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PROPAGATION OF MICROWAVES THROUGH A CYLINDRICAL METALLIC GUIDE FILLED COAXIALLY WITH TWO DIFFERENT DIELECTRICS—PART IV

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ABSTRACT

The field components E's and H's for the hybrid mode EH_{mn} or HE_{mn} in a cylindrical metallic guide filled completely with two coaxial dielectrics have been theoretically determined. By utilising the boundary conditions, the propagation characteristics of the EH_{mn} mode have been deduced. Losses suffered by the wave and hence the attenuation constant have also been calculated.

INTRODUCTION

In a recent paper (Chatterjee, 1953) it has been shown that a cylindrical metallic guide filled completely with two coaxial dielectrics can support a pure mode (TE or TM) when the wave is circularly electric or circularly magnetic. But for the propagation of higher modes it is necessary that both the modes, TE (H wave) and TM (E wave) should co-exist. This means that both the longitudinal components E_z and H_z should be present. The resultant wave is called the hybrid mode. Hybrid modes are designated by HE_{mn} or EH_{mn} according as H wave predominates over E wave or E wave predominates over H wave respectively. The order of the hybrid mode is assigned according to increasing cut-off frequencies. It should however be pointed out that the hybrid mode is not merely the sum of corresponding order and rank of TE and TM modes, but it possesses entirely different propagation characteristics. The object of the present paper is to determine the field components of the hybrid mode and also to calculate the propagation characteristics from the boundary conditions.

FIELD COMPONENTS OF THE HYBRID MODE

The field components of the TM mode in a cylindrical metallic guide filled completely with two coaxial dielectrics are given (Chatterjee, 1953) as follows when $\sigma = 0$

$$\begin{aligned}
H_r &= -\omega\epsilon \frac{m}{r}\,[AJ_m(kr) + BY_m(kr)] \tfrac{\sin}{\cos}\, m\theta e^{-\gamma z} \\
H_\theta &= -j\omega\epsilon k\,[AJ'_m(kr) + BY'_m(kr)] \tfrac{\sin}{\cos}\, m\theta e^{-\gamma z} \\
H_z &= 0 \\
E_r &= -j\gamma k\,[AJ'_m(kr) + BY'_m(kr)] \tfrac{\sin}{\cos}\, m\theta e^{-\gamma z} \\
E_\theta &= -\gamma \frac{m}{r}\,[AJ_m(kr) + BY_m(kr)] \tfrac{\sin}{\cos}\, m\theta e^{-\gamma z} \\
E_z &= k^2\,[AJ_m(kr) + BY_m(kr)] \tfrac{\sin}{\cos}\, m\theta e^{-\gamma z}
\end{aligned} \tag{1}$$

For $m > 0$, the modes are either non-circularly symmetric or asymmetric. If we impose boundary conditions and use the equation

$$k^2 = \omega^2\mu\epsilon + \gamma^2 \tag{2}$$

to find the unknowns A/B, k_1, k_2 and γ_1, γ_2, it is found that there are more number of equations than unknowns. In this case it is not correct to assume that only the TM mode exists. But if an auxiliary TE mode is superimposed on the TM mode, the boundary conditions may be satisfied and we then obtain a hybrid mode HE or EH. The field components for the TE mode are (Chatterjee, 1953)

$$\begin{aligned}
H_r &= -\gamma k\,[A'J'_m(kr) + B'Y'_m(kr)] \tfrac{\sin}{\cos}\, m\theta e^{-\gamma z} \\
H_\theta &= -\gamma \frac{m}{r}\,[A'J_m(kr) + B'Y_m(kr)] \tfrac{\sin}{\cos}\, m\theta e^{-\gamma z} \\
H_z &= \left(k^2 - \frac{2m^2}{r^2}\right)[A'J_m(kr) + B'Y_m(kr)] \tfrac{\sin}{\cos}\, m\theta e^{-\gamma z} \\
E_r &= -j\omega\mu \frac{m}{r}\,[A'J_m(kr) + B'Y_m(kr)] \tfrac{\sin}{\cos}\, m\theta e^{-\gamma z} \\
E_\theta &= j\omega\mu k\,[A'J'_m(kr) + B'Y'_m(kr)] \tfrac{\sin}{\cos}\, m\theta e^{-\gamma z} \\
E_z &= 0
\end{aligned} \tag{3}$$

If the two modes are linearly combined to form a hybrid mode, then the following conditions for the two media must be satisfied.

$$\gamma_1^{E} = \gamma_1^{H} \qquad \gamma_2^{E} = \gamma_2^{H}$$

$$k_1^{E} = k_1^{H} \qquad k_2^{E} = k_2^{H} \tag{4}$$

Thus the field components for the HE or the EH mode for the two media are obtained as follows from (1), (3) and (4). The polar axis for θ is adjusted so as to drop either the sine or the cosine term for any component (Schelkunoff, 1943).

First medium:

$$E_{r1}^{e,h} = -j\Big[\Big\{A_1\gamma_1 k_1 J'_m(k_1 r) + A'_1\,\omega\mu_1 \frac{m}{r} J_m(k_1 r)\Big\} + \Big\{B_1\gamma_1 k_1 Y'_m(k_1 r) + B'_1\omega\mu_1 \frac{m}{r} Y_m(k_1 r)\Big\}\Big] \cos m\theta e^{-\gamma_1 z}$$

$$E_{\theta 1}^{e,h} = \Big[\Big\{-A_1\gamma_1 \frac{m}{r} J_m(k_1 r) + jA'_1\omega\mu_1 k_1 J'_m(k_1 r)\Big\} + \Big\{-B_1\gamma_1 \frac{m}{r} Y_m(k_1 r) + jB'_1\omega\mu_1 k_1 Y'_m(k_1 r)\Big\}\Big] \sin m\theta e^{-\gamma_1 z}$$

$$E_{z1}^{e,h} = k_1^2\,[A_1 J_m(k_1 r) + B_1 Y_m(k_1 r)] \cos m\theta e^{-\gamma_1 z}$$

$$H_{r1}^{e,h} = -\Big[\Big\{A_1\omega\epsilon_1 \frac{m}{r} J_m(k_1 r) + A'_1\gamma_1 k_1 J'_m(k_1 r)\Big\} + \Big\{B_1\omega\epsilon_1 \frac{m}{r} Y_m(k_1 r) + B'_1\gamma_1 k_1 Y'_m(k_1 r)\Big\}\Big] \sin m\theta e^{-\gamma_1 z} \tag{5}$$

$$H_{\theta 1}^{e,h} = -\Big[\Big\{jA_1\omega\epsilon_1 k_1 J'_m(k_1 r) + A'_1\gamma_1 \frac{m}{r} J_m(k_1 r)\Big\} + \Big\{jB_1\omega\epsilon_1 k_1 Y'_m(k_1 r) + B'_1\gamma_1 \frac{m}{r} Y_m(k_1 r)\Big\}\Big] \cos m\theta e^{-\gamma_1 z}$$

$$H_{z1}^{e,h} = \Big(k_1^2 - \frac{2m^2}{r^2}\Big)\Big[A'_1 J_m(k_1 r) + B'_1 Y_m(k_1 r)\Big] \sin m\theta e^{-\gamma_1 z}$$

Second Medium:

Y'_ms are omitted as $Y'_m\text{s} \to -\infty$ near the axial region of the guide which is physically inadmissible.

$$\begin{aligned}
E_{r2}{}^{e,h} &= -j\left[A_2\gamma_2 k_2 J'_m(k_2 r) + A'_2\omega\mu_2 \frac{m}{r} J_m(k_2 r)\right]\cos m\theta e^{-\gamma_2 z} \\
E_{\theta 2}{}^{e,h} &= \left[-A_2\gamma_2 \frac{m}{r} J_m(k_2 r) + jA'_2\omega\mu_2 k_2 J'_m(k_2 r)\right]\sin m\theta e^{-\gamma_2 z} \\
E_{z2}{}^{e,h} &= [A_2 k_2{}^2 J_m(k_2 r)]\cos m\theta e^{-\gamma_2 z} \\
H_{r2}{}^{e,h} &= -\left[A_2\omega\epsilon_2 \frac{m}{r} J_m(k_2 r) + A'_2\gamma_2 k_2 J'_m(k_2 r)\right]\sin m\theta \theta^{-\gamma_2 z} \\
H_{\theta 2}{}^{e,h} &= -\left[jA_2\omega\epsilon_2 k_2 J'_m(k_2 r) + A'_2\gamma_2 \frac{m}{r} J_m(k_2 r)\right]\cos m\theta e^{-\gamma_2 z} \\
H_{z2}{}^{e,h} &= \left(k_2{}^2 - \frac{2m^2}{r^2}\right)[A'_2 J_m(k_2 r)]\sin m\theta e^{-\gamma_2 z}
\end{aligned} \qquad (6)$$

The constants A's and B's are all related by the boundary conditions at the interface between the two dielectrics and at the interface between the dielectric and the conducting wall of the guide. Their values are determined by the power flow in the guide.

BOUNDARY CONDITIONS

The following boundary conditions must be satisfied assuming the conducting wall of the guide and the enclosed dielectric to be perfect.

1. At the interface between the dielectric and the conducting wall of the guide ($r = r_1$)

$$\Sigma E_{z1} = 0 \qquad \Sigma E_{\theta 1} = 0$$
$$\frac{\partial}{\partial r}\Sigma H_{\theta 1} = 0 \qquad \frac{\partial}{\partial r}\Sigma H_{z1} = 0 \qquad (7)$$

2. At the interface between the two dielectrics ($r = r_2$)

$$\begin{aligned}
\Sigma E_{z1} &= \Sigma E_{z2} & \Sigma E_{\theta 1} &= \Sigma E_{\theta 2} \\
\Sigma H_{z1} &= \Sigma H_{z2} & \Sigma H_{\theta 1} &= \Sigma H_{\theta 2} \\
\epsilon_1 \Sigma E_{r1} &= \epsilon_2 \Sigma E_{r2} & \mu_1 \Sigma H_{r1} &= \mu_2 \Sigma H_{r2}
\end{aligned} \qquad (8)$$

EVALUATION OF CONSTANTS

Applying the boundary conditions, the following expressions are obtained from (5) and (6)

$$A_1 J_m(k_1 r_1) + B_1 Y_m(k_1 r_1) = 0$$
$$A'_1 J'_m(k_1 r_1) + B'_1 Y'_m(k_1 r_1) = 0$$

$$k_1^2 [A_1 J_m (k_1 r_2) + B_1 Y_m (k_1 r_2)] e^{-\gamma_1 z} - k_2^2 [A_2 J_m (k_2 r_2)] e^{-\gamma_2 z} = 0$$

$$\left(k_1^2 - \frac{2m^2}{r_2^2}\right) \left[A'_1 J_m (k_1 r_2) + B'_1 Y_m (k_1 r_2)\right] e^{-\gamma_1 z} - \left(k_2^2 - \frac{2m^2}{r_2^2}\right) e^{-\gamma_2 z} [A'_2 J_m (k_2 r_2)] = 0$$

$$\epsilon_1 \gamma_1 k_1 \left[A_1 J'_m (k_1 r_2) + B_1 Y'_m (k_1 r_2)\right] e^{-\gamma_1 z} + \omega \mu_1 \epsilon_1 \frac{m}{r_2} \left[A'_1 J_m (k_1 r_2) + B'_1 Y_m (k_1 r_2)\right] e^{-\gamma_1 z} - \epsilon_2 \gamma_2 k_2 \left[A_2 J'_m (k_2 r_2)\right] e^{-\gamma_2 z} - \omega \mu_2 \epsilon_2 \frac{m}{r_2} \left[A'_2 J_m (k_2 r_2)\right] e^{-\gamma_2 z} = 0$$

$$\omega \mu_1 \epsilon_1 \frac{m}{r_2} \left[A_1 J_m (k_1 r_2) + B_1 Y_m (k_1 r_2)\right] e^{-\gamma_1 z} + \mu_1 \gamma_1 k_1 \left[A'_1 J'_m (k_1 r_2) + B'_1 Y'_m (k_1 r_2)\right] e^{-\gamma_1 z} - \omega \mu_2 \epsilon_2 \frac{m}{r_2} \left[A_2 J_m (k_2 r_2)\right] e^{-\gamma_2 z} - \mu_2 \gamma_2 k_2 \left[A'_2 J'_m (k_2 r_2)\right] e^{-\gamma_2 z} = 0 \quad (9)$$

From the equations (9), the following relations are obtained

$$\begin{aligned} A_1 &= A_2 k_3^2 e^{\gamma_3 z} f(k_1, k_2, r_1, r_2) \\ B_1 &= A_2 k_3^2 e^{\gamma_3 z} g(k_1, k_2, r_1, r_2) \\ A'_1 &= A'_2 k_4^2 e^{\gamma_3 z} f(k_1, k_2, r_1, r_2) \\ B'_1 &= A'_2 k_4^2 e^{\gamma_3 z} g(k_1, k_2, r_1, r_2) \end{aligned} \quad (10)$$

where

$$k_3^2 = \frac{k_2^2}{k_1^2}, \; k_4^2 = \frac{k_2^2 - \frac{2m^2}{r_2^2}}{k_1^2 - \frac{2m^2}{r_2^2}}, \; \gamma_3 = \gamma_1 - \gamma_2 \quad (11)$$

$$f = \frac{J_m (k_2 r_2) Y_m (k_1 r_1)}{J_m (k_1 r_2) Y_m (k_1 r_1) - J_m (k_1 r_1) Y_m (k_1 r_2)} \quad (12)$$

$$g = \frac{J_m (k_2 r_2) J_m (k_1 r_1)}{J_m (k_1 r_1) Y_m (k_1 r_2) - J_m (k_1 r_2) Y_m (k_1 r_1)}$$

$$\frac{A_2}{A'_2} = - \frac{\omega \mu_1 \epsilon_1 \frac{m}{r_2} k_4^2 \left[f J_m (k_1 r_2) + g Y_m (k_1 r_2)\right] - \omega \mu_2 \epsilon_2 \frac{m}{r_2} J_m (k_2 r_2)}{\epsilon_1 \gamma_1 k_1 k_3^2 [f J'_m (k_1 r_2) + g Y'_m (k_1 r_2)] - \epsilon_2 \gamma_2 k_2 J'_m (k_2 r_2)} \quad (13)$$

$$= -\frac{\mu_1\gamma_1 k_1 k_4^2 [f J'_m(k_1 r_2) + g Y'_m(k_1 r_2)] - \mu_2\gamma_2 k_2 J'_m(k_2 r_2)}{\omega\mu_1\epsilon_1 \frac{m}{r_2} k_3^2 \left[f J_m(k_1 r_2) + g Y_m(k_1 r_2) \right] - \omega\mu_2\epsilon_2 \frac{m}{r_2} J_m(k_2 r_2)}$$

The value of A_2 can be found from the peak power flow given by

$$\hat{P}_z = \int_{r=r_2}^{r_1} \int_{\theta=0}^{2\pi} [E_{r1}^{e,h} H_{\theta 1}^{e,h*} - E_{\theta 1}^{e,h} H_{r1}^{e,h*}]\, r d\theta dr + \int_{r=0}^{r_2} \int_{\theta=0}^{2\pi} [E_{r2}^{e,h} H_{\theta 2}^{e,h*} - E_{\theta 2}^{e,h} H_{r2}^{e,h*}]\, r d\theta dr \tag{14}$$

The equation (14) when evaluated with the help of the field components (Equations 5 and 6) yields (Chatterjee, *loc. cit.*) the following result

$$A_2 = P \tag{15}$$

where

$$P = \left[\frac{\hat{P}_z}{2\pi\omega F} \right]^{\frac{1}{2}} \tag{15a}$$

where F represents some function of the constants and the radial dimensions of the two dielectric media.

FIELD COMPONENTS IN TERMS OF POWER FLOW

The field components of the hybrid mode in the two media are then obtained from (5), (6) and (10) to (13) and (15) as follows:

$$E_{r1}^{e,h} = jP \left[k_3^2 \gamma_1 k_1 \{g Y'_m(k_1 r) - f J'_m(k_1 r)\} + c_m k_4^2 \omega\mu_1 \frac{m}{r} \{g Y_m(k_1 r) - f J_m(k_1 r)\} \right] \cos m\theta e^{-\gamma_1 z}$$

$$E_{\theta 1}^{e,h} = P \left[- k_3^2 \gamma_1 \frac{m}{r} \{f J_m(k_1 r) + g Y_m(k_1 r)\} + j c_m \omega\mu_1 k_1 k_4^2 \{f J'_m(k_1 r) + g Y'_m(k_1 r)\} \right] \sin m\theta e^{-\gamma_1 z}$$

$$E_{z1} = P\, [k_1^2 k_3^2 \{f J_m(k_1 r) + g Y_m(k_1 r)\}] \cos m\theta e^{-\gamma_1 z}$$

$$H_{r1}^{e,h} = -P \left[\omega\epsilon_1 k_3^2 \frac{m}{r} \{f J_m(k_1 r) + g Y_m(k_1 r)\} + c_m \gamma_1 k_1 k_4^2 \{f J'_m(k_1 r) + g Y'_m(k_1 r)\} \right] \sin m\theta e^{-\gamma_1 z}$$

$$H_{\theta 1}^{e,h} = -P \left[j k_1 k_3^2 \omega\epsilon_1 \{f J'_m(k_1 r) + g Y'_m(k_1 r)\} + c_m \gamma_1 k_4^2 \frac{m}{r} \{f J_m(k_1 r) + g Y_m(k_1 r)\} \right] \cos m\theta e^{-\gamma_1 z}$$

$$H_{z1}^{e,h} = c_m P k_4^2 \left(k_1^2 - \frac{2m^2}{r^2}\right)[f J_m(k_1 r) + g Y_m(k_1 r)] \sin m\theta e^{-\gamma_1 z}$$

$$E_{r2}^{e,h} = -jP\left[\gamma_2 k_2 J'_m(k_2 r) + c_m \omega\mu_2 \frac{m}{r} J_m(k_2 r)\right]\cos m\theta e^{-\gamma_2 z} \quad (16)$$

$$E_{\theta 2}^{e,h} = P\left[-\gamma_2 \frac{m}{r} J_m(k_2 r) + jc_m\omega\mu_2 k_2 J'_m(k_2 r)\right]\sin m\theta e^{-\gamma_2 z}$$

$$E_{z2}^{e,h} = P[k_2^2 J_m(k_2 r)]\cos m\theta e^{-\gamma_2 z}$$

$$H_{r2}^{e,h} = -P\left[\omega\epsilon_2 \frac{m}{r} J_m(k_2 r) + c_m\gamma_2 k_2 J'_m(k_2 r)\right]\sin m\theta e^{-\gamma_2 z}$$

$$H_{\theta 2}^{e,h} = -P\left[j\omega\epsilon_2 k_2 J'_m(k_2 r) + c_m\gamma_2 \frac{m}{r} J_m(k_2 r)\right]\cos m\theta e^{-\gamma_2 z}$$

$$H_{z2}^{e,h} = Pc_m\left(k_2^2 - \frac{2m^2}{r^2}\right)[J_m(k_2 r)]\sin m\theta e^{-\gamma_2 z}$$

where c_m represents a constant involving the electrical properties and radial dimensions of the two media. c_m however, will vary with the order of the Bessel functions. That is c_m varies with the mode of excitation. c_m is given by the reciprocal of the right-hand expressions in (13).

Effect of $\mu\epsilon$ on Power Transmission

In some electronic devices using waveguides, it is necessary to slow down the wave and concentrate it in a certain region of the guide. This can be done in a guide containing two coaxial dielectrics by adjusting $\mu\epsilon$ or simply ϵ in the case of non-magnetic media. Considering γ_1 and γ_2 to be the same and equal to γ in the case of hybrid mode propagation, the following relations between k_1 and k_2 are obtained.

$$k_2^2 = k_1^2 - \omega^2(\mu_1\epsilon_1 - \mu_2\epsilon_2) \quad (17)$$

$$k_1^2 = k_2^2 - \omega^2(\mu_2\epsilon_2 - \mu_1\epsilon_1) \quad (18)$$

The following conclusions regarding power transmission may be drawn from (17) and (18).

1. If $\mu_1\epsilon_1 > \mu_2\epsilon_2$, $k_2^2 \rightarrow 0$ as $k_1^2 \rightarrow \omega^2(\mu_1\epsilon_1 - \mu_2\epsilon_2)$.

This critical frequency in this case is given by

$$f_c^{(1)} = \frac{1}{2\pi}\frac{k_1}{\sqrt{\mu_1\epsilon_1 - \mu_2\epsilon_2}} \quad (19)$$

If the frequency of excitation is increased beyond $f_c^{(1)}$ when $\mu_1\epsilon_1 > \mu_2\epsilon_2$, k becomes imaginary which means that there is no propagation in the second medium and all the energy flow is concentrated in the first medium ($r_1 \geqslant r \geqslant r_2$).

2. If $\mu_2\epsilon_2 > \mu_1\epsilon_1$, $k_1^2 \to 0$ as $k_2^2 \to \omega^2(\mu_2\epsilon_2 - \mu_1\epsilon_1)$. The critical frequency in this case is given by

$$f_c^{(2)} = \frac{k_2}{2\pi\sqrt{\mu_2\epsilon_2 - \mu_1\epsilon_1}} \tag{20}$$

If the frequency of excitation $> f_c^{(2)}$, k_1 becomes imaginary and there is no propagation in the first medium. Consequently, all the energy is concentrated in the second medium.

3. If the two media are non-magnetic it is evident that the energy flow will mostly take place through the medium having higher dielectric constant. The same result is also obtained in the case of pure mode transmission (Chatterjee, *loc. cit.*).

PROPAGATION CHARACTERISTICS OF THE HYBRID MODE

From (9) and (10) the following expression is obtained:

$$\frac{Y'_m(k_1r_1)}{Y_m(k_1r_1)} = \frac{J'_m(k_1r_1)}{J_m(k_1r_1)} \tag{21}$$

which yields

$$J_m(k_1r_1) = Y_m(k_1r_1) \tag{22}$$

This is possible for some discrete values of k_1r_1 where the curves of J_m's and Y_m's with respect to their arguments intersect. Let δ_{mn} represent the root of the equation (22) where m represents the order and n represents the rank of the root of the Bessel function. So, the following relations hold good

$$k_1^2 = \frac{\delta^2_{mn}}{r_1^2} = \omega^2\mu_1\epsilon_1 + \gamma^2_{mn} \tag{23}$$

$$k_2^2 = \frac{\delta^2_{mn}}{r_1^2} - \omega^2(\mu_1\epsilon_1 - \mu_2\epsilon_2) \tag{23 a}$$

$$\gamma_{mn} = \alpha_{mn} + j\beta_{mn} = \sqrt{\frac{\delta^2_{mn}}{r_1^2} - \omega^2\mu_1\epsilon_1} \tag{24}$$

In order that propagation may take place γ_{mn} must be imaginary. The equation (24) may be written as

$$\gamma_{mn} = j\sqrt{\omega^2\mu_1\epsilon_1 - \frac{\delta^2_{mn}}{r_1^2}} \tag{24 a}$$

which yields

$$\beta_{mn} = \sqrt{\omega^2\mu_1\epsilon_1 - \frac{\delta^2_{mn}}{r_1^2}} \tag{24 b}$$

For propagation to take place β_{mn} must be real and hence $\omega^2\mu_1\epsilon_1 > \frac{\delta^2_{mn}}{r_1^2}$.

The limiting frequency below which a condition of cut-off exists is

$$f_c = \frac{\delta_{mn}}{2\pi r_1 \sqrt{\mu_1 \epsilon_1}} \tag{24 c}$$

The corresponding cut-off wavelength is

$$\lambda_c = \frac{2\pi r_1}{\delta_{mn}} \tag{24 d}$$

where the free wave velocity $c_1 = 1/\sqrt{\mu_1 \epsilon_1}$

The phase velocity, group velocity and the guide wavelength are given respectively by the following expressions

$$(c_p)_{mn} = \omega/\sqrt{\omega^2 \mu_1 \epsilon_1 - \delta^2_{mn}/r_1^2} \tag{24 e}$$

$$(c_g)_{mn} = [\sqrt{\omega^2 \mu_1 \epsilon_1 - \delta^2_{mn}/r_1^2}]/\omega \mu_1 \epsilon_1 \tag{24 f}$$

$$(\lambda_g)_{mn} = 2\pi/\sqrt{\omega^2 \mu_1 \epsilon_1 - \delta^2_{mn}/r_1^2} \tag{24 g}$$

Substituting proper values of δ_{mn} (Jahnke, Emde, 1945) in (24 *a*) to (24 *g*), the propagation characteristics for different modes ($m \neq 0$) are obtained. The values for EH_{11}, EH_{22}, EH_{12} and EH_{21} are given in the following two tables.

TABLE I. *Propagation characteristics of EH_{11} and EH_{22} modes*

	EH_{11}	EH_{22}
γ	$j\sqrt{\omega^2\mu_1\epsilon_1 - \frac{9\cdot1204}{r_1^2}}$	$j\sqrt{\omega^2\mu_1\epsilon_1 - \frac{58\cdot1406}{r_1^2}}$
β	$\sqrt{\omega^2\mu_1\epsilon_1 - \frac{9\cdot1204}{r_1^2}}$	$\sqrt{\omega^2\mu_1\epsilon_1 - \frac{58\cdot1406}{r_1^2}}$
f_c	$\frac{3\cdot02}{2\pi r_1\sqrt{\mu_1\epsilon_1}}$	$\frac{7\cdot625}{2\pi r_1\sqrt{\mu_1\epsilon_1}}$
λ_c	$\frac{2\pi r_1}{3\cdot02}$	$\frac{2\pi r_1}{7\cdot625}$
λ_g	$\frac{2\pi}{\sqrt{\omega^2\mu_1\epsilon_1 - \frac{9\cdot1204}{r_1^2}}}$	$\frac{2\pi}{\sqrt{\omega^2\mu_1\epsilon_1 - \frac{58\cdot1406}{r_1^2}}}$
c_p	$\frac{\omega}{\sqrt{\omega^2\mu_1\epsilon_1 - \frac{9\cdot1204}{r_1^2}}}$	$\frac{\omega}{\sqrt{\omega^2\mu_1\epsilon_1 - \frac{58\cdot1406}{r_1^2}}}$
c_g	$\frac{\sqrt{\omega^2\mu_1\epsilon_1 - \frac{9\cdot1204}{r_1^2}}}{\omega\mu_1\epsilon_1}$	$\frac{\sqrt{\omega^2\mu_1\epsilon_1 - \frac{58\cdot1406}{r_1^2}}}{\omega\mu_1\epsilon_1}$

In the above table $\delta_{11} = 3\cdot02$ and $\delta_{22} = 7\cdot625$.

TABLE II

Propagation characteristics of EH_{12} and EH_{21} modes

	EH_{12}	EH_{21}
γ	$j\sqrt{\omega^2\mu_1\epsilon_1 - \frac{38\cdot8129}{r_1^2}}$	$j\sqrt{\omega^2\mu_1\epsilon_1 - \frac{18\cdot2756}{r_1^2}}$
β	$\sqrt{\omega^2\mu_1\epsilon_1 - \frac{38\cdot8129}{r_1^2}}$	$\sqrt{\omega^2\mu_1\epsilon_1 - \frac{18\cdot2756}{r_1^2}}$
f_c	$\frac{6\cdot23}{2\pi r_1\sqrt{\mu_1\epsilon_1}}$	$\frac{4\cdot275}{2\pi r_1\sqrt{\mu_1\epsilon_1}}$
λ_c	$\frac{2\pi r_1}{6\cdot23}$	$\frac{2\pi r_1}{4\cdot275}$
λ_g	$\frac{2\pi}{\sqrt{\omega^2\mu_1\epsilon_1 - \frac{38\cdot8129}{r_1^2}}}$	$\frac{2\pi}{\sqrt{\omega^2\mu_1\epsilon_1 - \frac{18\cdot2756}{r_1^2}}}$
c_p	$\frac{\omega}{\sqrt{\omega^2\mu_1\epsilon_1 - \frac{38\cdot8129}{r_1^2}}}$	$\frac{\omega}{\sqrt{\omega^2\mu_1\epsilon_1 - \frac{18\cdot2756}{r_1^2}}}$
c_g	$\frac{\sqrt{\omega^2\mu_1\epsilon_1 - \frac{38\cdot8129}{r_1^2}}}{\omega\mu_1\epsilon_1}$	$\frac{\sqrt{\omega^2\mu_1\epsilon_1 - \frac{18\cdot2756}{r_1^2}}}{\omega\mu_1\epsilon_1}$

In the above table $\delta_{12} = 6\cdot23$ and $\delta_{21} = 4\cdot275$.

LOSSES IN THE GUIDE

In the case of a guide filled with two dielectrics, the losses suffered by the wave is greater than that in the case of a hollow metallic waveguide due to the additional losses introduced by the two dielectric media, if $\sigma \neq 0$ for the dielectrics. This definite value of the conductivity introduces a complex dielectric constant given by the following relation

$$\nabla^2 E = -\omega^2\mu\epsilon$$

where $\epsilon = \epsilon' - j\epsilon''$ and $\sigma = \omega\epsilon' \tan\delta$, where, the loss tangent $\tan\delta$ of the dielectric is given by the ratio of the imaginary part ϵ'' and the real part ϵ' of the complex dielectric constant ϵ of the medium.

The peak power density of loss is $\sigma\,|\,\mathbf{E}.\mathbf{E}^*\,|$. The peak power $\hat{P}_d$ dissipated over the volumes of the two dielectric media is

$$\hat{P}_d{}^{mn} = \sigma \int_v |\mathbf{E}\cdot\mathbf{E}^*| \, dv$$

$$= \omega\epsilon'_1 \tan\delta_1 \Big[\int_{z=0}^{l} \int_{\theta=0}^{2\pi} \int_{r=r_2}^{r_1} (E_{r1}{}^{e,h}.E_{r1}{}^{e,h*} + E_{z1}{}^{e,h}.E_{z1}{}^{e,h*} + E_{\theta 1}{}^{e,h}.E_{\theta 1}{}^{e,h*}) \, r dr d\theta dz$$

$$+ \omega\epsilon'_2 \tan\delta_2 \Big[\int_0^{l} \int_0^{2\pi} \int_0^{r_2} (E_{r2}{}^{e,h}.E_{r2}{}^{e,h*} + E_{z2}{}^{e,h}.E_{z2}{}^{e,h*} + E_{\theta 2}{}^{e,h}.E_{\theta 2}{}^{e,h*}) \, r dr d\theta dz \Big] \quad (25)$$

where l is the length of the guide.

The equation (25) when evaluated with the help of (16) yields

$$\hat{P}_d{}^{mn} = \omega_2\epsilon'_1 \tan\delta_1 F'_1 + \omega\epsilon'_2 \tan\delta_2 F'_2 \quad (25\,a)$$

where F's represents some function of k, electrical constants and radial dimensions of the two dielectric media. The peak power loss suffered in the walls of the guide per unit length is

$$\hat{P}_w{}^{mn} = \eta \int_{\substack{\text{surface of}\\ \text{the guide}}} |H^2_{\tan}| \, r_1 d\theta = \eta r_1 \int_0^{2\pi} [\,|H_{r1}{}^{e,h}|^2 + |H_{\theta 1}{}^{e,h}|^2] \, d\theta \quad (26)$$

where

$$\mathrm{Re}\,\eta = [(\pi f \mu)/\sigma]^{\frac{1}{2}}$$

From (26) and (16), the following expression for $\hat{P}_w{}^{mn}$ is obtained

$$\hat{P}_w{}^{mn} = \pi\eta r_1 P^2 \Big[\Big\{ jk_1 k_3{}^2 \omega\epsilon_1 \Big(fJ'_m(k_1r_1) + gY'_m(k_1r_1) \Big) + c_m k_4{}^2 \gamma_1 \frac{m}{r_1} \Big(fJ_m(k_1r_1) + gY_m(k_1r_1) \Big\}^2 + \Big\{ c_m k_4{}^2 \Big(k_1{}^2 - \frac{2m^2}{r_1{}^2} \Big) \Big(fJ_m(k_1r_1) + gY_m(k_1r_1) \Big) \Big\}^2 \Big] \quad (26\,a)$$

Attenuation Constant

The attenuation constnat a_{mn} is obtained from the following relation

$$a_{mn} = -\frac{P_d{}^{mn} + P_w{}^{mn}}{2\,P_z} \quad (27)$$

where P_z represents the average power transmitted through the dielectric obtained from (14). P_d^{mn} and P_w^{mn} represent the average power loss per unit length in the dielectric and the guide walls respectively. The following conclusions regarding the attenuation constant may be drawn from (27).

1. The attenuation constant increases directly with the increasing loss tangent of the dielectric.

2. The attenuation constant increases as the square of the dielectric constants of the two media.

3. The attenuation constant increases with increasing frequency of excitation of the guide.

SPECIAL CASE

In the case of a hollow metallic waveguide, $\epsilon_1 = \epsilon_2 = \epsilon_0$ (air) and $\mu_1 = \mu_2 = \mu_0$ (air). The EH wave can be split up into two independent waves E and H, either of which can exist separately. In the case of a hollow guide, the terms containing Y_m's are omitted from the general solution of the wave equation. The propagation characteristics for the E and the H waves, in this case, as deduced from (24 *a*) to (24 *g*) with the following modifications are given in Table III.

TABLE III

Propagation characteristics of the E and the H wave in a hollow cylindrical metallic guide

	E_{mn}	H_{mn}
γ^{mn}	$j\sqrt{\omega^2\mu_0\epsilon_0 - \frac{X^2_{mn}}{r_1^2}}$	$j\sqrt{\omega^2\mu_0\epsilon_0 - \left(\frac{X'_{mn}}{r_1}\right)^2}$
β^{mn}	$\sqrt{\omega^2\epsilon_0\mu_0 - \frac{X^2_{mn}}{r_1^2}}$	$\sqrt{\omega^2\mu_0\epsilon_0 - \left(\frac{X'_{mn}}{r_1}\right)^2}$
f_c^{mn}	$X_{mn}/2\pi r_1 \sqrt{\mu_0\epsilon_0}$	$X'_{mn}/2\pi r_1 \sqrt{\mu_0\epsilon_0}$
λ_c^{mn}	$2\pi r_1/X_{mn}$	$2\pi r_1/X'_{mn}$
λ_g^{mn}	$2\pi \Big/ \sqrt{\omega^2\mu_0\epsilon_0 - \frac{X^2_{mn}}{r_1^2}}$	$2\pi\sqrt{\omega^2\mu_0\epsilon_0 - \left(\frac{X'_{mn}}{r_1}\right)^2}$
c_p^{mn}	$\omega \Big/ \sqrt{\omega^2\mu_0\epsilon_0 - \frac{X^2_{mn}}{r_1^2}}$	$\omega \Big/ \sqrt{\omega^2\mu_0\epsilon_0 - \left(\frac{X'_{mn}}{r_1}\right)^2}$
c_g^{mn}	$\sqrt{\omega^2\mu_0\epsilon_0 - \frac{X^2_{mn}}{r_1^2}} \Big/ \omega\epsilon_0\mu_0$	$\sqrt{\omega^2\mu_0\epsilon_0 - \left(\frac{X'_{mn}}{r_1}\right)^2} \Big/ \omega\mu_0\epsilon_0$

1. Replace δ_{mn} by χ_{mn} which is the root of the equation $J_m(kr) = 0$ in the case of the E wave.

2. Replace δ_{mn} by χ'_{mn} which is the root of the equation $J'_m(kr) = 0$ in the case of the H wave.

The results given in Table III agree with the results obtained by direct solution of the wave equation in the case of the E and the H wave in a hollow cylindrical metallic wave guide.

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(*To be continued*)