JOURNAL OF THE INDIAN INSTITUTE OF SCIENCE

SECTION B

VOLUME 36 JULY 1954 NUMBER 3

EVALUATION OF STRESSES IN A U-SHAPED MEMBER BY THE SHEAR DIFFERENCE METHOD

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Received April 25, 1954

SUMMARY

The theoretical stress analysis of a U-shaped member is quite complicated and such members find frequent application in the grips of riveters and punches.

This paper treats of the evaluation of stresses on a section of symmetry of the U-shaped member by the Shear Difference Method when loaded as shown in Fig. 1, as it is felt that the section of symmetry seems to be the highly stressed region.

In working out this problem, the isochromatic fringe pattern on dark and bright back-ground and the isoclinic sketch only are made use of; and the Shear Difference Method has been used. The values arrived at by the above method have been checked by the equilibrium and boundary conditions and the percentage error is only of the order of 1·5%. The authors feel that this is a quick and an accurate method by which the stresses across any section can be evaluated.

NOTATION

σ_1, σ_2 : Principal Stresses (σ_1 being algebraically greater than σ_2).

σ_x, σ_y: Normal components of the stresses parallel to X and Y-axes (*i.e.*, in the horizontal and vertical directions respectively).

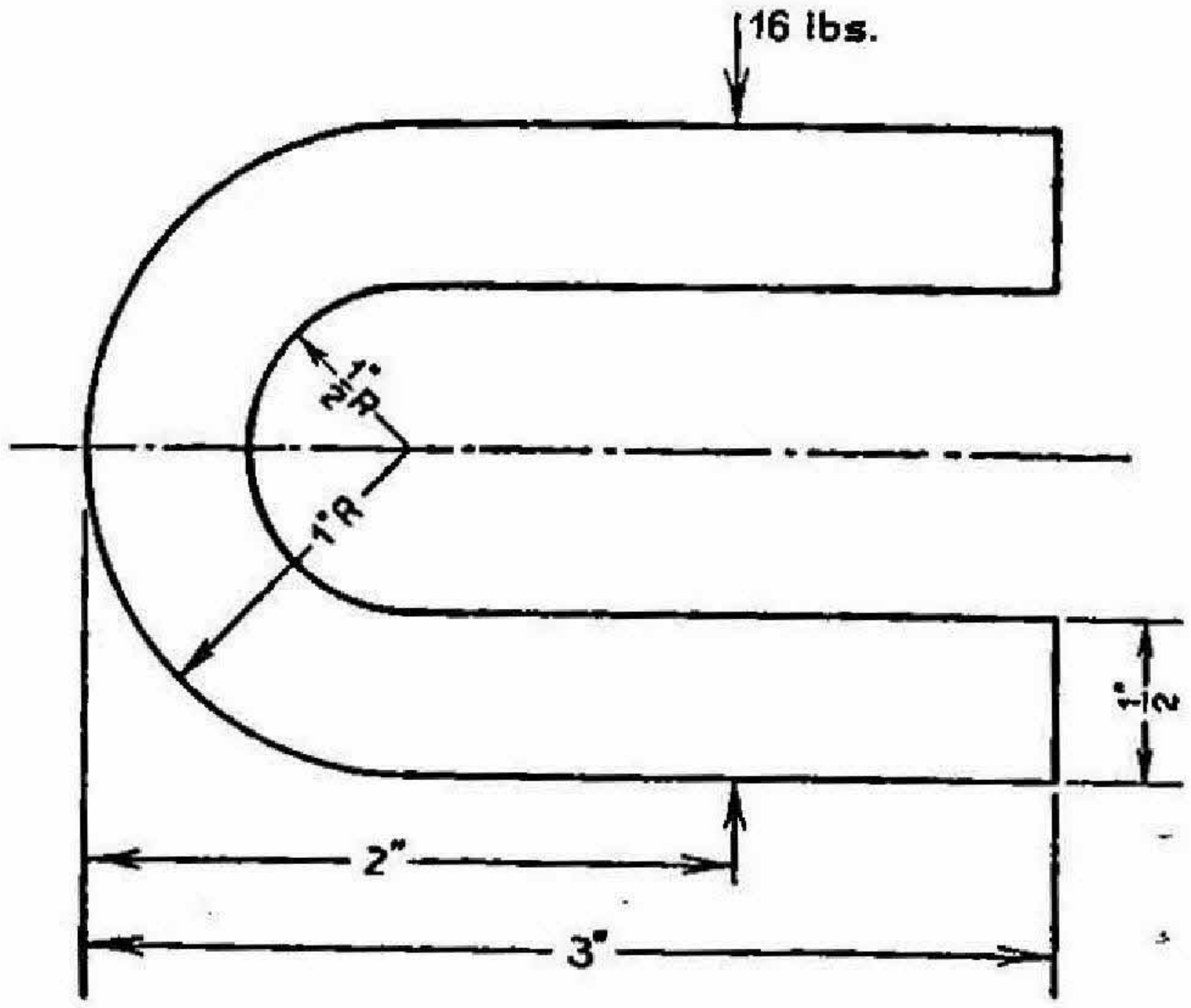

FIG. 1. U-shaped member.

θ : Angle made by either principal stress with X-axis (the angle is furnished by the parameter of the isoclinic passing through the point).

f : Fringe order.

τ : Shear Stress.

τ_{xy} : Shearing stress on the X, Y-plane.

C : Photoelastic material constant.

d : Thickness of specimen.

b : Width of section.

w : Distance from O to any point along XX′.

W : Concentrated load acting on the model.

PHOTOELASTIC APPARATUS

The apparatus used consists of an universal loading frame capable of being adapted to any sort of standard loading. The monochromatic source of light consists of a mercury vapour lamp with filters, condensing and collimating lenses. The other main parts of the polariscope are the polariser and the quarter-wave plate unit on either side of the loading frame. The parallel beam of light is collected by a condenser and converged through a lens system and projected on to the photographic plate or the ground glass screen. A powerful white source of light is used to determine the isoclinics, with the quarter-wave plates removed.

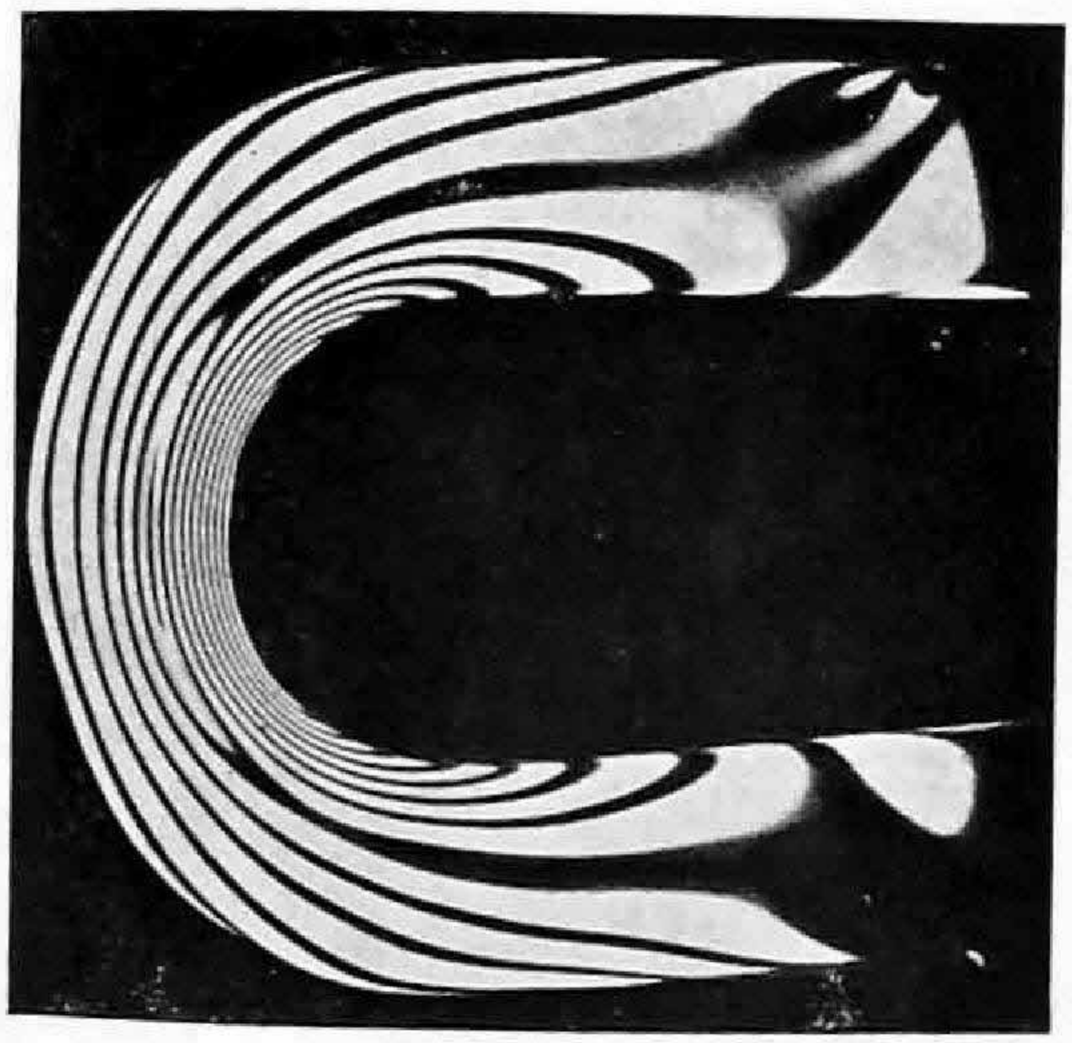

FIG. 2

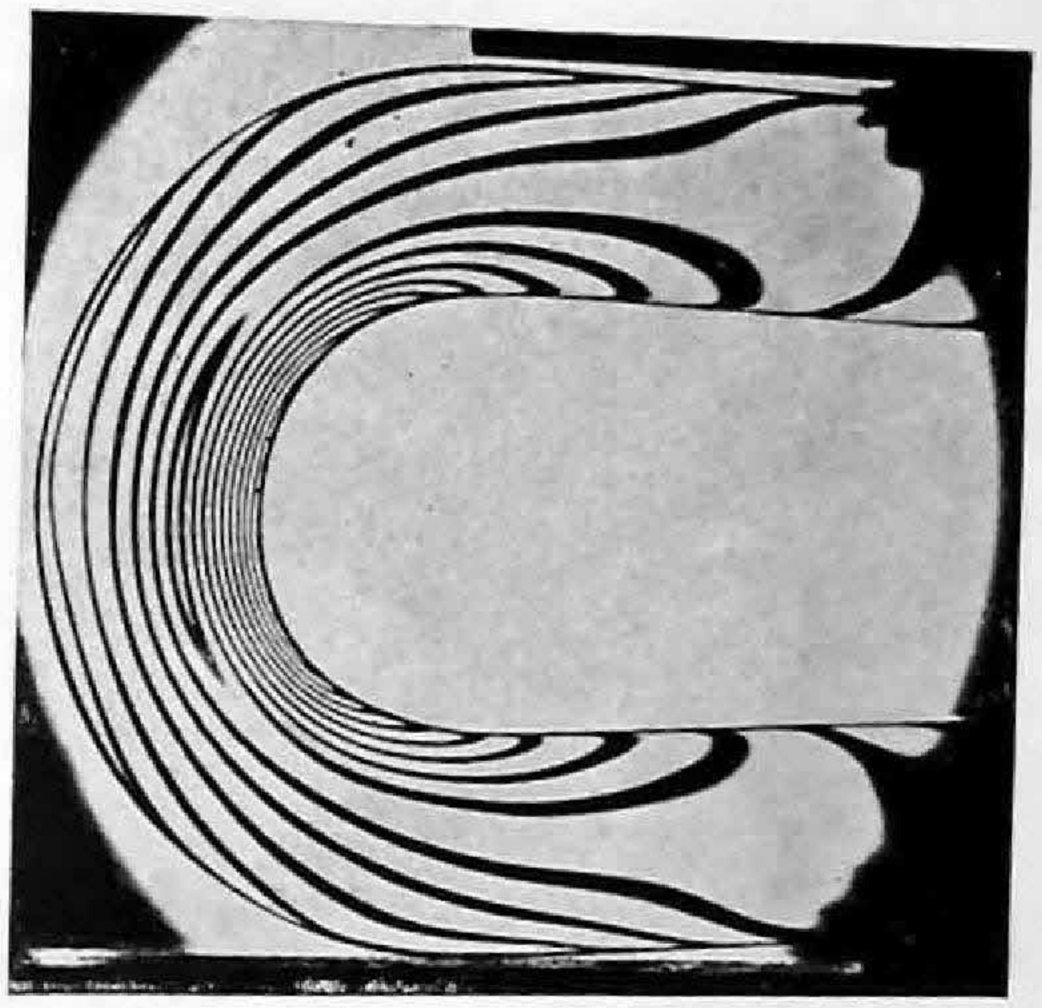

FIG. 3

FIG. 2. Isochromatic fringe photograph on a dark background; thickness = 0·234 in. Material fringe value = 94 lb./sq.in./fringe/in.

FIG. 3. Isochromatic fringe photograph on a bright background; thickness = 0·234 in. Material fringe value = 94 lb./sq.in./fringe/in.

EXPERIMENTAL TECHNIQUE

The isochromatic photographs as obtained on the dark (Fig. 2) and bright (Fig. 3) backgrounds respectively give the difference of principal stresses as integral fringe orders and as odd multiples of half the fringe orders. For obtaining clear boundaries, bright background pictures are to be preferred.

The isoclinics sketched on the screen (Fig. 4) indicate the directions of the principal stresses.

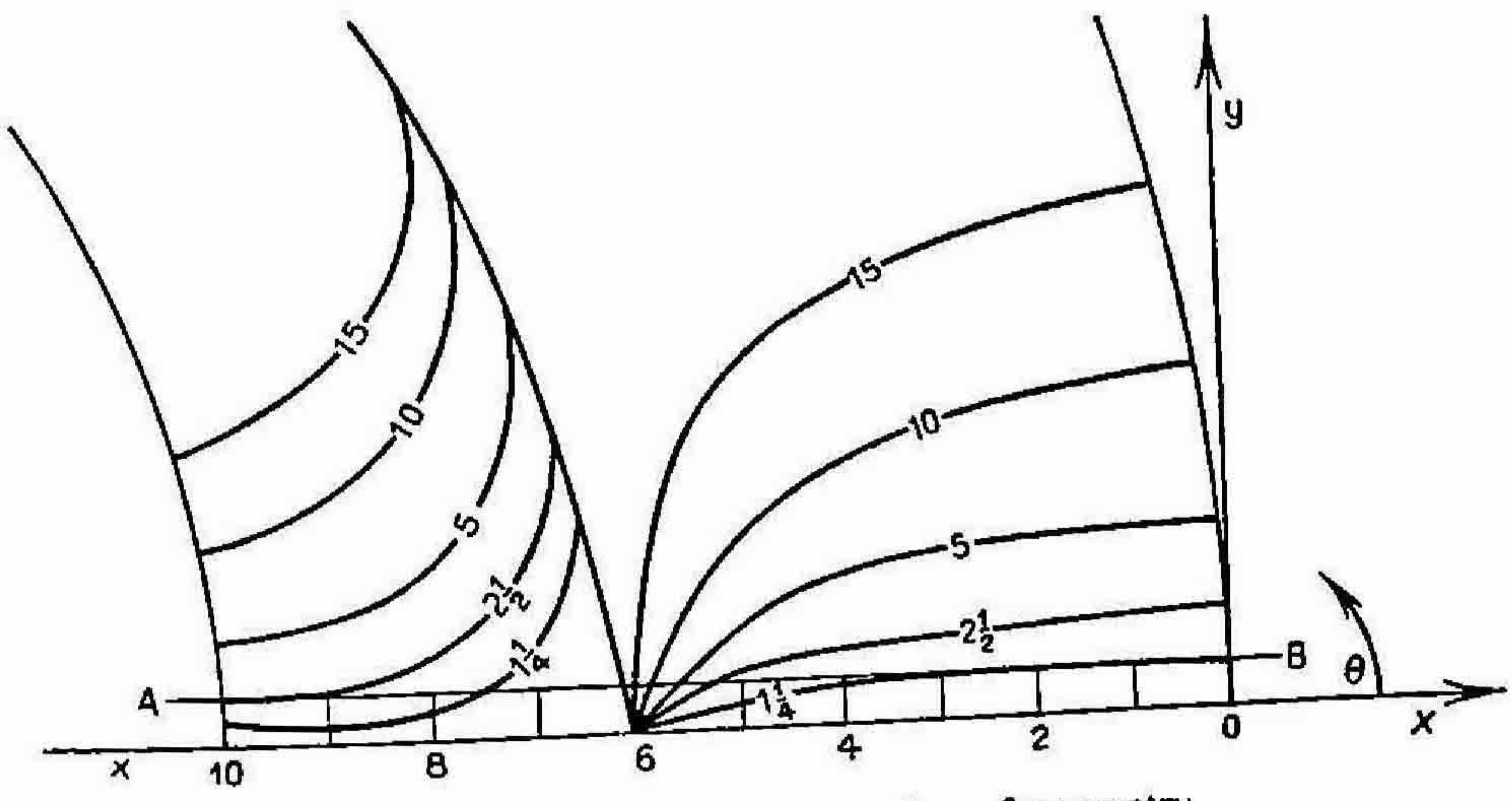

FIG. 4. Isoclinics sketch on section of symmetry.

The condensing lens used for the photographs is salvaged from an aircraft gun camera obtained from the disposals. Process plates and high contrast printing paper are used to obtain clear fringe photographs.

If the fringe order at any point on the model is known, then the value of $(\sigma_1 - \sigma_2)$ can be found out from the equation

$$\sigma_1 - \sigma_2 = f \times \frac{C}{d}. \tag{1}$$

THEORY OF THE SHEAR DIFFERENCE METHOD[1]

From a knowledge of Mohr's circle for stress, one can obtain the expression for shear stress τ_{xy} with the proper sign Fig. 6 (*a*) on any arbitrary section as

$$\tau_{xy} = \frac{\sigma_1 - \sigma_2}{2} \sin 2\theta. \tag{2}$$

From one of the equations of equilibrium (neglecting body forces) one obtains in the X-direction[3]

$$\frac{\partial \sigma_x}{\partial x} + \frac{\partial \tau_{xy}}{\partial y} = 0. \tag{3}$$

from which, by partial integration along X-axis

$$(\sigma_x)_i = (\sigma_x)_0 - \int_0^i \frac{\partial \tau_{xy}}{\partial y} dX \tag{4}$$

which reduces to

$$(\sigma_x)_i = (\sigma_x)_0 - \sum_0^i \frac{\Delta \tau_{xy}}{\Delta y} \Delta X \tag{4 a}$$

where

$(\sigma_x)_i =$ normal stress parallel to X-axis at any interior point.

$(\sigma_x)_0 =$ normal stress parallel to X-axis on any load-free boundary.

Further from Mohr's circle, it can be shown that

$$\sigma_y = \sigma_x \pm (\sigma_1 - \sigma_2) \cos 2\theta. \tag{5}$$

In all cases where $\sigma_y > \sigma_x$ algebraically, *i.e.*, one of the principal stress trajectories passes through the 45° angle formed by σ_y and the shear diagonal. Then

$$\sigma_y = \sigma_x + (\sigma_1 - \sigma_2) \cos 2\theta. \tag{6}$$

In a two-dimensional stress system acute angle θ is measured from the outward normal N on the plane on which shear stress acts to the algebraically maximum principal stress direction and that is the direction of the shear stress Fig. 6 (*b*).

Let one of the axes of the rectangular co-ordinate system (say X-axis) be parallel to the section XX′ and let X′ be the origin; a point on the load-free boundary. Divide XX′ into an arbitrary number of equal parts (say each $= \Delta$X). Consider two sections on either side of XX′ and parallel to it at a distance $\frac{\Delta X}{2}$ (Sections AB and CD).

The calculations that follow are based on the isoclinics (Fig. 4) and the isochromatics (Figs. 2 and 3) and in Table I, the values along the section AB are noted. In this problem

$$(\tau_{xy})_{AB} = -(\tau_{xy})_{CD}. \quad \text{Thus} \left[\Delta \tau_{XY} \frac{\Delta X}{\Delta Y}\right]_{XX'} = -[2\tau_{xy}]_{AB} \tag{7}$$

The values obtained from the last column of Table I are plotted in Fig. 5 and the mean values of τ_{xy} for the various divisions are read from the graph (Table II, Col. 2). In this problem, τ_{xy} along the section XX′ is zero as the zero degree isoclinic coincides with it. This immediately suggests that the normal stresses σ_x and σ_y are themselves the principal stresses.

$$\therefore \quad \sigma_y = \sigma_x \pm (\sigma_1 - \sigma_2) \tag{8}$$

The distribution of σ_x, σ_y and $\sigma_1 - \sigma_2$ on the section XX′ is shown in Fig. 5.

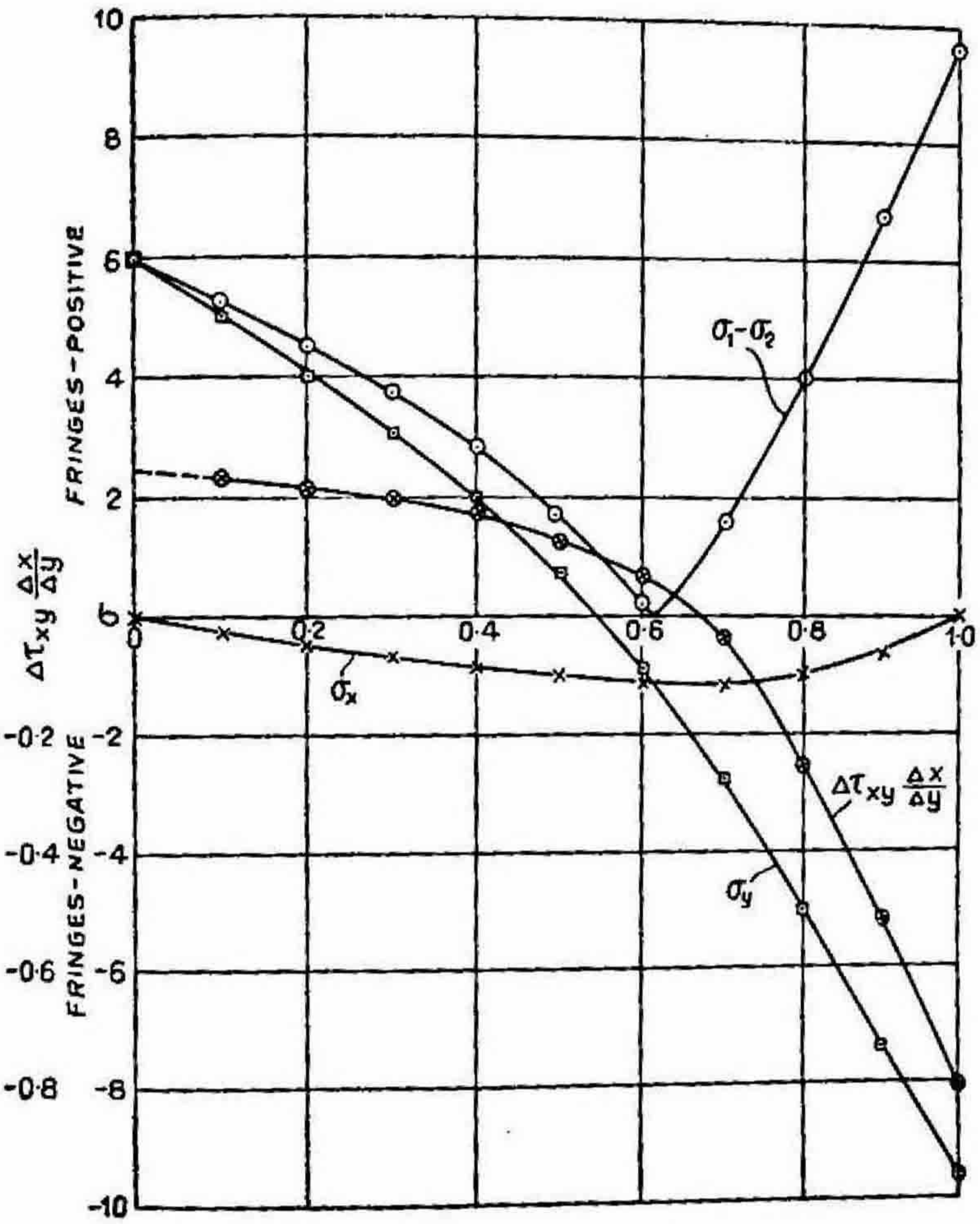

FIG. 5. Stress distribution on section of symmetry.

TABLE I

$\frac{w}{b}$ Station	$[\sigma_1-\sigma_2]_{AB}$ Fringes	$[\theta°]_{AB}$ Degrees	$[2\theta]_{AB}$ Degrees	$\sin 2\theta$	$\tau_{xy}=\frac{\sigma_1-\sigma_2}{2}\sin 2\theta$ Fringes	$\left[\Delta\tau_{xy}\frac{\Delta X}{\Delta Y}\right]$ Fringes
0	..	1·250	2·500	0·0436	..	..
0·1	5·30	1·250	2·500	0·0436	−0·1156	0·2311
0·2	4·50	1·375	2·750	0·0480	−0·1080	0·2160
0·3	3·80	1·500	3·000	0·0523	−0·0994	0·1987
0·4	2·90	1·750	3·500	0·0610	−0·0884	0·1769
0·5	1·80	2·000	4·000	0·0698	−0·0628	0·1256
0·6	0·25	8·000	16·000	0·2756	−0·0344	0·0689
0·7	1·50	0·750	1·500	0·0262	+0·0196	−0·0393
0·8	3·90	1·875	3·750	0·0654	+0·1276	−0·2551
0·9	6·60	2·250	4·500	0·0785	+0·2590	−0·5181
1·0	9·25	2·500	5·000	0·0872	+0·4033	−0·8066

TABLE II

$\frac{w}{b}$ Station	$\left[\Delta \tau_{xy}\frac{\Delta X}{\Delta Y}\right]_{XX'}$ Mean Value Fringes	$[\sigma_x]_{XX'}$ Fringes	$[\sigma_1-\sigma_2]_{XX'}$ Fringes	$[\sigma_y]_{XX'}$ Fringes	$[\sigma_x]_{XX'}$ Lb./sq.in.	$[\sigma_y]_{XX'}$ Lb./sq.in.
0	..	0	6·00	6·00	0	2410·26
0·1	+0·2383	−0·2383	5·25	5·0117	− 95·73	2013·25
0·2	+0·2235	−0·4618	4·50	4·0382	−185·51	1622·18
0·3	+0·2075	−0·6693	3·75	3·0807	−268·86	1237·55
0·4	+0·1862	−0·8555	2·85	1·9945	−343·66	801·21
0·5	+0·1540	−1·0095	1·75	0·7405	−405·53	297·47
0·6	+0·1040	−1·1135	0·22	−0·8935	−447·30	−358·93
0·7	+0·0242	−1·1377	1·60	−2·7377	−457·02	−1099·76
0·8	−0·1255	−1·0122	4·00	−5·0122	−406·61	−2013·45
0·9	−0·3880	−0·6242	6·75	−7·3742	−250·75	−2962·29
1·0	−0·6450	≈0	9·60	−9·6000	≈ 0	−3856·41

$$\text{Stress (in lb./sq.in.)} = C\,\frac{f}{d} = \frac{94}{0\cdot 234}\cdot f = 401\cdot 71\, f$$

CHECKS

(*a*) *Equilibrium Conditions.*—If a section is cut across XX′ it is evident that the forces in the Y direction equal to zero, or in other words

$$W = \int_{\text{Stn. 0}}^{\text{Stn. 10}} \sigma_Y d\, dx$$

$$= d \int_{\text{Stn. 0}}^{\text{Stn. 10}} \sigma_y\, dx \qquad \text{as } d \text{ is constant.}$$

The net area of the curve of σ_y *vs.* distance along X-axis gives an area of $\frac{15\cdot 75}{d}$ lb.

$$\therefore\ \%\ \text{error} = \frac{16 - 15\cdot 75}{16} + 100 = 1\cdot 56\%$$

This gives a static check on the accuracy of the results obtained in this problem, (*b*) *Isotropic Line.*—By referring to Fig. 6, the point at which the isotropic line crosses section XX' coincides with the point at which $\sigma_x = \sigma_y$ or $\sigma_1 = \sigma_2$. This acts as a second check.

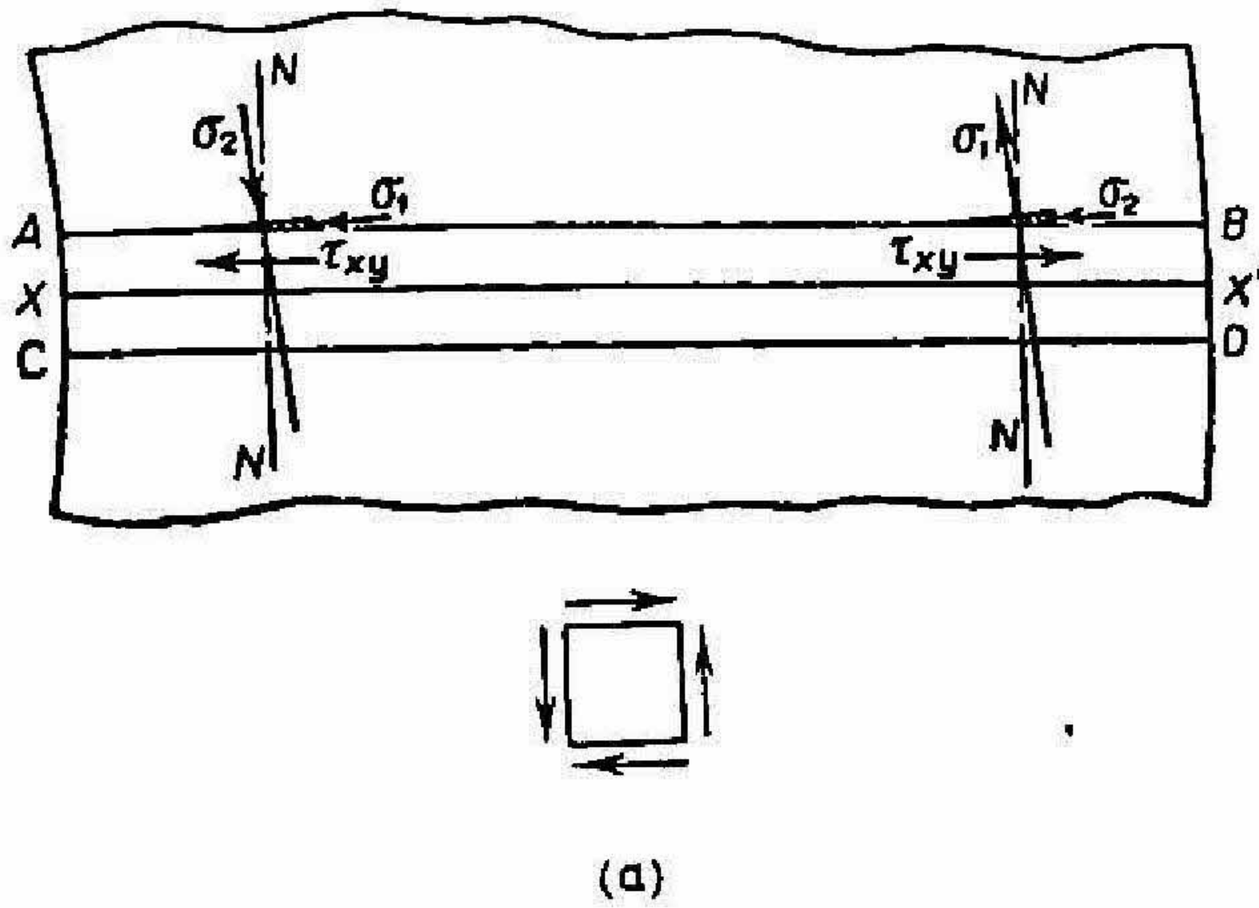

FIG. 6. Sketch showing—(*a*) Positive shear stress sign convention. (*b*) Distribution of horizontal shear stress.

(*c*) *Boundary Conditions.*—A third check is that in Table II, the values of σ_x become approximately equal to zero at the inner boundary which is obvious as it is a normal stress on a load-free boundary.

CONCLUSIONS

It is seen above that the error is of the order of 1·5% and is not very high, as the method employed is partly experimental and partly graphical integration. Part of the error creeps in while drawing the isoclinics, as it is difficult to get the isoclinics clearly due to the inadequacy of proper apparatus. Moreover, a glance at Fig. 2 and Fig. 3 show that time edge stresses have crept in, especially at the inner boundary of the member as is evident from the curving in of the fringes.[2]

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