A NOTE ON PARTIALLY FIXED LONG RECTANGULAR PLATES UNDER UNIFORMLY DISTRIBUTED LOADS

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INTRODUCTION

The solutions of cylindrical bending of uniformly loaded long rectangular plates reveal that maximum stress in the plate depends only on the intensity of load q, the width to thickness ratio l/h and the degree of fixture at both its edges. For a given maximum stress, it is found that a steel plate with clamped edges can carry considerably less load than the same plate with simply supported edges. This variation of load increases quite rapidly as width to thickness ratio, l/h increases. So it will be advantageous to know exactly what amount of load a rectangular plate can take before it is stressed to its allowable stress when its edges are neither simply supported nor rigidly clamped but partially fixed which is usually the case in most practical applications. The partial fixture is defined as in beams[1] and beam columns[2] and the same procedure is adopted in analysing partially fixed plates. For the sake of simplicity plates with equal edge fixtures at both edges of the plate have been considered.

Considering the bending of an elemental strip cut from a long uniformly loaded rectangular plate of uniform thickness by two planes perpendicular to the length of the plate and a unit distance apart as shown in Fig. 1, and following the same notation as Timoshenko[3] the equation for deflection w_x at any section of the strip can be obtained as

$$w_x = \frac{ql^4}{16u^4D} \frac{\tanh u - k(\tanh u - u)}{\tanh u} \left\{ \frac{\cosh u\left(1 - \frac{2x}{l}\right)}{\cosh u} - 1 \right\} + \frac{ql^2}{8u^2D} x(l - x), \tag{1}$$

where q is the intensity of load, K is the edge fixity coefficient, D is the flexural rigidity of the plate given by

$$D = \frac{Eh^3}{12(1-\nu^2)} \tag{2}$$

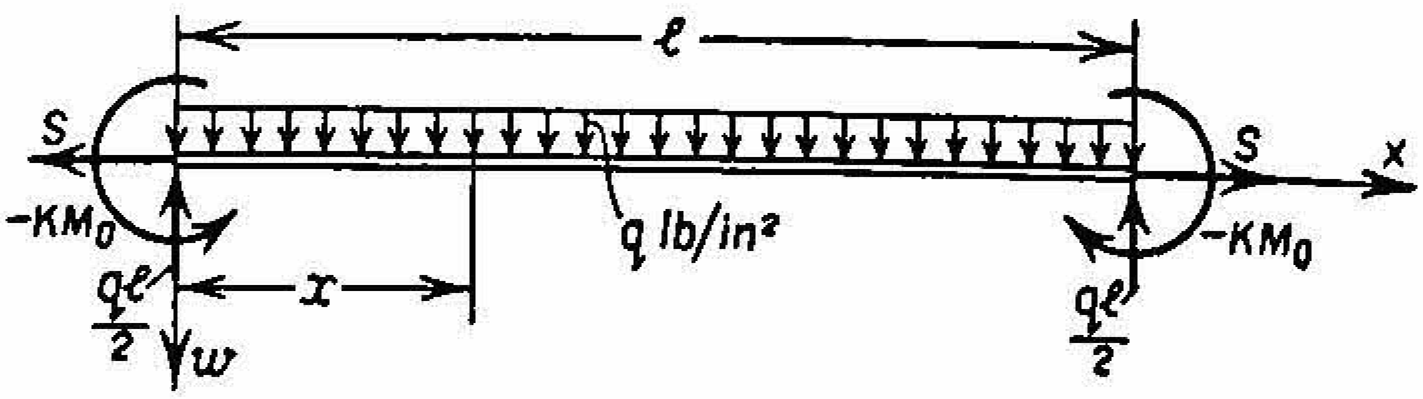

FIG. 1

E and ν being the modulus of elasticity and Poisson's ratio of the material of the plate, h its thickness and u is the parameter related to the tensile force S at the edges by

$$S = \frac{4u^2 D}{l^2} \tag{3}$$

Thus the deflections of the elemental strip depend upon the quantity u which is a function of the axial force S and also the edge fixity coefficient K. The relationship between the elastic constants of the material of the plate, its dimensions and the tensile parameter u and K can be got by calculating the extension of the strip produced by the forces S and equating it to the difference between the length of the arc along the deflection curve and chord length l. This condition leads to

$$\frac{S(1-\nu^2)\,l}{hE} = \frac{1}{2}\int_0^l \left(\frac{dw}{dx}\right)^2 dx.$$

Substituting equation (1) for w, integrating and setting (2) and (3) for D and S respectively, we get the final expression

$$\frac{E^2 h^8}{(1-\nu^2)^2 q^2 l^8} = (1-K)\,U_0 + KU_1 - (1-K)\,KU_2 \tag{4}$$

where

$$U_0 = \frac{135}{16}\,\frac{\tanh u}{u^9} + \frac{27}{16}\,\frac{\tanh^2 u}{u^8} - \frac{135}{16}\,\frac{1}{u^8} + \frac{9}{8}\cdot\frac{1}{u^6}$$

$$U_1 = -\frac{81}{16}\,\frac{1}{u^7 \tanh u} - \frac{27}{16}\,\frac{1}{u^6 \sinh^2 u} + \frac{27}{4}\cdot\frac{1}{u^8} + \frac{9}{8}\cdot\frac{1}{u^6}$$

and

$$U_2 = \frac{27}{16}\,\frac{(u-\tanh u)^2}{u^9 \tanh^2 u}\,(u\tanh^2 u - u + \tanh u)$$

and the values are tabulated in Table I, "Theory of Plates and Shells" by Timoshenko.

The deflection at the mid-section of the plate can be obtained from eq. (1).

$$w_{x=\frac{l}{2}} = \frac{ql^4}{D}\left[\frac{1}{16u^4}\left\{\frac{\tanh u - K(\tanh u - u)}{\tanh u}\right\}\left\{\frac{1}{\cosh u} - 1\right\} + \frac{1}{32u^2}\right]$$

Substituting the value of D, this can be rewritten as

$$\frac{Eh^3 w_{x=\frac{l}{2}}}{12(1-\nu^2)ql^4} = \frac{1}{16u^4}\left\{\frac{\tanh u - K(\tanh u - u)}{\tanh u}\right\}\left\{\frac{1}{\cosh u} - 1\right\} + \frac{1}{32u^2} \qquad (5)$$

The stress at any point of a cross-section of the strip consists of the constant tensile stress σ_1 and the bending stress σ_2. The maximum bending stress will act where the bending moment is largest, that is, either at the edges or at the mid-section of the element.

$$\sigma_1 = \frac{S}{h} = \frac{4u^2 D}{hl^2} = \frac{Eu^2}{3(1-\nu^2)}\left(\frac{h}{l}\right)^2 \qquad (6)$$

Bending moment at the edges $= -KM_0$

$$= K\frac{ql^2}{4}\left(\frac{u - \tanh u}{u^2 \tanh u}\right) \qquad (7)$$

Bending moment at the mid-section can be obtained as

$$M_{x=\frac{l}{2}} = \frac{ql^2}{8}\left(\frac{1-\operatorname{sech} u}{u^2/2}\right) - K\frac{ql^2}{4}\,\frac{u-\tanh u}{u^2\tanh u}\,\frac{1}{\cosh u}$$

$$= \frac{ql^2}{8}\psi_0(u) - K\frac{ql^2}{12}\psi_1(u)\,\frac{1}{\cosh u} \qquad (8)$$

where

$$\psi_0(u) = \frac{1-\operatorname{sech} u}{u^2/2}$$

$$\psi_1(u) = \frac{3(u-\tanh u)}{u^2\tanh u}$$

The values of $\psi_0(u)$ and $\psi_1(u)$ are tabulated in Table I of " Theory of Plates and Shells" by Timoshenko.

Then

$$\sigma_{2max.} = \frac{6}{h^2} M_{max.} \tag{9}$$

$M_{max.}$ being given by the greater of the two expressions (7) and (8) depending on the particular value of the edge fixity coefficient K. The maximum stress in the plate is then the sum of σ_1 and $\sigma_{2max.}$

It is seen from equation (4) that the tensile parameter u depends, for a given material of the plate, on the intensity of the load q, the width to thickness ratio l/h of the plate and the edge fixity coefficient K. From equations (6) and (9), it can be seen that σ_1 and σ_2 are also functions of u, q and l/h. Therefore, the maximum stress in the plate depends only on the load q, the ratio l/h and coefficient K. For any particular value of K, it is possible to plot curves giving maximum stress in terms of q, each curve in the set corresponding to a particular value of l/h.

It can be seen from equations (4) and (5) that the only unknowns are u and K provided we measure only one deflection at the middle of the given plate, for a given load q. Thus we get two independent equations in u and K from (4) and (5), which can then be solved, either directly or by graphical methods. Substituting the values of u and K in the expression for $\sigma_1 + \sigma_{2max.}$ we can get maximum stress in the plate.

Finally to study the effect of edge fixity-coefficient on maximum stress in plates, various curves of maximum stress in steel plates *versus* load q for l/h ratios ranging between 80 and 220 can be drawn for degrees of edge fixture ranging from 0 to 1. From these curves the variation of load q with the edge fixity coefficient K has been drawn in Fig. 2 for an assumed

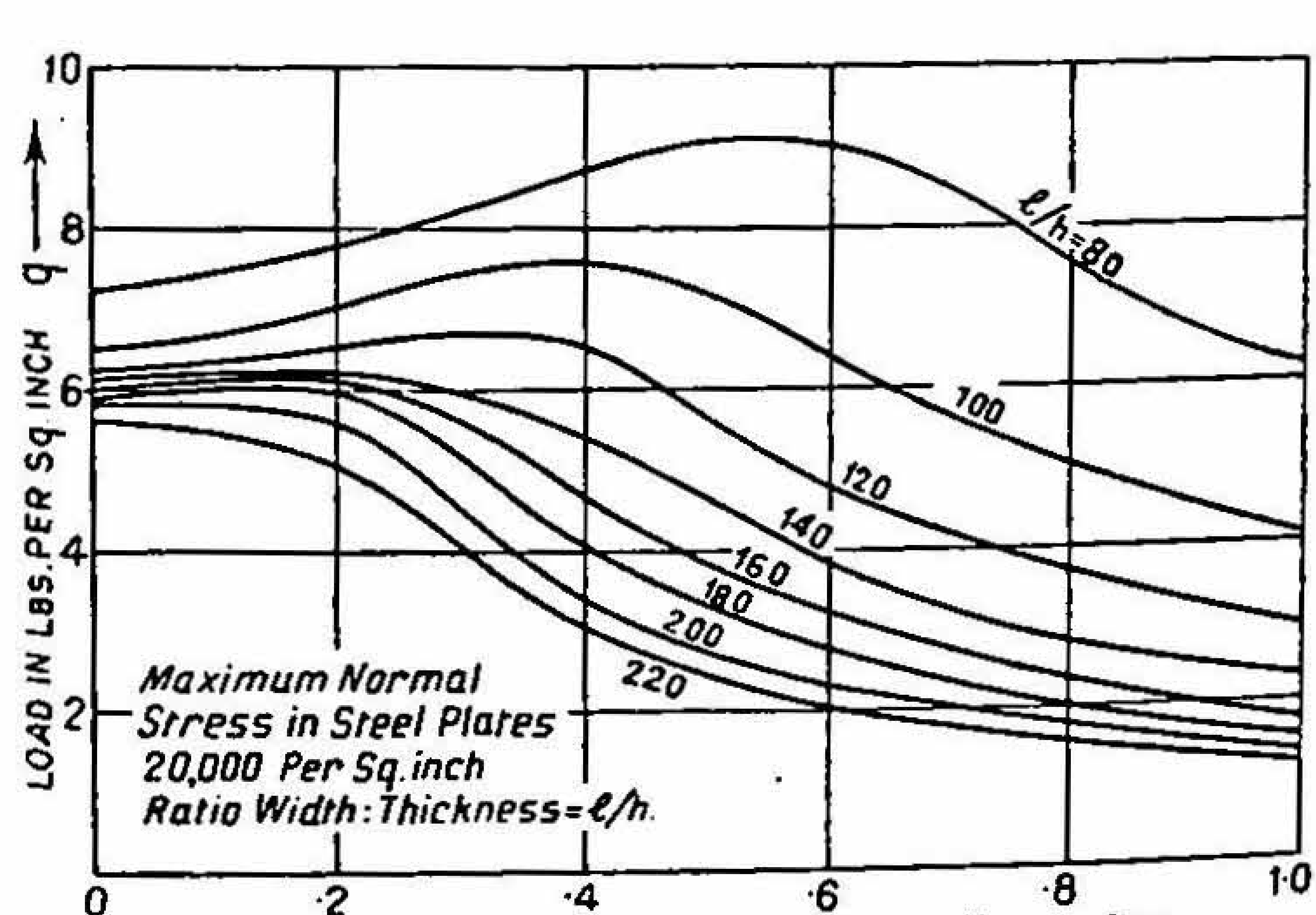

FIG. 2

maximum normal stress of 20,000 lb./sq. inch. It is evident from the curves that with some values of partial fixities, plates with some values of l/h will take greater load per sq. in. than either in simply supported or built-in cases, the plates being stressed to the same maximum stress in each case.

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