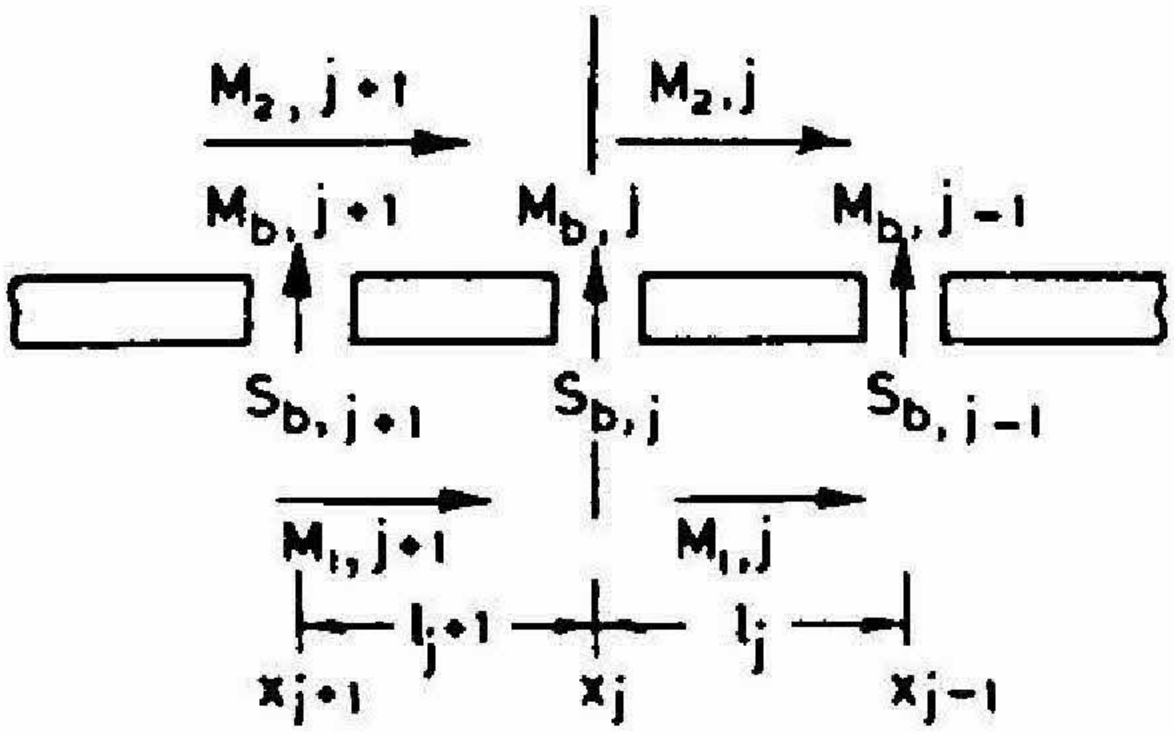


FIG. 4. Branch point model of perforate.

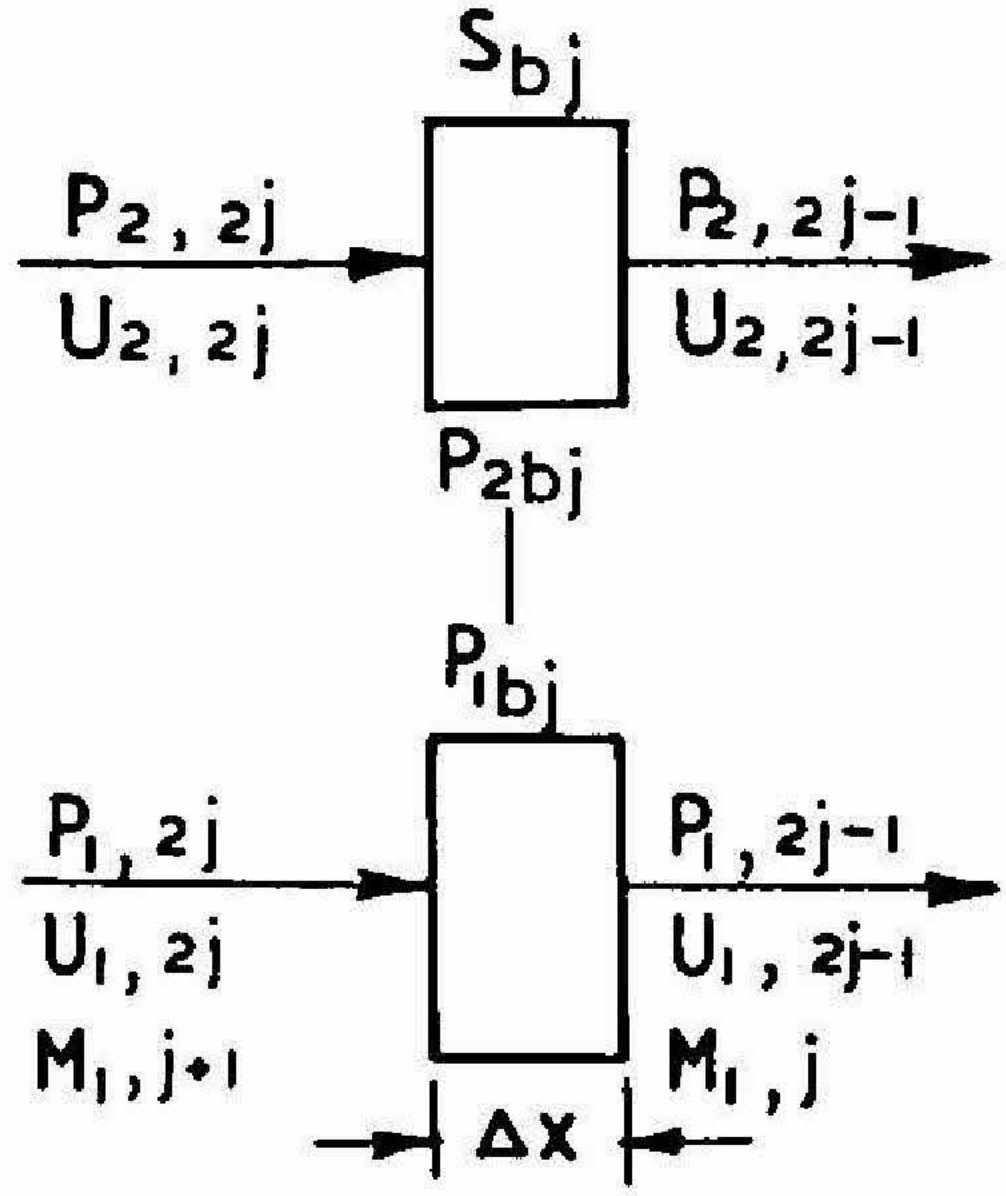


FIG. 5. Control volume representation of the J^{th} branch two-duct element.

$$G_{13} = B_3 B_5/E_1;\ G_{14} = B_3 C_1 B_5/E_1;$$
$$G_{21} = B_5/E_1;\ G_{22} = (1 + B_1(B_5 - B_2))/E_1;$$
$$G_{23} = -B_5/E_1;\ G_{24} = -C_1 B_5/E_1;$$
$$G_{31} = C_3 B_5/E_2;\ G_{32} = B_1 C_3 B_5/E_2;$$
$$G_{33} = (1 - C_3(C_4 + B_5))/E_2;$$
$$G_{34} = (C_1 - C_3(1 + C_1(C_4 + B_5 - C_2)))/E_2;$$
$$G_{41} = -B_5/E_2;\ G_{42} = -B_1 B_5/E_2;$$
$$G_{43} = B_5/E_2;\ G_{44} = (1 + C_1(B_5 - C_2))/E_2; \tag{2}$$

where

$$B_1 = M_{1,j} Z_1;\ B_2 = M_{1,j}/Z_1;$$
$$B_3 = M_{1,j+1} Z_1;\ B_4 = M_{1,j+1}/Z_1;$$
$$B_5 = A_j(1 - (M_{1,j+1} - M_{1,j})^2 Z_{bj}^2/Z_1^2); \tag{3}$$
$$C_1 = M_{2,j} Z_2;\ C_2 = M_{2,j}/Z_2;$$
$$C_3 = M_{2,j+1} Z_2;\ C_4 = M_{2,j+1}/Z_2; \tag{4}$$
$$E_1 = 1 - M_{1,j+1}^2;\ E_2 = 1 - M_{2,j+1}^2.$$

$$Z_1 = \frac{\rho_0 c}{S_1}\ ;\ Z_2 = \frac{\rho_0 c}{S_2}\ ;\ Z_{bj} = \frac{\rho_0 c}{S_{bj}} \tag{5}$$

$U = (uS)$ volume velocity; $S_{bj} = \pi d l_j$

d = diameter of perforated tube and l_j = length of the segment.

Assuming one-dimensional, convected flow between branch points, the pressure and volume velocity equations for harmonic motion may be rearranged to yield[1,5],

$$\begin{bmatrix} P_{1,2j+1} \\ U_{1,2j+1} \\ P_{2,2j+1} \\ U_{2,2j+1} \end{bmatrix} = [\,H_j\,] \begin{bmatrix} P_{1,2j} \\ U_{1,2j} \\ P_{2,2j} \\ U_{2,2j} \end{bmatrix} \tag{6}$$

where $[H_j]$ is a 4×4 transmission matrix between the j and $j+1$ branches, the elements of which are as follows.

$$H_{11} = F_1 \cos\alpha_1;\; H_{12} = iF_1 Z_1 \sin\alpha_1$$
$$H_{13} = H_{14} = 0;\; H_{21} = iF_1 Z_1^{-1} \sin\alpha_1$$
$$H_{22} = F_1 \cos\alpha_1;\; H_{23} = H_{24} = 0$$
$$H_{31} = H_{32} = 0;\; H_{33} = F_2 \cos\alpha_2$$
$$H_{34} = iF_2 Z_2 \sin\alpha_2;\; H_{41} = H_{42} = 0$$
$$H_{43} = iF_2 Z_2^{-1} \sin\alpha_2;\; H_{44} = F_2 \cos\alpha_2$$

where

$$\alpha_1 = Kl_j/E_1$$
$$\alpha_2 = kl_j/E_2$$
$$E_1 = 1 - M_{1,j+1}^2;\; E_2 = 1 - M_{2,j+1}^2$$
$$F_1 = \cos(M_{1,j+1}\alpha_1) - i \sin(M_{1,j+1}\alpha_1)$$
$$F_2 = \cos(M_{2,j+1}\alpha_2) - i \sin(M_{2,j+1}\alpha_2).$$

By successive multiplication of transmission matrices given by eqns (1) and (6) the pressure and volume velocity at $x = 0$ can be related to the pressure and volume velocity at $x = L$; *i.e.*,

$$\begin{bmatrix} P_{1,2N} \\ U_{1,2N} \\ P_{2,2N} \\ U_{2,2N} \end{bmatrix} = [\,T_{4\times4}\,] \begin{bmatrix} P_{1,1} \\ U_{1,1} \\ P_{2,1} \\ U_{2,1} \end{bmatrix} \tag{7}$$

where N is the number of segments or branches, and $[T]$ is the overall transmission matrix given by

$$[T] = [G_N] \prod_{K=1}^{N-1} [H_K][G_K] \,. \tag{8}$$

By specifying two of the four termination impedances in the basic element (fig. 3), we get a four-pole parameter representation of the remaining two terminals.

The boundary conditions for the flow-reversal element of fig. 2a are:

$$\frac{U_{1,1}}{P_{1,1}} = A_{1,1} = (i/Z_1) \tan kl_b, \tag{9}$$

$$\frac{U_{2,1}}{P_{2,1}} = A_{2,1} = (-i/Z_2) \tan kl_b. \tag{10}$$

With the help of these two boundary conditions, eqn (7) reduces to the desired transfer matrix relation

$$\begin{bmatrix} P_{1,2N} \\ U_{1,2N} \end{bmatrix} = [T'] \begin{bmatrix} P_{2,2N} \\ U_{2,2N} \end{bmatrix} \tag{11}$$

where the elements of the 2×2 transfer matrix $[T']$ are given by[5]

$$T'_{11} = (de-cf)/(ad-bc)$$

$$T'_{12} = (af-be)/(ad-bc)$$

$$T'_{21} = (dg-ch)/(ad-bc)$$

$$T'_{22} = (ah-bg)/(ad-bc)$$

with

$$a = T_{32}A_{1,1} + T_{31}\,;\ b = T_{34}A_{2,1} + T_{33}$$

$$c = T_{42}A_{1,1} + T_{41}\,;\ d = T_{44}A_{2,1} + T_{43}$$

$$e = T_{12}A_{1,1} + T_{11}\,;\ f = T_{14}A_{2,1} + T_{13}$$

$$g = T_{22}A_{1,1} + T_{21}\,;\ h = T_{24}A_{2,1} + T_{23}.$$

Now, getting out of the nomenclature of the segmentation approach, and referring to fig. 2a, we can recognize that

$$p_1 = p_{1,2N},\ v_1 = U_{1,2N},\ p_2 = p_{2,2N} \text{ and } v_2 = U_{2,2N}$$

and $[T']$ of eqn (11) is the required transfer matrix $[TM]$. Thus eqn (11) may be rewritten in the form

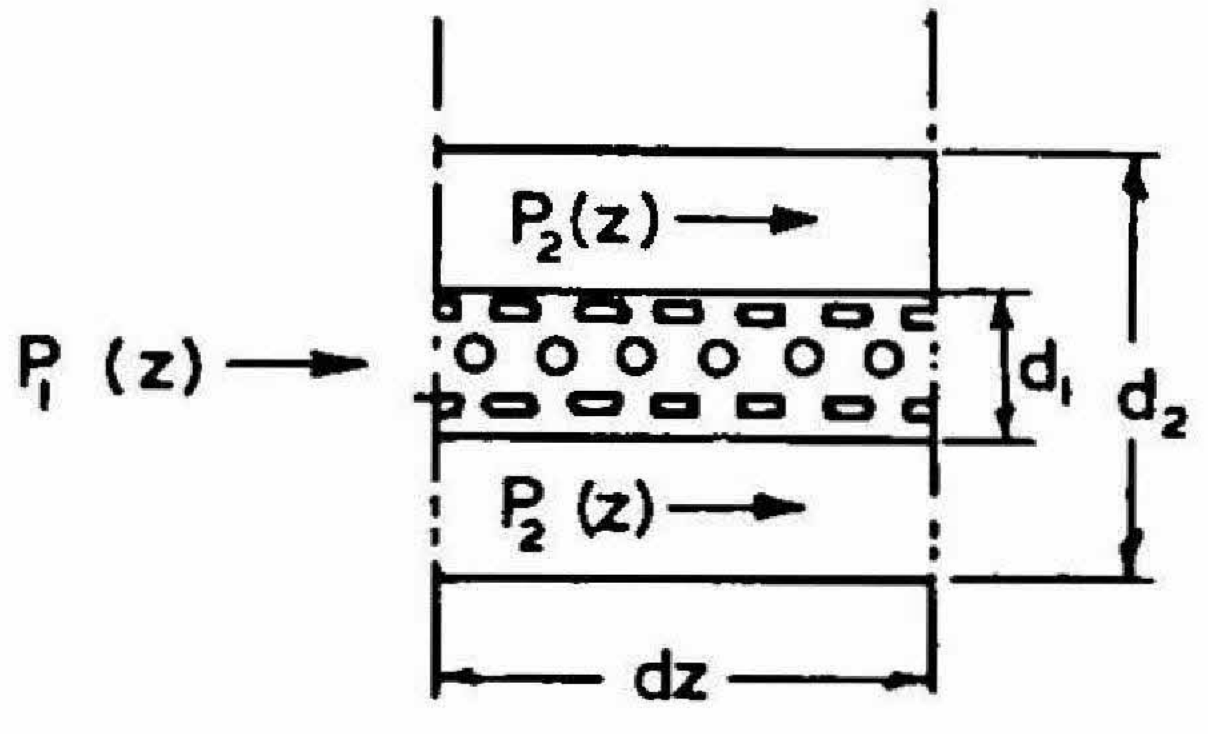


FIG. 6. Control volume of a two-duct perforated element.

$$\begin{bmatrix} p_1 \\ v_1 \end{bmatrix} = \begin{bmatrix} TM_{11} & TM_{12} \\ TM_{21} & TM_{22} \end{bmatrix} \begin{bmatrix} p_2 \\ v_2 \end{bmatrix} \tag{12}$$

where p and v are acoustic pressure and acoustic volume velocity. Points 1 and 2 refer to the points immediately upstream and immediately downstream of the flow-reversal element, respectively.

3. The distributed parameter method

In this method, the starting point is the partial differential equations describing mass continuity, momentum balance and isentropicity for a control volume of infinitesimal axial length dz in the inner tube as well as the outer annular tube (fig. 6). For sinusoidal time dependence these equations reduce to ordinary differential equations. Eliminating density and particle velocity terms, we get[6] two coupled equations in acoustic pressures p_1 and p_2. These may be written in the matrix notation as[3]

$$\begin{bmatrix} D^2+\alpha_1 D+\alpha_2 & \alpha_3 D+\alpha_4 \\ \alpha_5 D+\alpha_6 & D^2+\alpha_7 D+\alpha_8 \end{bmatrix} \begin{bmatrix} p_1(z) \\ p_2(z) \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \end{bmatrix} \tag{13a}$$

or

$$[A(D)]\,\{p\} = \{0\}\,, \tag{13b}$$

where

$$\alpha_1 = -\frac{iM_1}{1-M_1^2}\left(\frac{k_a^2+k^2}{k}\right);\ \alpha_2 = \frac{k_a^2}{1-M_1^2}\,;$$

$$\alpha_3 = \frac{iM_1}{1-M_1^2}\left(\frac{k_a^2-k^2}{k}\right);\ \alpha_4 = -\left(\frac{k_a^2-k^2}{1-M_1^2}\right);$$

$$\alpha_5 = \frac{iM_2}{1-M_2^2}\left(\frac{k_b^2-k^2}{k}\right);\ \alpha_6 = -\left(\frac{k_b^2-k^2}{1-M_2^2}\right);$$

$$\alpha_7 = -\frac{iM_2}{1-M_2^2}\left(\frac{k_b^2+k^2}{k}\right);\ \alpha_8 = \frac{k_b^2}{1-M_2^2};$$

$$k = \frac{\omega}{c_0},\ k_a^2 = k^2 - \frac{i4k}{d_1\zeta}$$

$$k_b^2 = k^2 - \frac{i4kd_1}{(d_2^2-d_1^2)\zeta} \quad \text{and} \quad D \equiv \frac{d}{dz}.$$

Defining

$$Dp_1 \equiv p_1' = y_1,\quad Dp_2 \equiv p_2' = y_2,\quad p_1 = y_3 \text{ and } p_2 = y_4, \tag{14}$$

eqns (13) reduce to a more convenient form as

$$\begin{bmatrix} -1 & 0 & D & 0 \\ 0 & -1 & 0 & D \\ D & 0 & \alpha_1 D+\alpha_2 & \alpha_3 D+\alpha_4 \\ 0 & D & \alpha_5 D+\alpha_6 & \alpha_7 D+\alpha_8 \end{bmatrix} \begin{bmatrix} y_1 \\ y_2 \\ y_3 \\ y_4 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ 0 \\ 0 \end{bmatrix} \tag{15a}$$

or

$$[\Delta]\{y\} = \{0\}. \tag{15b}$$

The characteristic polynomial of $[\Delta]$ has to be the same as the characteristic polynomial of $[A]$. Hence equations (15) are transformed to the principal variables Γ_1, Γ_2, Γ_3 and Γ_4 as

$$\begin{bmatrix} D-\beta_1 & 0 & 0 & 0 \\ 0 & D-\beta_2 & 0 & 0 \\ 0 & 0 & D-\beta_3 & 0 \\ 0 & 0 & 0 & D-\beta_4 \end{bmatrix} \begin{bmatrix} \Gamma_1 \\ \Gamma_2 \\ \Gamma_3 \\ \Gamma_4 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ 0 \\ 0 \end{bmatrix} \tag{16}$$

where βs are the roots of the fourth degree polynomial

$$|\Delta| = 0, \tag{17}$$

to be found numerically on computer by means of one of the standard subroutines.

Now, a modal vector $\{\psi_{1,j}, \psi_{2,j}, \psi_{3,j}, \psi_{4,j}\}$ can be calculated for each of the roots β_j. Equations (16) are the desired decoupled equations. The principal state variables Γ_1, Γ_2, Γ_3 and Γ_4 are related to the variables y_1, y_2, y_3 and y_4 through the eigenmatrix $[\psi]$ as

$$\{y\} = [\psi]\{\Gamma\} \tag{18}$$

or as

$$y_m = \frac{dp_m}{dz} = \sum_{n=1}^{4} \psi_{m,n}\Gamma_n, \ m = 1,2 \tag{19a}$$

$$y_{m+2} = p_m = \sum_{n=1}^{4} \psi(m+2), n\ \Gamma_n, m = 1,2 \tag{19b}$$

where

$$\psi_{1,n} = 1\,(\text{say}),$$

$$\psi_{2,n} = -\frac{\beta_n^2 + \alpha_1\beta_n + \alpha_2}{\alpha_3\beta_n + \alpha_4},$$

$$\psi_{3,n} = 1/\beta_n$$

and

$$\psi_{4,n} = \psi_{2,n}/\beta_n = \psi_{2,n}\psi_{3,n}, \ n = 1,2,3,4.$$

The general solution to eqns (16) can be written as

$$\Gamma_n(z) = c_n e^{\beta_n z}, n = 1,2,3,4. \tag{20}$$

Equations (18), (19) and (20) combined with the momentum equations yield[7]

$$\begin{bmatrix} p_1(z) \\ p_2(z) \\ \rho_0 c_0 w_1(z) \\ \rho_0 c_0 w_2(z) \end{bmatrix} = \begin{bmatrix} A_{m,n}(z) \end{bmatrix} \begin{bmatrix} C_1 \\ C_2 \\ C_3 \\ C_4 \end{bmatrix}, \tag{21}$$

where

$$A_{1,n} = \psi_{3,n} e^{\beta_n z}, \quad A_{2,n} = \psi_{4,n} e^{\beta_n z},$$

$$A_{3,n} = -\frac{e^{\beta_n z}}{ik + M_1\beta_n}, \quad A_{4,n} = -\frac{\psi_{2,n} e^{\beta_n z}}{ik + M_2\beta_n}$$

and $n = 1, 2, 3$ and 4 for the respective columns of $A_{m,n}(z)$. The state variables at $z = 0$ can be related to the state variables at $z = l$ through the transfer matrix relation

$$\begin{bmatrix} p_1(0) \\ p_2(0) \\ \rho_0 c_0 w_1(0) \\ \rho_0 c_0 w_2(0) \end{bmatrix} = [T_{m,n}] \begin{bmatrix} p_1(0) \\ p_2(l) \\ \rho_0 c_0 w_1(l) \\ \rho_0 c_0 w_2(l) \end{bmatrix} \tag{22}$$

where

$$[T_{m,n}] = [A_{m,n}(0)]\,[A_{m,n}(l)]^{-1} \quad m, n = 1,2,3,4. \tag{23}$$

The final two-by-two transfer matrix for a particular two-duct element may be obtained from $[T]$ making use of the appropriate boundary conditions of the element.

For reverse-flow expansion chamber (fig. 2a) the boundary conditions are:

$$z = l: \quad \frac{p_1(l)}{\rho_0 c_0 w_1(l)} = -i \cot (kl_b), \tag{24a}$$

$$z = l: \quad \frac{p_2(l)}{\rho_0 c_0 w_2(l)} = -i \cot (kl_b). \tag{24b}$$

Equations (22) and (24) yield the following transfer matrix relation[7]

$$\begin{bmatrix} p_1(0) \\ \rho_0 c_0 w_1(0) \end{bmatrix} = \begin{bmatrix} T_a & -T_b \\ T_c & -T_d \end{bmatrix} \begin{bmatrix} p_2(0) \\ \rho_0 c_0 w_2(0) \end{bmatrix}, \tag{25}$$

where

$$\begin{bmatrix} T_a & T_b \\ T_c & T_d \end{bmatrix} = \begin{bmatrix} A_1 & A_2 \\ A_3 & A_4 \end{bmatrix} \begin{bmatrix} B_1 & B_2 \\ B_3 & B_4 \end{bmatrix}^{-1} \tag{26}$$

$$A_1 = T_{1,1} + X_2 T_{1,3}, \quad A_2 = T_{1,2} + X_2 T_{1,4},$$

$$A_3 = T_{3,1} + X_2 T_{3,3}, \quad A_4 = T_{3,2} + X_2 T_{3,4},$$

$$B_1 = T_{2,1} + X_2 T_{2,3}, \quad B_2 = T_{2,2} + X_2 T_{2,4},$$

$$B_3 = T_{4,1} + X_2 T_{4,3}, \quad B_4 = T_{4,2} + X_2 T_{4,4}$$

and

$$X_2 = i \tan (kl_b).$$

Note that the minus sign with T_b and T_d in eqn (25) is due to the fact that w_2 here is positive in a direction opposite to the reference. This is needed to make up the transfer matrix adaptable to the transfer matrices of other downstream elements.

Finally, eqn (25) may be rearranged in the form of eqn (12) with the four-pole parameters of the desired transfer matrix given by

$$TM_{11} = T_a \tag{27a}$$

$$TM_{12} = -T_b \frac{\rho_0 c_0}{S_1} \tag{27b}$$

$$TM_{21} = T_c \frac{S_2}{\rho_0 c_0} \tag{27c}$$

$$TM_{22} = -T_d \frac{S_2}{S_1} \tag{27d}$$

4. Reverse-flow contraction element

The last two sections have dealt with derivation of the transfer matrix of the reverse-flow expansion element (fig. 2a). A look at figs 2a and b, the governing equations, and the boundary conditions would readily reveal that the analysis of the reverse-flow contraction element would run on identically similar lines except that points (or subscripts) 1 and 2 will be interchanged everywhere. Hence the transfer matrix of the contraction element would be simple inverse of the transfer matrix of the expansion element. It would therefore suffice to validate the transfer matrices of the expansion element.

5. Validation of the transfer matrices

In the foregoing sections, the transfer matrix of the reverse-flow expansion element has been derived by two methods, which differ from each other in substance as well as in detail. In the absence of experimental verification it would be instructive to compare values of TL computed from the transfer matrices derived using the two different methods.

TL is related to the four-pole parameters by[8]

$$TL = 20 \log_{10}\left[1/2 \left| TM_{11} + TM_{12} \frac{S_1}{\rho_0 c_0} + TM_{21} \frac{\rho_0 c_0}{S_2} + TM_{22} \frac{S_1}{S_2} \right| \right] \tag{28}$$

Impedance of the perforate, used in the computations, is given by the following formulae:

stationary medium[6]

$$\zeta = [6\times 10^{-3} + jk_0(t+0{\cdot}75\, d_h)]/\sigma \tag{29}$$

cross flow[1,2]

$$\zeta = \left[0{\cdot}514 \frac{d_1 M}{l\sigma} + j0{\cdot}95\, k_0 \left(t + 0{\cdot}75\, d_h\right)\right] \Big/ \sigma \tag{30}$$

where

σ = porosity of the perforate sample

M = mean flow Mach number along the perforated pipe upstream of the perforation

d_h = diameter of the holes

l = length of the perforated section

d_1 = inner diameter of the perforated pipe

t = thickness of the perforated pipe

f = wave frequency, Hz

k_0 = wave number, $2\pi f/c_0$.

Making use of the four-pole parameters of the reverse-flow expansion element evaluated from the segmentation method (eqn 11) and the distributed parameter method (eqn 27), TL was calculated from eqn (28) by means of a general computer program at various frequencies covering the entire range of pure plane wave propagation.

Figures 7–9 compare the predictions by the two methods for mean flow Mach numbers of 0·0 (stationary medium), 0·05 and 0·15, respectively. Near complete agreement

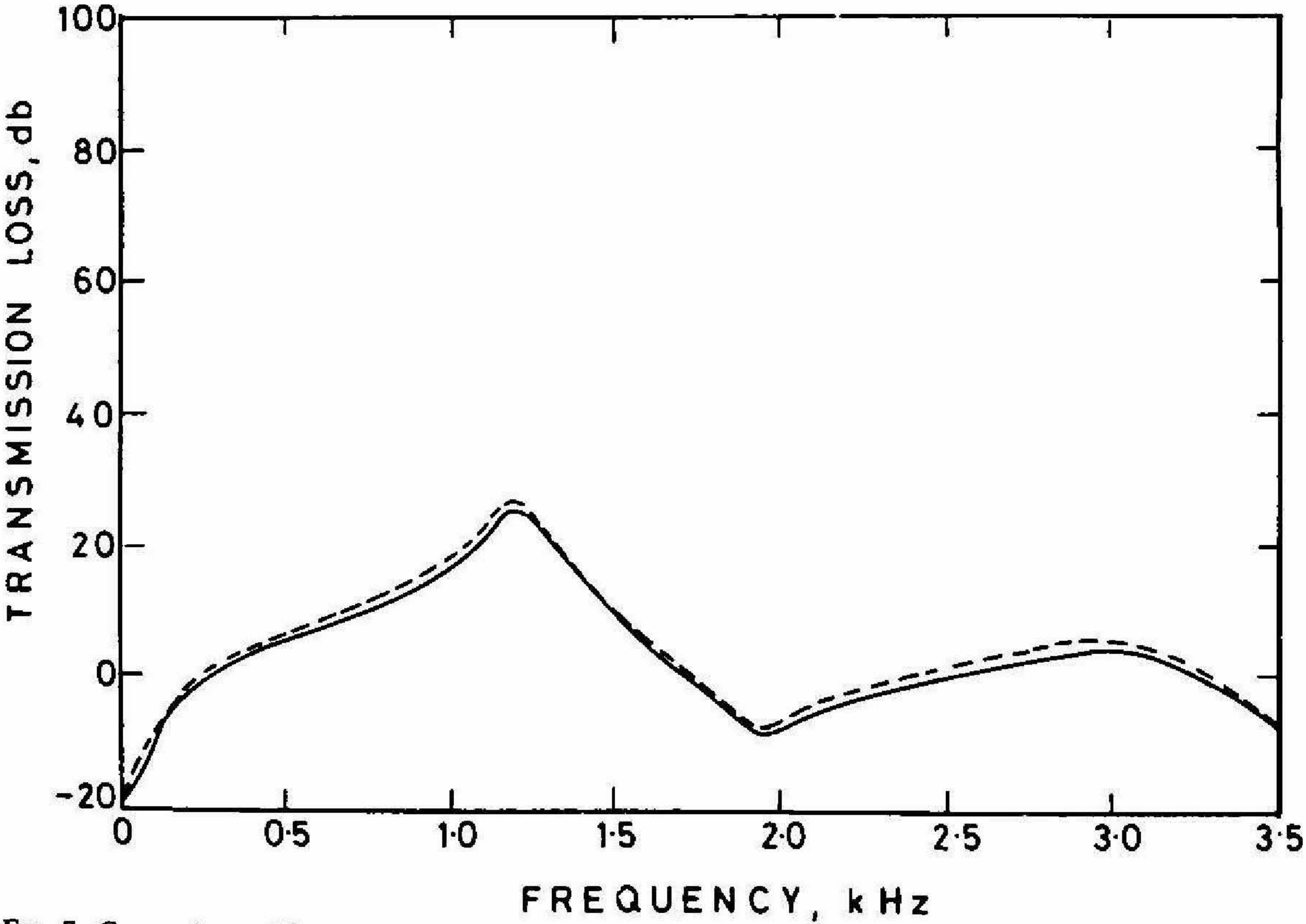


FIG. 7. Comparison of TL stationary medium $M = 0$. ----segmentation method; ——distributed parameter method.

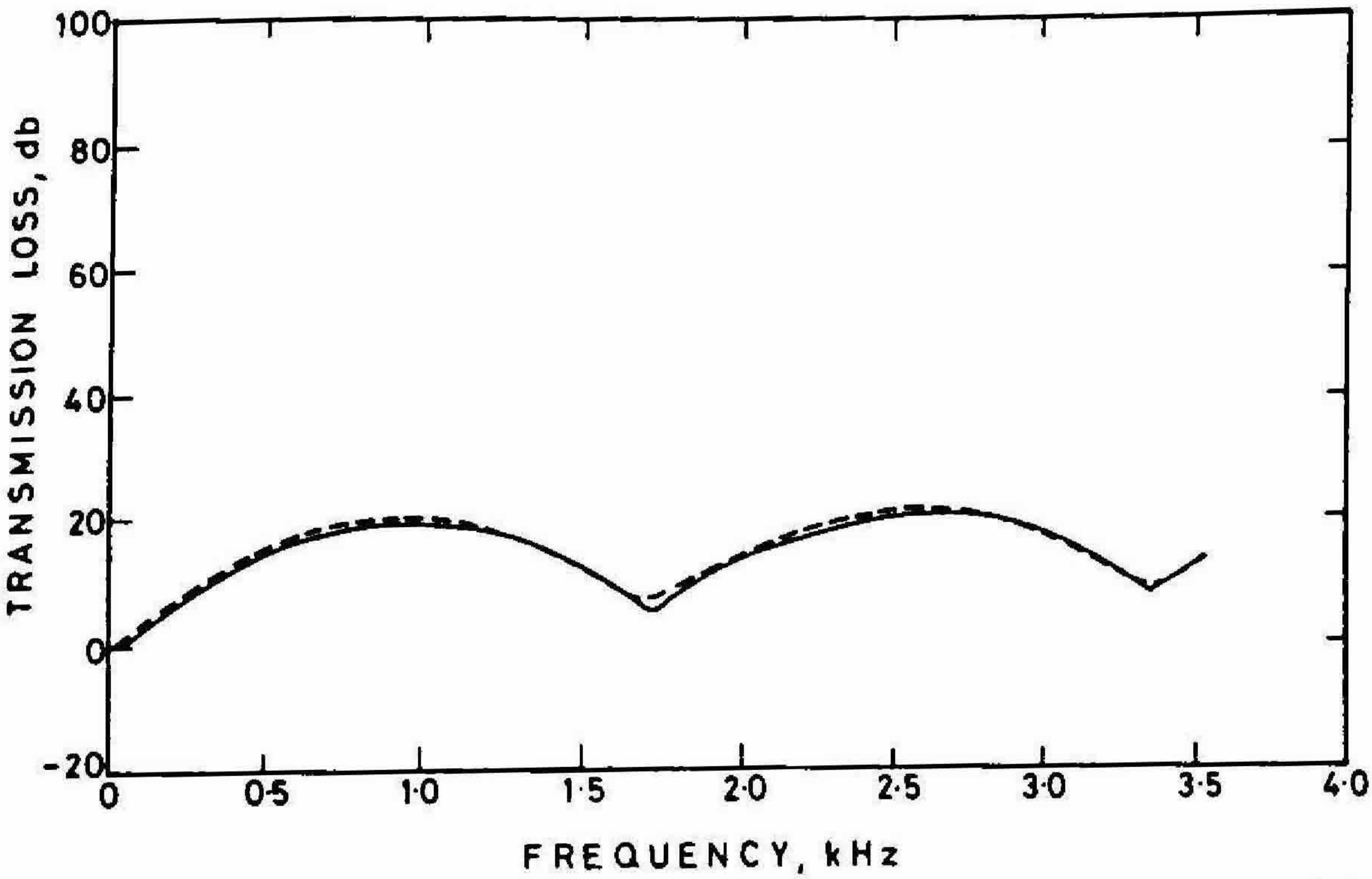


Fig. 8. Comparison of TL for mean flow Mach number $M = 0.05$. ----segmentation method; ——distributed parameter method.

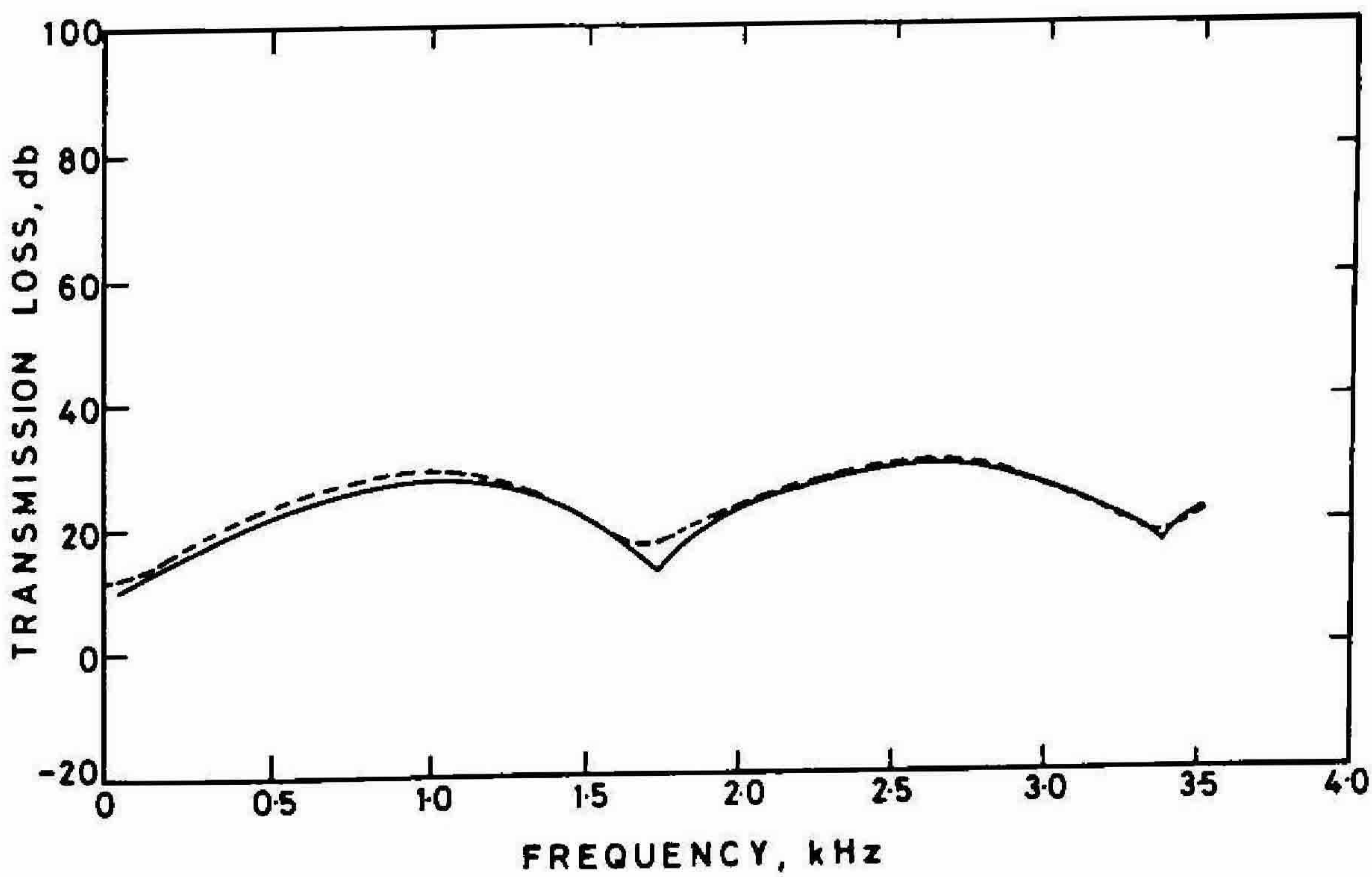


Fig. 9. Comparison of TL for mean flow Mach number $M = 0.15$. ----segmentation method; ——distributed parameter method.

between the two for all Mach numbers and at all frequencies indicates that expressions of four-pole parameters of the reverse-flow expansion element derived by the two methods must be correct.

As indicated earlier, no separate validation of the four-pole parameters of the reverse-flow contraction element is called for, as the transfer matrix of the contraction element is simple inverse of that of the expansion element.

These transfer matrices may be combined readily with the other muffler elements (upstream and downstream of the two-duct reverse-flow element) in order to evaluate the overall performance of the exhaust system[9].

6. Comparison of the two methods

The distributed parameter method is more elegant than the segmentation method as it treats the continuous system as such. However, the segmentation method is more suitable for modelling cross-flow elements because it allows for the variation of convective mean flow Mach number along the tube length. However, flow velocity through the perforations has been assumed to be independent of the axial coordinates in both the methods. But the perforate impedance depends only on this radial velocity as is implied in the resistive component of eqn (30) where M is mean flow Mach number upsteam of the perforation (not the local value). Out of these two effects of mean flow (*i.e.* the convective effect and dissipative effect), the latter plays a primary role as is indicated by Munjal[8], whereas the former has only a marginal role to play. This is why the agreement between predictions of the two methods tallies not only for the stationary case ($M = 0$) but also for the moving medium ($0 < M \leqq 0{\cdot}2$).

Thus, the conceptual advantage of the segmentation method over the distributed parameter method is not significant.

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