Proc. Indian Acad.Sci.A, 33, 199-215, (March, 1951)
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PHOTO-ELASTIC CONSTANTS OF SODIUM CHLORATE

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Received December 20, 1950

1. INTRODUCTION

SODIUM chlorate is a cubic crystal belonging to the tetrahedrite (T) class and therefore exhibits optical activity. The determination of the photo-elastic constants of this substance is of interest, because of two reasons. Firstly, Bhagavantam (1942) has shown that crystals of this class (T) and of the T_h class must have four independent piezo-optic constants, q_{11}, q_{12}, q_{13} and q_{44}, as against only three constants possessed by crystals of the other classes of the cubic system. Bhagavantam and Suryanarayana have also experimentally shown this to be the case for crystals of the T_h class, such as alums (1947 *a*, 1949) and barium nitrate (1948). It is interesting to verify the theory for a crystal belonging to the T class also. Secondly, unlike the other cubic crystals for which photoelastic constants have been determined, sodium chlorate exhibits optical activity. This requires that special techniques have to be used for the determination of the birefringence produced by stress.

The theory of the propagation of light in a crystal exhibiting both birefringence and optical activity has been worked out by several authors. The results of the theory are summarized in Pockels' Lehrbuch (1906). We shall make use of this theory in this paper, with slight modifications in notation. The theory has been applied to quartz for determining its rotatory power in different directions and also for the effect of birefringence on the Faraday rotation (Bruhat, 1935; Ramaseshan, 1950). The rotation in the case of quartz is small compared with birefringence, and Bruhat's formulæ have been developed only for this particular case. His experimental methods are therefore not applicable in the present study. Consequently, general methods are described whereby rotation and birefringence can be separately determined when both are present and are of similar orders of magnitude.

Measurements of the birefringence of the crystal for different orientations can only lead to the values of three piezo-optic constants, namely $q_{11}-q_{12}$, $q_{11}-q_{13}$ and q_{44}. To determine q_{11}, q_{12}, q_{13} separately it is necessary to make at least one determination of the change in absolute path of the light beam through the crystal. Attempts to obtain interference fringes failed and consequently the ratios p_{12}/p_{11} and p_{13}/p_{11} were determined from the study of ultrasonic diffraction patterns (Mueller, 1938) and thus all the constants were evaluated. It is found that $(q_{11} - q_{12})$ and $(q_{11} - q_{13})$ differ by nearly 70% for this crystal, which is larger than what is found for any crystal of the T_h class.

2. THEORY OF LIGHT PROPAGATION IN OPTICALLY-ACTIVE BIREFRINGENT CRYSTALS

In this paper, we shall use throughout Poincaré's representation of an elliptic vibration by a point on a sphere (Pockels, *loc. cit.*). Fig. 1 shows

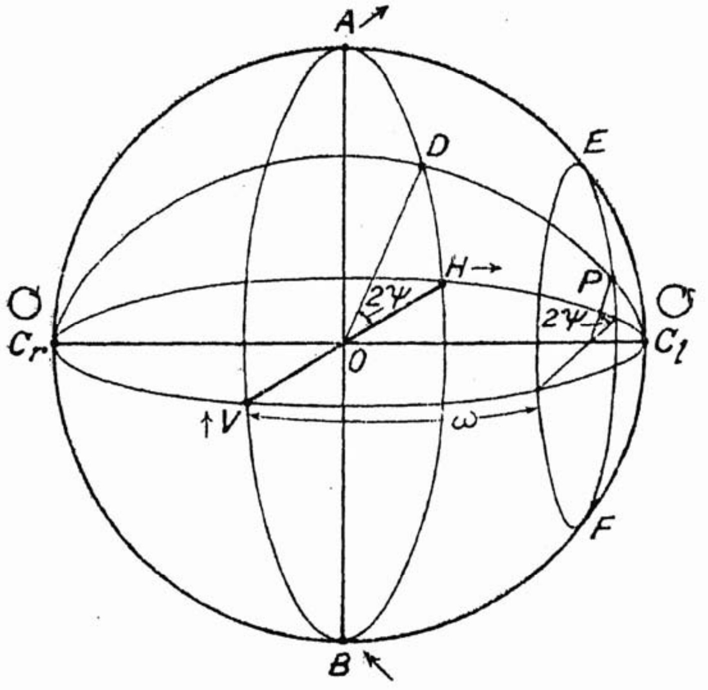

FIG. 1.

such a sphere and the notation followed in designating a point on it. C_l and C_r are called the two poles of the sphere and represent respectively left- and right-rotating circular vibrations. The great circle HAVB is the equator, and points on this represent plane vibrations of varying azimuth. The point H stands for a horizontal plane vibration and a plane vibration making an angle ψ measured anticlockwise in the usual way, will be represented by the point D on the equator, such that $D\hat{O}H = 2\psi$, 2ψ being measured anticlockwise from O H when looking from C_l. Thus the point V diametrically opposite to H represents vertical vibration and A and B represent respectively vibrations at $+45°$ and $-45°$. The latitude circle E P F of latitude ω represents all elliptic vibrations, whose ratio of axes is given by tan $(\omega/2)$. The azimuth 2ψ (or longitude) gives the angle (ψ) between the horizontal and the direction of the major axis. ω is taken to be positive for the hemisphere containing C_l and negative for the other hemisphere. All elliptic vibrations in the former are taken to be left-rotating and in the latter right-rotating. Obviously C_l and C_r are the limiting circular vibrations, for which $|\omega| = \pi/2$ and the ratio of the axes is unity.

To represent the Poincaré sphere in two dimensions the stereographic projection is used in this paper. It is most convenient to project the sphere on a plane parallel to C_l HC_r V. The co-ordinates 2ψ and ω of the point P are shown in the projection in Fig. 2. With reference to this figure, the following statements can be made about the propagation of light in an optically rotating and/or birefringent crystal. Many of these are well-known results (Pockels, *loc. cit.*), but the signs and senses of rotation are precisely defined here.

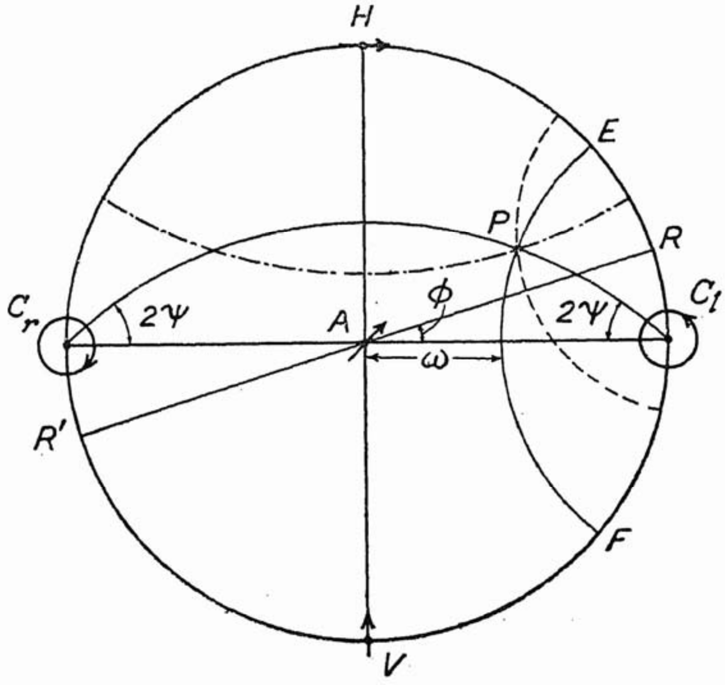

FIG. 2

Suppose that the state of polarisation of the incident light is represented by P. If the crystal is not optically active, then the point P is rotated about the axis H V (along the dot-dashed line in Fig. 2), the angle of rotation being given by the phase retardation $\delta = \delta_o t$ where δ_0 is the phase retardation (vertical — horizontal) per cm. and t is the distance traversed in cm. The rotation δ is anticlockwise looking from H to V if it is positive and *vice-versa.*

Similarly, for a rotating crystal having no birefringence, the point is rotated about the axis C_r C_l (circle E F), the amount of rotation in the Poincaré sphere being equal to twice the optical rotation ρ of the crystal, *i.e.*, $2\rho = 2\,\rho_0 t$, where ρ_0 is the rotation per cm. Looking from C_l to C_r the rotation is anti-clockwise for a left-rotating crystal and *vice-versa*. When both birefringence and rotation are simultaneously present, the point rotates about an axis RR′ lying in the plane C_lH C_rV, which makes an angle ϕ with C_r C_l, where $\tan\phi = \delta_0/2\rho$ (dashed line Fig. 2). For a left-rotating crystal, ϕ is positive if the horizontally polarised ray is the faster one and ϕ is negative if the vertical is faster. The reverse conditions hold for a right-rotating crystal. In all cases, the angle by which the point P has to be rotated is given by Δ, where

$$\Delta = \sqrt{\delta^2 + (2\rho)^2} \tag{2}$$

and the sense of rotation is counterclockwise about that point R (or R′) which lies between the faster of the two circular components (C_l or C_r) and the faster of the two birefringent components (H or V). Using this construction, the state of polarisation of the emergent light Q can be determined when that of the incident light P is known.

3. DETERMINATION OF BIREFRINGENCE AND ROTATION SEPARATELY WHEN BOTH ARE PRESENT

In principle the problem is the reverse of what was stated in the last section and can be solved very simply with stereographic nets. Allow light having the state of polarisation represented by an arbitrary point P (Fig. 3) to fall

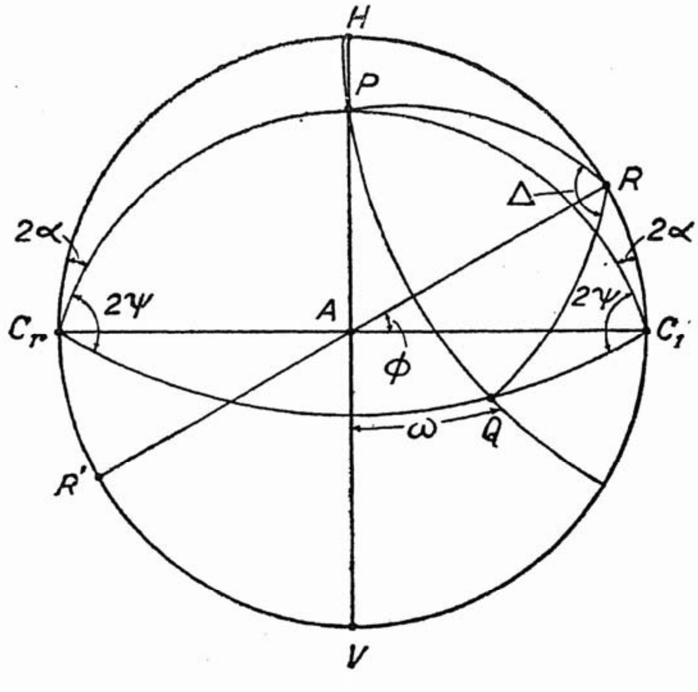

FIG. 3

on a parallel plate of thickness t and determine its state (Q) on emergence. Plotting P and Q in a stereographic projection it is only necessary to find the axis RR′, such that PR = QR. This can be readily done by using the Wulff net and rotating it about A until the latitudes of P and Q are the same. Then the angle $R\hat{A}C_l$ gives ϕ and the difference in longitude between P and Q, *i.e.*, $R\hat{P}Q = \Delta$. Knowing Δ and ϕ we have

$$\delta = \Delta \sin \phi \; ; \; 2\rho = \Delta \cos \phi. \tag{3}$$

An analytical expression for the most general case is too complicated and it is best to use the stereographic net and find Δ and ϕ in such a case. However, when P lies on the plane HAVB, *i.e.*, when the incident light is plane polarized at an arbitrary azimuth, then the problem can be solved analytically. This particular case is used throughout the present investigation and we shall quote the final formulæ below.

Suppose the plane of vibration of the incident light makes an angle α with one of the principal vibration directions (H). Suppose that the emergent light is an ellipse, whose major axis makes an angle ψ with vibration direction of the incident light and that the ratio of its axes is $b/a = \tan \omega/2$. Then we have the conditions shown in Fig. 3. Both ψ and ω can be directly measured as described below (§5). In terms of these quantities, it can be shown that

$$\left.\begin{aligned} \tan \phi &= \frac{\sin \omega}{\cos 2\alpha - \cos \omega \cos 2(\alpha + \psi)} \\ \cos \Delta &= 1 - \frac{1 - \cos \omega \cos 2\psi}{1 - \sin^2 \phi \cos^2 2\alpha} \end{aligned}\right\} \tag{4}$$

In the particular cases, when $2\alpha = 0$ or $\pi/2$, *i.e.*, when the light is polarised parallel to H, or at 45° to it, we have the following simplified formulæ:

$$\begin{array}{ll} (a)\ 2\alpha = 0 & (b)\ 2\alpha = \pi/2 \\ \tan \phi = \sin \omega/(1 - \cos \omega \cos 2\psi) & \tan \phi = \tan \omega/\sin 2\psi \\ \sin \Delta = \cos \omega \sin 2\psi/\cos \phi & \cos \Delta = \cos \omega \cos 2\psi \end{array} \tag{5}$$

It is clear that one has to know, in addition to the measured values of ψ and ω, also the value of α, the inclination of the plane of vibration of the incident light to one of the principal vibration directions. This requires a knowledge of the principal directions, which in a rotating birefringent crystal is not easy to determine. However, it can be done by determining the so-called azimuth of minimum, a method due to Bruhat (1935). In this method, crossed nicols are used just as for a non-rotating birefringent crystal, and the azimuth when the transmitted intensity is a minimum gives the principal vibration directions. However, the method is not accurate unless $2\,\rho/\delta$ is small for otherwise the minimum is not marked.

4. THE ELLIPTIC ANALYSER

If the principal vibration directions are known, then a very simple method can be used for determining the magnitude of the birefringence. The method uses the technique described below, which may be called the "elliptic analyser". It is clear from Fig. 2 that, if the incident light has a state of polarisation denoted by the point R (or R′) on the Poincaré sphere, then it is propagated unchanged through the crystal. For different positions of R, *i.e.*, for varying ratios of $\delta/2\,\rho$ the ellipses all have their major and minor axes parallel to H and V, but differ in their axial ratio. It is therefore enough to determine the ratio of the axes (B/A) of the ellipse that is propagated unchanged through the crystal. Then the angle ϕ is given by

$$\tan\tfrac{1}{2}\left(\frac{\pi}{2}-\phi\right)=\frac{\mathrm{B}}{\mathrm{A}} \tag{6}$$

and δ can be calculated, knowing 2ρ, the natural rotation, from the formula

$$\delta=2\,\rho\tan\phi. \tag{7}$$

The experiment consists in the following. Two quarter wave plates are mounted, one before and the other after the crystal, such that their axes are parallel to the axes of birefringence of the crystal. The first one is mounted with its faster axis horizontal and the second with its faster axis vertical. The polarising nicol before the first quarter wave plate and the analysing nicol after the second are kept crossed and they can be rotated together in the crossed position. As the crossed nicols are rotated, the first combination transmits elliptically polarised light of varying axial ratio and, in the absence of the crystal, this is extinguished by the analyser. When the crystal is interposed, the ellipse is in general altered and there is a restoration of light. Crossing occurs only for the particular setting when the incident ellipse is propagated unchanged. The angle ξ made by the polariser with the horizontal when this happens gives the axial ratio $\mathrm{B/A}=\tan\xi$. Thus $\phi=\pi/2-2\,\xi$. Thus, when $0<\xi<\pi/4$, δ is positive for a left-rotating crystal and negative for a right-rotating crystal and *vice-versa* when $\pi/4<\xi<\pi/2$.

In actual practice, extinctions are found once every 90°, so that a mean value can be obtained. In these studies, care was not taken to set the fast and slow axes of the quarter wave plates correctly. The measurements were used only to determine the magnitude of δ, the sign being obtained with the Babinet compensator.

5. EXPERIMENTAL SET-UP

Since it was necessary to measure not only the path retardation of the light passing through the crystal, but also various other constants of the elliptic vibration emerging from it, the stressing apparatus was so designed that a petrological microscope could be used together with it to make the optical measurements. The stress was applied by means of a lever arrangement designed such that the stress on the crystal could be made zero, when required, by means of counterpoise weights. Lead and paper spacers were used to render the stress uniform on the crystal, and by making the length of the specimens roughly thrice their lateral dimensions, the stress in the central portion was kept practically constant. The crystals were subjected to a maximum stress of 100 kg./sq. cm. and were found to return to their original state of no birefringence when unstressed. Initially a large stress was applied so that the crystal sunk a little into the lead and thereafter its orientation was maintained at all loads.

The crystals were grown from a saturated solution in water and after two or three crops had been grown and redissolved, a number of very clear and large specimens were obtained. They were all of the { 100 } form and crystals of other orientation were cut from these. The crystals kept remarkably well in dry weather and could be left in the stressing apparatus for several days. Measurements were made for three different cuts. The specimens used for final measurements had the dimensions and orientations given in Table I.

TABLE I

Crystal No.	Sense of rotation	Length		Breadth		Thickness	
		Parallel to	Dimns. mm.	Parallel to	Dimns. mm.	Parallel to	Dimns. mm.
1	R	[100]	**16·8**	[010]	5·30	[001]	**4·86**
2	R	[100]	**17**	[001]	**11·0**	[010]	**5·6**
3	L	[100]	**28**	[001]	**15·6**	[010]	**4·9**
4*a*	L	[110]	**19**	$[1\bar{1}0]$	**7·77**	[001]	**7·30**
4*b*	L	[110]	**18**	[001]	**6·78**	$[1\bar{1}0]$	**6·61**
5	R	[111]	**11**	$[11\bar{2}]$	**4·30**	$[1\bar{1}0]$	**3·99**

Measurements were made with three specimens for stress along the cubic direction. Of these, crystal 1 was used for a detailed study, making measurements of 2ψ and ω, as well as of the ellipse that is propagated unchanged. Readings were taken only with the elliptic analyser in the case of crystals 2 and 3. Crystal 4 was affected by a large humidity in the atmosphere after observations were finished along [1$\bar{1}$0]. The crystal was therefore reground to the dimensions shown against 4*b* and observations were made along [001]. For both directions, detailed measurements were made. With crystal 5, measurements were taken only with the elliptic analyser.

6. ORIENTATION OF THE X, Y, Z AXES

With the usual notation, the stress-optical equations for sodium chlorate (belonging to class T) are:

$$\begin{aligned}\Delta a_{11} &= q_{11} X_x + q_{12} Y_y + q_{13} Z_z\\ \Delta a_{22} &= q_{13} X_x + q_{11} Y_y + q_{12} Z_z\\ \Delta a_{33} &= q_{12} X_x + q_{13} Y_y + q_{11} Z_z\\ \Delta a_{23} &= q_{44} Y_y\\ \Delta a_{31} &= q_{44} Z_z\\ \Delta a_{12} &= q_{44} X_x\end{aligned} \tag{8}$$

As a consequence, q_{12} and q_{13} are unequal, and for stress along the X-axis, the birefringence for light proceeding along the Y and Z axes are different, being proportional to $(q_{11} - q_{12})$ and $(q_{11} - q_{13})$ respectively. To define the photoelastic constants, therefore, one has to define the relative orientation of the X, Y and Z axes. None of the crystals used in this study exhibited {210} faces, all having only the form {100}. Consequently, the external form could not be used for defining the axes and X-rays were used for this purpose. A thin plate, about 1 mm. thick, was cut parallel to one of the cube faces. The thickness direction was arbitrarily defined as [100] and a Laue pattern was photographed with the X-ray beam in this direction. One quarter of the Laue pattern is shown diagrammatically in Fig. 4. The sizes of the spots are made so as roughly to correspond to their intensities. That direction for which the 130 spot is more intense and 150 is less intense is defined as the Z-axis and the third orthogonal direction is Y. This corresponds to the indexing of X-ray sopts adopted by Zachariasen (1929).

This definition uniquely defines the relative orientations of the X, Y, Z axes. However, a cyclic permutation is allowed, whereby any one might be called the X-axis and the other two are then automatically fixed. This method of defining the axes is independent of the fact whether the crystal is right- or left-handed, as it is based directly on the structure. In fact, for

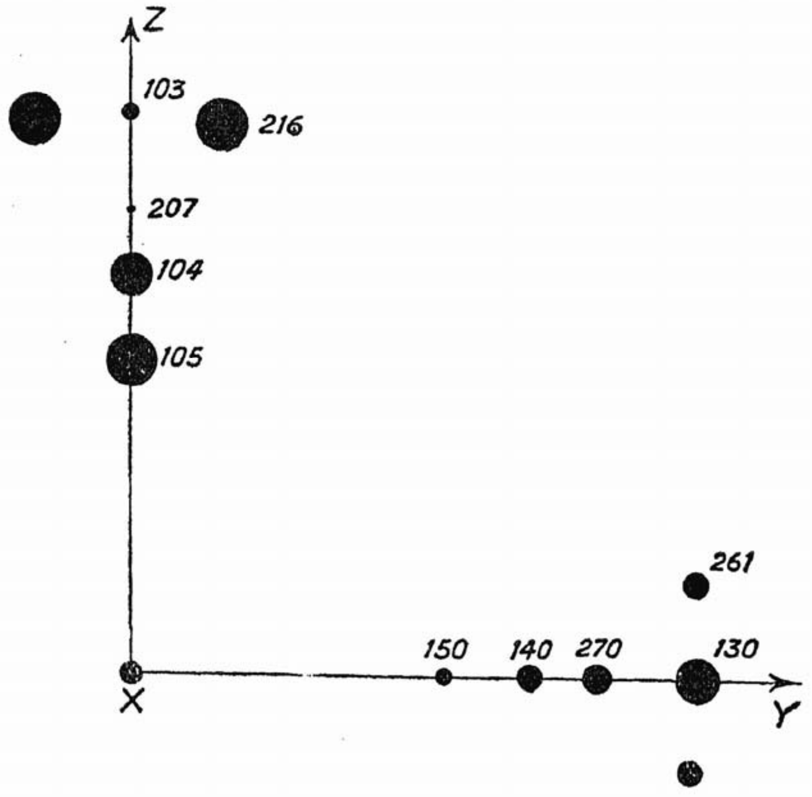

FIG. 4

both types of crystal, the X-, Y- and Z- axes defined in this way gave the same values for $q_{11} - q_{12}$ and $q_{11} - q_{13}$ (see next section).

Bhagavantam and co-workers (*loc. cit.*) have not defined their X-, Y-, Z- axes either with respect to the morphology or X-ray pattern of the crystals (of the T_h class) studied by them, although they have shown that q_{12} is different from q_{13}. It is to be noted that in the present definition for sodium chlorate, the sense of the axes has not been defined—in fact it is impossible to define it by means of X-rays and the external form will have to be used for this—but the sense is irrelevant in photoelasticity.

The relative retardations between the vertically and horizontally polarised components for various orientations have been calculated and are given in Table II. The relative retardation δ per unit thickness of the crystal is

$$\delta_0 = \frac{n^3}{2} \cdot f(q). \mathrm{P}, \qquad (9)$$

where $f(q)$ is a linear function of the constants q_{rs} and P is the stress, $f(q)$ is given in Table II for different settings.

TABLE II

Stress direction	Direction of observation	$f(q)$
[100]	[010]	$q_{11}-q_{12}$
[100]	[001]	$q_{11}-q_{13}$
[110]	$[1\bar{1}0]$	$\frac{1}{4}[(q_{11}-q_{12})+(q_{11}-q_{13})+2q_{44}]$
[110]	[001]	$-q_{43}/\sin 2\theta$*
[111]	$[1\bar{1}0]$	q_{44}
[111]	$[11\bar{2}]$	γ_{44}

* See text.

In all the orientations, except the one marked with an asterisk, the directions of vibration are parallel and at right angles to the stress direction. In this case alone, they are inclined at an angle θ to the X- and Y- axes, where

$$\tan 2\,\theta = 2\,q_{44}/(q_{12} - q_{13}) \tag{10}$$

The $f(q)$ shown in Table II for this orientation is actually the retardation between the two principal components, as this is the one measured in the experiment.

6. MEASUREMENT OF $q_{11} - q_{12}$ AND $q_{11} - q_{13}$

As mentioned earlier, because of the presence of rotation, the birefringence produced by stress can only be measured indirectly. The two quantities measured are the co-ordinates $2\,\psi$ and ω of Fig. 3 for azimuths α near about 0, $\pi/2$, $+\pi/4$ and $-\pi/4$. Since the principal axes of birefringence cannot be determined exactly, the azimuth (α') was measured with respect to the horizontal and four sets of measurements were taken with the incident light polarised at angles $-45°$, $0°$, $+45°$ and $90°$ to the horizontal. In each case, the rotation of the major axis of the emergent ellipse was determined by means of a half-shade and this gave the value of ψ. A Babinet compensator was then kept with its axes at 45° to the major axis of the ellipse. The shift of the Babinet fringes under these conditions would directly give the quantity ω. If this shift is s and the fringe width is f, then

$$\omega = 2\pi\, s/f.$$

The pair of values ψ, ω was determined for three or four values of the stress upto a maximum stress of about 100 kg./cm.^{2} Each measurement was an average of ten readings. It is clear from the Poincaré sphere that the measurements with $\alpha' = 0°$ and $90°$ are equivalent and similarly those with

$\alpha' = \pm 45°$. Consequently the mean of these pairs of values was taken. The results obtained in this way for crystal 1 are shown in Table III. The values for the load given in the first column are the actual weights put on the pan. They have to be multiplied by a factor 7, the ratio of the lever, to give the total force on the crystal.

TABLE III

Mean values of ψ *and* ω *for different stresses for crystal* 1

Load in kg.	Obsn.	$\alpha' = 0°, 90°$		$\alpha' = \pm 45°$	
		ψ	ω	ψ	ω
0	[001]	−17·35	0·00	−17·5	0·0
0·6	[001]	−17·2	− 6·20	−17·85	8·3
1·0	[001]	−17·0	− 9·35	−18·6	18·7
2·0	[001]	−15·1	−17·70	−22·5	36·1
3·0	[001]	−12·3	−25·40	−31·55	51·9
0	[010]	−19·25	0·0	−19·3	0·0
0·6	[010]	−18·7	− 9·55	−20·4	20 2
1·0	[010]	−17·55	−15·7	−22·55	36·3
2·0	[010]	−13·75	−27·5	−41·75	58·9
3·0	[010]	− 6·2	−33·2	−73·55	45·5

These data enable one to calculate the angle θ which one of the principal vibration directions makes with the vertical, and hence the relative retardation δ. α would then be equal to $\alpha' + \theta$. θ is determined by trial and error, that value being chosen for which measurements with $\alpha' = 0$ and 90° and $\alpha' = \pm 45°$ give the best agreement. θ can be obtained roughly by determining the azimuth of minimum and further refinements are made by calculation. Thus, for crystal 1, the azimuth of minimum gave $\theta = -5°$, while calculations gave it as $-4\frac{1}{2}°$. The errors introduced by taking the pressure to be exactly vertical is shown in Table IV, which contains the calculated values of ϕ, 2ρ and δ for various stresses assuming $\theta = 0$ and $-4\frac{1}{2}°$. It will be noticed that the two sets of measurements for the same load lead to different values of ϕ and δ if θ is taken as zero, and further the value of 2ρ appears to increase with load. Both these discrepancies disappear when the correct value is taken for θ.

The same procedure was adopted for the other observation direction for this crystal. Only the final values for this are shown in Table V.

TABLE IV

Calculated values of ϕ, 2ρ and δ in degrees for crystal 1, *observation along* [001]—*with θ assumed to be* 0° *and* $-4\frac{1}{2}$°

Load	$\theta = 0$				$\theta = -4\frac{1}{2}$°				δ per *kg*
	$2a$	ϕ	2ρ	δ	$2a$	ϕ	2ρ	δ	
0·6	0	−30·8	−35·2	21·0	− 9	−22·0	−35·1	14·2	19·8
	90	−13·8	−35·4	8·7	81	−15·0	−35·3	9·5	
1·0	0	−41·6	−36·2	32·1	− 9	−31·1	−35·1	21·2	21·3
	90	−29·2	−35·8	21·0	81	−31·3	−35·2	21·4	
2·0	0	−60·0	−36·4	62·9	− 9	−50·7	−33·9	42·4	21·2
	90	−45·9	−38·4	39·6	81	−49·8	−35·9	42·5	
3·0	0	−67·6	−40·0	94·3	− 9	−61·3	−35·2	64·3	21·6
	90	−55·1	−42·2	60·4	81	−61·3	−35·8	65·4	
									mean 21·3

TABLE V

Calculated values of ϕ, 2ρ and δ in degrees for crystal 1, *observation along* [010]: $\theta = -2\frac{1}{2}$°

Load	$2a$	ϕ	2ρ	δ	δ per kg.
0·6	− 5	31·6	38·7	23·8	38·8
	85	30·5	38·6	22·8	
1·0	− 5	48·3	38·8	43·6	42·4
	85	48·1	37·1	41·2	
2·0	− 5	61·8	40·4	75·2	38·1
	85	63·4	39·0	77·0	
3·0	− 5	70·2	40·7	111·2	38·6
	85	71·3	40·6	120·1	
					mean 38·9

These measurements lead to values for the piezo-optic constants:

$$q_{11} - q_{12} = -2\cdot42 \times 10^{-13},\ q_{11} - q_{13} = -1\cdot41 \times 10^{-13}\ \text{cm.}^2\ \text{dyne}^{-1}$$

The measurements with the elliptic analyser and the values of $\delta/2\rho$ calculated from them for the same crystal are shown in Table VI.

TABLE VI

Load kg.	Observation along [001]			Observation along [010]		
	ξ	$\delta/2\rho$ $= \tan(90^\circ - 2\xi)$	$\delta/2\rho$ per kg.	ξ	$\delta/2\rho$ $= \tan(90^\circ - 2\xi)$	$\delta/2\rho$ per kg.
0	45·2	0·007	..	45·1	0·004	..
0·6	35·2	0·356	0·593	28·9	0·630	1·05
1·0	30·3	0·564	0·564	22·4	1·007	1·01
2·0	21·1	1·103	0·521	12·4	2·164	1·08
3·0	14·6	1·789	0·596	8·6	3·231	1·08
		Weighted mean	0·576		Weighted mean	1·06

The degree of constancy of the entries under $\delta/2\rho$ per kg. indicates the accuracy of the measurements. The values deduced from these for the photo-elastic constants are

$$q_{11} - q_{12} = -2 \cdot 36 \times 10^{-13},\ q_{11} - q_{13} = -1 \cdot 43 \times 10^{-13}\ \text{cm.}^2\ \text{dyne}^{-1}$$

Elliptic analyser measurements with crystals 2 and 3 led respectively to the values:

$$q_{11} - q_{12} = -2 \cdot 41 \times 10^{-13},\ q_{11} - q_{13} = -1 \cdot 41 \times 10^{-13}$$

$$q_{11} - q_{12} = -2 \cdot 34 \times 10^{-13},\ q_{11} - q_{13} = -1 \cdot 43 \times 10^{-13}$$

Taking the mean of the four independent measurements for these constants, we have

$$q_{11} - q_{12} = -2 \cdot 39 \times 10^{-13},\ q_{11} \times q_{13} = -1 \cdot 42 \times 10^{-13}\ \text{cm.}^2\ \text{dyne}^{-1}.$$

7. MEASUREMENTS WITH CRYSTALS 4 AND 5

With stress along [110] and observation along $[1\bar{1}0]$ (4*a*) the method is identical with that for crystal 1. The measurements and the values of 2ρ and δ deduced from them are summarized in Table VII.

Using the dimensions of the crystal shown in Table I, we have

$$\tfrac{1}{4}\{(q_{11} - q_{12}) + (q_{11} - q_{13}) + 2\, q_{44}\} = -1 \cdot 68 \times 10^{-13}\ \text{cm}^2\ \text{dyne}^{-1}.$$

Measurements with the elliptic analyser for the same crystal gave a value $1 \cdot 76 \times 10^{-13}$ for this constant, so that the mean is $1 \cdot 72 \times 10^{-13}$. Using the values of $(q_{11} - q_{12})$ and $(q_{11} - q_{13})$ determined earlier, q_{44} is thus equal to $1 \cdot 54 \times 10^{-13}$.

TABLE VII

Measurements with Crystal 4a

Load in kg.	2α	ψ	ω	ϕ	2ρ	δ	δ per kg.
0	− 7 83	28·3 28·4	.. ..	.. ..	56·6 56·8	.. ..	
2	− 7 83	27·4 31·5	−11·3 24·6	32·5 33·4	56·5 56·3	36·0 37·1	18·3
5	− 7 83	21·3 58·0	−27·7 52·1	59·5 59·8	54·6 57·8	92·6 99·6	17·8
8	− 7 83	8·9 84·0	−44·0 26·2	67·9 68·9	61·7 55·6	152·1 144·2	18·5

Weighted mean 18·6

When the observation direction is along [001] (4 *b*), the principal vibration directions are not parallel and perpendicular to the stress direction. This did not lead to any difficulty as the principal directions are themselves determined in the experiment, as mentioned earlier. They were approximately determined using the elliptic analyser, and the polariser was set parallel or at 45° to them. However, they were off by 2°, as may be seen from the results in Table VIII. These lead to value for $q_{44}/\cos 2\theta$ — $= -1 \cdot 70 \times 10^{-13}$. Measurements with the elliptic analyser of the same quantity gave a value $-1 \cdot 66 \times 10^{-13}$, so that the mean is $-1 \cdot 68 \times 10^{-13}$.

TABLE VIII

Load in kg.	2α	ψ	ω	ϕ	2ρ	δ	δ per kg.
0	− 4 86	24·2 24·2	.. ..	.. ..	48·4 48·4	.	
2	− 4 86	22·4 29·0	−14·2 30·1	42·9 38·1	47·7 47·8	44·3 37·5	20·5
4	− 4 86	18·2 51·2	−23·2 54·0	58·8 57·6	47·5 51·7	78·4 81·5	20·0
7	− 4 86	8·7 85·1	−30·6 28·0	72·6 73·4	46·0 42·9	146·7 143·8	20·7

Weighted mean 20·5

The angle 2θ was determined experimentally by finding the azimuth of minimum with the crystal direct and with it turned around and calculating the difference. This gave 2θ as 74°, from which $q_{44} = -1 \cdot 61 \times 10^{-13}$, The value of 2θ calculated from Eq. (10), assuming this value of q_{44} is 73° 20′. The agreement between the two is fairly good.

In the case of crystal 5 only measurements with the elliptic analyser were made. These led to a value $-1 \cdot 60 \times 10^{-13}$ for q_{44}.

The mean of the three independent determinations of q_{44} is thus $-1 \cdot 58 \times 10^{-13}$.

8. DETERMINATION OF THE ELASTO-OPTIC CONSTANTS

From the mean values of the piezo-optic constants given above, namely, $q_{11} - q_{2} = -2 \cdot 39 \times 10^{-13}$, $q_{11} - q_{13} = -1 \cdot 42 \times 10^{-13}$ and $q_{44} = -1 \cdot 58 \times 10^{-13}$, the following values are found for the elasto-optic constants, using for the elastic constants the values $C_{11} = 5 \cdot 00$, $C_{12} = 1 \cdot 47$, $C_{44} = 1 \cdot 18 \times 10^{11}$, which are the averages of the measurements of Mason (1946) and of Bhagavantam and Suryanarayana (1947 *b*)

$$p_{11} - p_{12} = -0 \cdot 0502, \quad p_{11} - p_{13} = -0 \cdot 0842, \quad p_{44} = -0 \cdot 0187.$$

To determine p_{11}, p_{12}, p_{13} separately, it is necessary to measure the change in refractive index for the individual components that are propagated unchanged through the crystal. The interference method adopted by one of the authors (Ramachandran, 1947) in the case of diamond could not be used for this crystal as several attempts made to obtain interference fringes were not successful. Consequently, the ratios p_{12}/p_{11} and p_{13}/p_{11} were determined from a study of the polarisation of ultrasonic diffraction patterns provided by the crystal (method due to Mueller, 1938). At the suggestion of one of us (G. N. R.) these measurements were made by Mr. Vedam in this Laboratory. We are grateful to him for handing over to us the results of his measurements. In the calculation of the results, allowance was made for the fact that the crystal was optically active. The following values were found:

$$\frac{p_{12}}{p_{11}} = 1 \cdot 49 \qquad \frac{p_{13}}{p_{11}} = 1 \cdot 285$$

It may be noticed that $(p_{11} - p_{12})/(p_{11} - p_{13})$ from these is $1 \cdot 70$ as compared with $1 \cdot 68$, obtained from birefringence measurements.

Thus we obtain the complete set of photoelastic constans for sodium chlorate as:

$$p_{11} = 0 \cdot 173, \; p_{12} = 0 \cdot 258, \; p_{13} = 0 \cdot 223, \; p_{44} = -0 \cdot 0187$$

$$q_{11} = 1 \cdot 48, \; q_{12} = 3 \cdot 88, \; q_{13} = 2 \cdot 89, \; q_{44} = -1 \cdot 58, \; \times 10^{-13} \text{ cm.}^2 \text{ dyne}^{-1}.$$

9. DISCUSSION OF THE RESULTS

The most interesting result is the large difference between q_{12} and q_{13} which leads to a difference of about 70% between $q_{11}-q_{12}$ and $q_{11}-q_{13}$. Thus with stress along the X-axis, the birefringence observed when light is propagated along the Y-axis is very different from that when it is along the Z-axis. This is obviously because of the fact that the structure is asymmetric as is evidenced by the optical activity exhibited by it.

Sodium chlorate thus behaves as an optically biaxial crystal when compressed along a cubic axis. Calculations with the above constants show that it behaves like a *negative* biaxial crystal under compressive stresses, that the optic axial angle $2\,V = 80°$ and that the acute bisectrix is the stress direction. These characteristics are independent of the magnitude of the stress.

This is the first crystal of class T whose photoelastic constants have been determined and the measurements support Bhagavantam's results (1942) that it should have four independent constants. The values of the constants are similar to those for rock-salt, except for the fact that $p_{12}=p_{13}$ for rock-salt (*cf.* $p_{11}=+0\cdot137$, $p_{12}=+0\cdot178$, $p_{44}=-0\cdot0108$ for rock-salt). This is not surprising in view of the fact that the structure of sodium chlorate is based on a slightly distorted sodium chloride lattice. Further, the refractive index of sodium chlorate ($1\cdot514$) is also very near to the refractive index of rock-salt ($1\cdot544$).

The question arises whether stress produces any effect on the rotatory power. The present measurements do not reveal any such effect, as $2\,\rho$ is sensibly constant for different stresses in all the crystal specimens studied. The measurements, however, were not accurate enough to detect small deviations, since the main interest in the work was to determine the birefringence produced by stress.

ACKNOWLEDGMENT

We are grateful to Mr. S. Ramaseshan for the discussions we had with him and to Professor R. S. Krishnan for his interest in the work.

SUMMARY

Sodium chlorate is the first crystal belonging to the tetrahedrite class (T) of the cubic system for which photoelastic constants have been measured. Since the crystal exhibits optical activity and no birefringence in the absence of stress, special techniques have to be adopted for measuring the birefringence introduced by stress. This has been done by the use of a petrological microscope in conjunction with the stressing apparatus and measur-

ing the ellipticity and other characteristics of the light transmitted by the crystal. A particularly simple method is to use the "elliptic analyser". Since the crystal does not possess four-fold axes, the relative orientation of the X- Y- and Z-axes was determined by means of X-rays. From observations on crystals compressed along [100], [110] and [111] directions, $q_{11}-q_{12}$, $q_{11}-q_{13}$ and q_{44} were evaluated. All the four constants were independently obtained by combining these with polarisation measurements of light diffracted by ultrasonic waves in the crystal. The values are: $q_{11}=1\cdot 48$, $q_{12}=3\cdot 88$, $q_{13}=2\cdot 89$, $q_{44}=-1\cdot 58\times 10^{-13}$ cm.^{2} dyne^{-1}; $p_{11}=0\cdot 173$, $p_{12}=0\cdot 258$, $p_{13}=0\cdot 223$, $p_{44}=-0\cdot 0187$. q_{12} and q_{13} are different as is to be expected from Bhagavantam's theory for crystal classes T and T_h. For stress along X-axis, the values of birefringence for observation along the Y- and Z- axes differ by as much as 70%, which is the largest observed so far for a cubic crystal.

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128-51—Printed at The Bangalore Press, Bangalore City, by G. Srinivasa Rao, Superintendent
and Published by The Indian Academy of Sciences, Bangalore