A METHOD FOR THE MEASUREMENT OF CONDUCTIVITY OF METALS AT MICROWAVE FREQUENCIES

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Abstract

A theory for a new method of determining the conductivities of metals at microwave frequencies from the measurements of reflection coefficients has been worked with the help of field equations. An expression for the reflection coefficient in terms of voltage standing wave ratio, attenuation constant, scattering coefficient of any waveguide discontinuity present in the system and phase factors has been derived. The values of conductivity for several metals obtained by this method agree well with the values obtained by other existing methods.

Introduction

In investigating the electromagnetic field patterns within a waveguide or a cavity, it is usually assumed that the metallic boundaries behave as perfect conductors. This assumption is an idealisation of actual metallic conductors having finite conductivities. In practice, the electromagnetic field penetrates from the dielectric into the conductor. But the skin effect at microwave frequencies effectively concentrates all currents and fields within the metal into a very narrow layer near the surface. The surface currents result in power dissipation which varies inversely as the conductivity of the metal. So, a wave travelling through a waveguide suffers attenuation due to the finite conductivity of the metallic boundary. Therefore a predetermined knowledge of the conductivity of metals is important for the construction of microwave components. Conductivities of metals have been measured at microwave frequencies by Maxwell (1947), Beck and Dawson (1950).

The paper presents a report of theoretical and experimental investigations on a new method of measurement of conductivities of metals at microwave frequencies. The method involves essentially the determination of reflection coefficient of a metallic plate terminating a waveguide. The measurement of the reflection coefficient, the attenuation suffered by a wave in travelling through the guide, etc., enables an accurate calculation of the conductivity of the terminating plate. It is well known that when two waveguides having even the same characteristic impedance are connected together, a considerable mismatch may occur. Consequently, reflection from the junction of two waveguides will be invariably present due to the geometrical discontinuities inherently present in a microwave system. The theory takes into account the scattering suffered by the wave from discontinuities. The scattering coefficient of the waveguide junctions has also been experimentally evaluated and taken into account in the measurement of the true reflection coefficient of the terminating plate. This enhances considerably the accuracy of the measurement.

THEORETICAL

1. *Conductivity*

A standing wave is set up in a rectangular waveguide by terminating it with a metallic plate whose conductivity is to be measured. The waveguide is excited by H_{01} wave by the usual method. In this mode, the electric field is perpendicular to the long edges of the cross-section and the magnetic loops are in planes parallel to the broader pair of faces. This particular mode has been selected as it represents the dominant mode in a rectangular guide. If the propagation of the wave takes place in the x direction and b represents the width of the guide in the z direction, the field components of the H_{01} wave are given by the following expressions

$$E_x = E_z = H_y = 0$$

$$E_y = C \frac{j\omega\mu_1}{\gamma_{01}^2 + \omega^2\mu_1\epsilon_1} \frac{\pi}{b} \sin\left(\frac{\pi z}{b}\right) e^{-\gamma_{01}x}$$

$$H_x = C \cos\left(\frac{\pi z}{b}\right) e^{-\gamma_{01}x} \tag{1}$$

$$H_z = C \frac{\gamma_{01}}{\gamma_{01}^2 + \omega^2\mu_1\epsilon_1} \frac{\pi}{b} \sin\left(\frac{\pi z}{b}\right) e^{-\gamma_{01}x}$$

where μ_1, ϵ_1 refer to the constants of the dielectric medium inside the guide; C is a constant depending on the initial excitation of the guide. The propagation constant γ_{01} is

$$\gamma_{01} = \sqrt{\left(\frac{\pi}{b}\right)^2 - \omega^2\mu_1\epsilon_1} = \alpha_{01} + j\beta_{01} \tag{2}$$

where, α_{01} and β_{01} represent the attenuation constant and the phase constant respectively for the H_{01} wave. The time variation exp $(j\omega t)$ has been omitted from (1) for convenience.

Taking the real part and maximising eq. (1) reduces to

$$E'_{y1} = C\,\frac{\omega\mu_1 b}{\pi}\,\sin\left(\frac{\pi z}{b}\right) e^{-\alpha_{01}x}$$

$$H'_{x1} = C\cos\left(\frac{\pi z}{b}\right) e^{-\alpha_{01}x} \tag{3}$$

$$H'_{z1} = C\,\frac{\beta_{01} b}{\pi}\,\sin\left(\frac{\pi z}{b}\right) e^{-\alpha_{01}x}$$

The subscript 1 refers to the dielectric medium inside the guide. The primes on E's and H's indicate maximum values. Assuming that there is no discontinuity in the waveguide system, the power incident on the terminating plate is

$$P_x = \frac{1}{2}\,\frac{C^2\beta_{01}\,\omega\mu_1 b^2}{\pi^2}\,e^{-2\alpha_{01}x}\int\limits_{z=0}^{z=b}\int\limits_{y=0}^{y=a}\sin^2\frac{\pi z}{b}\,dy\,dz$$

$$= \frac{C^2\beta_{01}\omega\mu_1 b^2}{4\pi^2}\,ab\,e^{-2\alpha_{01}x} \tag{4}$$

where a represents the height of the guide in the y direction.

To satisfy the condition $\mathbf{n}\times\mathbf{E}=0$ at the surface of the terminating plate, the resultant $\mathbf{E}$ from the incident and the reflected waves must be very small, so that the electric field of these two waves must almost exactly cancel. As a result the magnetic fields must be in the same phase and must add. If a component of electric field intensity exists at the surface of the metal termination, then E_2 (in the metal) $= \eta_2 H_2$, where $\eta_2 = \sqrt{j\omega\mu_2/\sigma_2}$ is the intrinsic impedance of the metal and μ_2, σ_2 refer to the constants of the terminating plate. Using the Poynting vector $S = \frac{1}{2}\,Re\,(E\times H)$ and the real part of the intrinsic impedance, the average power lost in the termination is

$$P_{Ter} = \tfrac{1}{2}\,C^2\beta_{01}^{\,2}\,\frac{ab^2}{\pi^2}\,e^{-2\alpha_{01}x}\sqrt{\frac{\pi f\mu_2}{\sigma_2}}\int_0^b \sin^2\frac{\pi z}{b}\,dz$$

$$= \frac{C^2}{4}\,\beta_{01}^{\,2}\,b^2\,\frac{ab}{\pi^2}\,e^{-2\alpha_{01}x}\sqrt{\frac{\pi f\mu_2}{\sigma_2}} \tag{5}$$

The power reflected from the termination is (6)

$$\frac{C^2 b^2\beta_{01}}{4\pi^2}\,ab\,e^{-2\alpha_{01}x}\left[\omega\mu_1 - \beta_{01}\sqrt{\frac{\pi f\mu_2}{\sigma_2}}\right]$$

The reflection coefficient of the terminating plate is therefore

$$\rho = \frac{\omega\mu_1 - \beta_{01}\sqrt{\frac{\pi f \mu_2}{\sigma_2}}}{\omega\mu_1} \tag{7}$$

which yields the conductivity of the terminating plate as

$$\sigma_2 = \frac{\pi f \mu_2 \beta_{01}^2}{\omega^2 \mu_1^2 (1-\rho)^2} \ \frac{mho}{\text{meter}} \tag{8}$$

If the terminating plate is non-magnetic $\mu_1 = \mu_2 = \mu$. Substituting, $\beta_{01} = 2\pi/\lambda_g$ where λ_g represents the guide wavelength, eq. (8) reduces to

$$\sigma_2 = \frac{\pi}{\mu f \lambda_g^2 (1-\rho)^2} \ \frac{mho}{\text{meter}} \tag{9}$$

2. *Reflection Coefficient: Effect of Attenuation*

The true reflection coefficient ρ can be determined by measuring the voltage standing wave ratio (v.s.w.r.) in the transmission system at a point very near the terminating plate. As $\rho \simeq 1$ for metals at microwave frequencies, it requires an arrangement by which a very high v.s.w.r. of the order of 10^4 can be measured. High standing wave ratios of the order of 600 can be measured with an accuracy of $\pm$ 4% by the 'Phase' method and with an accuracy of $\pm$ 6% by the modified 'Hipple' method (Buchanan, 1952). The measurement of higher standing wave ratios by a direct comparison of maximum and minimum voltages (*i.e.,*) by a slotted section needs special technique and which may not be accurate as it will require a knowledge of the law of the detector response over a very large range of power. So, the method adopted in the present investigation is to terminate a long section of a waveguide and measure the v.s.w.r. at the input end of the waveguide. The reflection coefficient thus determined from the v.s.w.r. measurements needs correction both as regards magnitude and phase in order to give the true reflection coefficient which may be calculated in the following way.

If the terminating plate (Fig. 1) is considered as the reference plane ($l = 0$) and V_i represents the complex voltage incident on the termination, the total complex voltage at the slotted section is

$$V = V_i e^{\gamma l} [1 + |\rho| e^{j\phi} e^{-2\gamma l}] \tag{10}$$

where γ is the propagation constant. The true reflection coefficient $\rho = |\rho| e^{j\phi}$ of the terminating plate can be easily determined from (10) and is as follows

$$\rho = \rho' e^{2\alpha l} \cos (2\beta x_{min} \pm \pi) \tag{11}$$

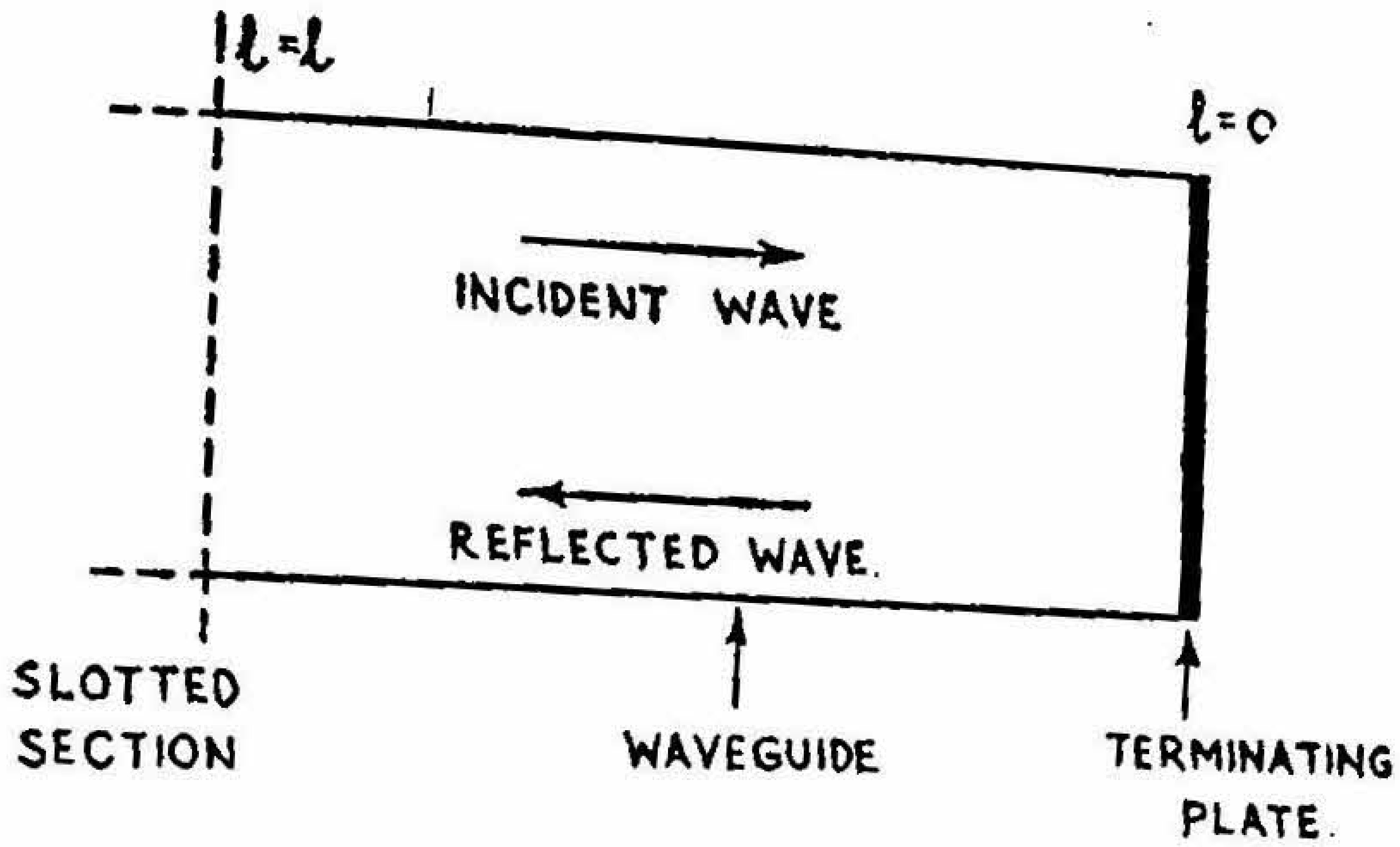

FIG. 1. Reflections in Waveguide.

where ρ' is the measured reflection coefficient given in terms of the measured v.s.w.r. (r) as $\rho' = (r - 1)/(r + 1)$ and β represents the phase constant and x_{min} indicates the first nodal position from the terminating plate.

The cosine term in (11) involves x_{min} for the following reason. If the probe of the slotted section is not perfectly tuned or if there is frequency drift of the generator during the measurement, there will be a residual susceptance component of the probe. Moreover, if the load has any conductive component, the slotted section probe admittance will affect the v.s.w.r. and shift the positions of the maxima and the minima from their true positions. But the shift in minima is much less than that of the maxima. Hence it is essential that the location of the minimum rather than the maximum should be considered in establishing the phase of the reflection coefficient of the terminating plate.

3. *Reflection Coefficient: Effect of Scattering*

In deducing the relation (11) it has been assumed that the slotted section is far away from any waveguide discontinuity, so that the effect of evanescent modes generated due to scattering is negligible. But any waveguide structure is composed of uniform as well as discontinuity regions. For a standard waveguide, the discontinuity mostly existing at the junctions creates a number of higher order evanescent modes. The discontinuity can be represented by a lumped element in shunt or in series with the main waveguide structure according as the scattering of the electric field is symmetrical or antisymmetrical. Consequently, due to the change in the admittance or impedance of the guide in the region of discontinuity, there will be a change in the v.s.w.r. and hence in the measured reflection coefficient. The lumped element can be treated as an obstacle in the path of the wave, producing scattering.

So, the effect of discontinuity can be studied as follows with the help of a scattering coefficient s. The following two cases will be considered.

(i) *Discontinuity very near the terminating plate.*—Fig. 2 shows the incident, scattered and reflected waves when the discontinuity ($l=0$) is at a

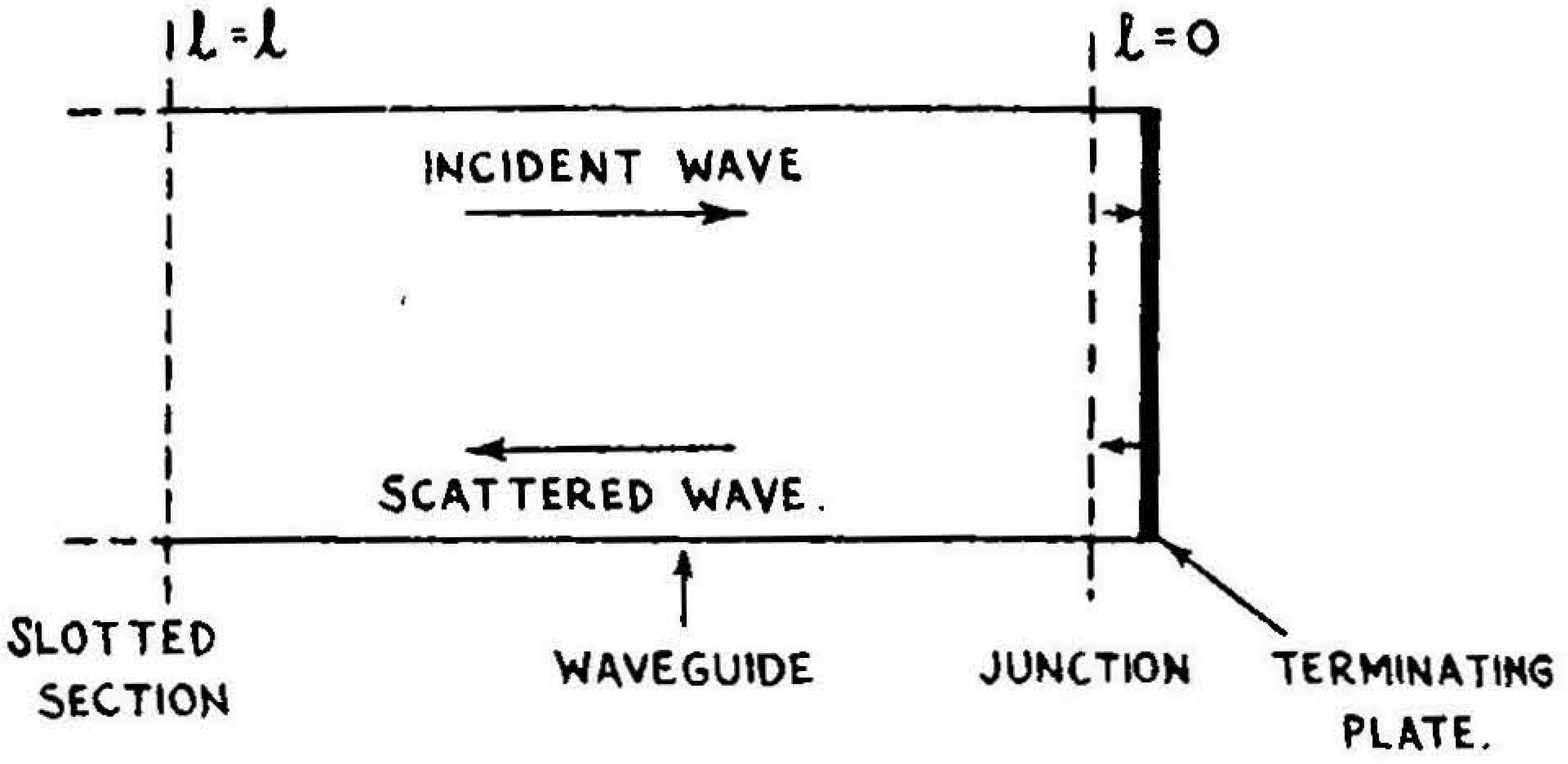

FIG. 2. Reflection and scattering in waveguide junction which is very near the plate.

distance l from the slotted section and the terminating plate is very close to $l=0$, so that the attenuation suffered by the wave between the waveguide junction and the terminating plate is negligible. If the amplitude of the incident wave is unity and if it is assumed that the reflecting and the scattering points are non-dissipative, the amplitudes of the wave after successive scattering at the junction and reflection at the termination are as follows.

Amplitudes of the scattered waves from the discontinuity towards the slotted section:

$$se^{-al},\ \rho(1-s^2)e^{-al},\ \rho^2 s(1-s^2)e^{-al},\ \ldots.$$

Amplitudes of scattered waves from the discontinuity towards the termination:

$$(1+s)e^{-al},\ \rho s(1+s)e^{-al},\ \ldots.$$

Amplitudes of waves reflected from the termination:

$$\rho(1+s)e^{-al},\ \rho^2 s(1+s)e^{-al},\ldots.$$

Amplitudes of waves at the slotted section:

$$se^{-2al},\ \rho(1-s^2)e^{-2al},\ \rho^2 s(1-s^2)e^{-2al},\ldots.$$

where the complex scattering coefficient is $s=|s|e^{j\theta}$ and the true reflection coefficient of the terminating plate is $\rho=|\rho|e^{j\psi}$ where, θ and ψ are the phases of the scattering and the reflection coefficients respectively. The resultant reflection coefficient $R_m=|R_m|e^{j\phi}$ is obtained from the vector summation of the amplitudes of the waves at the slotted section, ϕ being the phase of R_m.

$$R_m = se^{-2al} + \rho(1 - s^2)e^{-2al} + \rho^2 s(1 - s^2)e^{-2al} + \ldots \quad (12)$$

which yields

$$\rho = \frac{R_m - se^{-2al}}{R_m s + (1 - 2s^2)e^{-2al}} \quad (13)$$

(ii) *Discontinuity very near the slotted section.*—Fig. 3 shows the incident and the scattered waves. The amplitudes of the different waves are as follows:—

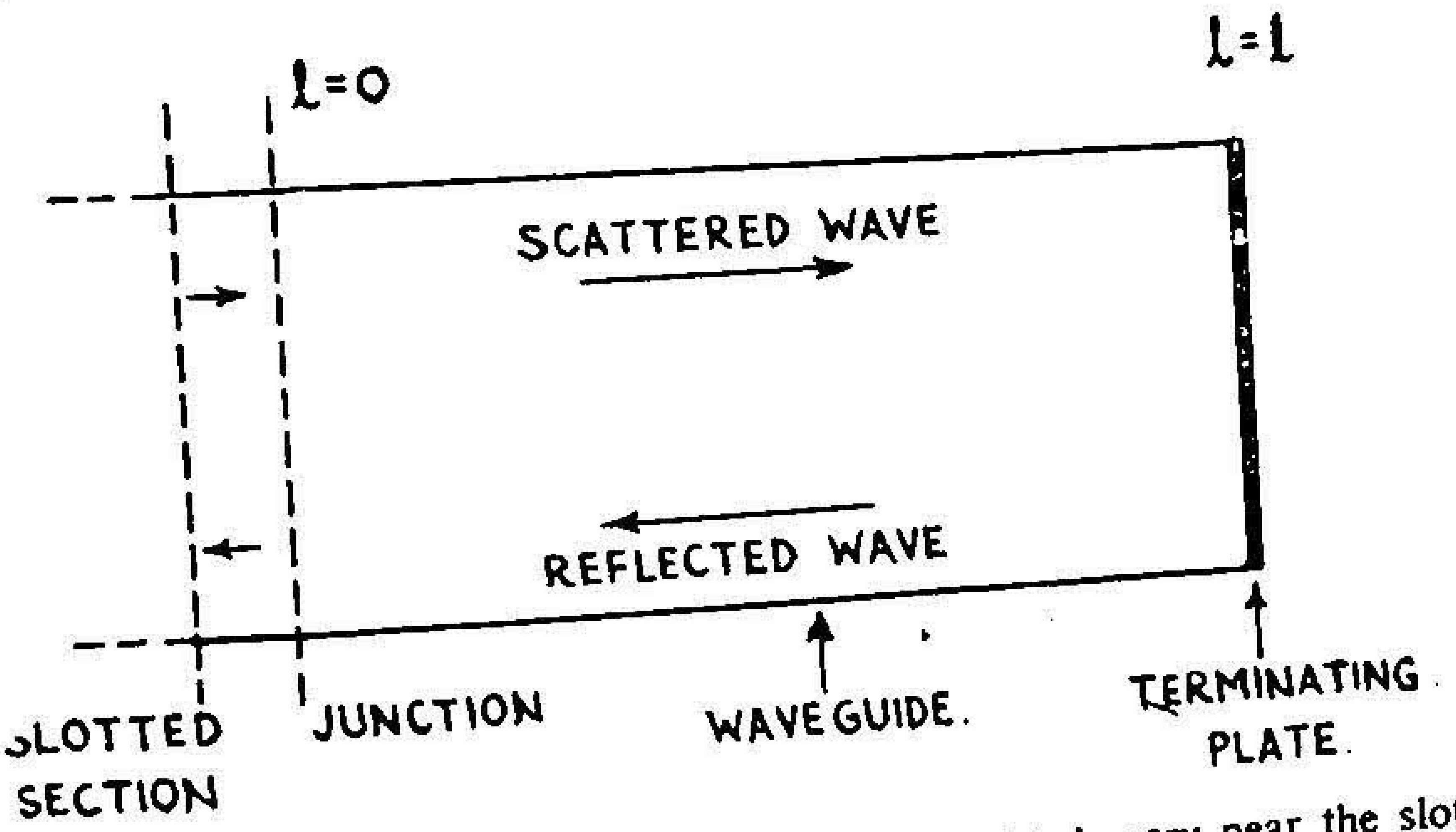

FIG. 3. Reflection and scattering in waveguide junction which is very near the slotted section.

Amplitudes of scattered waves from the junction towards the slotted section which is the same as the amplitudes of the waves at the slotted section:

$$s, \ \rho(1 - s^2)e^{-2al}, \ \rho^2 s(1 - s^2)e^{-4al}, \ldots$$

Amplitudes of waves scattered from the junction towards the termination:

$$(1 + s), \ \rho s(1 + s)e^{-2al}, \ldots$$

Amplitudes of waves reflected from the termination:

$$\rho(1 + s)e^{-al}, \ \rho^2 s(1 + s)e^{-3al}, \ldots$$

The resultant reflection coefficient is

$$R_m = s + \rho(1 - s^2)e^{-2al} + \rho^2 s(1 - s^2)e^{-4al} + \ldots \quad (14)$$

which yields

$$\rho = \frac{(R_m - s)e^{2al}}{R_m s + (1 - 2s^2)} \quad (15)$$

The above analysis can similarly be extended to the case where there are more than one junction. In the present investigation only one junction has been used.

EXPERIMENTAL

The experimental arrangement for the measurement of the reflection coefficient is shown by the block schematic (Fig. 4) and by the photograph (Fig. 5).

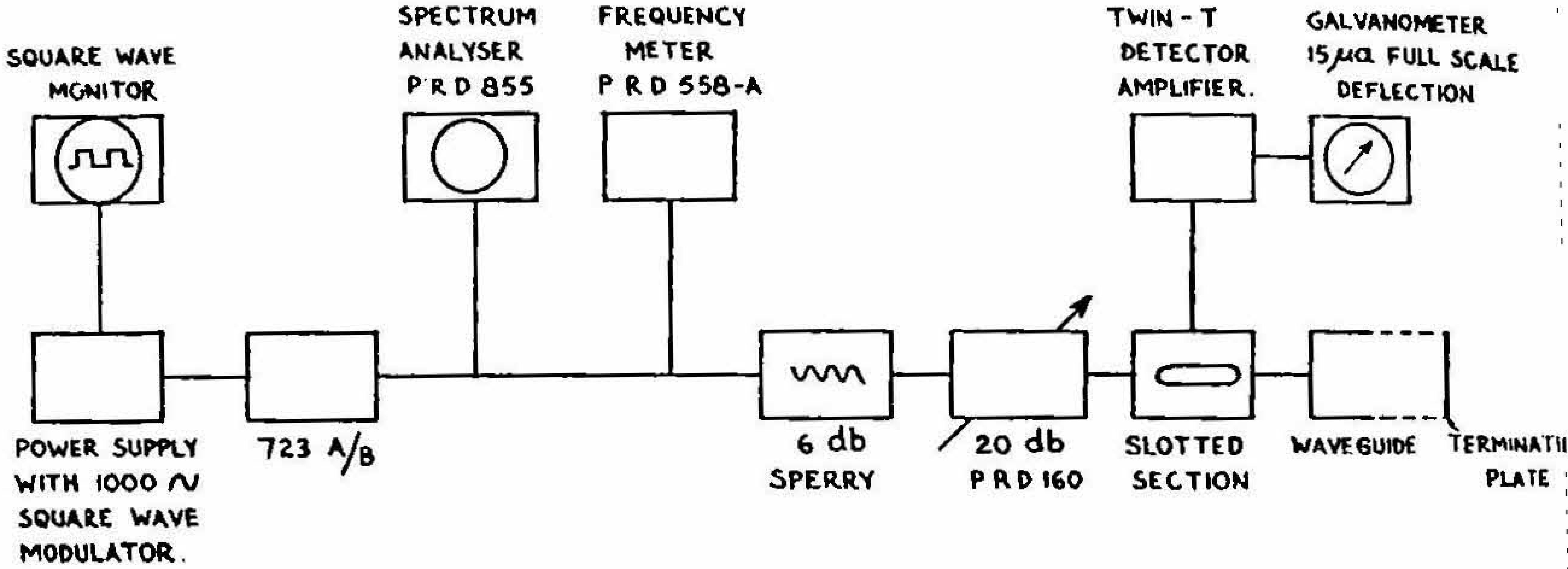

FIG. 4. Block schematic of the experimental set-up.

Power supply for the klystron.—As a reflex klystron is very frequency sensitive to reflector and beam voltages, it is necessary that the power supply system should be well regulated to a fraction of a volt. The power supply system consists of the usual combination type of stabiliser, giving a voltage stability of $\pm 0\cdot 05$ volts in the operating reflector and beam voltages. A variation of $\pm 0\cdot 05$ volts in the reflector voltage changes the operating carrier frequency 9697·9 mc./s. of 723 A/B by $\pm 0\cdot 11$ mc./s. This frequency shift causes a change of $\pm 0\cdot 000035$ cm. in the operating wavelength. The voltage of the modulator unit is stabilised by VR tubes. The complete unit is supplied from a constant voltage transformer.

Detector amplifier system.—The detector amplifier consists of a high gain (85 db. at the operating gain control) frequency selective (1000 c.p.s.) amplifier of 130 cycles full bandwidth at half response, feeding into a bridge network having two 1N34 in two of its arms. The indicating meter in the bridge is a galvanometer (15 μa full scale deflection). The amplifier consists of six RC coupled stages. The fourth stage is provided with twin RC feed back network (Fig. 6), so that regeneration occurs at one frequency and the amplifier exhibits a selective characteristic (Fig. 7). The phase curve and the linearity characteristics of the amplifier are shown in Fig. 8 and Fig. 9

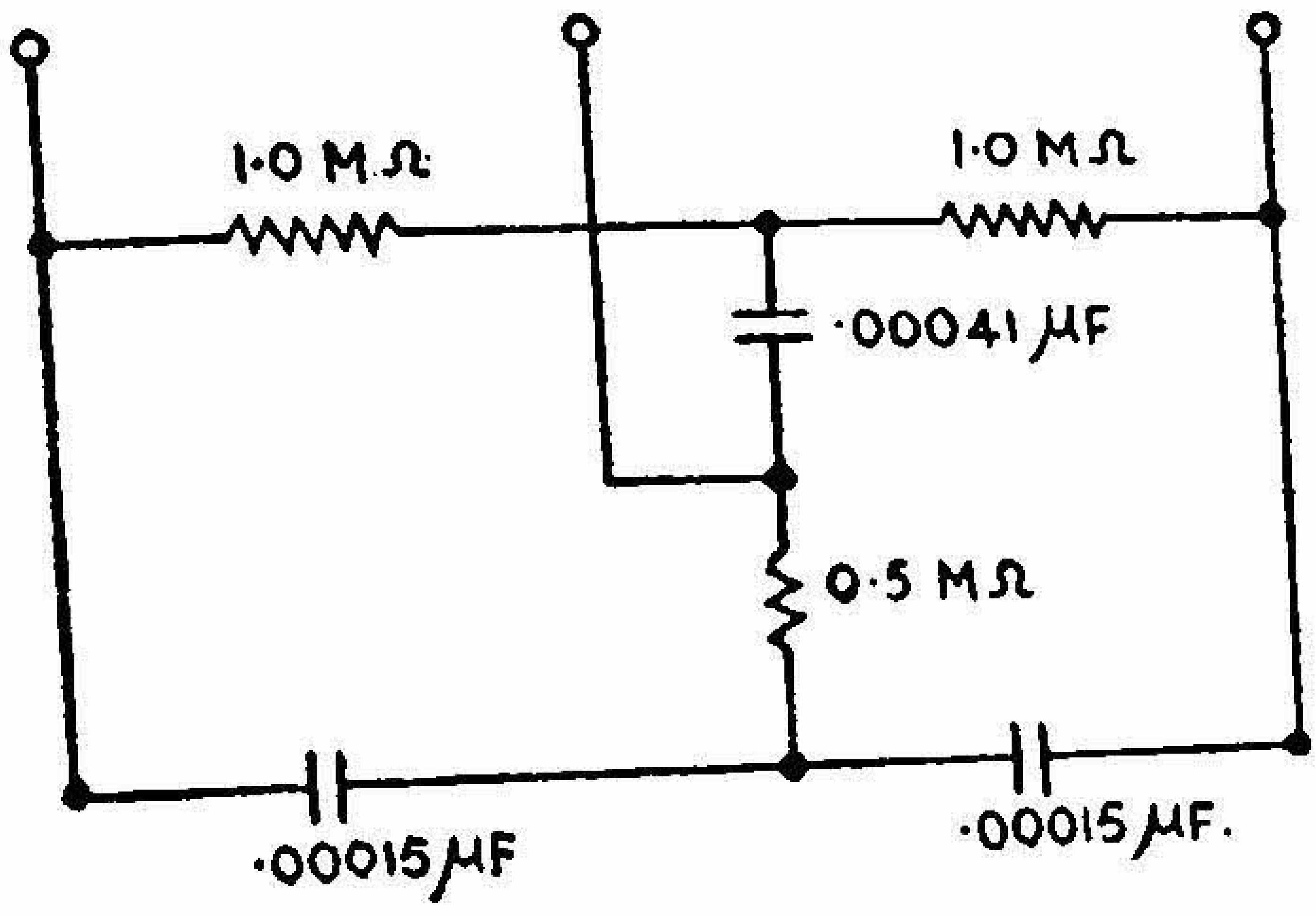

FIG. 6. Twin-T-Network.

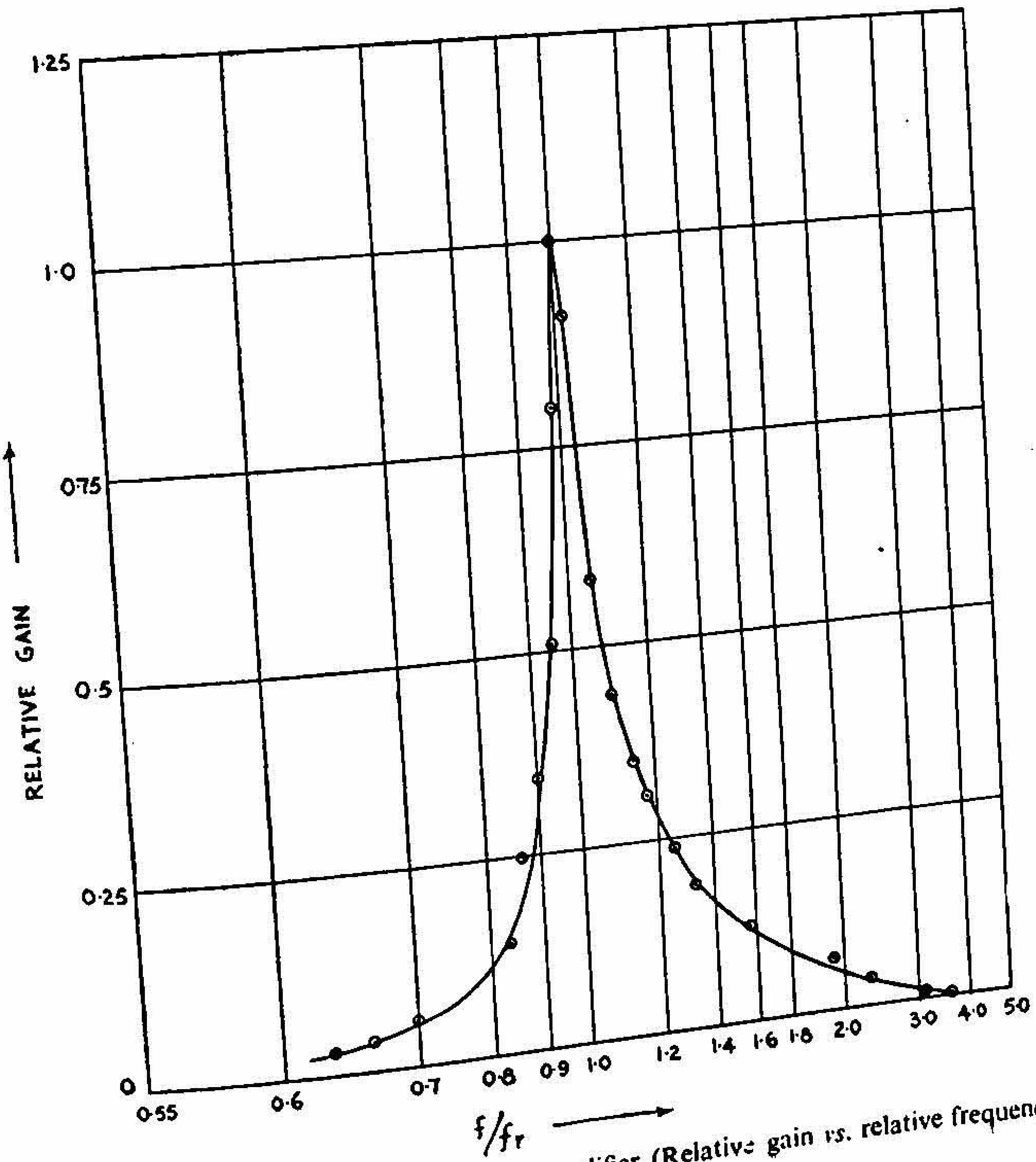

FIG. 7. Resonance curve of the detector amplifier (Relative gain *vs.* relative frequency).

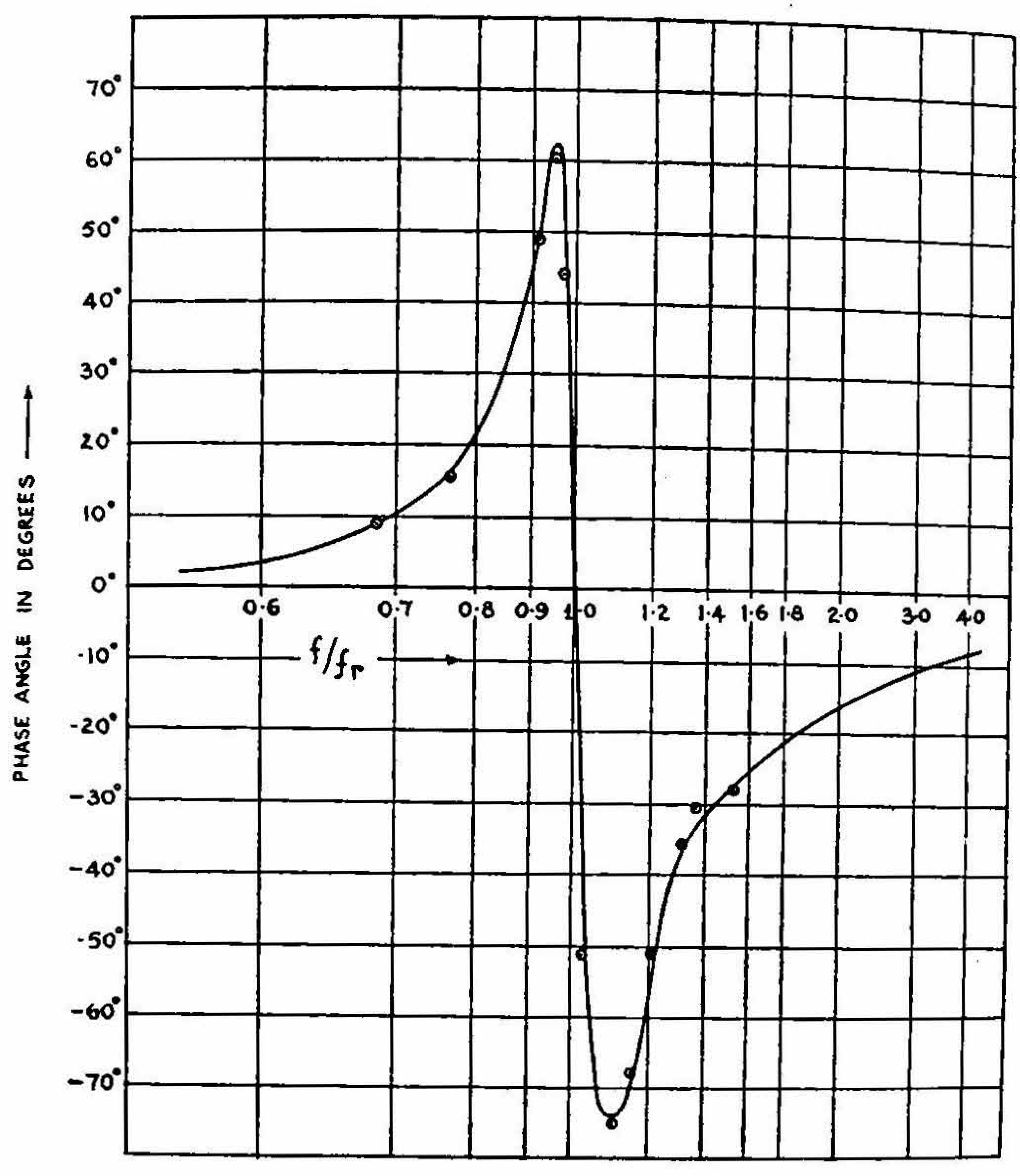

FIG. 8. Phase curve of the detector amplifier.

respectively. A tuned amplifier is essential for the standing wave measurements for the following reasons. For standing wave measurements, frequency modulation of the source should be avoided, otherwise a frequency change of the carrier displaces the standing wave pattern along the waveguide system. As it is difficult to obtain amplitude modulation without undesired frequency modulation, the klystron is operated with on-off modulation with a square wave modulation envelope, so that the reflector voltage is shifted from the correct operating point to a value that does not allow the tube to oscillate. This is achieved by adjusting the amplitudes of the reflector and modulating voltages. So, the tuned amplifier, not only eliminates any spurious signal present in the system but also reduces the effect

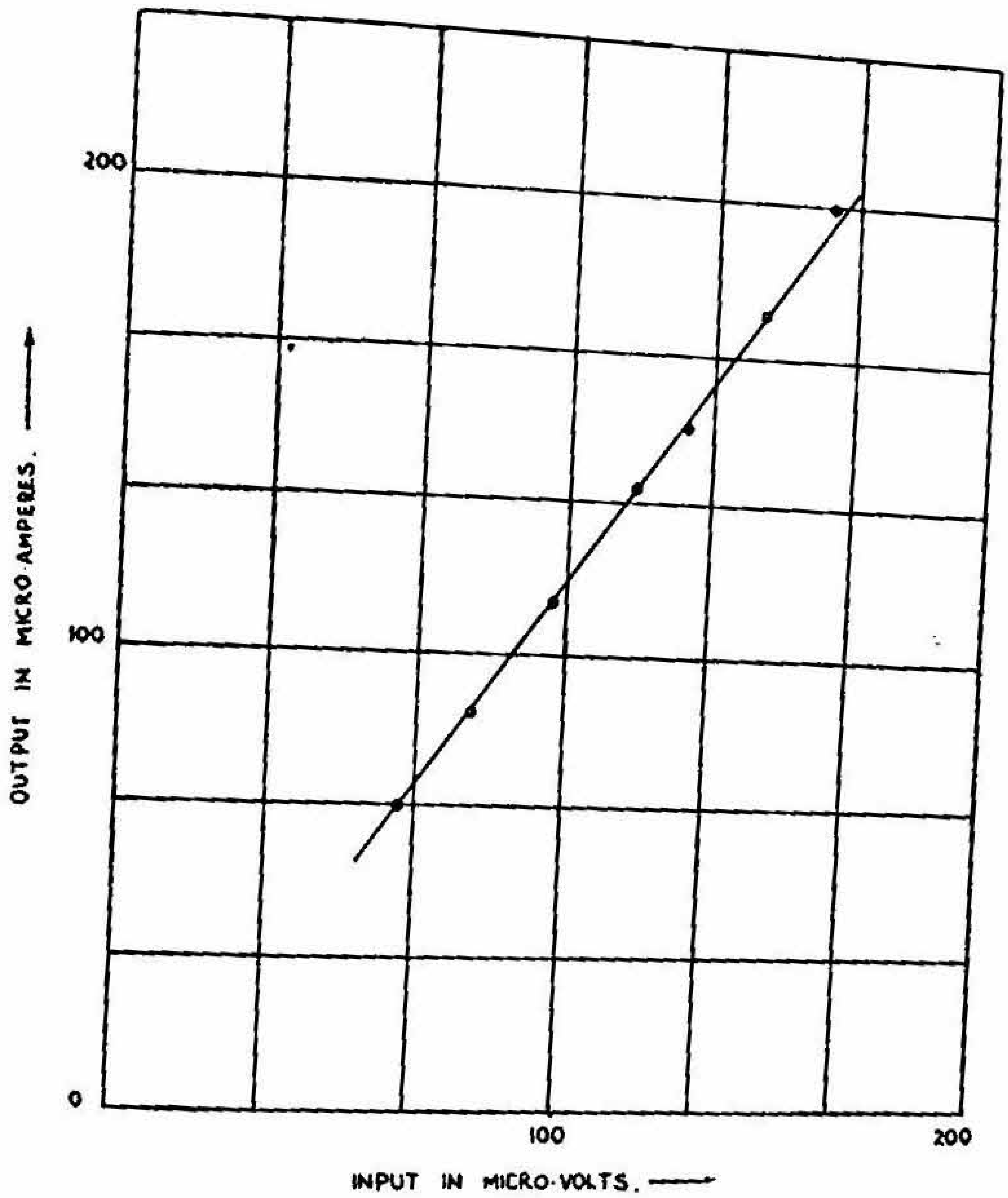

FIG. 9. Detector amplifier input *vs.* output curve.

of any frequency modulation that may occur during the rise and fall of the modulating square wave if the leading and the trailing edges of the modulation envelope do not occur instantaneously. A photograph of the square wave used for modulating the reflector voltage is shown in Fig. 10. The modulating square wave is monitored continuously during the measurement, by means of an oscilloscope in order to insure the constancy of the modulation envelope and hence that of the output from the slotted section. The monitoring of the square wave also helps to check the rise and fall of the modulation pulse so as to insure the minimum frequency modulation of the r.f. source.

For an accurate measurement of the v.s.w.r. the detector amplifier response law should be correctly known. When the v.s.w.r. is nearly unity, the response may be assumed to follow square law without introducing much error. But the error in the above assumption increases with the increase of the v.s.w.r. In general, when the response does not follow a square law, the v.s.w.r. (r) is given by

$$\log r = \frac{1}{n_1} \log (\text{maximum response}) - \frac{1}{n_2} \log (\text{minimum response}) \qquad (16)$$

where n_1 and n_2 are the laws of the detecting system at the maximum and the minimum reading of the slotted section respectively. The values of n_1 and n_2 can be found by shorting the slotted section and taking the maximum reading (I_0) and the corresponding full width ($d = 2x$) of the response curve at half power points (I) for various r.f. power input levels. Under this condition the law of the detector follows the relation

$$n = \frac{\log I/I_0}{\log \cos \frac{2\pi x}{\lambda g}} \tag{17}$$

The calibration curve of the detector system response law is shown in Fig. 11. For the convenience of obtaining n directly for any reading I of the detector, the second curve is given in Fig. 12.

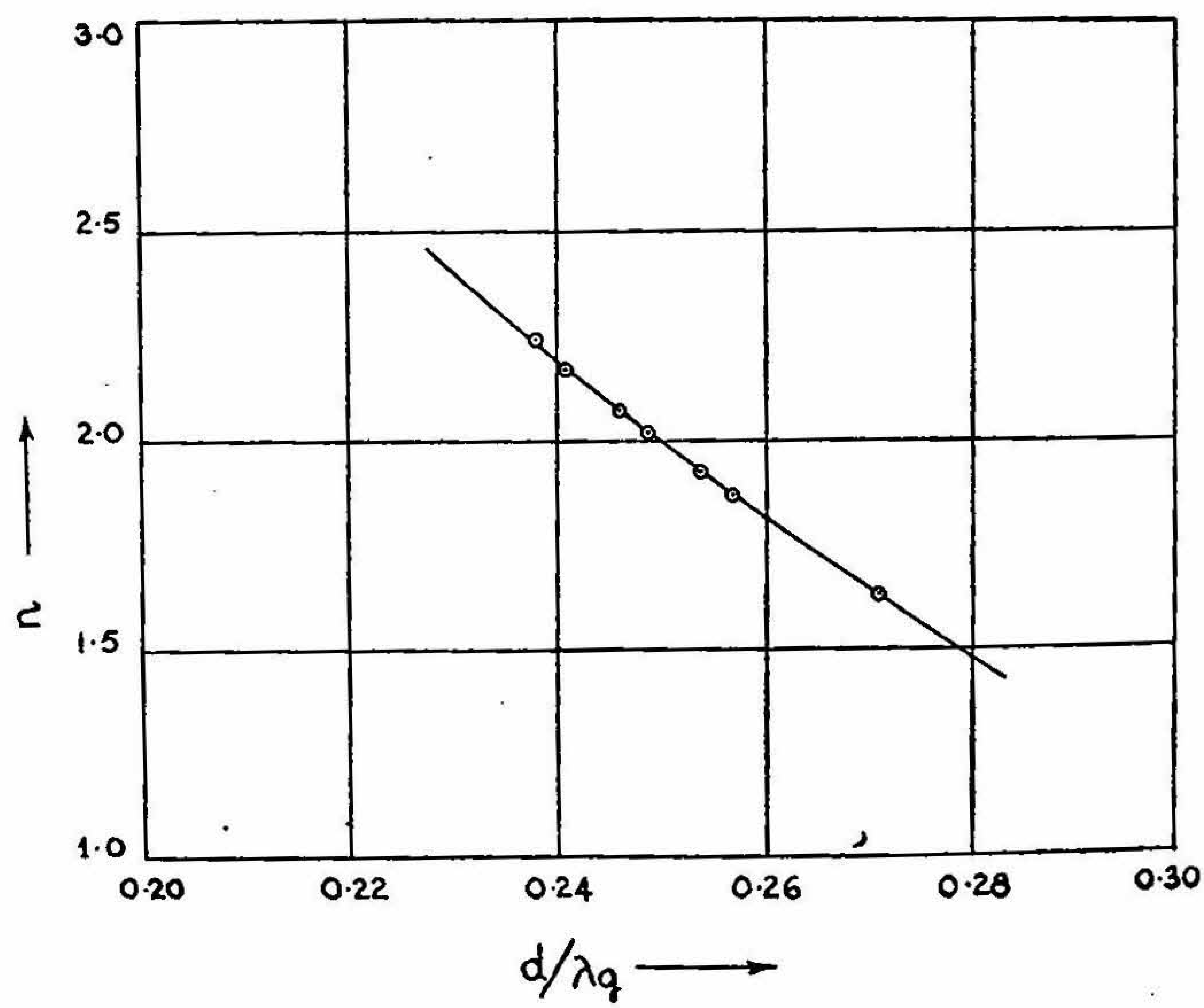

FIG. 11. Detector law n vs. full width d/λ_g at half response.

V.S.W.R.—When a standing wave is set up in a guide of length l, the frequency sensitivity of the phase ζ of the standing wave pattern is given (Brown, 1946) by the following relation

$$\frac{d\zeta}{d\lambda_g} = - \frac{2\pi l}{\lambda_g^2} \tag{18}$$

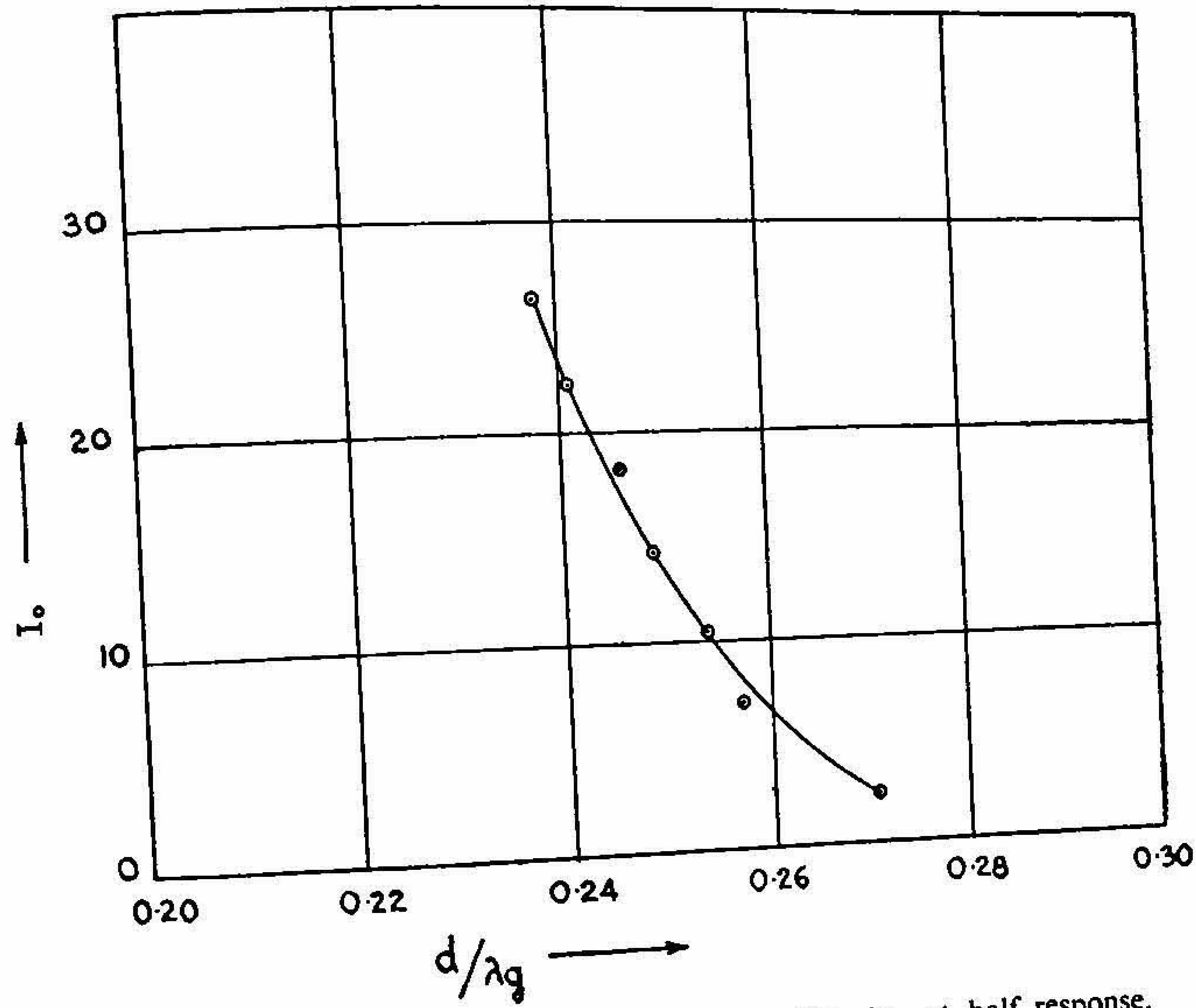

FIG. 12. Detector maximum response I_0 *vs* full width d/λ_g at half response.

As the relative phases of the waves will vary with frequencies, the v.s.w.r. becomes a function of the frequency. The frequency stability of the system is checked during the measurement with the help of a spectrum analyser P.R.D. type 855.

Scattering coefficient.—For measuring s, the terminating plate is replaced by a waveguide termination P.R.D. type 114. In this case, $\rho=0$ in equations (13) and (15) and hence R_m obtained from the measurement of v.s.w.r. is equal to s. Under perfectly matched condition $s=R_m$ holds good. But the waveguide termination (type 114) has a v.s.w.r. of 1·06, so the true value of s is $R_m/1\cdot06$. The measurement of s has been done with three waveguide terminations and the mean value of s has been taken. The formula for ρ shows that it is extremely sensitive to changes in s. As a change of $\pm1\%$ in s changes the value of ρ by $\pm$ 0·01%, it is essential that the measurement of s be carefully done.

Phase factors.—The values of θ, ψ and ϕ are obtained from the electrical distances between nodal planes in the respective cases and the theoretical value of λ_g. The determination of the phase factors, especially ψ should be done with great care as a change of $\pm$ 1° in ψ will alter the value of ρ by $\pm0\cdot35\%$.

Attenuation constant.—The attenuation constant α is determined from the measurement of v.s.w.r. as follows:

$$\alpha = -\frac{1}{2l} \ln \left(\frac{r-1}{r+1}\right) \text{ nepers per meter} \tag{19}$$

where l is the length of the waveguide in meters. It may be pointed out that $\alpha = \alpha_r$ as given by (19) is different from $\alpha = \alpha_m$ measured under matched condition. The relation between α_r and α_m is

$$\frac{\alpha_r}{\alpha_m} = \frac{r^2+1}{2r} \tag{20}$$

So, for every R_m, α is calculated from the measured r in order to obtain the true value of the reflection coefficient for different plates.

RESULTS AND DISCUSSION

The results of measurements for some metals are given in Table I. There is usually some variations in conductivity among different specimens of the same material. So, in each case a number of plates were tested which is indicated by the subscripts. The conductivities as obtained by Maxwell (1947) and Beck and Dawson (1950) are also given in the same table for the sake of comparison.

TABLE I

Conductivity of metals at microwave frequencies

Material	σ expl. in $10^7 \frac{\text{mho}}{\text{meter}}$ at 9697·9 mc./s.	σ (Beck) in $10^7 \frac{\text{mho}}{\text{meter}}$ at 9000 mc./s. for wires	σ (Maxwell) in $10^7 \frac{\text{mho}}{\text{meter}}$ at K band for machined surface
Pure silver (Electrolytic) ..	$5{\cdot}92_2$	6·155	2·66
Copper (Commercial) ..	$5{\cdot}52_4$	5·970	4·65
Duralumin (Commercial) ..	$2{\cdot}58_4$	..	..
Brass (Commercial) ..	$1{\cdot}36_4$	1·566	1·11

The ultimate accuracy of σ is determined by the accuracies with which λ_g, f and ρ can be measured. In order to determine ρ as accurately as possible, the measurements of r and nodal points have been done by following the same procedure as suggested by Sorrows, etc. (1951). The maximum error introduced in the determination of ρ is due to the difficulty of reading correctly the minimum signal in the presence of the residual noise of the detector system. The noise of the detector system at the operating gain control could not be reduced below $1\cdot7\,\mu a$ at the output which remained practically constant during the measurements. It is believed that a greater accuracy can be attained by using a detector system of higher gain and lower noise level. The maximum probable error that may be introduced in the measurements of λ_g, f and ρ has been experimentally evaluated (Table II).

TABLE II

Probable error in λ_g, f and ρ

λ_g	f	ρ
$\pm 0\cdot0094$ cm.	$\pm 0\cdot46$ mc./s.	$\pm 0\cdot004$

The maximum accuracy with which σ can be determined by this method is about 3% as experimentally evaluated.

As a check of the measured values of r, the attenuation constant α of the H_{01}-wave in a standard silver plated brass guide (RG-51 U) has been calculated with the help of (19) when the guide is terminated by different plates. Table III gives some of the experimental values and also the corresponding values under matched condition. The theoretical values of α as calculated from the following formula is also given in the same table.

$$\alpha_{Theo} = \frac{1}{2a}\left[\frac{1}{1-\left(\frac{\lambda_0}{\lambda_c}\right)^2}\right]^{\frac{1}{2}}\left[1+\frac{2a}{b}\left(\frac{\lambda_0}{\lambda_c}\right)^2\right]\left[\frac{4\pi}{\lambda_0\mu c\sigma}\right]^{\frac{1}{2}} \quad \frac{\text{neper}}{\text{meter}} \qquad (21)$$

where, λ_c, λ_0, μ, c represent the cut-off wavelength, free-space wavelength, permeability of the free-space and velocity of electromagnetic waves in free-space respectively. For a guide of inner dimensions $1\cdot122'' \times 0\cdot497''$ (RG-51 U), and taking the experimental value of conductivity for silver as

$$\sigma_{Ag} = 5\cdot92 \times 10^7 \; \frac{\text{mho}}{\text{meter}} \text{ at a frequency of } 9697\cdot9 \text{ mc./s., the theoretical value} \qquad (22)$$

of α reduces to

$$\alpha_{Theo} = 0\cdot008 \text{ nepers per meter}$$

TABLE III

Values of attenuation constant for the H_{01}*-wave in silver plated guide* (RG-51 U)

Termination	α expl. db. per meter	α matched db. per meter	α theo. db. per meter	α (Maxwell) db. per meter at K band
Silver	1·302	0·348	..	..
Copper ..	0·497	0·099	0·069	0·51
Duralumin ..	0·770	0·124	..	..
Brass	0·743	0·115	..	..

It will be observed from the above table that the values of $\alpha_{matched}$ calculated from $\alpha_{expl.}$ obtained with terminations of copper, duralumin and brass are practically the same. The values of α obtained with silver termination is about five times that of the theoretical value and differs widely from the other values, which remains to be explained. The experimental values ($\alpha_{matched}$) are in all the cases higher than the theoretical values. This difference may be due to the fact that the value of σ taken for the calculation of $\alpha_{Theo.}$ is that of pure silver. Also, the fact that the silver plating of commercially available silver plated waveguide being crystalline and porous may contribute to the increase of the observed value of α (Vogelman, 1953). The surface roughness of the guide may also be responsible for the increased value of the experimental α (Benson, 1953).

It may be pointed out that in the present investigations, no attempt has been made to take into account the error introduced due to the surface conditions of the terminating plate. Especially, in the case of copper, a layer of oxide coating which is easily formed when exposed to air, will alter the value of σ significantly.

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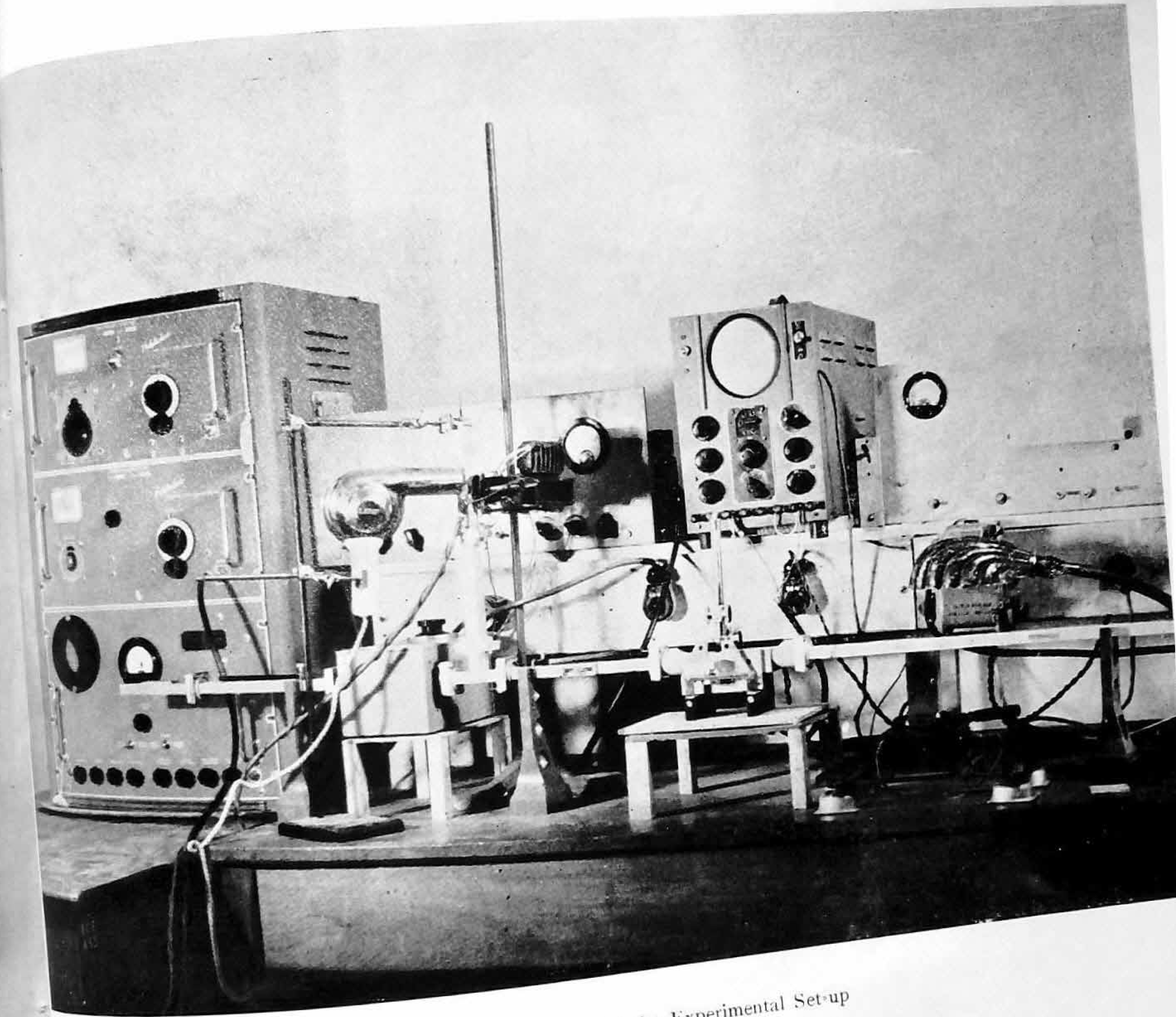

FIG. 5. Photograph of the Experimental Set-up

...tograph of the Modulating Square Wave