PROPAGATION OF MICROWAVES THROUGH A CYLINDRICAL METALLIC GUIDE FILLED COAXIALLY WITH TWO DIFFERENT DIELECTRICS—PART V

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ABSTRACT

The general conditional equation from which the propagation constant of the hybrid mode can be derived has been formulated by utilising the field components and boundary conditions given in a previous paper (Chatterjee, 1954). The propagation characteristics have been calculated in the case when the cylindrical guide containing two coaxial dielectrics is reduced to a simple dielectric rod. The dielectric rod behaves as a guide or as an aerial depending on a certain limiting value of the radius of the rod. The critical value of the radius depends on the mode in which the rod is excited.

INTRODUCTION

In a recent paper (Chatterjee, *loc. cit.*) the propagation characteristics of several hybrid modes have been derived from the field theory with some approximations. The object of the present paper is to formulate the general conditional equation from which the propagation characteristics of any particular mode can be derived accurately. The paper also presents a report of the calculation of the propagation characteristics in the case when by making suitable assumptions the cylindrical guide with two coaxial dielectrics is reduced to a simple dielectric rod.

CONDITIONAL EQUATION

Applying proper boundary conditions, the following equations are obtained from equations (5) and (6) of the previous paper (Chatterjee, *loc. cit.*).

$$A_1 J_m (k_1 r_1) + B_1 Y_m (k_1 r_1) = 0 \tag{1 a}$$

$$A'_1 J'_m (k_1 r_1) + B'_1 Y'_m (k_1 r_1) = 0 \tag{1 b}$$

$$k_1^2 [A_1 J_m (k_1 r_2) + B_1 Y_m (k_1 r_2)] e^{-\gamma_1 z} - k_2^2 [A_2 J_m (k_2 r_2)] e^{-\gamma_2 z} = 0 \tag{1 c}$$

$$\left(k_1^2 - \frac{2m^2}{r_2^2}\right) [A'_1 J_m (k_1 r_2) + B'_1 Y_m (k_1 r_2)] e^{-\gamma_1 z}$$
$$- \left(k_2^2 - \frac{2m^2}{r_2^2}\right) e^{-\gamma_2 z} A'_2 J_m (k_2 r_2) = 0 \qquad (1\,d)$$

$$\epsilon_1 \gamma_1 k_1 [A_1 J'_m (k_1 r_2) + B_1 Y'_m (k_1 r_2)] e^{-\gamma_1 z} + \omega\mu_1 \epsilon_1 \frac{m}{r_2} [A'_1 J_m (k_1 r_2)$$
$$+ B'_1 Y_m (k_1 r_2)] e^{-\gamma_1 z} - \epsilon_2 \gamma_2 k_2 [A_2 J'_m (k_2 r_2)] e^{-\gamma_2 z}$$
$$- \omega\mu_2 \epsilon_2 \frac{m}{r_2} [A'_2 J_m (k_2 r_2)] e^{-\gamma_2 z} = 0 \qquad (1\,e)$$

$$\omega\mu_1 \epsilon_1 \frac{m}{r_2} [A_1 J_m (k_1 r_2) + B_1 Y_m (k_1 r_2)] e^{-\gamma_1 z} + \mu_1 \gamma_1 k_1 [A'_1 J'_m (k_1 r_2)$$
$$+ B'_1 Y'_m (k_1 r_2)] e^{-\gamma_1 z} - \omega\mu_2 \epsilon_2 \frac{m}{r_2} [A_2 J_m (k_2 r_2)] e^{-\gamma_2 z}$$
$$- \mu_2 \gamma_2 k_2 [A'_2 J'_m (k_2 r_2)] e^{-\gamma_2 z} = 0 \qquad (1\,f)$$

Equations (1 *c*) and (1 *d*) can be written respectively as follows:

$$k_1^2 \frac{J_m (k_1 r_2)}{J_m (k_1 r_1)} [A_1 J_m (k_1 r_1) + B_1 Y_m (k_1 r_1)] e^{-\gamma_1 z} = 0 \qquad (2\,a)$$

and

$$\left(k_1^2 - \frac{2m^2}{r_2^2}\right) \frac{J_m (k_1 r_2)}{J'_m (k_1 r_1)} [A'_1 J'_m (k_1 r_1) + B'_1 Y'_m (k_1 r_1)] e^{-\gamma_1 z} = 0 \qquad (2\,b)$$

It is evident that the equations (1 *a*) and (1 *b*) are contained in the equations (2 *a*) and (2 *b*). So, the equations (1) can be reduced to the following four:

$$A_1 J_m (k_1 r_1) + B_1 Y_m (k_1 r_1) = 0 \qquad (3\,a)$$

$$A'_1 J'_m (k_1 r_1) + B'_1 Y'_m (k_1 r_1) = 0 \qquad (3\,b)$$

$$A_1 \epsilon_1 \gamma_1 k_1 J'_m (k_1 r_2)$$
$$+ B_1 \Big[\epsilon_1 \gamma_1 k_1 Y'_m (k_1 r_2) - \epsilon_2 \gamma_2 \frac{k_2}{k_3^2} \xi_m (k_2 r_2) \{Y_m (k_1 r_2)$$
$$- \chi_m (k_1 r_1) J_m (k_1 r_2)\} \Big]$$
$$+ A'_1 \omega\mu_1 \epsilon_1 \frac{m}{r_2} J_m (k_1 r_2)$$
$$+ B'_1 \frac{\omega m}{r_2} \Big[\mu_1 \epsilon_1 Y_m (k_1 r_2) - \frac{\mu_2 \epsilon_2}{k_4^2} \{Y_m (k_1 r_2)$$
$$- \psi_m (k_1 r_1) J_m (k_1 r_2)\} \Big] = 0 \qquad (3\,c)$$

$$A_1 \omega \mu_1 \epsilon_1 \frac{m}{r_2} J_m(k_1 r_2)$$

$$+ B_1 \left[\mu_1 \epsilon_1 Y_m(k_1 r_2) - \frac{\mu_2 \epsilon_2}{k_3^2} \{ Y_m(k_1 r_2) - \chi_m(k_1 r_1) J_m(k_1 r_2) \} \right] \omega \frac{m}{r_2}$$

$$+ A'_1 \mu_1 \gamma_1 k_1 J'_m(k_1 r_2)$$

$$+ B'_1 \left[\mu_1 \gamma_1 k_1 Y'_m(k_1 r_2) - \frac{\mu_2 \gamma_2 k_2}{k_4^2} \{ Y_m(k_1 r_2) \right.$$

$$\left. - \psi_m(k_1 r_1) J_m(k_1 r_2) \} \xi_m(k_2 r_2) \right] = 0 \tag{3 d}$$

In order that A's and B's in eq. (3) may have non-zero values, the following condition must be fulfilled:

$$\begin{vmatrix} A_1^{(a)} & B_1^{(a)} & 0 & 0 \\ 0 & 0 & A'^{(b)}_1 & B'^{(b)}_1 \\ A_1^{(c)} & B_1^{(c)} & A'^{(c)}_1 & B'^{(c)}_1 \\ A_1^{(d)} & B_1^{(d)} & A'^{(d)}_1 & B'^{(d)}_1 \end{vmatrix} = 0 \tag{4}$$

The A's and B's in the array indicate their respective coefficients in eq. (3). The determinant when solved gives the following transcendental equation.

$$\frac{Y_m(k_1 r_1)}{Y'_m(k_1 r_1)} = \frac{Y_m(k_1 r_2) J_m(k_1 r_1) \left\{ \mu_1 \epsilon_1 - \frac{\mu_2 \epsilon_2}{k_3^2} \right\} + \frac{\mu_2 \epsilon_2}{k_3^2} Y_m(k_1 r_1) J_m(k_1 r_2)}{Y_m(k_1 r_2) J'_m(k_1 r_1) \left\{ \mu_1 \epsilon_1 - \frac{\mu_2 \epsilon_2}{k_4^2} \right\} + \frac{\mu_2 \epsilon_2}{k_4^2} Y'_m(k_1 r_1) J_m(k_1 r_2)} \tag{5}$$

In the above equations (3) and (5), the following abbreviations have been used.

$$k_3^2 = \frac{k_2^2}{k_1^2} \qquad k_4^2 = \frac{k_2^2 - \frac{2m^2}{r_2^2}}{k_1^2 - \frac{2m^2}{r_2^2}} \tag{6 a}$$

$$\chi_m(k_1 r_1) = \frac{Y_m(k_1 r_1)}{J_m(k_1 r_1)}, \qquad \psi_m(k_1 r_1) = \frac{Y'_m(k_1 r_1)}{J'_m(k_1 r_1)},$$

$$\xi_m(k_2 r_2) = \frac{J'_m(k_2 r_2)}{J_m(k_2 r_2)} \tag{6 b}$$

The above equation is very much involved and a general solution is difficult. But as the propagation of hybrid modes requires that γ_1 be equal to γ_2, the eqn. (5) in conjunction with the relation $k_1^2 - k_2^2 = \omega^2(\mu_1 \epsilon_1 - \mu_2 \epsilon_2)$ can be solved graphically for only some definite modes to give k_1 and k_2 in terms of the radial dimensions and the electrical constants of the dielectric. The other propagation characteristics can then be found from the values of k's. The validity of the above equation can be tested as follows:

For a single dielectric (say air) bounded by perfectly conducting metallic boundaries

$$\mu_1 = \mu_2 = \mu_0 \quad , \quad \epsilon_1 = \epsilon_2 = \epsilon_0$$
$$k_3 = k_4 = 1 \qquad r_2 = r_1 = r_0$$

The eqn. (5) reduces to the following identity:

$$\frac{Y_m(k_1 r_0)}{Y'_m(k_1 r_0)} = \frac{Y_m(k_1 r_0)}{Y'_m(k_1 r_0)} \tag{7}$$

which shows that the general transcendental equation derived above is correct.

Dielectric Rod

If the boundary wall of the metallic guide is shifted to infinity ($r_1 = \infty$), the guide with two coaxial dielectrics is reduced to the case of a dielectric rod having dielectric constant ϵ_2 immersed in a dielectric medium having dielectric constant ϵ_1. In this case, in order to account for the exponential decay of the field intensity to a vanishingly small value at infinity, the functions J_m's, Y_m's and their derivatives for the first medium are to be replaced by the Hankel functions $H_m^{(1)}$, $H_m^{(2)}$ and their derivative $H_m^{(1)'}$, $H_m^{(2)'}$ respectively. Applying continuity conditions at the interface of the two media and introducing the above changes in notations in equations (5) and (6) of the previous paper (Chatterjee, *loc. cit.*) the following are obtained:

$$\begin{aligned}
&A_1 \left[\epsilon_1 \gamma_1 k_1 H_m^{(1)'}(k_1 r_2) - \frac{\epsilon_2 \gamma_2 k_2}{k_3^2} \frac{J'_m(k_2 r_2)}{J_m(k_2 r_2)} H_m^{(1)}(k_1 r_2)\right] \\
&+ B_1 \left[\epsilon_1 \gamma_1 k_1 H_m^{(2)'}(k_1 r_2) - \frac{\epsilon_2 \gamma_2 k_2}{k_3^2} \frac{J'_m(k_2 r_2)}{J_m(k_2 r_2)} H_m^{(2)}(k_1 r_2)\right] \\
&+ A'_1 \left[\omega \mu_1 \epsilon_1 \frac{m}{r_2} H_m^{(1)}(k_1 r_2) - \frac{\omega \mu_2 \epsilon_2}{k_4^2} \frac{m}{r_2} H_m^{(1)}(k_1 r_2)\right] \\
&+ B'_1 \left[\omega \mu_1 \epsilon_1 \frac{m}{r_2} H_m^{(2)}(k_1 r_2) - \frac{\omega \mu_2 \epsilon_2}{k_4^2} \frac{m}{r_2} H_m^{(2)}(k_1 r_2)\right] = 0
\end{aligned} \tag{8 a}$$

$$\begin{aligned}
&A_1 \left[\omega \mu_1 \epsilon_1 \frac{m}{r_2} H_m^{(1)}(k_1 r_2) - \frac{\omega \mu_2 \epsilon_2}{k_3^2} \frac{m}{r_2} H_m^{(1)}(k_1 r_2)\right] \\
&+ B_1 \left[\omega \mu_1 \epsilon_1 \frac{m}{r_2} H_m^{(2)}(k_1 r_2) - \frac{\omega \mu_2 \epsilon_2}{k_3^2} \frac{m}{r_2} H_m^{(2)}(k_1 r_2)\right]
\end{aligned}$$

$$+ A'_1 \left[\mu_1\gamma_1 k_1 H_m^{(1)\prime}(k_1r_2) - \frac{\mu_2\gamma_2 k_2}{k_4^2} \frac{J'_m(k_2r_2)}{J_m(k_2r_2)} H_m^{(1)}(k_1r_2)\right]$$

$$+ B'_1 \left[\mu_1\gamma_1 k_1 H_m^{(2)\prime}(k_1r_2) - \frac{\mu_2\gamma_2 k_2}{k_4^2} \frac{J'_m(k_2r_2)}{J_m(k_2r_2)} H_m^{(2)}(k_1r_2)\right] = 0 \quad (8\,b)$$

$$A_1 \left[j\omega\epsilon_1 k_1 H_m^{(1)\prime}(k_1r_2) - j\omega\epsilon_2 \frac{k_2}{k_3^2} \frac{J'_m(k_2r_2)}{J_m(k_2r_2)} H_m^{(1)}(k_1r_2)\right]$$

$$- B_1 \left[j\omega\epsilon_1 k_1 H_m^{(2)\prime}(k_1r_2) + j\omega\epsilon_2 \frac{k_2}{k_3^2} \frac{J'_m(k_2r_2)}{J_m(k_2r_2)} H_m^{(2)}(k_1r_2)\right]$$

$$+ A'_1 \left[\gamma_1 \frac{m}{r_2} H_m^{(1)}(k_1r_2) - \gamma_2 \frac{m}{r_2} \frac{1}{k_4^2} H_m^{(1)}(k_1r_2)\right]$$

$$- B'_1 \left[\gamma_1 \frac{m}{r_2} H_m^{(2)}(k_1r_2) + \gamma_2 \frac{m}{r_2} \frac{1}{k_4^2} H_m^{(2)}(k_1r_2)\right] = 0 \quad (8\,c)$$

$$A_1 \left[\omega\mu_1\epsilon_1 \frac{m}{r_2} H_m^{(1)}(k_1r_2) - \omega\mu_2\epsilon_2 \frac{m}{r_2} \frac{1}{k_3^2} H_m^{(1)}(k_1r_2)\right]$$

$$- B_1 \left[\omega\mu_1\epsilon_1 \frac{m}{r_2} H_m^{(2)}(k_1r_2) + \omega\mu_2\epsilon_2 \frac{m}{r_2} \frac{1}{k_3^2} H_m^{(2)}(k_1r_2)\right]$$

$$+ A'_1 \left[\mu_1\gamma_1 k_1 H_m^{(1)\prime}(k_1r_2) - \gamma_2 k_2 \frac{\mu_2}{k_4^2} \frac{J'_m(k_2r_2)}{J_m(k_2r_2)} H_m^{(1)}(k_1r_2)\right]$$

$$- B'_1 \left[\mu_1\gamma_1 k_1 H_m^{(2)\prime}(k_1r_2) + \gamma_2 k_2 \frac{\mu_2}{k_4^2} \frac{J'_m(k_2r_2)}{J_m(k_2r_2)} H_m^{(2)}(k_1r_2)\right] = 0 \quad (8\,d)$$

where,

$$A_2 = \frac{1}{k_3^2} e^{-\gamma_2 z} \left[A_1 \frac{H_m^{(1)}(k_1r_2)}{J_m(k_2r_2)} + B_1 \frac{H_m^{(2)}(k_1r_2)}{J_m(k_2r_2)}\right] \quad (9\,a)$$

and

$$A'_2 = \frac{1}{k_4^2} e^{-\gamma_2 z} \left[A'_1 \frac{H_m^{(1)}(k_1r_2)}{J_m(k_2r_2)} + B'_1 \frac{H_m^{(2)}(k_1r_2)}{J_m(k_2r_2)}\right] \quad (9\,b)$$

In order that A_1, B_1, A'_1, B'_1 may not vanish the following conditions must hold good.

$$\begin{vmatrix} A_1^{(a)} & B_1^{(a)} & A'^{(a)}_1 & B'^{(a)}_1 \\ A_1^{(b)} & B_1^{(b)} & A'^{(b)}_1 & B'^{(b)}_1 \\ A_1^{(c)} & -B_1^{(c)} & A'^{(c)}_1 & -B'^{(c)}_1 \\ A_1^{(d)} & -B_1^{(d)} & A'^{(d)}_1 & -B'^{(d)}_1 \end{vmatrix} = 0 \quad (10)$$

where A's and B's refer to their coefficients in the respective equations.

Solving the above determinant, the following equations are obtained:

$$H^{(1)}(k_1r_2)\,H^{(2)}(k_1r_2) = 0 \tag{11 a}$$

$$\Big[H^{(1)\prime} H^{(2)\prime} \left\{\frac{\epsilon_1\gamma_1{}^3k_1{}^2\epsilon_2}{k_3{}^2} - \frac{\epsilon_2\gamma_2\epsilon_1\gamma_1{}^2k_1{}^2}{k_3{}^2k_4{}^2}\right\}$$
$$+ H^{(1)\prime} H^{(2)} \frac{J'}{J} \left\{\frac{\epsilon_1{}^2\gamma_2{}^2\gamma_1k_1k_2}{k_4{}^2} - \frac{\epsilon_1{}^2\gamma_1{}^2k_1k_2\gamma_2}{k_4{}^2}\right\}$$
$$+ H^{(1)} H^{(2)\prime} \frac{J'}{J} \left\{\frac{\epsilon_2{}^2\gamma_2{}^2k_2\gamma_1k_1}{k_4{}^2k_3{}^4} - \frac{\epsilon_2{}^2\gamma_1{}^2\gamma_2k_1k_2}{k_3{}^4}\right\}$$
$$+ H^{(2)} H^{(1)} \frac{J'^2}{J^2} \left\{\frac{\epsilon_2\gamma_2{}^2k_2{}^2\epsilon_1\gamma_1}{k_3{}^2k_4{}^2} - \frac{\epsilon_1\gamma_2{}^3\epsilon_2k_2{}^2}{k_3{}^2k_4{}^4}\right\}\Big] = 0 \tag{11 b}$$

$$\Big[H^{(1)\prime} H^{(2)\prime} \left\{\frac{\mu_2\epsilon_2{}^2\epsilon_1\gamma_1k_1{}^2}{k_3{}^2k_4{}^2} - \frac{\mu_1\epsilon_1{}^2\epsilon_2\gamma_1k_1{}^2}{k_3{}^2}\right\}$$
$$+ H^{(1)\prime} H^{(2)} \frac{J'}{J} \left\{\frac{\mu_1\epsilon_1{}^3k_1k_2\gamma_2}{k_4{}^2} - \frac{\epsilon_1{}^2k_1\gamma_2\mu_2\epsilon_2k_2}{k_4{}^4}\right\}$$
$$+ H^{(1)} H^{(2)\prime} \frac{J'}{J} \left\{\frac{\mu_1\epsilon_1\epsilon_2{}^2k_2k_1\gamma_1}{k_3{}^4} - \frac{\mu_2\epsilon_2{}^3k_2k_1\gamma_1}{k_4{}^2k_3{}^4}\right\}$$
$$+ H^{(1)} H^{(2)} \frac{J'^2}{J^2} \left\{\frac{\mu_2\gamma_2k_2{}^2\epsilon_1\epsilon_2{}^2}{k_3{}^2k_4{}^4} - \frac{\gamma_2k_2{}^2\epsilon_2\epsilon_1{}^2\mu_1}{k_4{}^2k_3{}^2}\right\}\Big] = 0 \tag{11 c}$$

where the subscript m and the arguments of H's and J's have been omitted for convenience. Substituting the following approximate values for the Hankel functions in (11 *a*) and (11 *b*)

$$\begin{aligned} H_m{}^{(1)}(k_1r_2) &\simeq \sqrt{\frac{2}{\pi k_1r_2}}\, e^{j\left(k_1r_2-\frac{\pi}{4}-\frac{m\pi}{2}\right)} \\ H_m{}^{(2)}(k_1r_2) &\simeq \sqrt{\frac{2}{\pi k_1r_2}}\, e^{-j\left(k_1r_2-\frac{\pi}{4}-\frac{m\pi}{2}\right)} \end{aligned} \tag{12}$$

and after simplification and rearrangement of the terms, the equations (11) are reduced to

$$H_m{}^{(1)}(k_1r_2)\,H_m{}^{(2)}(k_1r_2) = 0 \tag{13 a}$$

$$\frac{J'_m(k_2r_2)}{J_m(k_2r_2)} = \frac{\vartheta \pm \sqrt{\vartheta^2 - 4\rho\tau}}{2\rho} \tag{13 b}$$

$$\frac{J'_m(k_2r_2)}{J_m(k_2r_2)} = \frac{\delta \pm \sqrt{\delta^2 - 4\Omega\Theta}}{2\Omega} \tag{13 c}$$

where.

$$\rho = \frac{\epsilon_2\gamma_2{}^2k_2{}^2\epsilon_1\gamma_1}{k_3{}^2k_4{}^2} - \frac{\epsilon_1\gamma_2{}^3\epsilon_2k_2{}^2}{k_3{}^2k_4{}^2}$$

$$\vartheta = \frac{1}{2k_1r_2}\left[\frac{\epsilon_1{}^2\gamma_2{}^2\gamma_1k_1k_2}{k_4{}^2} - \frac{\epsilon_1{}^2\gamma_1{}^2k_1k_2\gamma_2}{k_4{}^2} + \frac{\epsilon_2{}^2\gamma_2{}^2k_2k_1\gamma_1}{k_4{}^2k_3{}^4} - \frac{\epsilon_2{}^2\gamma_1{}^2\gamma_2k_1k_2}{k_3{}^4}\right]$$

$$\tau = \left(1 + \frac{1}{4k_1{}^2r_2{}^2}\right)\left(\frac{\epsilon_1\gamma_1{}^3k_1{}^2\epsilon^2}{k_3{}^2} - \frac{\epsilon_2\gamma_2\epsilon_1\gamma_1{}^2k_1{}^2}{k_3{}^2k_4{}^2}\right)$$

$$\Omega = \frac{\mu_2\gamma_2k_2{}^2\epsilon_1\epsilon_2{}^2}{k_3{}^2k_4{}^4} - \frac{\gamma_2k_2{}^2\epsilon_2\epsilon_1{}^2\mu_1}{k_3{}^2k_4{}^2}$$

$$\delta = \frac{1}{2k_1r_2}\left[\frac{\mu_1\epsilon_1{}^3k_1k_2\gamma_2}{k_4{}^2} - \frac{\epsilon_1{}^2k_1\gamma_2\mu_2\epsilon_2k_2}{k_4{}^4} + \frac{\mu_1\epsilon_1\epsilon_2{}^2k_2k_1\gamma_1}{k_3{}^4} - \frac{\mu_2\epsilon_2{}^3k_2k_1\gamma_1}{k_4{}^2k_3{}^4}\right]$$

$$\Theta = \left(1 + \frac{1}{4k_1{}^2r_2{}^2}\right)\left(\frac{\mu_2\epsilon_2{}^2\epsilon_1\gamma_1k_1{}^2}{k_3{}^2k_4{}^2} - \frac{\mu_1\epsilon_1{}^2\epsilon_2\gamma_1k_1{}^2}{k_3{}^2}\right)$$

The first condition $H_m^{(1)}H_m^{(2)} = 0$ is satisfied when the argument is large. It is observed from (12) that in the case of $H_m^{(1)}$, only the expression under the root sign tends to vanish for large arguments. But in the case of $H_m^{(2)}$, the expression under the root as well as the exponential one tend towards zero as the argument is increased. As the arguments become very large, it is evident from $H_m^{(2)} \to 0$ that the following conditions hold good:

$$\cos\left(k_1r_2 - \frac{\pi}{4} - \frac{m\pi}{2}\right) = 0 \qquad (14\,a)$$

$$\sin\left(k_1r_2 - \frac{\pi}{4} - \frac{m\pi}{2}\right) = 0 \qquad (14\,b)$$

or,

$$k_1 = \frac{1}{r_2}\left[\frac{\pi}{2}(m+n) + \frac{\pi}{4}\right] \qquad (15)$$

where n is considered to have non-zero integral values in order to account for the propagation of the hybrid mode. The above value of k_1 yields the following value for the propagation constant of the EH_{mn} mode in the first medium

$$\therefore \quad \gamma_1 = j\sqrt{\omega^2\mu_1\epsilon_1 - \frac{1}{r_2{}^2}\left[\frac{\pi}{2}(m+n) + \frac{\pi}{4}\right]^2} \qquad (15\,a)$$

As the propagating mode is hybrid, $\gamma_1 = \gamma_2$, in which case, ρ in (13 *b*) vanishes. As $J'_m(k_2 r_2) \neq \infty$, $J_m(k_2 r_2)$ becomes evanescent, which holds good if the following condition is satisfied.

$$k_2 = \frac{s_{mn}}{r_2} \tag{16}$$

where s_{mn} is the root of $J_m(k_2 r_2) = 0$ where m and n represent the order of the Bessel function and the rank of the root respectively. The condition (16) together with the relation $k_2^2 = \gamma_2^2 + \omega^2 \mu_2 \epsilon_2$ give the following values of the propagation characteristics of the hybrid mode in the second medium:

$$\gamma_2 = j\sqrt{\omega^2 \mu_2 \epsilon_2 - \frac{s^2_{mn}}{r_2^2}}, \qquad \beta_2 = \sqrt{\omega^2 \mu_2 \epsilon_2 - \frac{s^2_{mn}}{r_2^2}}$$

$$f_2^c = \frac{s_{mn}}{2\pi r_2 \sqrt{\mu_2 \epsilon_2}}, \qquad \lambda_2^c = \frac{2\pi r_2}{s_{mn}} \tag{17}$$

$$c_2^p = \frac{\omega}{\sqrt{\omega^2 \mu_2 \epsilon_2 - \frac{s^2_{mn}}{r_2^2}}}, \qquad c_2^g = \frac{\sqrt{\omega^2 \mu_2 \epsilon_2 - \frac{s^2_{mn}}{r_2^2}}}{\omega \mu_2 \epsilon_2}$$

When the propagation constant in (15) under the assumption of H_m's $\rightarrow 0$ is substituted for γ in the expressions of field components, it will be observed that there is spreading out of the field to a great distance. Or, in other words, the dielectric rod acts as an aerial under the condition H_m's $\rightarrow 0$. Whereas, when the propagation constant derived in (17) under the condition that J_m's $\rightarrow 0$ is substituted for γ in the expressions for field components, it will be evident that the dielectric rod acts as a guide supporting a non-radiating mode.

The following points regarding the phase and the group velocities are significant.

(i) The phase velocity increases and the group velocity decreases as the radius of the dielectric rod decreases and *vice versa*.

(ii) The phase velocity decreases and the group velocity increases with the increase of the dielectric constant of the rod and *vice versa*.

It will be evident from (15 *a*) that the value of γ_1 becomes zero, real or imaginary according as r_2 is equal, less or greater than

$$\left\{\left[\frac{\pi}{2}(m+n) + \frac{\pi}{4}\right] \Big/ \omega\sqrt{\mu_1 \epsilon_1}\right\} \tag{18}$$

Or, in other words, there will be no energy flow in the first medium and the energy flow will be wholly concentrated in the second medium unless r_2 is greater than the expression (18). It is also obvious from (17) that there will be no propagation in the second medium unless r_2 is greater than $\frac{s_{mn}}{\omega\sqrt{\mu_2\epsilon_2}}$. It may, therefore, be concluded that the dielectric rod will behave as a guide or as an aerial depending on the radius of the rod. The limiting values of the radii of the rod in the above two cases are found from the following expressions:

$$r_2 = \frac{1}{\omega\sqrt{\mu_1\epsilon_1}}\left[\frac{\pi}{2}(m+n)+\frac{\pi}{4}\right] \tag{18 a}$$

and

$$r_2 = \frac{s_{mn}}{\omega\sqrt{\mu_2\epsilon_2}} \tag{18 b}$$

The limiting values for some of the modes are tabulated below:

Modes	EH_{11}	EH_{22}	EH_{12}	EH_{21}
First medium ..	$0\cdot625\ \lambda_1$	$1\cdot125\ \lambda_1$	$0\cdot875\ \lambda_1$	$0\cdot875\ \lambda_1$
Second medium ..	$0\cdot61\ \lambda_2$	$1\cdot34\ \lambda_2$	$1\cdot12\ \lambda_2$	$0\cdot82\ \lambda_2$
	$(0\cdot386\ \lambda_1)$	$(0\cdot848\ \lambda_1)$	$(0\cdot708\ \lambda_1)$	$(0\cdot519\ \lambda_1)$

If the second medium possesses a dielectric constant of 2·5 and if the first medium is air, then $\lambda_2 \simeq 0\cdot64\ \lambda_1$ and the critical dimensions of the rod for several modes are given in the above table inside parantheses. The following conclusions may be drawn from the above table.

(i) If the radius of the rod is less than approximately quarter wavelength $\left(r_2 < \frac{\lambda_1}{4}\right)$, none of the above modes can be propagated either in the first or the second medium.

(ii) The energy flow is wholly concentrated in the second medium when the radius of the rod lies between 0 386 λ_1 to $0\cdot625\ \lambda_1$ in the case when the rod is excited in the EH_{11} mode. The limiting values for the other modes can be similarly found from the above table.

(iii) When the rod is excited in the EH_{11} mode, if the radius of the rod is increased beyond $0\cdot625\ \lambda_1$, the energy flow will be distributed in both the media. Similarly, for the EH_{22} mode when $r_2 > 1\cdot125\ \lambda_1$ and so on for the other modes the energy will flow in both the media.

RADIAL SPREAD OF THE FIELD

The radial spread of the field in the case of the dielectric rod can be obtained from the radial components of the electric and the magnetic fields (Chatterjee, 1954). These components when modified by the introduction of Hankel functions so as to ensure the proper decrease of the field at infinity and also to give the proper direction of energy flow when k becomes complex are

$$E_r = jP\Big[k_3^2 k_1 \gamma_1 \{g H_m^{(2)\prime}(k_1 r) - f H_m^{(1)\prime}(k_1 r)\} + c_m k_4^2 \omega \mu_2 \frac{m}{r}\{g H_m^{(2)}(k_1 r) - f H_m^{(1)}(k_1 r)\}\Big]\cos m\theta\, e^{-\gamma_1 z} \quad (19\,a)$$

$$H_r = -P\Big[\omega \epsilon_1 k_3^2 \frac{m}{r}\{f H_m^{(1)}(k_1 r) + g H_m^{(2)}(k_1 r)\} + c_m \gamma_1 k_1 k_4^2 \{f H_m^{(1)\prime}(k_1 r) + g H_m^{(2)\prime}(k_1 r)\}\Big]\sin m\theta\, e^{-\gamma_1 z} \quad (19\,b)$$

where, g, f and c_m in the altered form are given by the following expressions

$$f = \frac{J_m(k_2 r_2) H_m^{(2)}(k_1 r_1)}{H_m^{(1)}(k_1 r_2) H_m^{(2)}(k_1 r_1) - H_m^{(1)}(k_1 r_1) H_m^{(2)}(k_1 r_2)}$$

$$g = \frac{J_m(k_2 r_2) H_m^{(1)}(k_1 r_1)}{H_m^{(1)}(k_1 r_1) H_m^{(2)}(k_1 r_2) - H_m^{(1)}(k_1 r_2) H_m^{(2)}(k_1 r_2)} \quad (19\,c)$$

$$c_m = -\frac{\epsilon_1 \gamma_1 k_1 k_3^2 [f H_m^{(1)\prime}(k_1 r_2) + g H_m^{(2)\prime}(k_1 r_2)] - \epsilon_2 \gamma_2 k_2 J'_m(k_2 r_2)}{\omega \mu_1 \epsilon_1 \frac{m}{r_2} k_4^2 [f H_m^{(1)}(k_1 r_2) + g H_m^{(2)}(k_1 r_2)] - \omega \mu_2 \epsilon_2 \frac{m}{r_2} J_m(k_2 r_2)}$$

The values of k_1, γ_1 and k_2, γ_2 being known, the radial fields at any point can be calculated in terms of the total power flow P along the guide. The power flow outside the guide ($\infty > r > a$) is different from P. The peak power flow through the space surrounding the rod ($\infty > r > a$) is

$$P_0 = Re\Big[\int_{\theta=0}^{2\pi} \int_{r>a}^{<\infty} r E_r H_\theta^* \, dr\, d\theta\Big] \quad (20)$$

where, H_θ is given in the modified form as follows:

$$H_\theta = -P\Big\{jk_1 k_3^2 \omega \epsilon_1 [f H_m^{(1)\prime}(k_1 r) + g H_m^{(2)\prime}(k_1 r)] + c_m k_4^2 \gamma_1 \frac{m}{r}[f H_m^{(1)}(k_1 r) + g H_m^{(2)}(k_1 r)]\Big\}\cos m\theta\, e^{-\gamma_1 z} \quad (20\,a)$$

The numerical evaluation of the above integral is rather cumbersome. But, the evaluation is much simplified if the asymptotic expansions for H's and J's are used for large arguments in the case of the lower order modes.

CONCLUSIONS

The results of the theoretical investigations (Chatterjee, 1953, 1954) on the propagation of microwaves through a cylindrical guide filled completely with two coaxial dielectrics lead to the following conclusions:

1. All the modes which can be supported by such a guide except the TM_0 are hybrid.

2. The phase velocity for a given mode can be adjusted to a pre-assigned value by a suitable choice of the dielectric constants and radii of the two dielectric media.

3. The calculation of the power flowing through the guide in the case of the TE_{01} as well as the other hybrid modes shows that most of the power flow is located in the medium having higher dielectric constant when the media are non-magnetic.

4. Depending on the relative values of $\mu\epsilon$ for the two media, the frequency of excitation may be adjusted so as to concentrate the power flow either in the first or the second medium.

5. The attenuation constant is higher in such a guide than a hollow wave guide due to the additional losses introduced by the dielectrics.

6. The attenuation constant increases directly with the increasing loss tangents of the dielectrics.

7. The attenuation constant increases as the square of the dielectric constants of the two media.

8. The attenuation constant increases with the increasing frequency of excitation of the guide.

9. Hollow wave guides, dielectric guides are special cases of the metallic guide filled with two coaxial dielectrics.

REFERENCES

1. Chatterjee, S. K. .. *Jour. Ind. Inst. Sci.*, 1953, 35, No. 1, 1; *Ibid.*, No. 3, 103; *Ibid.*, No. 4, 149; *Ibid.*, 1954, 36, No. 1, 1.

(Concluded)