

THE DISPERSION OF THE STRESS OPTICAL COEFFICIENT OF VITREOUS SILICA

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ABSTRACT

Accurate determinations of the stress optical coefficient C of vitreous silica and its variation with wavelength have been made over the range from λ 230 $m\mu$ to λ 500 $m\mu$. Filon's spectroscopic method has been used with suitable modifications. Using the data obtained by the author and those of Filon (which are confined to the visible region), a formula for the dispersion of C is given which is valid from λ 230 $m\mu$ to λ 650 $m\mu$.

1. INTRODUCTION

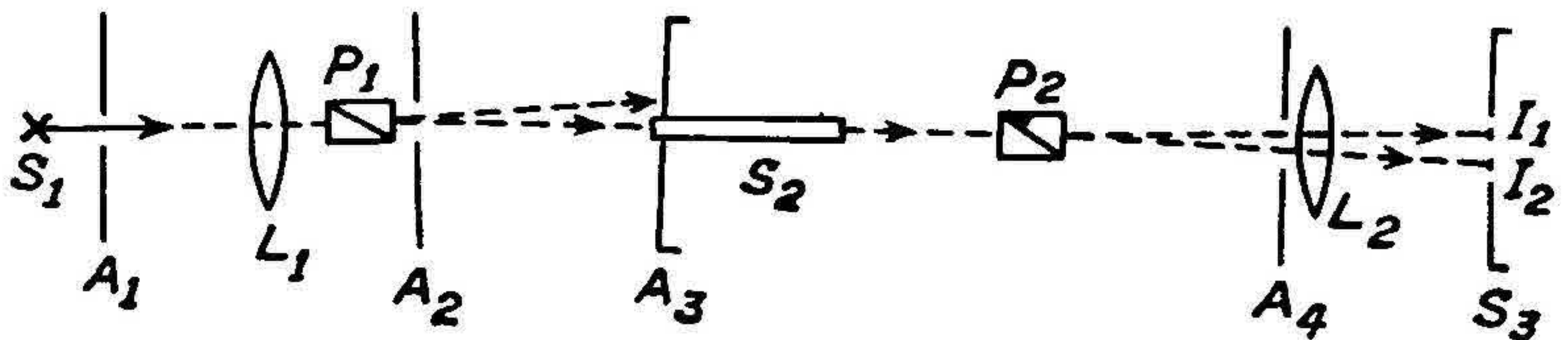
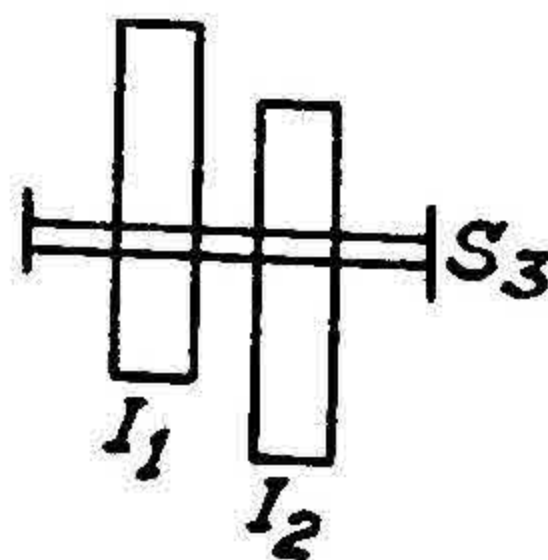
The importance of investigating the dispersion of the photoelastic constants of solids is obvious. Discrepancies between the observed and the theoretically calculated intensities of thermal scattering of light in solids (Krishnan, 1953) are often attributed to the existence of the dispersion of these constants. A knowledge of the dispersion enables one to investigate the variation of the quantity dn/dp with wavelength. In view of the paucity of data it was decided to make a systematic investigation of the dispersion of the photoelastic constants of solids. Among amorphous substances, fused silica is the simplest since it is made up of a single chemical constituent. It is therefore best suited for a thorough investigation of the dispersion of the photoelastic constants over a wide range of wavelength. In the present paper the investigation of the stress optical coefficient of fused silica carried out by the author is described.

The stress optical coefficient, $C = n^3 (q_{12} - q_{11})/2$, has been measured for fused silica by Heymans and Allis (1923) for one wavelength, whereas Filon and Harris (1931) measured the same coefficient over the range of wavelengths 460–650 $m\mu$. No work has so far been carried out in the ultraviolet region where one might expect an increased dispersion. Vedam (1950) measured the individual piezo-optic constants of fused silica for λ 589.3 $m\mu$. The value of C obtained from these constants does not agree with that of Filon. The author has now extended the measurements of C upto 230 $m\mu$. A preliminary note describing the results has been communicated to *Nature*.

2. EXPERIMENTAL DETAILS

The experimental set-up used by the author for measuring the dispersion of C for fused silica was similar to the one employed by Filon and Harris. The

optical arrangement is illustrated in Fig. 1 *a*. The light from the source S_1 passed through a vertical slit A_1 . A quartz lens L_1 focussed the image of A_1 on the specimen S_2 . A double image prism P_1 was placed in such a way as to polarise the incident beam at 45° to the direction of pressure namely vertical. A_2 was another vertical slit. The positions of P_1 and A_2 were adjusted such that the two mutually perpendicular components were well separated before falling on the specimen. One of the components was cut off with the aid of black paper A_3 stuck to the specimen. This also cut off the stray illumination if any. The front surface of the specimen was slightly ground thereby serving as an extended source. The light coming out of the specimen was passed through a second double image prism P_2 kept with its principal axes inclined at 45° to the vertical. It was then focussed by the quartz lens L_2 on the slit S_3 of a Hilger small quartz spectrograph. The aperture of L_2 was restricted by A_4 . The spectrograph was mounted in a vertical plane in order to photograph the horizontal section of the specimen. The two images I_1 and I_2 resulting from P_2 were separated both laterally and vertically with respect to each other. They fell on the slit of the spectrograph which was horizontal as indicated in Fig. 1 *b*. When the specimen was loaded and viewed through the camera side of the spectrograph the two spectra corresponding to I_1 and I_2 would be crossed by interference bands.

FIG. 1 *a*.FIG. 1 *b*.

A white incandescent lamp, a copper arc and an iron arc were used as sources of illumination for any one photograph; all the three sources were used one after the other. The Wollaston prisms were oriented as accurately as possible. Any error in their setting would not, however, affect the positions of the minima but would only increase the width of the band. For the identification of the wave-

length corresponding to the dark band the line spectrum of mercury was superimposed on the two spectra. Measurements of these lines together with some previously identified copper lines were made with by the aid of a Hilger high resolution comparator, and a calibration curve was drawn. For determining the wavelength of the band a low resolution microscope as the band width was not small was used. The eye-piece of this microscope had two parallel cross-wires which were set up parallel to the lines of the two spectra. With such an eye-piece it was easier to determine the position of the band. The band position was read four times with respect to some two identified lines and the mean value was used for the identification of the wavelength of the band from the calibration curve.

The stressing apparatus is essentially the same as that used and described by Ramachandran (1947). Thick lead sheets were used for padding purposes. For each photograph the specimen was mounted several times and the weights on each hanger were adjusted till the image seen through the prism P_2 had a uniform tint of passage. This tint as described by Filon was a sensitive test. Besides, as the horizontal section of the specimen was photographed the slight non-uniformity of the pressure along the direction of the section would show tapering effects in both the spectra I_1 and I_2 close to the bands, indicating the existence of greater stress to one side. Care was taken to eliminate this source of error and photographs which showed appreciable tapering were rejected. Thus the negatives actually used for measurements were those taken with practically uniform stress distribution.

The observations were made on a specimen A of vitreous silica supplied by the Thermal Syndicate Ltd., England, having dimensions $20 \times 6.8 \times 1.285$ mm. The direction of pressure was along its length while the direction of the incident light was perpendicular to the plane (20×1.285) mm. Four photographs corresponding to four different loads 13.782, 15.085, 16.077, 17.027 kgm. were taken for measurements. The maximum pressure applied was about 2 kg./mm.²

In the present measurements the following modifications have been introduced to the spectroscopic method used by Filon.

(i) Instead of visual observations the photographic technique was adopted. It is well known that the accurate determination of the wavelength corresponding to the zero intensity in any band system is more objectively carried out by the photographic method than visually. Besides by suitably adjusting the time of exposure the minima can be made well defined. The photographic method has another added advantage that the measurements could be extended to the ultraviolet.

(ii) In place of the polarising nicols used by Filon, the double image prisms were employed here. This enables one to photograph and compare the two images simultaneously. The advantage of this will be clear from the discussion to follow.

(iii) The horizontal sections of the stressed specimen were studied instead of the vertical sections as studied by Filon. This enables one to test the uniformity of pressure along the section as mentioned above.

3. THE THEORY OF THE METHOD

The intensities I_1' and I_2' of the two beams corresponding to I_1 and I_2 are given by

$$I_1' = r^2 \sin^2 \pi \frac{\mu_2 - \mu_1}{\lambda} b, \quad (\phi = \pi/2) \quad (1)$$

$$I_2' = r^2 \left(1 - \sin^2 \pi \frac{\mu_2 - \mu_1}{\lambda} b \right), \quad (\phi = 0) \quad (2)$$

where r is the amplitude of the incident wave on P_1 , ϕ is the angle between the planes of vibrations of the incident beam on the specimen and those of the transmitted beams by the prism P_2 . $(\mu_2 - \mu_1) b = R$ is the retardation due to the stressed specimen having the length b in the direction of the light beam, λ is the wavelength of light, and the loss of intensity due to effects such as reflection, etc., does not affect the following discussion and hence can be neglected.

By putting I_1' and I_2' equal to zero we get the retardation corresponding to a dark band as

$$R = (\mu_2 - \mu_1) \cdot b = N' \lambda \quad (3)$$

where

$$\begin{aligned} N' &= (N + 1) \text{ for } I_1' \\ &= (N + \frac{1}{2}) \text{ for } I_2' \text{ and} \\ N &= 0, 1, 2, 3, \text{ etc.} \end{aligned}$$

It is seen that both the spectra are crossed by dark bands. By determining the wavelength λ and identifying N' corresponding to a dark band, the stress optical coefficient C can be determined from the relation

$$C = \frac{R}{b \cdot P} \quad (4)$$

where P is the applied pressure. It is interesting to note that the first order dark band for the parallel position of the nicols or the polarising prisms occur at half the load for which it appears for the commonly used crossed positions. This fact may be of great use in the case of crystals which cannot be strained much. It is seen that as I_1' and I_2' are complementary, the spectrum due to I_1 would show a maximum intensity for a wavelength for which the other spectrum due to I_2 shows a minimum; and *vice-versa*. As the two spectra were photographed side by side simultaneously the position of the band and hence the corresponding wavelength could be determined accurately.

4. RESULTS AND DISCUSSION

As already explained in the previous section care was taken to select only those photographs for which the pressure distribution in the sections studied was practically uniform. An examination of Fig. 1 *b* will show that the horizontal

sections of the specimen responsible for the two spectra photographed are not the same. In the actual case the error arising from this was practically negligible as the portions responsible for the two spectra were restricted to about one-third portion of the specimen around the centre where the stress distribution is expected to be uniform.

In the stressing apparatus used, the beam to which the load is applied moves over two guiding pillars. If there was any friction between these pillars and the beam, the stress effective on the specimen would be less than the applied load. By suitable manipulation and by keeping the maximum load less than 18 kg., the error arising from this cause was reduced to a minimum. This was further tested out by adding a small load of say 100 gm. or less to one of the hangers when the beam is fully loaded and noting the change in the uniform tint of passage. The error if any is estimated to be less than 0.5%.

The main source of error lies in the determination of the wavelength corresponding to the zero intensity. The position of the centre of the dark band could be determined correct to the one-twentieth of the band width and the error in the estimation of the wavelength was found to be less than 0.5%. As the dispersion of the spectrograph is not linear an error to the extent of 0.5% would be introduced in the region of 400 $m\mu$, since the determination of the wavelength is based on the measurements of the centre of the dark band. In the ultraviolet region the error is very much reduced.

It will be evident from equations (3) and (4) that C depends on λ and as such the percentage error in the evaluation of C will be greater than that in λ . The errors in the values of C for the various wavelengths given in Table I are estimated to be less than 2%.

The observed values in $m\mu$ of the wavelengths corresponding to zero intensities and the corresponding values of C calculated by using equation (4) are given in Table I. The average values of C from Filon and Harris data for four different wavelengths are also included in Table I. The variation of C with wavelength is graphically represented in Fig. 2. The existence of a definite dispersion of the

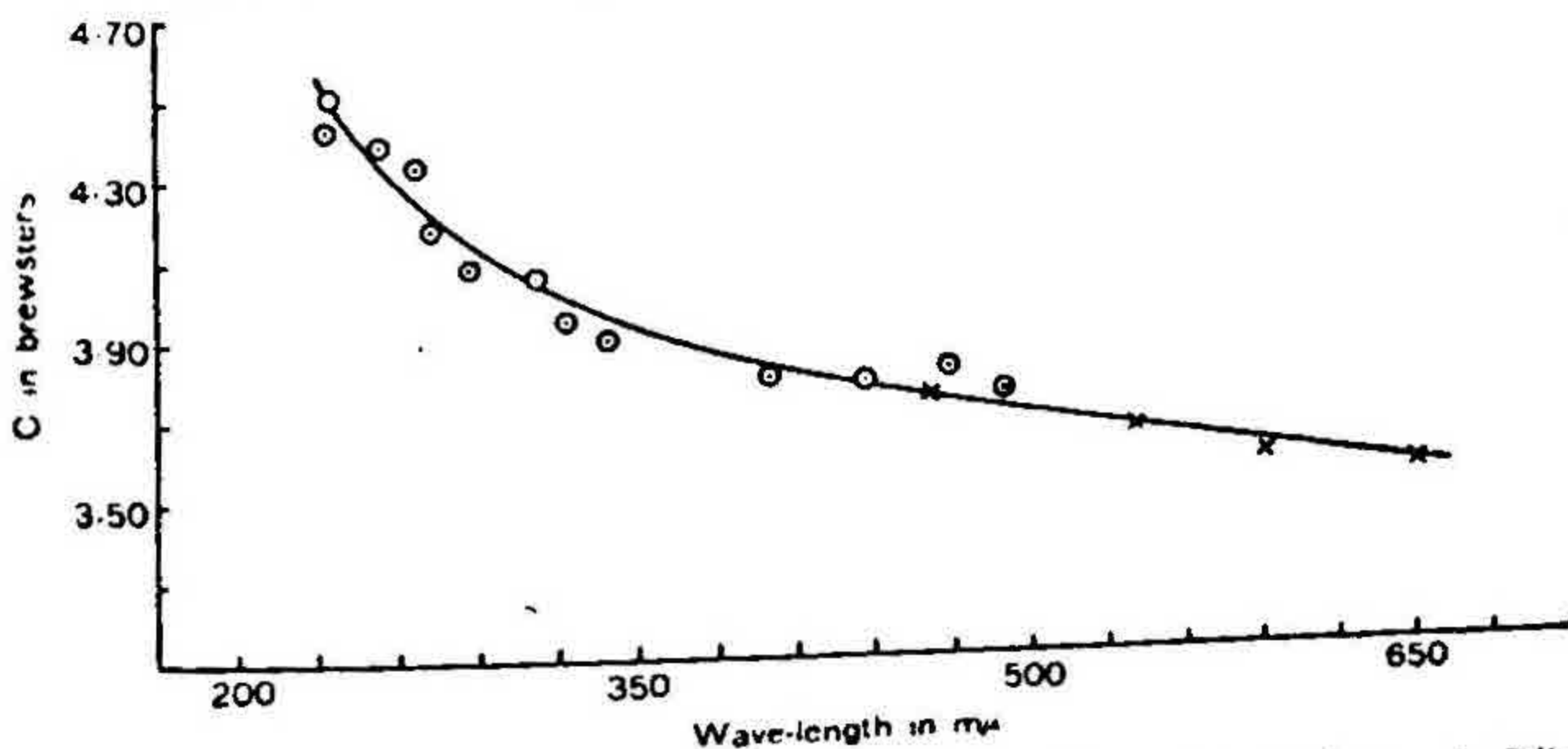


FIG. 2. The stress optical coefficient of vitreous silica. \circ Author. \times Filon's mean values.

TABLE I

Observation No.	N'	Plate No.	Wave-length in $m\mu$	C	C'	C'-C
				in brewsters		
Filon and Harris data (mean values)						
			649.5	3.55	3.55	.00
			589.6	3.58	3.61	+ .03
			539.7	3.65	3.66	+ .01
			460.3	3.75	3.75	.00
Author's observations						
1	1	4	488	3.76	3.72	- .04
2	1	3	468	3.82	3.74	- .08
3	1	2	436	3.79	3.78	- .01
4	1	1	399	3.80	3.83	+ .03
5	3/2	4	338	3.90	3.96	+ .06
6	3/2	3	323	3.95	4.00	+ .05
7	3/2	2	311	4.06	4.04	- .02
8	3/2	1	286	4.08	4.14	+ .06
9	2	4	272	4.19	4.20	+ .01
10	2	3	266	4.34	4.24	- .10
11	2	2	252	4.38	4.33	- .05
12	5/2	4	234	4.51	4.49	- .02
13	2	1	233	4.43	4.51	+ .08

Room Temperature (25 ± 2)° C.

stress optical coefficient is obvious from the figure. The dispersion of C increases with decreasing wavelength.

According to Martens (1904) the refractive index for fused silica may be represented by the relation

$$n^2 = m + \frac{m_1 \lambda^2}{\lambda^2 - \lambda_1^2} - k\lambda^2 \quad (5)$$

where λ (in μ) is the wavelength in vacuum.

$$m = 1.36112, m_1 = .74655, \lambda_1 = .107044 \mu \text{ and } k = .01350.$$

The variation of the refractive index per unit change of temperature dn is given by

$$2ndn = (dm) + \frac{\lambda^2}{\lambda^2 - \lambda_1^2} (dm_1) + \frac{m_1 \cdot \lambda^2 \cdot 2\lambda_1}{(\lambda^2 - \lambda_1^2)^2} (d\lambda_1) - (dk)\lambda^2 \quad (6)$$

The values of dm , dm_1 and dk were determined by Martens using relation (6). For expressing the stress optical coefficient one can use a formula similar to that of (6). This is given below:

$$2nC_x = (dm)_x + \frac{\lambda^2}{\lambda^2 - \lambda_1^2} (dm_1)_x + \frac{m_1 \lambda^2 \cdot 2\lambda_1}{(\lambda^2 - \lambda_1^2)^2} (d\lambda_1)_x - (dk)_x \lambda^2 \quad (7)$$

where m_1 and λ_1 are same in relations (6) and (7), $x = 1, 2$ for polarisations parallel and perpendicular with respect to the pressure direction respectively, C_1 and C_2 are the changes in the refractive indices per unit pressure change.

From (7) the stress optical coefficient C is given by:

$$2nC = \delta(dm) + \frac{\lambda^2}{\lambda^2 - \lambda_1^2} \delta(dm_1) + \frac{m_1 \lambda^2 \cdot 2\lambda_1}{(\lambda^2 - \lambda_1^2)^2} \delta(d\lambda_1) - \delta(dk)\lambda^2 \quad (8)$$

where

$$C = [C_1 - C_2]$$

and

$$\delta(y) = [(dy)_1 - (dy)_2]$$

The constants involved in equation (8) could be evaluated from the data given in Table I. In the present paper it was assumed that

$$\delta(dm_1) = 0 \quad (9)$$

The expression for the dispersion over the entire range, *i.e.*, from 230 $m\mu$ to 650 $m\mu$ obtained by using the method of least squares and considering all the observed values of C given in Table I is

$$C = \frac{1}{2n} \left[10.87 + \frac{m_1 \lambda^2 \cdot 2\lambda_1}{(\lambda^2 - \lambda_1^2)^2} \cdot (0.613) - 1.86\lambda^2 \right] \quad (10)$$

where n is the refractive index and C is the stress optical coefficient in brewster corresponding to the wavelength λ in μ . m_1 and λ_1 are same as in (5). The smooth curve showing the variation of C with λ given in Fig. 2 is obtained by using equation (10). The calculated values of C using equation (10) are also included in Table I under the column C' . The last column gives the values of $C' - C$. The deviations of the observed value of C for any wavelength from the corresponding theoretical value C' is about $\pm 2\%$ or less. It follows therefore that equation (10) represents the dispersion of C in a satisfactory manner.

The smooth curve also shows the close agreement between the Filon and Harris data and the author's measurements, although the specimens used in the two cases are of entirely different origin. Thus it can be inferred that the stress optical coefficient C for vitreous silica may not vary much for different specimens. If at all there exists such a variation it may be of the order of 1% or less.

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