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Faraday Effect and Birefringence. I
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FARADAY EFFECT AND BIREFRINGENCE—I

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1. INTRODUCTION

IT is well known that the presence of birefringence in a solid, both accidental and inherent, considerably decreases the magnitude of its magneto-optic rotation (Schutz, 1936). In consequence, very great care has to be exercised in selecting perfect crystals for the measurement of Faraday rotation. It was therefore considered that an investigation of the influence of birefringence on the Faraday effect in a solid would be of some interest. Although the complete theory of magnetic and optical rotation in birefringent solids has been worked out (Chauvin, 1890; Wiener, 1888; Pockels, 1906) this particular problem of the magnitude of the decrease in the rotation has not been specifically treated before. In the present investigation the general theories have been used to derive simple expressions correlating the decrease in the magnetic rotation with birefringence. The preliminary report of this has already been published in *Current Science* (Ramaseshan and Chandrasekharan, 1951). These expressions have been verified by experiments on strained glass and strained plastics. It is found that birefringences corresponding to path retardations of the order of $\lambda/30$ for the total length of the solid do not affect the magnetic rotation by more than 1%. The rotation actually becomes zero for a path retardation of about $\lambda/2$ and has a negative value when it is between $\lambda/2$ and λ. Further, it is found that the Verdet constant in birefringent solids can be measured accurately, provided that specimens are taken such that the total birefringence is not very large. In fact the decrease in the Faraday rotation in a strained solid can itself be used to calculate the birefringence very accurately.

2. ROTATION FOR LOW BIREFRINGENCE

When plane polarised light is incident on an isotropic solid placed in a magnetic field, it is split up into two circular vibrations of opposite senses which travel with different velocities inside the medium. Since these two vibrations are coherent, they combine at every point in the medium to produce a plane vibration which is rotated with respect to the plane of polarisation of the incident light, the rotation increasing directly as the thickness

traversed. By measuring the value of the rotation ρ for the whole thickness t of the solid for a known field, the Verdet constant of the substance can be calculated. Let us now assume that the isotropic solid is made birefringent by the application of a stress. In this case the incident plane polarised light splits up into two elliptic vibrations of opposite senses lying crossed to each other, which travel with different velocities. These being coherent, they combine at every point to produce an elliptic vibration whose major axis is rotated with respect to the plane of polarisation of the incident light. The magnitude of the rotation of the major axis (ψ) depends on the thickness and the birefringence of the solid. It is possible to measure the angle ψ very accurately with a half shade if the ellipticity of the emergent light is not very large. ψ can be considered to be the apparent rotation, and the so-called decrease in the rotation in a strained solid is due to the fact that ψ is less than ρ, if the incident light is polarised parallel to a principal direction, as would happen if the solid were originally set for minimum restoration of light under crossed nicols. However, this is not always true and the apparent rotation would actually increase if the incident light is polarised at 45° to the principal directions. We shall consider only the former case as measurements are best made in that setting. An expression connecting ψ with the birefringence and the rotation would enable one to calculate the true Verdet constant of a stressed or birefringent solid.

Let δ_0 be the phase retardation per unit length in the absence of rotation and ρ_0 the rotation per unit length when there is no birefringence. Then the phase retardation per unit length Δ_0 between the two elliptic vibrations that are propagated unchanged inside the anisotropic medium is given by

$$\Delta_0 = \sqrt{\delta_0^2 + (2\rho_0)^2} \tag{1}$$

and the ratio of the axes of the ellipse is given by $\tan \gamma$, where

$$\tan 2\gamma = 2\rho_0/\delta_0 \tag{2}$$

and the total phase difference for the thickness t is

$$\Delta = \Delta_0 t = t\sqrt{\delta_0^2 + (2\rho_0)^2}. \tag{3}$$

If the incident light is polarised with its plane parallel to one of the two principal vibration directions, then ψ is given by the relation (Pockels)

$$\tan 2\psi = \frac{\sin 2\gamma \sin \Delta}{\cos^2 2\gamma + \sin^2 2\gamma \cos \Delta}. \tag{4}$$

The total rotation for the solid when there is no birefringence is $\rho = t\rho_0$ and the total phase retardation when there is no rotation is $\delta = t\delta_0$.

From (1) and (2) we have

$$\sin 2\gamma = 2\rho_0/\Delta_0 = 1/m \text{ (say)} \tag{5}$$

or

$$\Delta = 2m\rho \tag{6}$$

Equation (4) could be written as

$$\tan 2\psi = \frac{\sin 2\gamma \sin \Delta}{1 - \sin^2 2\gamma (\cos \Delta - 1)}. \tag{7}$$

Substituting (5) and (6) in (7) and expanding sin $2m\rho$ and cos $2m\rho$ we have

$$\tan 2\psi = 2\rho \left[1 - \frac{m^2 (2\rho)^2}{3!} + \frac{m^4 (2\rho)^4}{5!} - \frac{m^6 (2\rho)^6}{7!} \cdots \right]$$

$$\times \left[1 + \frac{(2\rho)^2}{2!} - \frac{m^2 (2\rho)^4}{4!} + \frac{2\rho^4}{[2!]^2} + \frac{2\rho^6}{[2!]^3} \cdots \right] = L$$

$$\therefore \quad 2\psi = \tan^{-1} L = L - \frac{L^3}{3} + \frac{L^5}{5} \cdots$$

substituting the values of L we have

$$2\psi = 2\rho \left[1 - \frac{m^2 - 1}{3!} (2\rho)^2 + \frac{(m^2 - 1)(m^2 + 6)}{5!} (2\rho)^4 \cdots \right]$$

but $(m^2 - 1)(2\rho)^2 = \delta^2$

$$\therefore \quad 2\psi = 2\rho \left(1 - \frac{\delta^2}{3!} + \frac{(m^2 + 6)}{5!} (2\rho)^2 \delta^2 \cdots \right) \tag{8 a}$$

This series is convergent when ρ is not very large and δ is less than one. When δ is very small

$$2\psi = 2\rho (1 - \delta^2/3!). \tag{8}$$

This formula can be very easily verified by experiment using the usual Faraday effect apparatus with the half-shade mounted at the analyser end. The magnetic rotation ρ in an isotropic rectangular block of glass is measured for a known value of the magnetic field. The glass is then strained by applying pressure. The plane of polarisation of the incident light is made parallel to the axis of strain. The birefringence introduced (δ) is measured with the aid of a Babinet compensator and finally the value of ψ for the strained glass is measured for the same value of the magnetic field. In practice ρ and ψ are taken as the mean of the rotations obtained for positive and negative magnetic fields.

Fig. 1 gives the theoretical curve together with the experimental points. The agreement between theory and experiment is satisfactory. It is to be

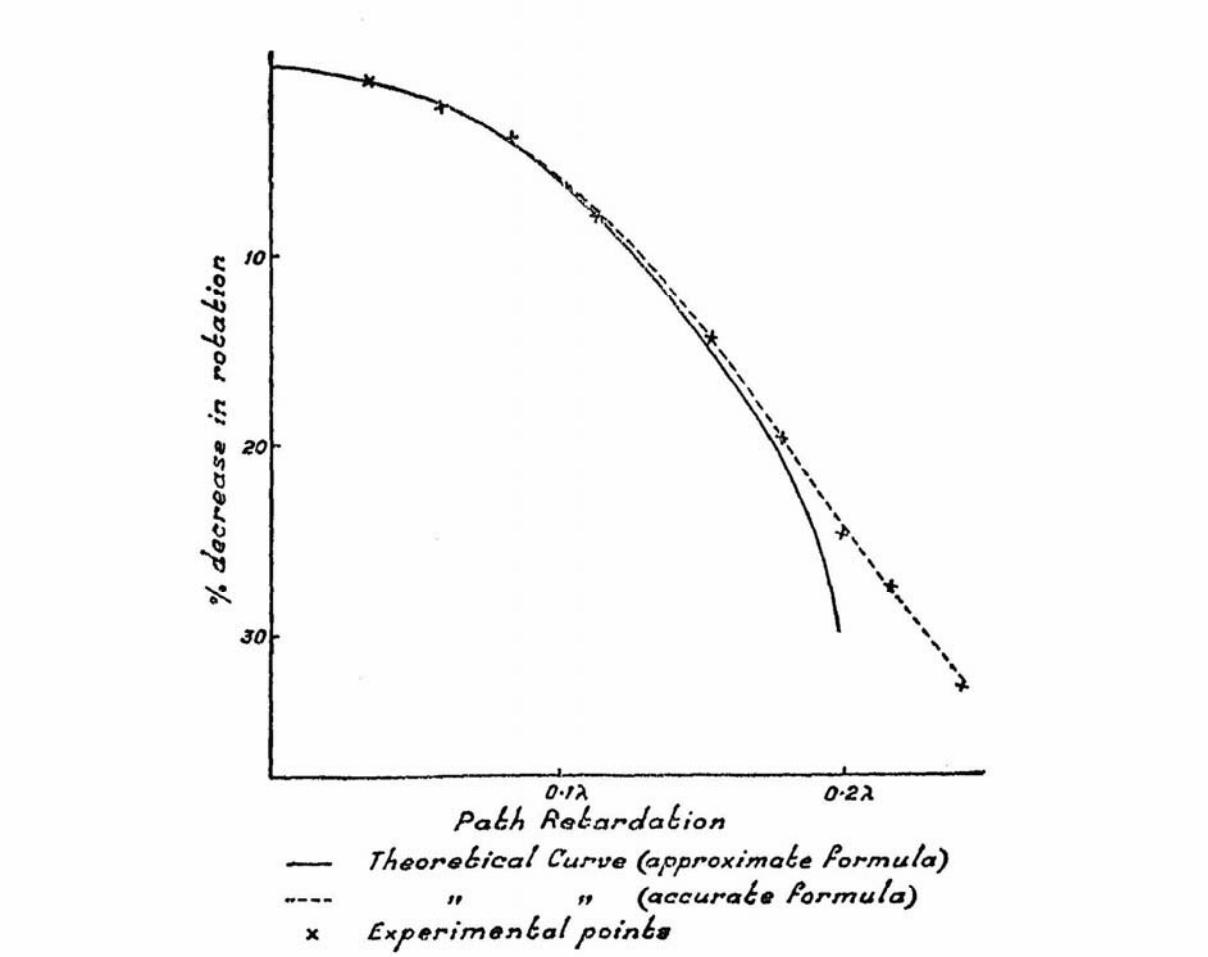

FIG. 1. Variation of rotation with birefringence (for small values of birefringence)

noticed that even when δ is nearly $\lambda/6$, the deviation from the approximate formula is not more than 3%. If the second term in expression (8 *a*) is also taken, a greater accuracy is obtained.

The formula (8) is approximately true for both inherent and accidental birefringence. Since the decrease in the rotation is proportional to $\delta^2/3$!, small birefringences of the order of $\lambda/30$ do not affect the results by more than 1%. This is quite obvious from Fig. 1. Further the decrease is dependent on the *total phase retardation* ($\delta = t\delta_0$) and not on δ_0 alone, and the Faraday rotation is not affected by the presence of birefringence if $\delta = t\delta_0 \simeq 0$. This can be achieved by two methods. The first is to have $\delta_0 = 0$, *i.e.*, to choose crystals that show no traces of birefringence. The second is to make the thickness t small, even though $\delta_0 \neq 0$ so that $t\delta_0$ is very nearly zero. Obviously if the birefringence δ_0 is large, the thickness t must be very small. This would necessitate the use of very accurate methods of measurements since the magnitude of the rotation would also be correspondingly reduced. But it is advantageous to reduce the thickness provided such accurate methods are available, because the *percentage* error introduced by the birefringence is

appreciably reduced thereby. Indeed by using larger thicknesses it may be possible to increase the inherent accuracy of the measurement of 2ψ, but it would differ from the true value 2ρ by a sensible amount due to the *accidental* birefringence that is present (the magnitude of which cannot be readily estimated).

These inferences from theory find direct confirmation in the measurements made on diamond and alumina (Ramaseshan, 1946). As is well known, although birefringence is not an inherent property of diamond, most diamonds exhibit accidental birefringence. It was invariably found that the values of Verdet constant obtained with thin specimens were always higher and more consistent than those obtained with thicker specimens.

3. Birefringence Large Compared to Rotation

A second formula can be derived in the case when ρ_0 the rotation is much smaller than δ_0 the birefringence. Equation (4) of the last section may be written as

$$\tan 2\psi = \frac{\sin 2\gamma \sin \Delta}{1 - 2 \sin^2 2\gamma \sin^2 \Delta/2}$$

putting $\sin 2\gamma = 2\rho_0/\Delta_0 = \epsilon$ and expanding we get

$$\tan 2\psi = \epsilon \sin \Delta (1 + 2\epsilon^2 \sin^2 \Delta/2 + 4\epsilon^4 \sin^4 \Delta/2 \ldots)$$

and when $2\rho_0/\delta_0$ is small

$$\tan 2\psi = \epsilon \sin \Delta = \frac{2\rho_0}{\Delta_0} \sin \Delta$$

or

$$\tan 2\psi = \frac{2\rho}{\Delta} \sin \Delta. \qquad (9)$$

While the formula in the previous section could only be used when the total phase difference is small, the formula (9) can be used for all values of birefringence provided the ratio $2\rho_0/\delta_0$ is small. One notices that ψ is an oscillating function and is zero when $\Delta = 0$ and $\Delta = \pi$ and has a positive maximum at $\Delta \simeq \pi/2$ and a negative one at $\Delta \simeq 3\pi/2$. The maximum value that ψ can take is when

$$\cos \Delta = - \tan^2 2\gamma$$

and

$$\psi_{max.} = \tfrac{1}{2} \text{ arc sin } \frac{2\rho_0}{\delta_0} \qquad (10)$$

The equation (9) has been verified by experiments on strained plastic plates. Most transparent colourless plastic sheets that are sold in the market

exhibit permanent strains probably due to the calendering during manufacture and behave like uniaxial crystals with very small birefringence. The direction of the optic axis is perpendicular to the large extensive surface of the plate. From such a plate, a large number of pieces were cut with lengths varying from 0·5 cm. to 8 cm. The sides were polished and due care was taken to see that no undue strains were introduced during the cutting and polishing processes. The rotatory power ρ_0 in the absence of birefringence was obtained by measuring the rotation along the optic axis. Each plastic piece was then placed with its optic axis perpendicular to the magnetic field and its birefringence in the absence of the field was measured with the aid of a Babinet compensator. It was found that the path retardation was proportional to the thickness of the piece. The incident light (which is an accurately parallel beam) had its plane of vibration parallel to one of the principal vibration directions of the plastic piece. The value of ϕ was determined by means of a half-shade at the analyser end, it being taken as the mean of values obtained when the magnetic field was put on and reversed. The same field was used for all the specimens.

Tables I and II give the results of the experiment for the two wavelengths λ 5461 and λ 4358 and Fig. 2 represents the variation of ϕ with the thickness of the specimens for λ 5461 and λ 4358. In the plastics as $2\rho_0/\delta_0$ is constant and as $2\rho_0 \ll \delta_0$, Λ can be taken to be equal to δ. Therefore from (9 *a*) tan ϕ is proportional to sin Λ which is exactly what the curves in Fig. 2 depict.

TABLE I

Magnetic rotation in strained plastic plates for λ 5461

Magnetic field — 4050 Oersteds ρ_0 — 60 minutes

Thickness	Phase difference Λ	Rotation 2ϕ		V^* / V
		Calculated	Observed	
0·54	0·12	31′	21′	0·68
1·21	0·34	63′	55′	0·70
1·70	0·40	70′	73′	0·75
2·26	0·50	84′	80′	0·81
3·05	0·70	71′	70′	0·40
3·61	0·80	51′	48′	0·24
4·10	0·95	18′	22′	0·10
4·78	1·10	23′	18′	0·08
5·60	1·30	62′	55′	0·17
6·50	1·50	83′	82′	0·22
7·19	1·60	77′	68′	0·17

TABLE II

Magneto-optic rotation in strained plastic plates for λ 4358

Magnetic field = 4050 Oersteds $\rho_0 = 101{\cdot}2$ minutes

Thickness	Phase difference $X\pi$	Rotation 2ψ		$\frac{V^*}{V}$
		Calculated	Observed	
0·54	0·15	53′	41′	0·76
1·21	0·36	94′	85	0·71
1·70	0·50	108.	102′	0·61
2·26	0·66	100′	96′	0·42
3·05	0·85	50′	54	0·18
3·51	1·00	0′	8′	0·12
3·10	1·15	− 47′	− 52′	−0·13
4·78	1·35	− 95′	− 90′	−0·09
5·60	1·60	−104′	−100′	0·18
6·50	1·85	− 55′	− 60′	−0·09
7·10	2·00	0′	8′	−0·00

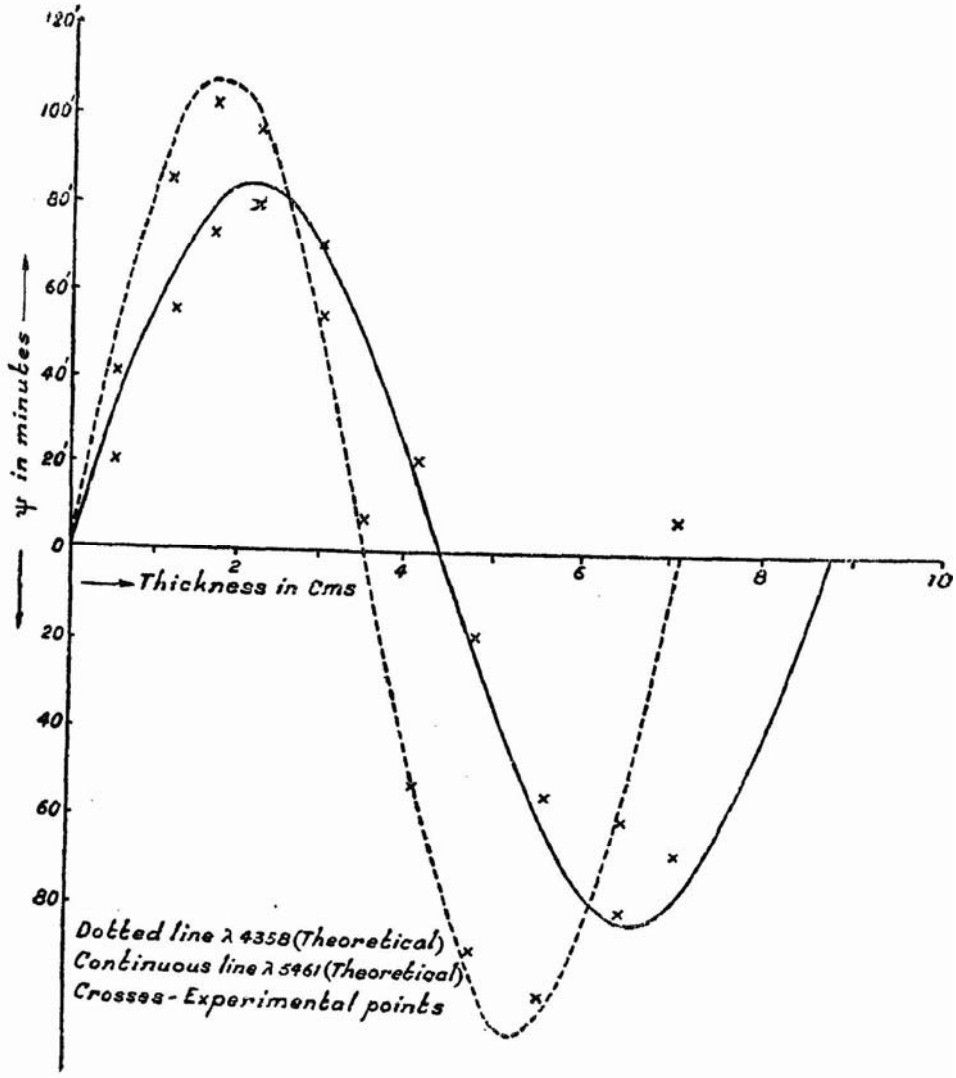

FIG. 2. The variation of ψ with thickness in strained plastics

From the curves one notices that for small values of thickness, the rotation is proportional to t, *i.e.*, the magnitude of the rotation is not affected when the birefringence is small. Another point of interest is that for values of thickness between 3·5 and 4·5 cm., ψ for λ 5461 is positive while that for λ 4358 is negative. The maximum value of the rotation is found to obey formula (10) within about 3%.

The apparent Verdet constant V* can be defined as

$$V^* = \frac{\psi}{Ht}$$

where H is the magnetic field. If ψ is small

$$\frac{V^*}{V} = \frac{\psi}{R} = \frac{\sin \Delta}{\Delta}$$

The experimental value of V*/V are given in the last column of Tables I and II. Fig. 3 represents the theoretical curve showing the variation of V*/V

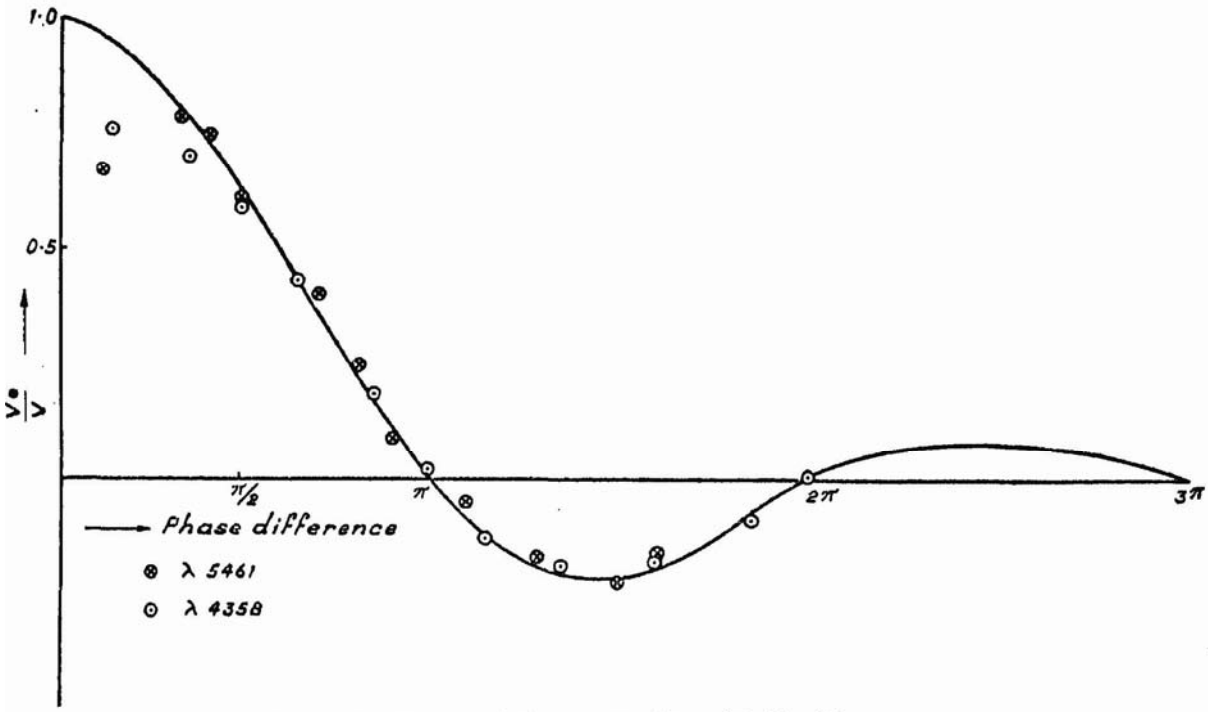

FIG. 3. The variation of V*/V with birefringence

with birefringence together with the experimental points. The agreement between theory and experiment is quite satisfactory. It may be mentioned that the magnetic rotations observed in most solids are not very great and the formulæ (8) and (9) together cover practically all the cases that are likely to occur in experimental observations of magneto-optic rotation in strained solids.

4. Measurement of Birefringence by the Magneto-Optic Method

From Fig. 3 one notices that the value of the magnetic rotation decreases as the birefringence increases. The decrease is first slow till the birefringence has a value of $\lambda/8$. From $\lambda/8$ to $\lambda/2$ the decrease in rotation is almost proportional to the increase in the birefringence. The decrease in the magnetic rotation can, therefore, be used as a fairly sensitive method for the measurement of the birefringence in a solid and the sensitivity is greatest when the value of the birefringence is between $\lambda/8$ and $\lambda/2$. This method has been used to measure the photoclastic constants in some glasses and crystals, the results of which will be reported in a later paper.

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Summary

Approximate formulæ correlating the decrease in Faraday rotation with birefringence have been derived and these have been verified by experiments on strained glasses and plastics. It is found that for small values of birefringence, the decrease in the Faraday rotation is proportional to the square of the total birefringence. For values of the birefringence of $\lambda/30$, the decrease in the rotation is less than 1%. The Verdet constant in birefringent solids can be accurately determined provided sufficiently thin specimens are taken such that the total birefringence is small. Finally it is shown that the decrease in the Faraday rotation can itself be used as an accurate measure of the birefringence in strained solids.

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