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STARTING OF INDUCTION MOTORS AT FREQUENCIES OTHER THAN NORMAL

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ABSTRACT

The conflicting requirements of starting and running of induction motors efficiently are discussed. The effect of higher frequency voltages on the starting of induction motors designed for operation at normal commercial frequency is analysed theoretically and test results on a commercial motor are given. The problem is dealt with both for the case of a single higher frequency voltage wave and for voltages of different frequencies applied simultaneously. The equivalent cage resistance and reactance values for a double cage motor as adopted in standard design procedures are considered and a simple expression is developed whereby a double cage motor can be represented by an equivalent circuit of a single-cage motor with suitable expressions for the rotor resistance and reactance. Test results on a commercial 11 H.P. double cage induction motor are given to indicate the similarity of results obtained theoretically and by experiment.

Though the polyphase squirrel cage induction motors have become the most popular drive for outputs less than 10 H.P. their application to larger loads is seriously limited because of their inherent poor starting characteristics. The requirements of a good starting characteristic are based on criteria which are in conflict with that of a good running characteristic. A motor designed and constructed to develop a large starting torque runs at a lower power factor and at a lower efficiency.

While a large starting torque is usually required to enable the motor to rapidly accelerate at its normal load to full speed in a few seconds, a low starting current is required to prevent overload of the line, dip in the supply voltage and overheating of the motor, before it has picked up speed.

The conflicting requirements in designing a squirrel cage motor with both good starting and running performances have led designers to design and construct motors with excellent running characteristics and to use special devices for starting.[1, 2, 3]

The resistance, reactance and flux density of a motor are dependent on the frequency of the power supply.

However, it is not always practicable to change the supply frequency and this drawback has discouraged research workers so far not only from designing variable frequency starters but also from studying the performance of the motors at frequencies other than that for which they were designed.[4]

The characteristics of operation at starting and when running of commercial motors at frequencies other than the design frequency are investigated theoretically in this paper both for the case when a single frequency is applied as also when more than one frequency are applied simultaneously. A method of simplifying the equivalent circuit in the analysis of double-cage squirrel-cage motor is developed and experimental data on tests carried out on a 11 H.P., 50 cycle motor are given. Based on the theoretical analysis and experimental data obtained the practicability of devising suitable variable frequency starters with a view to improve the starting characteristics is discussed.

The e.m.f. equation for a motor is given by the relation

$$e \propto 4 \cdot 44 f N \phi$$

Thus an increase in the frequency of supply at constant voltage

(1) reduces the mutual (or working) flux

(2) increases the reactance or winding

(3) increases the resistances due to skin effect.

The reduction of the working flux reduces the core, surface and pulsation losses. The extent of this effect is not exactly known and has to be investigated both theoretically and experimentally.

The increase in reactance lowers the motor current as well as the torque. The increase in resistance decreases the current and increases the torque; when a motor is designed with deep bar rotors, it is possible to compensate the increase in torque due to increase in resistance against the decrease in torque due to increase in reactance. With this end in view motors have been designed with deep bar rotors, but their operation at different frequencies has not yet been investigated.[5]

Using the approximate equivalent circuit, for an induction motor, the current is given by:

$$I = \frac{E}{\left(r_1 + \frac{r_2}{s}\right) + j\,(x_1 + x_2)} \tag{1}$$

Remembering that $x \propto f$, the variation of the current and power factor with frequency, can be calculated. Figures 1 and 2 indicate these variations for a slip $= 1 \cdot 0$, in terms of per unit values.

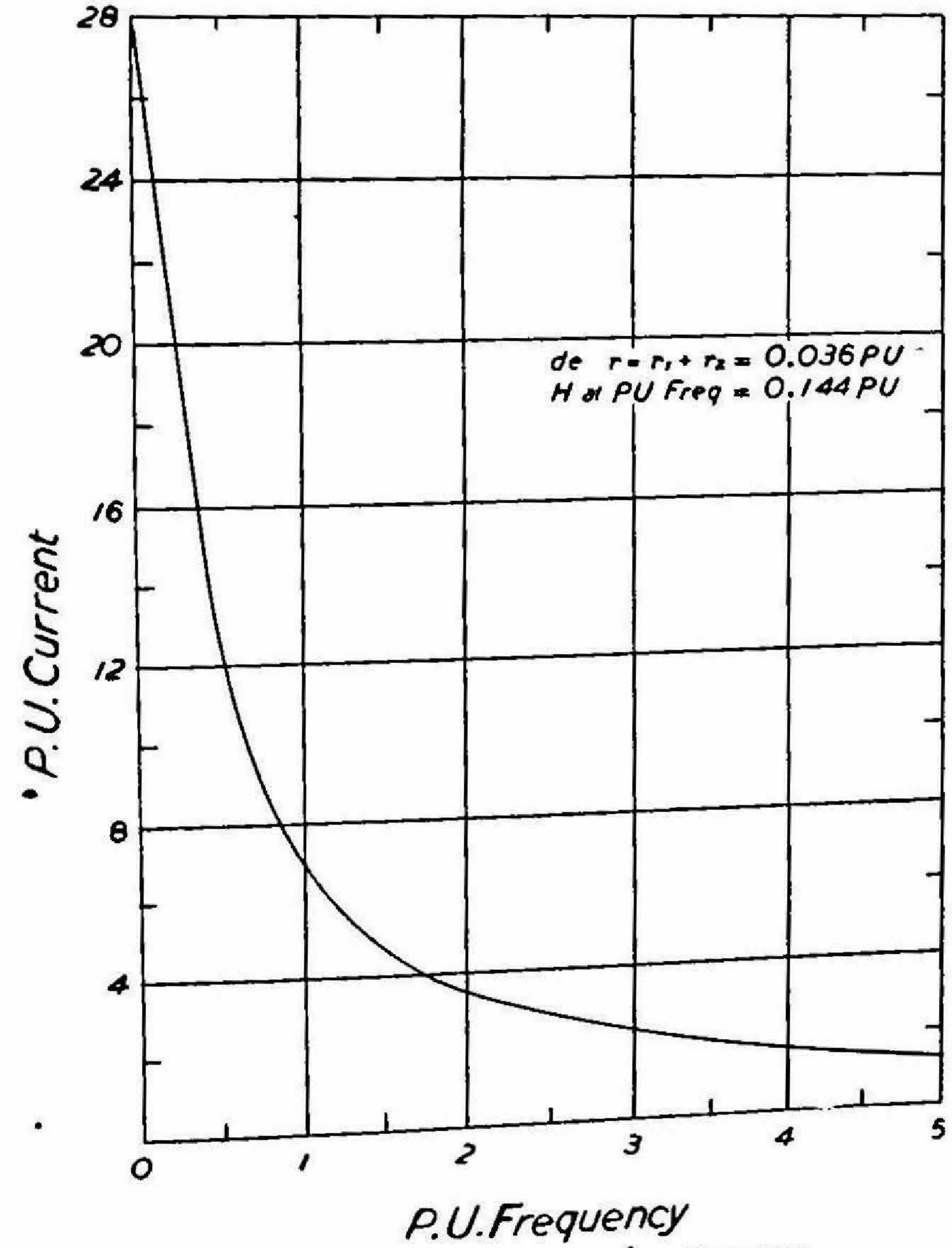

FIG. 1. Current in Locked Motor-Frequency.

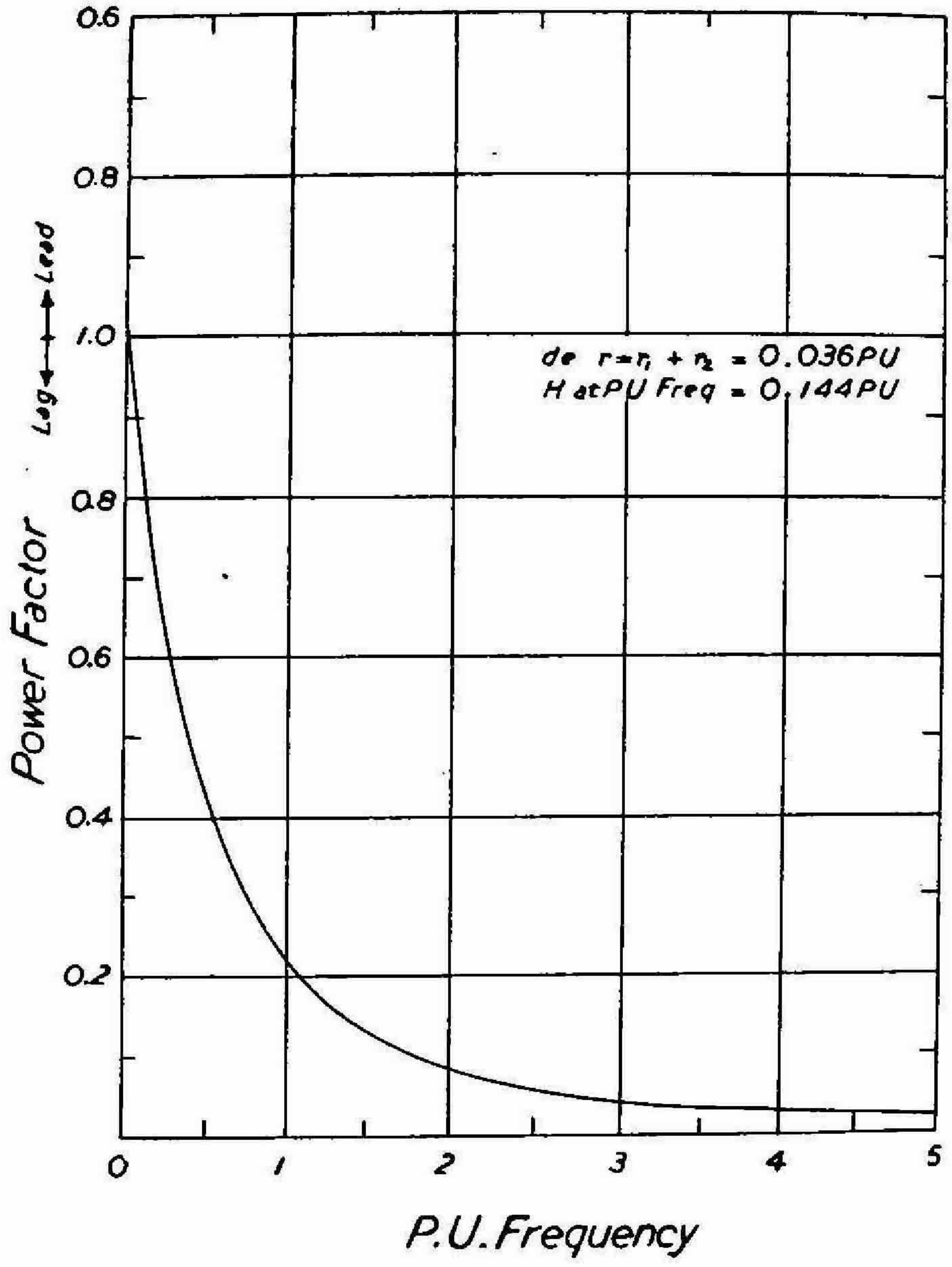

FIG. 2. Power Factor-Frequency.

The torque of a motor is given by

$$T = \frac{r_2}{s\left[\left(r_1 + \frac{r_2}{s}\right) + j\,(x_1 + x_2)\right]^2} \qquad (2)$$

Curve in Fig. 3 shows the variation of the starting torque with frequency in per unit values, skin effect being neglected. All the curves have been drawn on a per unit basis to enable general conclusions to be drawn.

From the curves it can be seen that a motor drawing a normal locked current of 6·75 P.U. at power factor of 0·243 at normal frequency draws

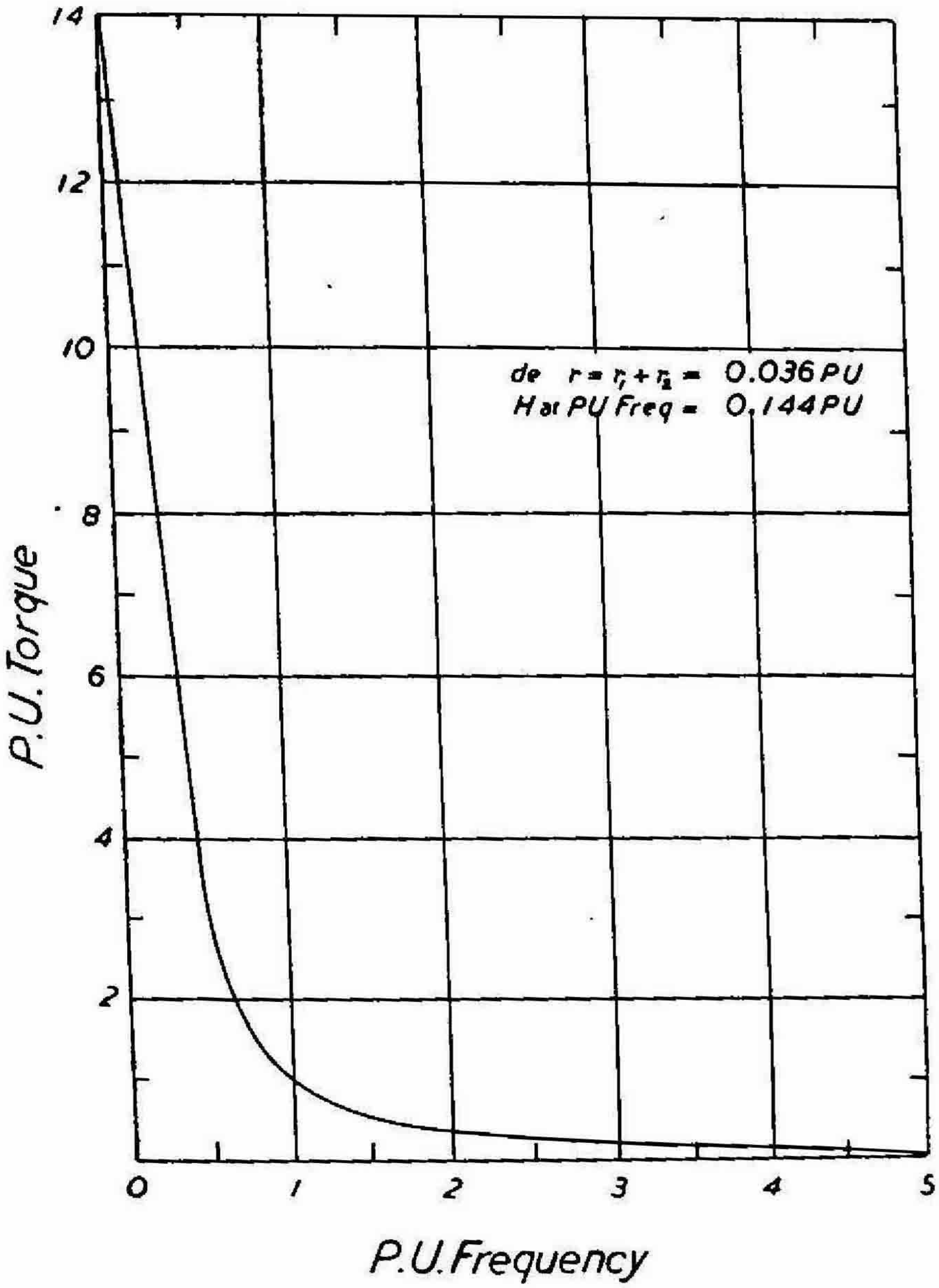

FIG. 3. Starting Torque-Frequency.

a current of 2·31 P.U. at a power factor of 0·083 at triple frequency, so that the wattage dissipation at start with triple frequency is roughly $\frac{1}{9}$ of the dissipation at normal frequency. The starting torque is also seen to fall very rapidly with the increase in frequency.

Thus on the basis of the discussion above and from a study of the curves in Figs. 1 to 3 one can draw the following conclusions:

(1) When the motor starts on no load and draws its supply from a small capacity line, the motor has to be started on a high frequency.

(2) When the motor starts on load and draws its supply from a large capacity line, it has to be started on a low frequency.

(3) When the starting current as well as the starting torque has to be considered, the motor has to start on its normal frequency.

The above discussions are based on the consideration of a single-frequency voltage applied to the motor. It is, however, known that in the commercial supply there are harmonics present in the voltage wave. There are also harmonics generated due to slots in machines.[6, 7, 8]

A distorted voltage wave can be analysed and seen to be the resultant of two or more sine waves of different frequencies superimposed over each other.[9, 10] The performance of a motor subjected simultaneously to voltages of different frequencies can therefore be studied by using a voltage with distorted wave form, whose components are known. In the following investigations it is assumed that there is no appreciable saturation of the core material.

In the general expression for the rotor current, equation (1), slip for the mth harmonic

$$s_m = \frac{mN_s - N}{mN_s}$$

$$= \frac{m + s_1 - 1}{m} \qquad (3)$$

Let the supply voltage E consist of a voltage E_1 at the fundamental frequency, a third harmonic voltage of E_3, a fifth harmonic voltage of E_5, etc.

Let

$$\frac{E_3}{y_3} = \frac{E_5}{y_5} = \frac{E_7}{y_7} = \frac{E_1}{y_1} = E_0 \text{ (say)}$$

Then

$$E = (E_1^2 + E_3^2 + E_5^2 + \ldots.)^{\frac{1}{2}}$$

$$= E_0 (y_1^2 + y_2^2 + y_3^2 + \ldots.)^{\frac{1}{2}}$$

$$= E_0 \left[\sum_{3,5,\ldots}^{\infty} y^2 \right]^{\frac{1}{2}} \qquad (4)$$

Hence

$$\frac{E_1}{y_1} = \frac{E_3}{y_3} = \frac{E_5}{y_5} \cdots\cdots\cdots = \frac{E}{(\Sigma y^2)^{\frac{1}{2}}} \qquad (5)$$

As the percentage of the harmonic voltages in the wave form are varied, keeping the effective voltage E constant, the corresponding variations in the several currents are obtained as follows:—

I_1 = current due to $E_1 = E_1/Z_1$

I_3 = current due to $E_3 = E_3/Z_3$

I_5 = current due to $E_5 = E_5/Z_5$

where

$$Z_m = \left[r_1 + \frac{r_2}{s_m}\right] + jm\,(x_1 + x_2)$$

assuming that the resistances remain unchanged.

Then the total current flowing $= I = (I_1^2 + I_3^2 + \ldots.)^{\frac{1}{2}}$

Substituting for I_1, I_3, I_5, etc.

$$I = \frac{E}{[\Sigma y^2]^{\frac{1}{2}}} \cdot \left[\frac{y_1^2}{Z_1^2} + \frac{y_3^2}{Z_3^2} + \frac{y_5^2}{Z_5^2} + \ldots\ldots\right]^{\frac{1}{2}} \quad (6)$$

If there is only one harmonic of considerable magnitude besides the fundamental, as is very often the case, the expression is considerably simplified.

Expression (6) then simplifies to

$$I = \frac{E}{\sqrt{1 + y_m^2}} \left[\frac{1}{Z_1^2} + \frac{y_m^2}{Z_m^2}\right]^{\frac{1}{2}} \quad (7)$$

Considering voltages of two waveforms, one containing Y_{m_1} P.U., and the other Y_{m_2} P.U., respectively, of the *m*th harmonic, referred to the fundamental, the slip referred to the fundamental at which the motor will draw the same current from both the voltages is:—

$$\frac{E}{\sqrt{1 + y_{m_1}^2}} \left[\frac{1}{Z_1^2} + \frac{y_{m_1}^2}{Z_m^2}\right]^{\frac{1}{2}} = \frac{E}{\sqrt{1 + y_{m_2}^2}} \left[\frac{1}{Z_1^2} + \frac{y_{m_2}^2}{Z_m^2}\right]^{\frac{1}{2}} \quad (8)$$

Whence

$$\frac{1}{Z_1^2}(y_{m_1}^2 - y_{m_2}^2) = \frac{1}{Z_m^2}(y_{m_1}^2 - y_{m_2}^2) \quad (9)$$

$$Z_1 = Z_m \quad (10)$$

The slip at which the currents are equal is, therefore, given by the relation

$$\left(r_1 + \frac{r_2}{s_1}\right)^2 + (x_1 + x_2^2) = \left(r_1 + \frac{r_2}{s_m}\right)^2 + m^2\,(x_1 + x_2)^2 \quad (11)$$

which is independent of Y_m, the harmonic content of the voltage wave.

Hence, all curves relating I with *s*, for different harmonic contents, will pass through a common point indicating that the net current for different proportions of harmonic voltage content will be the same at a particular speed.

Neglecting

$$\left(r_1 + \frac{r_2}{s_m}\right)^2 \text{ as } << m^2x^2$$

From (11)

$$s_1 = \frac{r_2}{x\sqrt{m^2 - 1} - r_1} \tag{12}$$

The value of s_1 is found to be 0·05 in the case of the 11 H.P. motor discussed later.

It may therefore be concluded that the harmonic voltage, if artificially applied during starting, can be cut off when the motor has attained a speed of $(1 - s_1)\, N_s$ without affecting the current of the motor. The presence of impurities in the commercial supply tends to reduce the current drawn by the motor while starting only.

A similar relation can be derived for the torque at standstill.

$$T_1 = \frac{E_1{}^2 r_2}{s_1\left[\left(r_1 + \frac{r_2}{s_1}\right)^2 + x^2\right]} \text{ P.U.} \tag{13}$$

$$T_m = \frac{E_m{}^2 r_2}{s_m\left[\left(r_1 + \frac{r_2}{s_m}\right)^2 + (mx)^2\right]} \text{ P.U.} \tag{14}$$

If the mth harmonic is switched off at a slip of s_m so as not to result in any jump in torque

$$T_1 = T_m$$

If

$$E_1 = E_m,$$

$$\frac{r_2}{s_1\left[\left(r_1 + \frac{r_2}{s_1}\right)^2 + x^2\right]} = \frac{r_2}{s_m\left[\left(r_1 + \frac{r_2}{s_m}\right)^2 + m^2x^2\right]} \tag{15}$$

i.e.,

$$r_1{}^2 (s_1 - s_m) + x^2 (s_1 - m^2 s_m) + r_2{}^2\left(\frac{1}{s_1} - \frac{1}{s_m}\right) = 0 \tag{16}$$

Neglecting $\left(r_1 + \frac{r_2}{s_m}\right)$ and re-arranging

$$s_1{}^2\, [r_1{}^2 + (1 - m)\, x^2] + s_1\, [2r_1r_2 - m\,(m - 1)\, x^2] + r_2{}^2 = 0 \tag{17}$$

Neglecting $r_1{}^2$ and r_1r_2 as $<< (1 - m)\, x^2$ and $m\,(m - 1)\, x^2$

$$s_1{}^2\, (1 - m)\, x^2 - s_1 m\, (m - 1)\, x^2 + r_2{}^2 = 0$$

i.e.,

$$s_1 = -\frac{mx}{2} \pm \frac{m}{2}\left[1 + \frac{2r_2^2}{m(m-1)x^2}\right]^{\frac{1}{2}} \tag{18}$$

But

$$\frac{2r_2^2}{m(m-1)x^2} \text{ is very small.}$$

$$\therefore\ s_1 = \frac{r_2^2}{2(m-1)x^2}. \tag{19}$$

For the 11 H.P. motor considered the value of s_1 works out to be 0·0045.

Starting of Induction Motors

In considering the problem of starting of an induction motor, one has first to determine how a motor of standard design will behave at different frequencies and secondly to find out how the design can be altered to take advantage of variable frequency starting.

From standard design procedures actually followed by motor manufacturers, one can deduce that for a double-cage induction motor, the equivalent cage reactance

$$x_2 = \frac{x_3(x_4-x_5)^2+x_4(x_3-x_5)^2+2x_5(x_3-x_5)+r_3^2x_4+r_4^2x_3+2r_3r_4x_5}{(x_3+x_4-2x_5)^2+(r_3+r_4)^2} \tag{20}$$

and the equivalent cage resistance

$$r_2 = \frac{r_3(x_4-x_5)^2+r_4(x_3-x_5)^2+r_3r_4(r_3+r_4)}{(x_3+x_4-2x_5)^2+(r_3+r_4)^2} \tag{21}$$

where

r_3 = Top cage resistance

r_4 = Bottom cage resistance

x_3 = Top cage reactance

x_4 = Bottom cage reactance

x_5 = mutual reactance.

Hence using the usual equivalent circuit of Fig. 4, one obtains an expression for the stator current

$$I_s = \frac{E}{\frac{(r_2+jx_2)jx_m}{r_2+j(x_2+x_m)} + r_1 + jx_1} \tag{22}$$

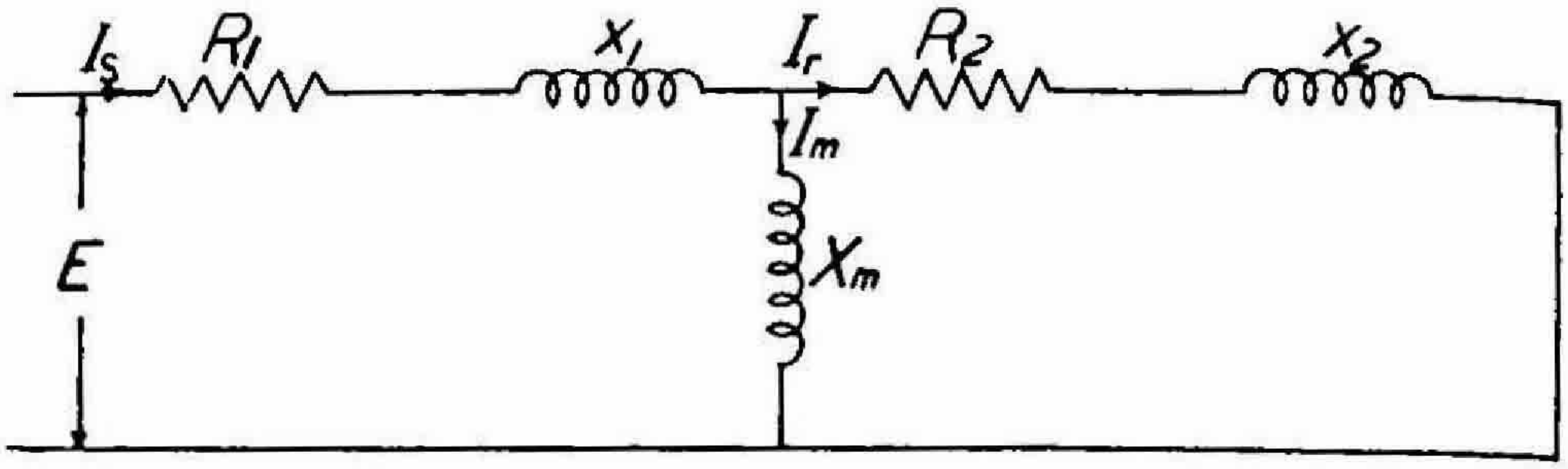

FIG. 4.

By use of the Equation (22) the locked current for the 11 H.P. motor, described in the appendix, was calculated and the results are plotted in Fig. 7.

DOUBLE CAGE INDUCTION MOTORS

Double cage induction motors present a curious problem in that the way, in which the two cages share the currents and combine the torques, is very uncertain and depends on a large number of factors such as conductor sizes, their location, leakage of flux at the ends, saturation of core, laminar thickness, speed, voltage impressed, etc.[11, 12]

However, a simple expression has been derived below to obtain an equivalent single cage rotor. While the values obtained by the use of this expression cannot be very accurate during normal running, they have been found to be surprisingly close enough at starting and under locked condition when the slip is nearly unity.

If the phase angle between the currents drawn by top cage and bottom cage be α, Fig. 5,

$$I_t^2 + I_b^2 + 2I_tI_b \cos \alpha = I_e^2 \tag{23}$$

At starting all the losses are in the top cage.

and

$$\left.\begin{aligned} I_t^2R_t &= I_e^2R_e \\ I_t^2X_t &= I_e^2X_e \end{aligned}\right\} \tag{24}$$

The voltage drop along the vector of the bottom cage voltage is

$$I_tR_t \cos \alpha - I_tX_t \sin \alpha = I_bR_b - I_tX_m \sin \alpha \tag{25}$$

But I_bR_b is very small.

Hence

$$I_tR_t \cos \alpha - I_tX_t \sin \alpha = - I_tX_m \sin \alpha \tag{26}$$

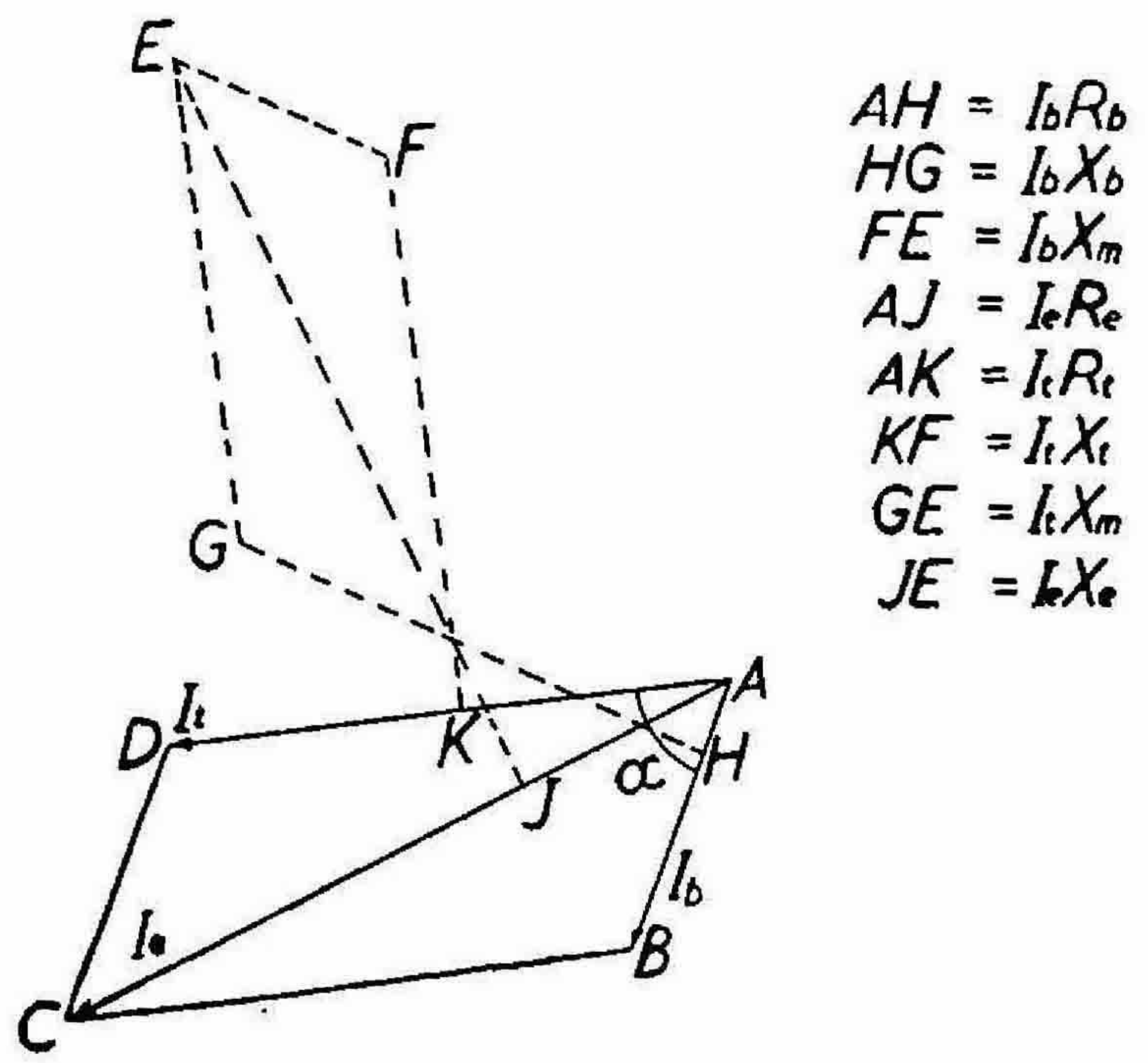

FIG. 5.

Resolving at right angles to I_b

$$I_bX_b + I_tX_m \cos\alpha = I_bX_m + I_tX_t \cos\alpha + I_tR_t \sin\alpha \tag{27}$$

From (26)

$$\sin\alpha = \frac{I_tR_t \cos\alpha}{I_t(x_t - x_m)} \tag{28}$$

From (27)

$$\sin\alpha = \frac{I_b(x_b - x_m) - I_t(x_t - x_m)\cos\alpha}{I_tR_t} \tag{29}$$

Equating (28) and (29)

$$I_t^2R_t \cos\alpha = I_tI_b(x_t - x_m)(x_b - x_m) - I_t^2(x_t - x_m)^2 \cos\alpha$$

or

$$\cos\alpha = \frac{I_b}{I_t} \cdot \frac{(x_t - x_m)(x_b - x_m)}{R_t^2 + (x_t - x_m)^2} \tag{30}$$

Substituting for $\cos\alpha$ from Equation (30) in Equation 29.

$$\sin\alpha = \frac{I_b(x_b - x_m) - I_t(x_t - x_m) \cdot \frac{I_b}{I_t} \cdot \frac{(x_t - x_m)(x_b - x_m)}{\{R_t^2 + (x_t - x_m)^2\}}}{I_tR_t}$$

which reduces to

$$\sin a = \frac{I_b R_t (x_b - x_m)}{I_t [R_t^2 + (x_t - x_m)^2]} \tag{31}$$

From (30) and (31)

$$\left[\frac{I_b}{I_t} \cdot \frac{(x_t - x_m)(x_b - x_m)}{R_t^2 + (x_t - x_m)^2}\right]^2 + \left[\frac{I_b}{I_t} \cdot \frac{R_t (x_b - x_m)}{R_t^2 + (x_t - x_m)^2}\right]^2 = 1$$

$$\therefore \frac{I_b^2}{I_t^2} \cdot \frac{(x_b - x_m)^2}{\{R_t^2 + (x_t - x_m)^2\}^2} \cdot [R_t^2 + (x_t - x_m)^2] = 1 \tag{32}$$

or

$$\left(\frac{I_b}{I_t}\right)^2 = \frac{R_t^2 + (x_t - x_m)^2}{(x_b - x_m)^2} \tag{32}$$

Substituting in (23)

$$I_e^2 = I_t^2 + 2I_t \cdot I_b \cdot \frac{I_b}{I_t} \cdot \frac{(x_t - x_m)(x_b - x_m)}{R_t^2 + (x_t - x_m)^2} + I_b^2$$

Neglecting I_b^2, since I_b is very small at start, and using equation (32)

$$I_e^2 = I_t^2 + 2I_t^2 \cdot \frac{R_t^2 + (x_t - x_m)^2}{(x_b - x_m)^2} \cdot \frac{(x_t - x_m)(x_b - x_m)}{R_t^2 + (x_t - x_m)^2}$$

$$I_e^2 = I_t^2 \left[1 + 2 \cdot \frac{x_t - x_m}{x_b - x_m}\right] \tag{33}$$

In virtue of (24)

$$R_e = R_t \cdot \frac{x_b - x_m}{(x_b - x_m) + 2(x_t - x_m)} \tag{34}$$

$$X_e = X_t \cdot \frac{x_b - x_m}{(x_b - x_m) + 2(x_t - x_m)} \tag{35}$$

Equations (34) and (35) give the values of equivalent resistance and equivalent reactance, for a double cage rotor which can be used as a single cage rotor.

Hence it is evident that a double cage winding can be replaced during starting by a single cage winding such that the impedance of the single cage is the impedance of the top cage shunted with an impedance equal to $\frac{x_b - x_m}{2(x_t - x_m)}$ times the impedance of the top cage.

The equivalent circuit is shown in Fig. 6 *a*.

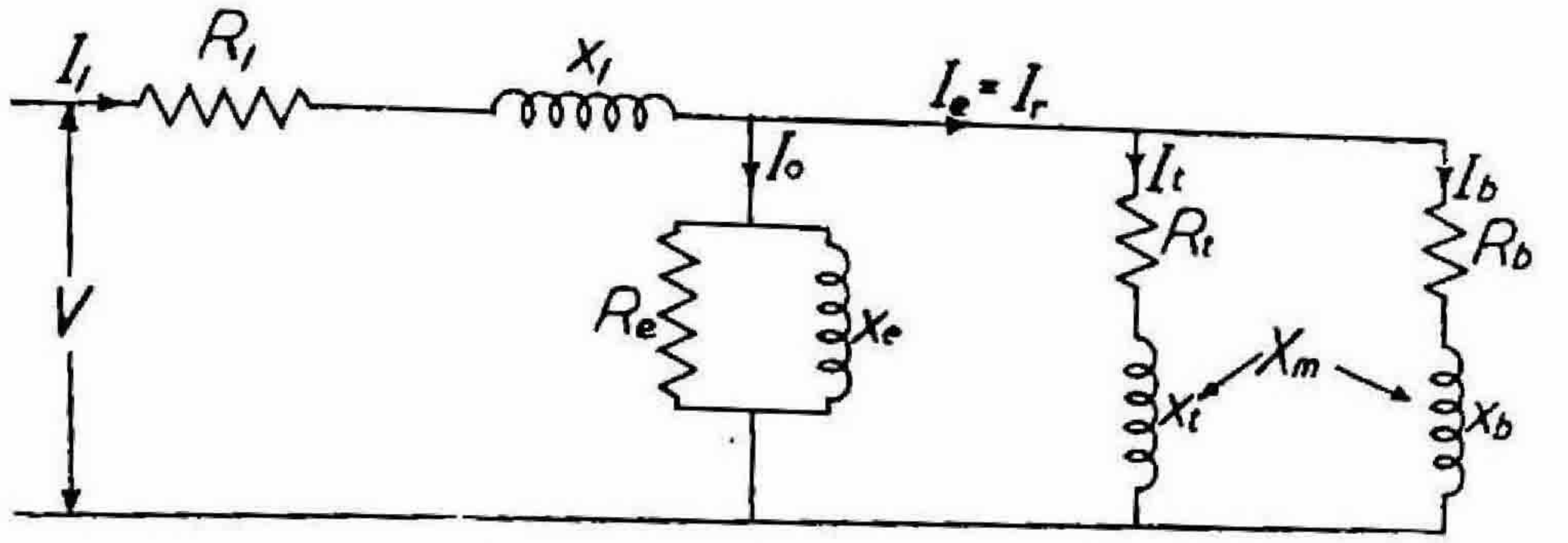

FIG. 6 *a*.

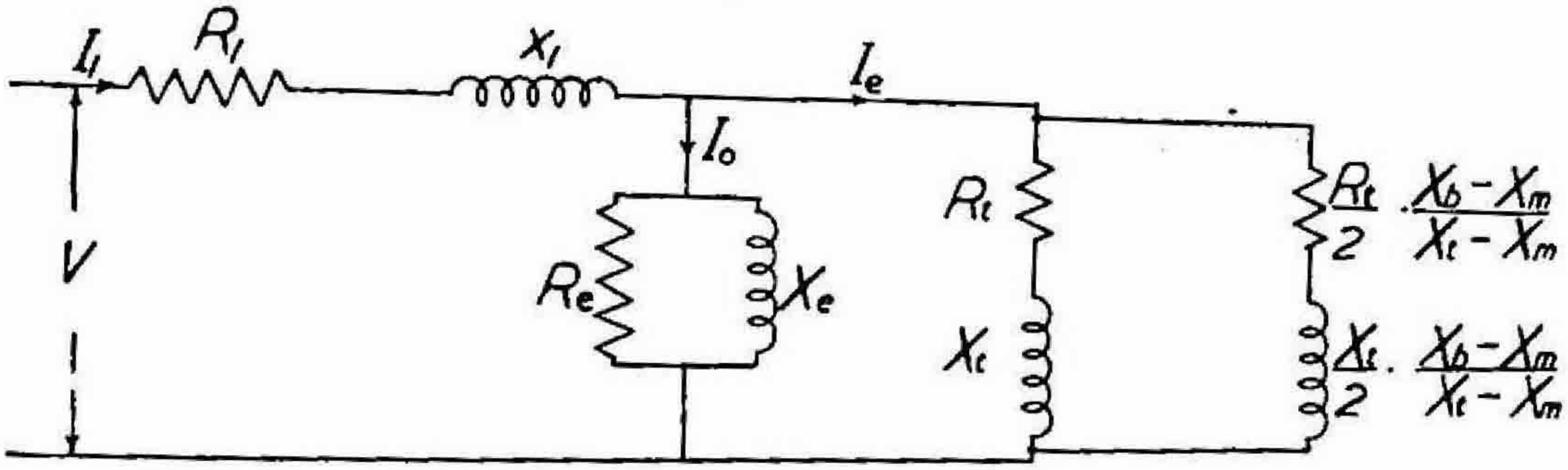

FIG. 6 *b*.

The above rigorous circuit with the mutual reactance x_m between cages has been replaced by a single cage in Fig. 6 *b* in terms of the results deduced above.

EXPERIMENTAL RESULTS

A 11 H.P., 3 phase, double cage, 4-pole, 420 V, 50 cycles induction motor with data as given in the appendix was tested in the laboratory from a frequency of 8·33 cycles to 100·00 cycles in steps of 8·33 cycles.

The readings obtained at 25, 50, 75 and 100 cycles are shown by the curves in Figs. 9–12. The same data are given in a different form in Figs. 13–15.

It will be noticed that the reactance of the winding and power factor are not constant. Hence the usual method of drawing the circle diagram by conducting short circuit test at lower voltages and linearly increasing the current at constant power factor for normal voltage is a very erroneous procedure, tending to destroy what little merit the circle diagram has.

The results of open circuit test conducted are drawn in Fig. 8.

The rise of current at lower voltages extends to as high as 400 volts at 100 cycles. At low voltages, the core loss is small. But the friction

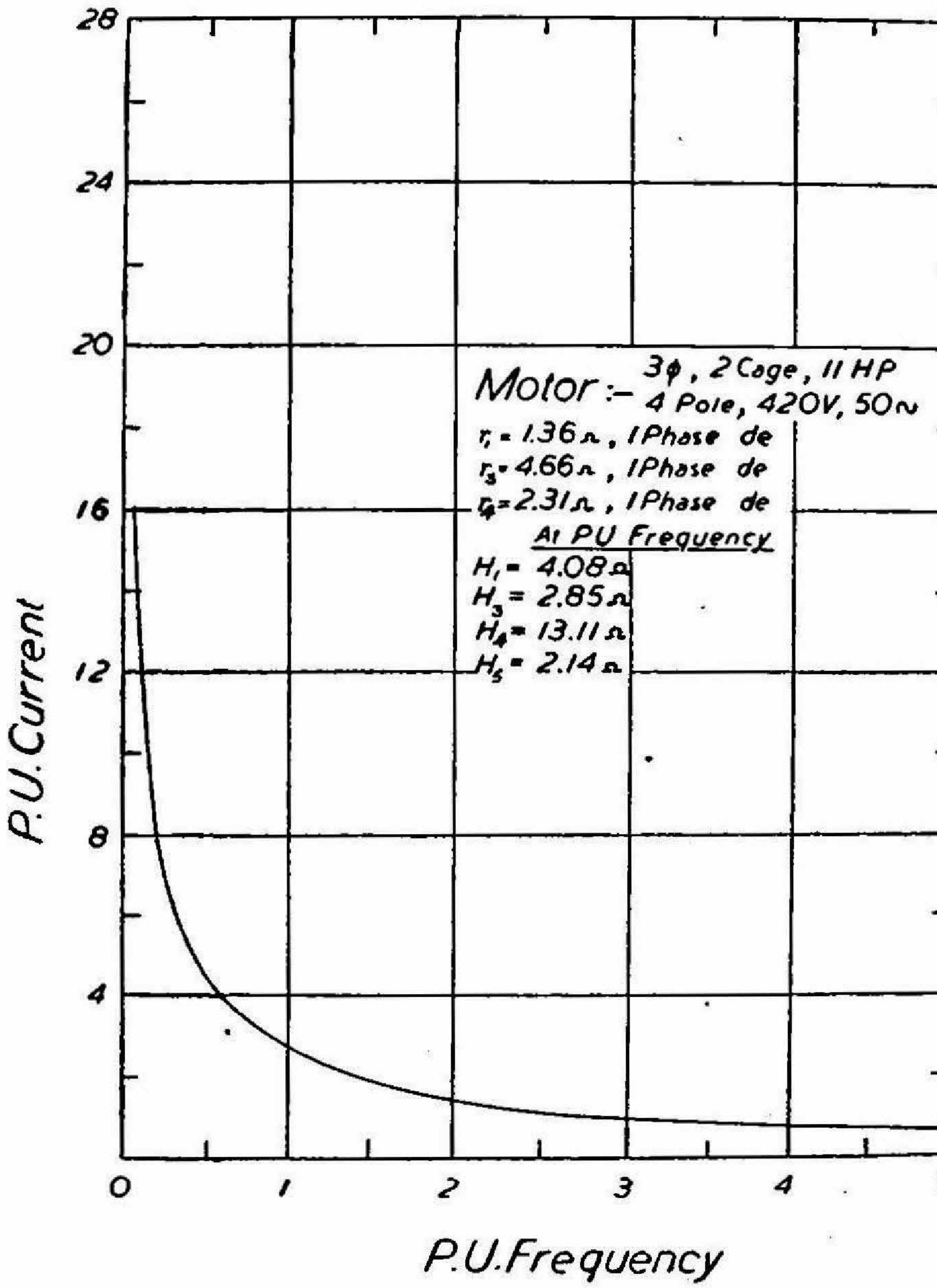

FIG. 7. Current in Locked Motor-Frequency.

and windage losses are almost constant. Hence the motor begins to draw an increased current to react with the decreased flux to produce the almost constant loss.

The increase in short circuit impedance so strikingly brought out at lower currents in Fig. 13 is because of the increased leakage reactance due to the desaturation of the teeth. The parasitic losses are increased and hence the resistance must rise with decrease of current, which we do not observe. Further readings at very low current densities with low scale instruments is required before this contradiction is established.

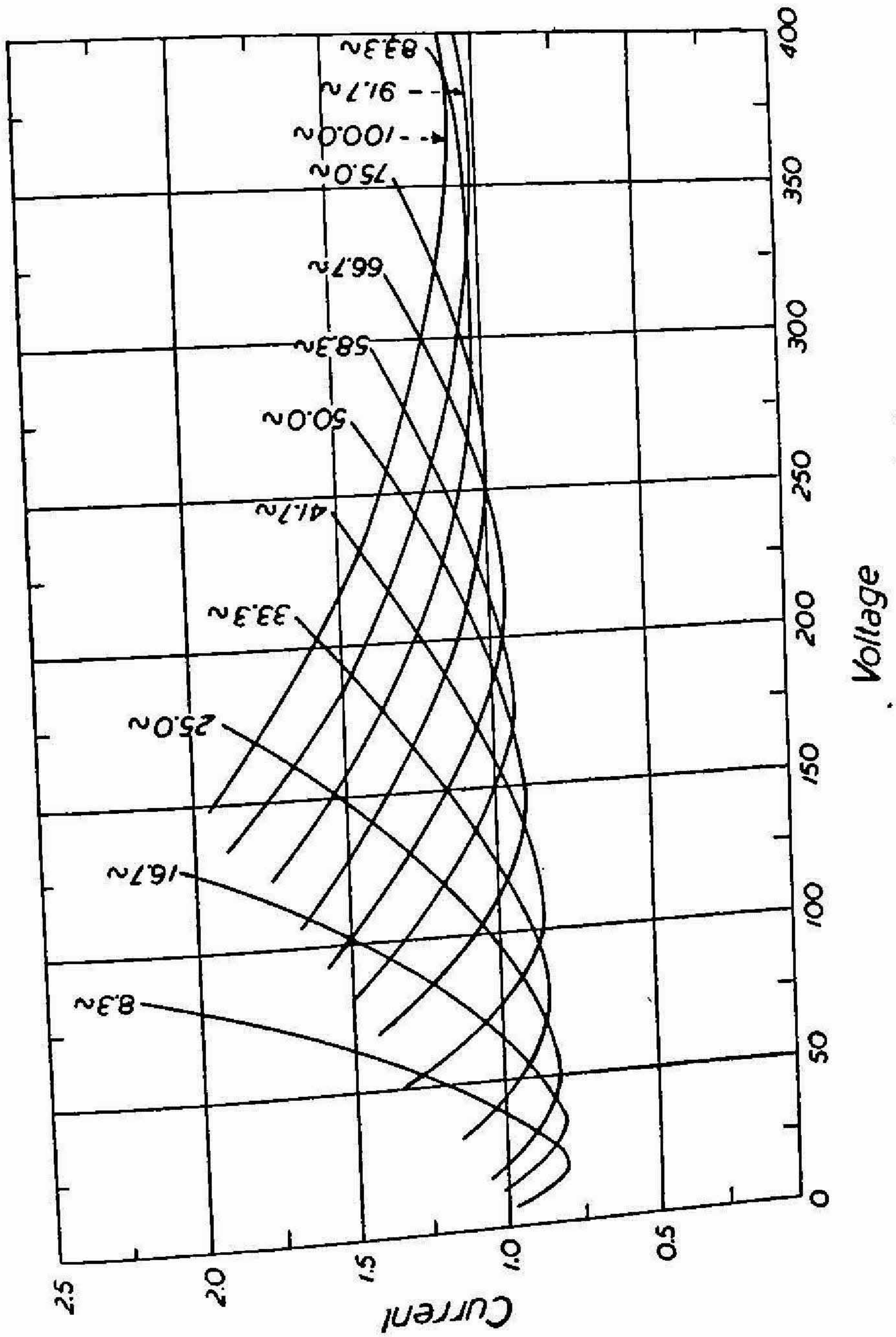

FIG. 8. Current-Voltage in Open Circuit.

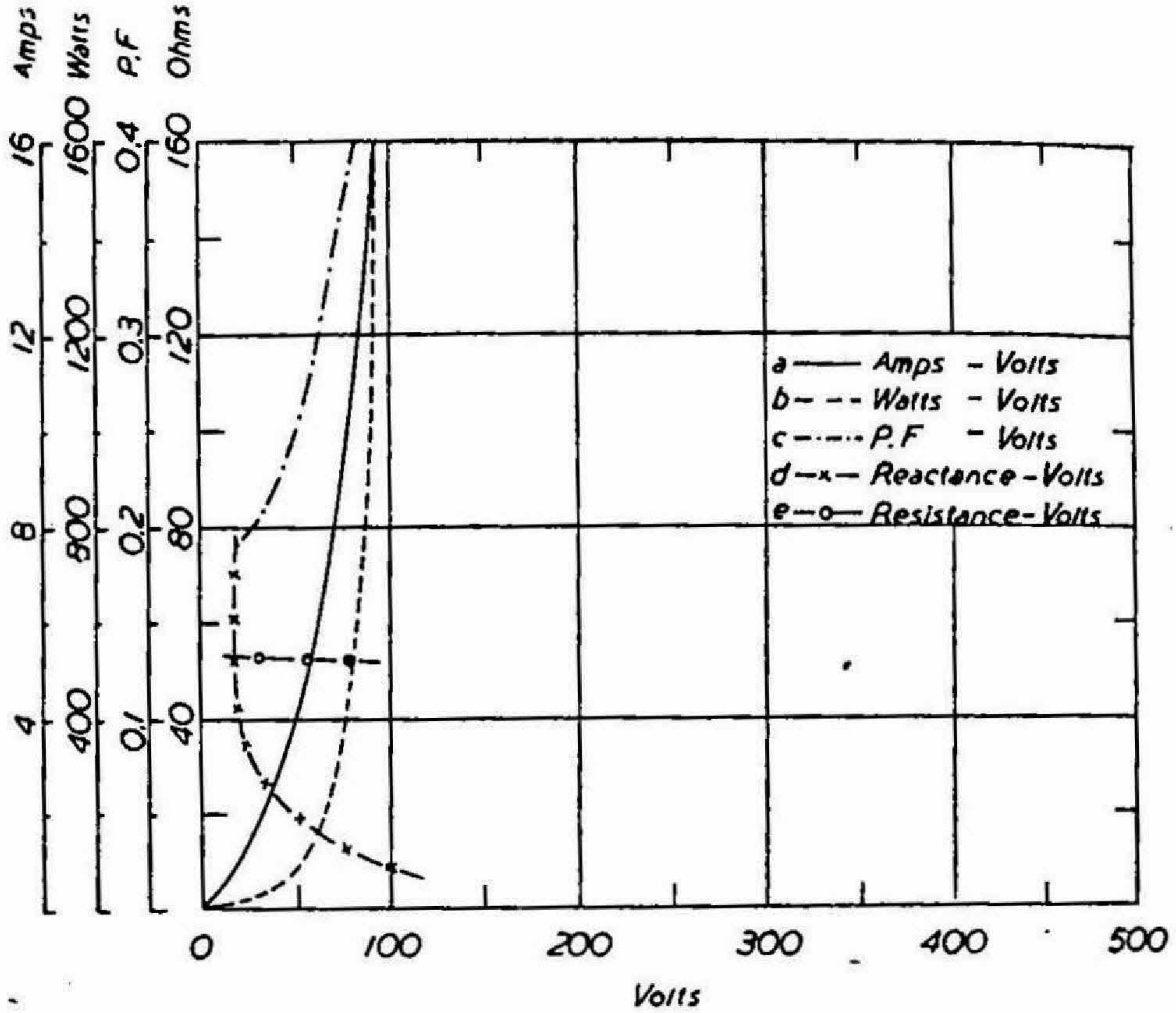

FIG. 9. 25 Cycles / Sec.

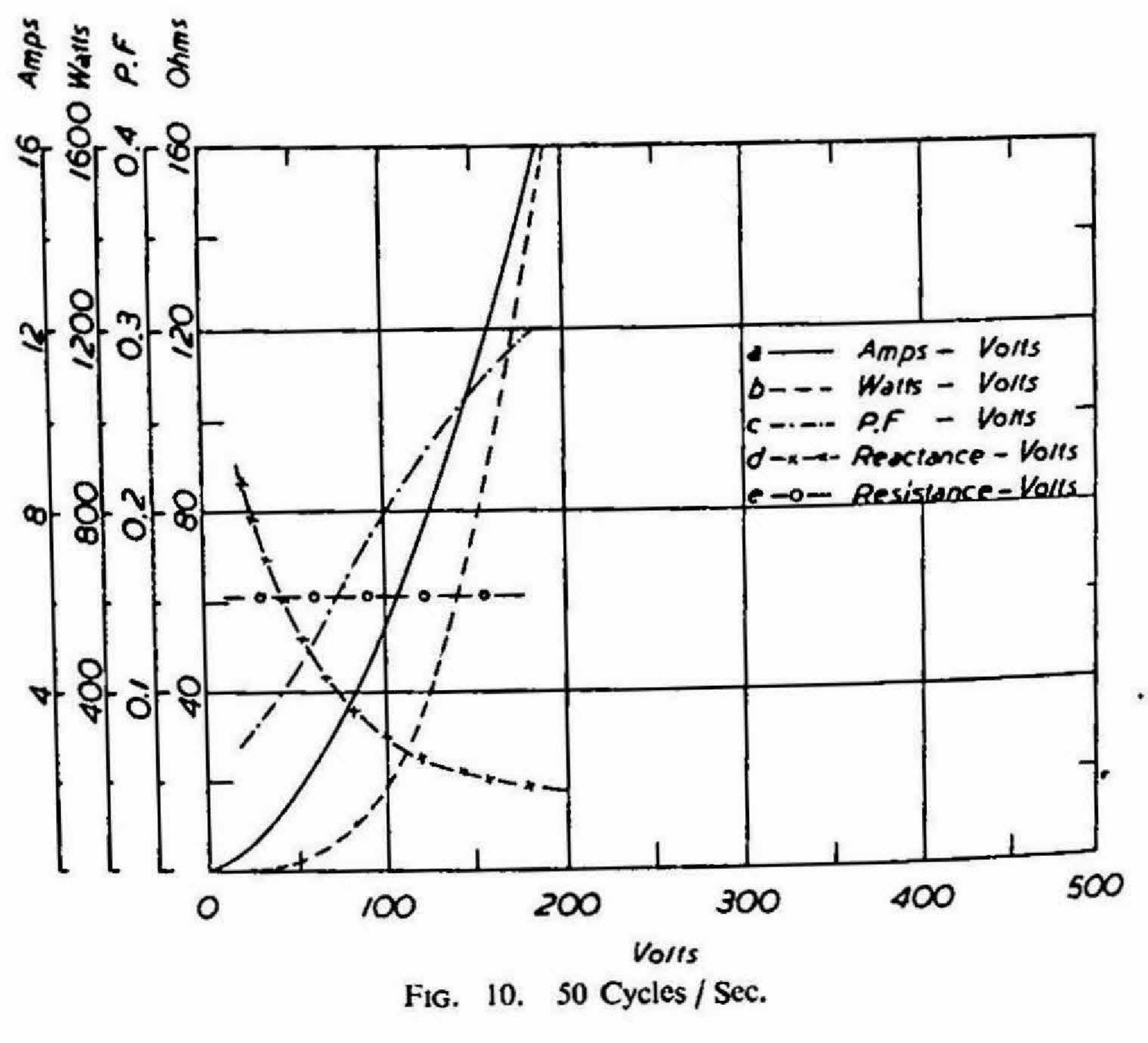

FIG. 10. 50 Cycles / Sec.

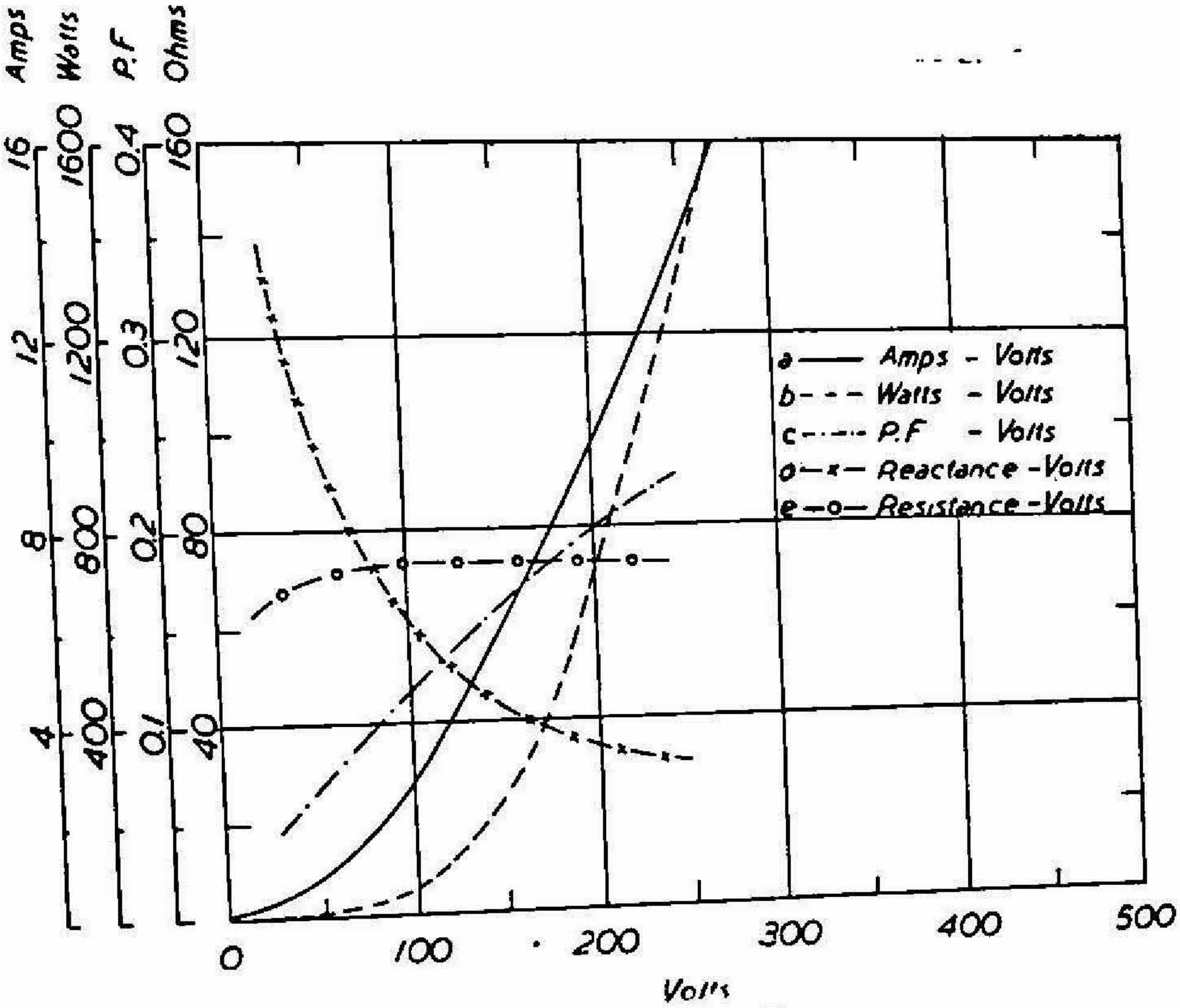

FIG. 11. 75 Cycles / Sec.

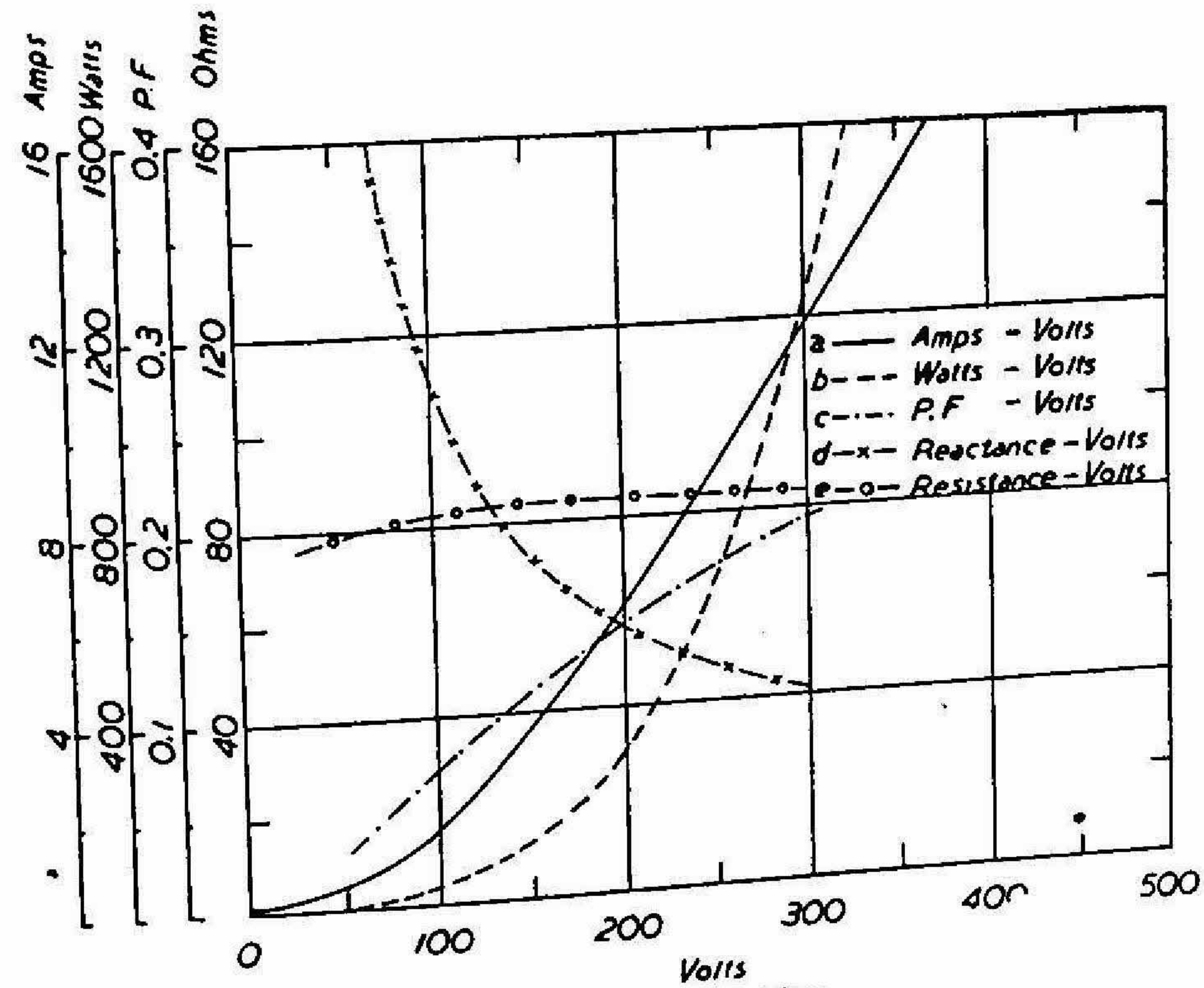

FIG. 12. 100 Cycles / Sec.

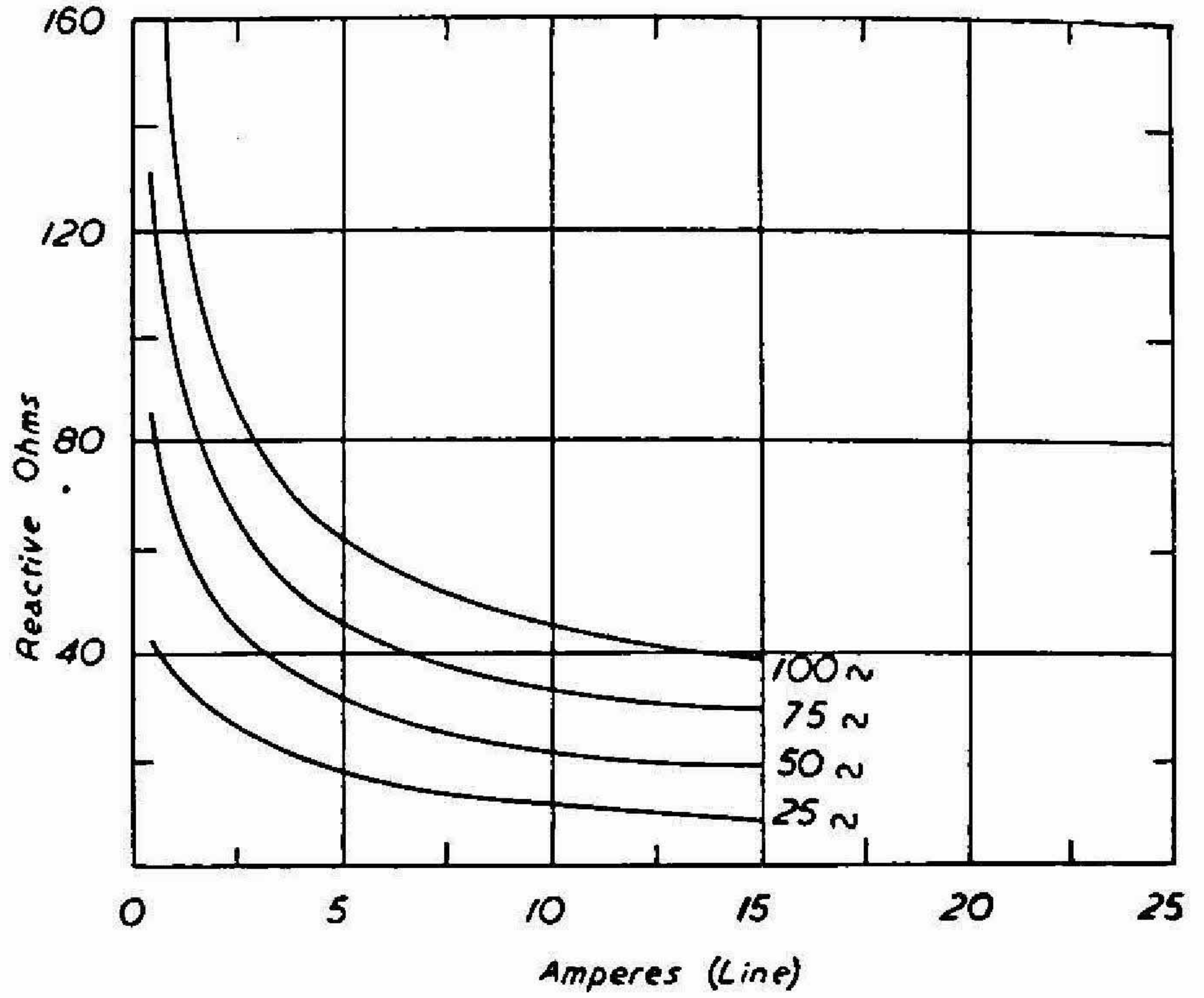
160
120
80
40
Reactive Ohms
0
5
10
15
20
25
Amperes (Line)
100 ~
75 ~
50 ~
25 ~

FIG. 13.

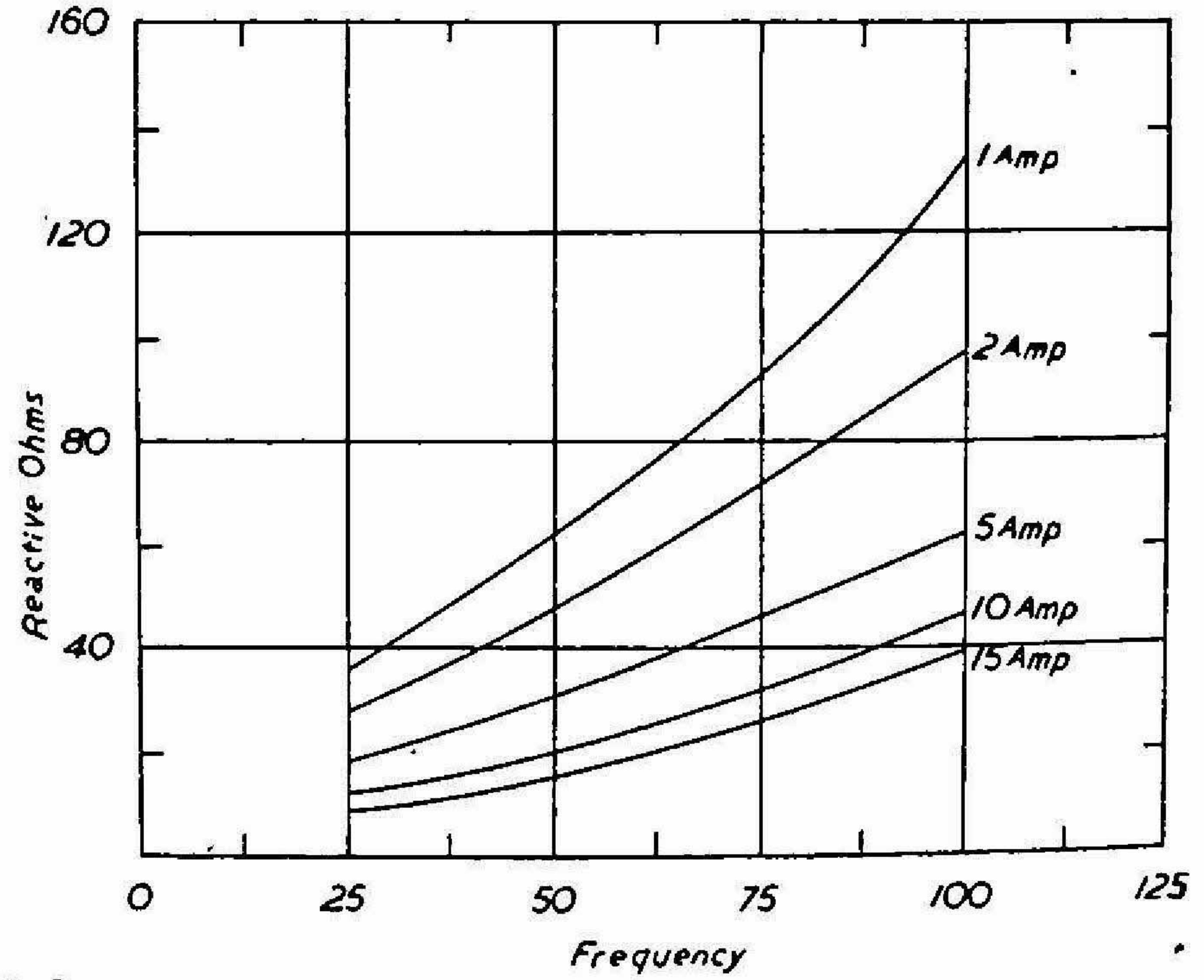
160
120
80
40
Reactive Ohms
0
25
50
75
100
125
Frequency
1 Amp
2 Amp
5 Amp
10 Amp
15 Amp

FIG. 14.

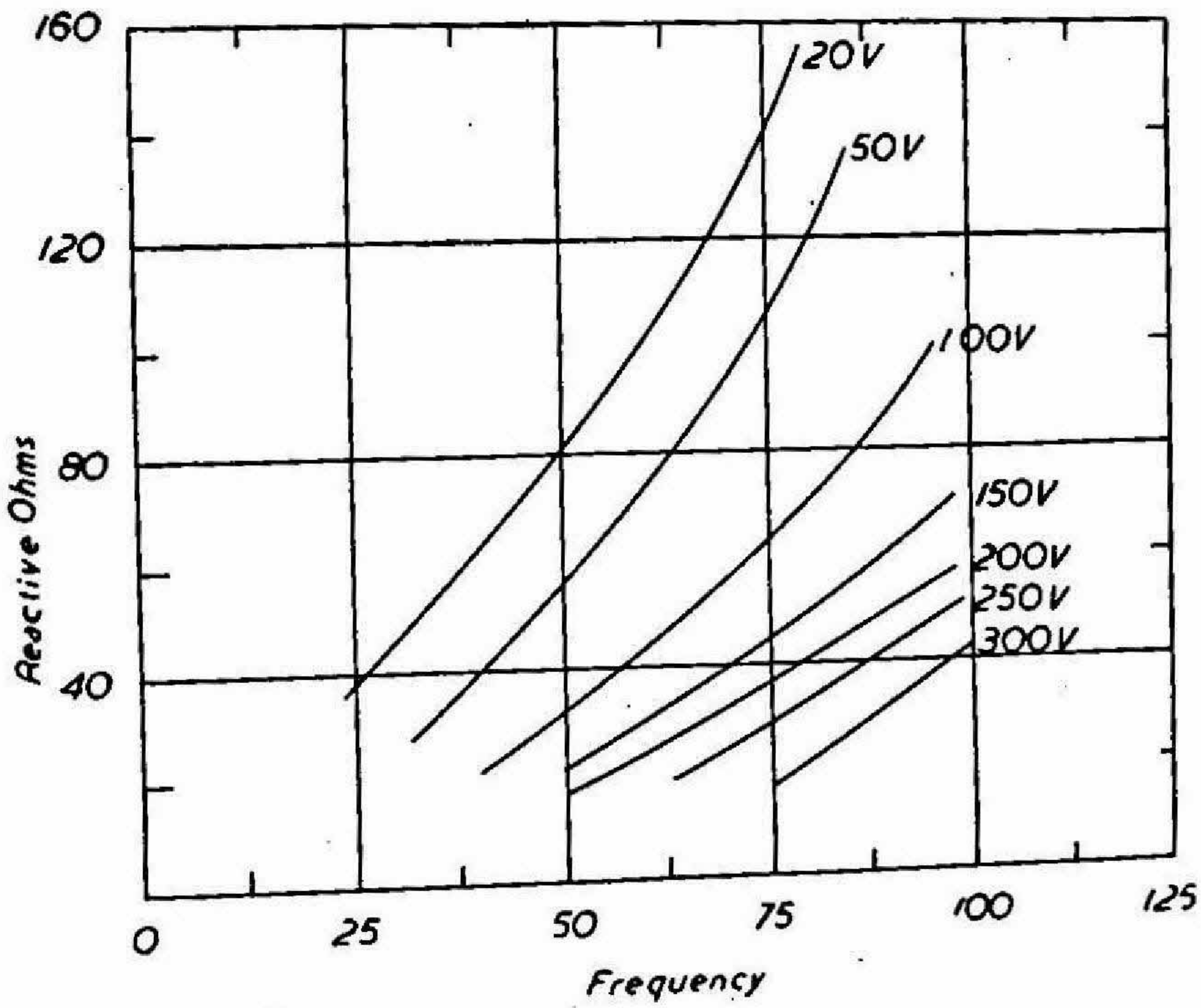
160
120
80
40
Reactive Ohms
0
25
50
75
100
125
Frequency
20V
50V
100V
150V
200V
250V
300V

Fig. 15.

APPENDIX I

LIST OF SYMBOLS USED

B = flux density: suffix g—gap, m—maximum, r—remnant, s—saturated.

E = voltage per phase.

q = number of phases.

f = frequency

H = magnetising force: suffix e—coercive, m—maximum.

I = current/phase: suffix m—magnetising, r—rotor, s—stator.

$j = \sqrt{-1}$.

k = constant.

m = number of phases.

N = speed: suffix s—synchronous speed

P = number of poles.

r, R = resistance/phase

s = slip: suffix indicates for which harmonic.

T = torque.

x, X = reactance/phase.

Z = impedance/phase.

μ = slope: suffix 0—initial, 1—final.

ϕ = flux

W_1 = turns × slots.

Note.—(1) For r, x, Z suffix (s, 1)—stator, (e, 2)—equivalent rotor, t—top cage, b—bottom cage, m—mutual.

(2) For I, E, T, s suffix indicates the harmonic.

APPENDIX II

The design data of a 3-phase, double cage, 11 horse-power, 4-pole, 420 V, 50 cycles induction motor is given below and its performance at frequencies other than 50 cycles is investigated.

Stator: CK5–204.

Mesh winding series delta.

33 turns of 2/·056 DCC.

Pitch 1–10.

Rotor: Top cage: 12″ long, 6 g. cu rods and $\frac{5''}{32} \times \frac{31''}{64} \times 21\frac{3}{8}''$ cu rings with outer diameter of 6·95″

Bottom cage: $\frac{5''}{32} \times \frac{5''}{16} \times 15\frac{3}{8}''$ cu rings with outer diameter of 5·14″.

Air gap: 0·02″.

Flux Area: Primary tooth = 13·40 sq. inch.

Primary core = 9·78 sq. inch.

Secondary tooth = 48·02 sq. inch.

Secondary core = 17·70 sq. inch.

Air gap = 35·20 sq. inch.

Length of Flux Path: Primary tooth length = 1·375″

Secondary tooth length = 1·113″

Primary core length = 3·830″

Secondary core length = 1·410″

Effective gap length = 0·02504″

Distribution Factor: DF for 3 phase, 60 degrees phase belt for two coils per group = 0·967.

Chording Factor: $CF = \sin \frac{\text{throw}}{\text{slots/pole}} \cdot \pi/2 = \sin \frac{9}{12} \cdot \pi/2 = 0{\cdot}924$

Primary Unchorded Slot Leakage Factor $= K_1 = 1{\cdot}782$

Secondary Unchorded Slot Leakage Factor $= K_2$

Primary Chording Leakage Factor $= Ks_1 = 0{\cdot}856$

Secondary Chording Leakage Factor $= K_{S_2}$

Air Gap Flux $= \phi = [(E - I_m x_1)\, q\, 10^8]/[2\cdot 22 f (DF)(CF) W_1]$

$$= \frac{E_1}{420} \cdot \frac{420 \times \cdot 965 \times 3 \times 10^8}{2\cdot 22 \times 50 \times \cdot 967 \times \cdot 924 \times (33 \times 48)} \cdot \frac{50}{f}$$

$$= 7\cdot 75 \times 10^5 \cdot \frac{E'}{f_1}$$

where $E' = \frac{E_1}{420}$ and $f_1 = f/50$.

Total Ampere Turns: Since E is considerably smaller than one and f is considerably greater than one, the flux for higher frequency will be far less and for the low flux densities, the (AT) for the iron path will be negligible.

Maximum Gap Density $Bg = \frac{\phi}{Ag} \times \frac{\pi}{2} = \frac{7\cdot 75 \times 10^5}{35\cdot 2} \frac{E'}{f_1} \times \frac{\pi}{2}$

$$= \frac{E'}{f_1} \cdot 34600 \text{ lines/sq. in.}$$

Magnetising current $= I_m = \frac{2\cdot 22\,(AT)\,P}{W_1\,(DF)\,(CF)}$

$$= 1\cdot 707\, \frac{E'}{f_1} \text{ amp./phase.}$$

Primary Slot Leakage Reactance $= x_{s_1} = 1\cdot 60 f_1\ \Omega$

Primary End Leakage Reactance $= x_{l_1} = 1\cdot 17 f_1\ \Omega$

Zig Zag Leakage Reactance $= x_{z_1} = 2\cdot 62 f_1\ \Omega$

Primary Resistance per Phase $= r_1 = 1\cdot 36\ \Omega$

Top Cage Slot Constant $= K_3 = 1\cdot 803$

Bottom Cage Slot Constant $= K_4 = 16\cdot 479$

Mutual Slot Constant $= K_5 = 1\cdot 250$

Top Cage Reactance per Phase $= x_3 = 2\cdot 85 f_1\ \Omega$

Bottom Cage Reactance per Phase $= x_4 = 13\cdot 11 f_1\ \Omega$

Mutual Reactance $= x_5 = 2\cdot 14 f_1\ \Omega$

Turn Ratio $= 24 \cdot 8$

Top Cage Resistance per Phase $r_3 = 4 \cdot 66\ \Omega$

Bottom Cage Resistance per Phase $= r_4 = 2 \cdot 31\ \Omega$.

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