SPACE-CHARGE EFFECTS IN A PROPORTIONAL COUNTER

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Abstract

From the known characteristics of the space charge, an expression for the gas multiplication in a proportional counter is derived as a function of the primary ionisation. The results are applied to the case of α and β particle pulses.

1. Introduction

It is known that in a proportional counter, when the amplification reaches high values ($\sim 10^4$ or more), true proportionality is lost, the multiplication factor decreasing with increase of the primary ionization. This is, of course, different from another disturbing factor that sets in under these circumstances, *viz.*, ultraviolet photons emitted from the multiplying avalanche. These quanta emit electrons from the cathode wall, causing a large increase in the multiplication factor and also help to propagate the discharge along the wire by the photo-ionization of the gas molecules. In particular, if γ is the probability of production of photoelectrons from an ion pair, the gas multiplication is increased from M in the absence of this effect to a value (1)

$$A = \frac{M}{1 - \gamma M}.$$

But even here the counter remains proportional as the amplification is independent of the size of the initial ionising event.

The fall in proportionality at high A values is due to space-charge effects. The positive ions are practically at rest, in the vicinity of the anode during the formation of the avalanche and this positive charge density results in a reduction of the field strength near the wire with a consequent decrease in the multiplication by electron collision. Added to this, is the axial spread (along the wire) of the discharge initially radial in the counter. This spread again helps to obliterate the variation in avalanche size with the magnitude of the initial ionization until at the Geiger threshold, the discharge has spread across the whole tube, and the output pulse is independent of the primary ionizing event.

The magnitude of this effect is dependent not only on the applied voltage but also on (n/x), the number of ions collected per cm. of the length x of the wire collecting electrons. Thus the amplification at which proportionality is lost depends also on the ionizing power of the incident particle and its track inside the counter.

2. CHARACTERISTICS OF THE SPACE CHARGE

Before discussing how the space charge of one avalanche inhibits the full development of the next, the chief characteristics of the space charge can be summarized, following the discussion due to Wilkinson[2] on the theory of the Geiger discharge.

(a) As each avalanche undergoes diffusion in the course of its formation, the positive ions from neighbouring avalanches overlap, so that the space charge sheath is uniform in structure.

(b) As the electron mobility is about 10^3 times greater than the positive ion mobility, the sheath is practically stationary during the discharge.

(c) As the sheath is formed at a radial distance of a few mean free paths, it holds by induction practically all the electrons in the discharge.

3. EFFECT OF SPACE CHARGE

Suppose that at any instant n avalanches have taken place and $N(n, r)$ is the ionization produced by the nth avalanche between r_c and r, where r_c is the critical radius for the onset of gas multiplication. Then, if the space charges are uniformly distributed over an effective length x, then we have for the field of the $(n+1)$th avalanche,

$$\xi = \frac{2}{r}\left[Q - \frac{e}{x}\int_1^n N(n, r)\, dn\right]$$

where $Q = CV$, V being the applied voltage and $C = \left(\frac{1}{2\, ln \frac{b}{a}}\right)$ the capacitance/unit length of the counter, while the multiplication of this avalanche is given by

$$dN(n, r) = N(n, r)\, sp \,.\, dr$$

where $(s.p)$ = specific ionization at pressure p.

Further, making use of an expression for (sp)

$$sp = \left(\frac{pI}{E_i}\right)^{\frac{1}{2}} \xi^{\frac{1}{2}}$$

where E_i= ionization potential, I = constant characteristic of the gas, and ξ = electric field strength, the multiplication in the absence of space charge which is practically the same as N (1, r) is given by (3)

$$ln\, N(1, r) = 2\left(\frac{2QpI}{E_i}\right)^{\frac{1}{2}}(r_c^{\frac{1}{2}} - a^{\frac{1}{2}})$$

$$= \mu\,(r_c^{\frac{1}{2}} - a^{\frac{1}{2}})$$

where a is the radius of the counter wire.

Now, if, following Wilkinson, we assume the form $N(n, r) = A(n)\, N(1, r)$ the field strength for the $(n + 1)$th avalanche becomes

$$\frac{2Q}{r}\left[1 - \frac{e\bar{A}}{xQ}\exp.\,\mu\,(r_c^{\frac{1}{2}} - a^{\frac{1}{2}})\right]$$

where

$$\bar{A} = \int A(n).dn$$

Hence, for a definite value of n,

$$ln\,N(n, r) = \mu \int \left[1 - \frac{e\bar{A}}{xQ}\exp.\,\mu\,(r_c^{\frac{1}{2}} - r^{\frac{1}{2}})\right]\frac{dr}{r^{\frac{1}{2}}}$$

Making the final assumption that the proportional reduction of the field due to space charge effects is small (and this is sufficient to reduce the multiplication by an appreciable factor), the expression for the total multiplication of the nth avalanche becomes

$$ln\,N(n) = \mu\,(r_c^{\frac{1}{2}} - a^{\frac{1}{2}}) - \frac{e\bar{A}\mu}{2xQ}\exp.\,(\mu r_c^{\frac{1}{2}})\int_{r_c}^{a}\exp.\,(-\mu r^{\frac{1}{2}})\, r^{-\frac{1}{2}}\, dr$$

$$= ln\,N(1) - \frac{e\bar{A}}{xQ}\,[\exp.\,\mu\,(r_c^{\frac{1}{2}} - a^{\frac{1}{2}}) - 1]$$

$$= ln\,N(1) - \beta\bar{A}$$

where

$$\beta = \frac{e}{xQ}\cdot N(1)$$

Differentiating w.r.t. n,

$$\frac{1}{A}\frac{dA}{dn} = -\,\beta A$$

Since $A(n) = 1$ for $n = 1$, this gives

$$A(n) = 1/(1 + \beta n)$$

Now the output pulse for a primary ionization of m ion pairs, is

$$N(m) = N(1)\cdot\int\frac{dn}{1 + \beta n}$$

$$= N(1)\cdot\frac{ln\,(1 + \beta m)}{\beta}$$

$$= N(1)\cdot m\cdot\frac{ln\,(1 + \beta m)}{\beta m}$$

4. DISCUSSION

The ratio $ln(1 + \beta m)/\beta m$ gives the departure from proportionality of the multiplication factor. From the tables of natural logarithms, it is seen that a 10% drop occurs at a value of $(\beta m) = 0 \cdot 2$ nearly.

As a practical case, we can consider a counter of dimensions—cathode = 2 cm., anode wire = 2×10^{-3} cm., operating voltage = 1000 V., operated in the proportional region with a gas multiplication of a high value. Considering an α-particle track parallel to the axes, of length 1 cm. giving an ionization 10^5 ion pairs, we have

$$Q = \frac{10^3 \cdot 10^{-11}}{41 \cdot 4 \cdot 3} \simeq 8 \times 10^{-11} \text{ coulomb}$$

so that for $\beta m = 0 \cdot 2$, $N(1) = 10^3$.

The proportionality hence drops by 10% for a value of gas multiplication $N = 10^3$. Next, for a β-particle track parallel to the counter wire, the specific ionization = 50 ion pairs/cm. so that

$$(\beta m) = \frac{50\, N(1).e}{8.10^{-11}} = 0 \cdot 2, \text{ for a } 10\%$$

drop in proportionality.

Hence the permissible maximum amplification taking into account the space charge limitation alone is $2 \cdot 0 \times 10^5$. Hence the departure from proportionality is small for β-particles almost up to the Geiger threshold. Near about this region, however, the phenomena of avalanche breeding commence, so that the positive ion sheath becomes dense enough to reduce the field strength at the wire to stop further multiplication, while the motion of the positive ion sheath yields the Geiger pulse.

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