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DETERMINATION OF TRANSIENT STABILITY LIMITS OF A SYNCHRONOUS MOTOR DUE TO SUDDEN LOAD VARIATION*—PART II

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III. NUMERICAL SOLUTION OF THE PROBLEM

The problem of the determination of the stability limits of a synchronous motor due to sudden load variation has led to the solution of the non-linear differential equation[1]

$$\frac{dv}{d\delta} = \frac{T - Kv(1 - \cos 2\delta) - \sin\delta - T_r \sin 2\delta}{v}$$

with the initial condition $\delta = \delta_0$, $v = 0$; δ is the torque angle in electrical degrees and $v = d\delta/d\tau$, where τ is the time variable given[1] by $\tau = t\sqrt{P_m/P_j}$.

The solution of the above equation subject to the conditions above-mentioned gives all the information regarding the stability limits. The numerical method employed here is the Runge-Kutta method which can be briefly summarized as follows. Let $dy/dx = f(x, y)$ be the equation to be solved with $x = x_0$, $y = y_0$ and let h be a small increment in x.

$$\left(\frac{dy}{dx}\right)_{\substack{x=x_0\\ y=y_0}} = f(x_0, y_0).$$

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Let

$$K_1 = f(x_0, y_0)\, h$$
$$K_2 = f(x_0 + h/2, y_0 + K_1/2)\, h$$
$$K_3 = f(x_0 + h/2, y_0 + K_2/2)\, h$$
$$K_4 = f(x_0 + h, y_0 + K_3)\, h$$

Then

$$K = \tfrac{1}{6}(K_1 + 2K_2 + 2K_3 + K_4)$$

gives the corresponding increment in y.

The same procedure is adopted to continue the process. By choosing h sufficiently small, this method is found to give accurate results, though it involves somewhat lengthy computation. It can be shown[2] that the accuracy of this method is of the order of h^4. This method is first applied to the non-salient pole machine investigated first by Lyon and Edgerton,[3] and later by McLachlan.[4] The damping coefficient for this machine is ·05 and $P_m = 16$ K.W. Total load $P = P_0 + P_1$ is taken as 14·6 K.W. so that $T = 14{\cdot}6/16 = {\cdot}9125$. The saddle singularity δ_s corresponding to this point is given by $(180^\circ - \delta_e)$ where $\sin \delta_e = {\cdot}9125$, which gives $\delta_s = 180^\circ - 78^\circ{\cdot}5$; The starting point therefore is $114^\circ{\cdot}2$.

To determine the value of $dv/d\delta$ at this point, it is necessary to express v as a power series in δ in the neighbourhood of this point since $dv/d\delta$ becomes indeterminate. Let $\xi = \delta - \delta_s$ so that the singularity is now the origin. Let $v = a_1\xi + a_2\xi^2 + a_3\xi^3 + \ldots\ldots$ be the solution in the neighbourhood of the origin.

Substituting from above for v and $dv/d\xi$ in the equation

$$\frac{dv}{d\xi} = \frac{{\cdot}9125 - {\cdot}05v - \sin \xi}{v}$$

we get

$$(a_1\xi + a_2\xi^2 + a_3\xi^3 + \ldots\ldots)(a_1 + 2a_2\xi + \ldots\ldots)$$
$$= {\cdot}9125 - {\cdot}05(a_1\xi + a_2\xi^2 + \ldots\ldots) - \sin(\xi + \delta_s)$$
$$= {\cdot}9125 - {\cdot}05(a_1\xi + a_2\xi^2 + \ldots\ldots)$$
$$+ \sqrt{1 - {\cdot}9125}\left(\frac{\xi}{57{\cdot}3} - \frac{\xi^3}{57{\cdot}3^3} + \ldots\right)$$
$$- {\cdot}9125\left(1 - \frac{\xi^2}{57{\cdot}3^2 \times 2!} + \ldots\ldots\right)$$

Equating the coefficients on both sides, we get

$$a_1^2 = -{\cdot}05a_1 + \sqrt{\frac{1 - {\cdot}9125^2}{57{\cdot}3}}$$

$$a_1 = -\frac{{\cdot}05}{2} \pm \sqrt{\frac{{\cdot}05}{2} + \frac{{\cdot}410}{57{\cdot}3}} = -{\cdot}1132.$$

There are clearly two values of the slope at the saddle point, but we are interested only in the negative value, as we can easily see from the figures.

$$\left(\frac{dv}{d\delta}\right)_{\substack{\delta=\delta_s \\ v=0}} = -\cdot 1132.$$

Starting from the point (114°·2, 0) and the slope determined above, the separatrix is plotted from the numerically computed values by means of the Runge-Kutta process. It is found to intersect the δ-axis at $\delta_0 = 4^\circ\cdot 82$ which means that $T_0 = \sin 4^\circ\cdot 82 = \cdot 0812$.

Thus

$$P_0 = \cdot 0812 \times 16 = 1\cdot 2992 \text{ K.W.}$$

$$P_1 = 14\cdot 6 - 1\cdot 3 = 13\cdot 3 \text{ K.W.}$$

These results agree very closely with the results obtained by Lyon and Edgerton with the differential analyzer.

Before applying this method to the salient pole machine in the laboratory, it is tried for the salient pole synchronous machine investigated by Ku.[5] The machine constants are $K = \cdot 0068$, $b = 1$, $T_r = \cdot 338$.

Let $T = 1$; the saddle singularity is now given by

$$\sin \delta_s + \cdot 338 \sin 2\delta_s = 1, \quad \textit{i.e.}, \quad \delta_s = 90^\circ;$$

The slope at the saddle point is[1]

$$\frac{dv}{d\delta} = -\frac{\cdot 0068\,(1+1)}{2} - \sqrt{(\cdot 0068)^2 + \frac{0\cdot 676 - 0}{57\cdot 3}} = -\cdot 1156.$$

The curve shown in Fig. 7 gives the separatrix obtained by the numerical computation. This intersects the δ-axis again at $\delta_0 = 16^\circ\cdot 2$ which agrees well with the result 16°·5 obtained by Ku.

Thus

$$T_0 = \sin 16^\circ\cdot 2 + \cdot 338 \sin 32^\circ\cdot 4 = \cdot 46.$$

The method is now applied to the machine, whose machine constants are measured in the laboratory. The exact details of the measurement of these constants and the verification of the results by experiment are given in the next section. The constants of the machine are $K = \cdot 053$, $T_r = \cdot 25$ and $b = \cdot 67$. P_m for this particular machine is 16·73 K.W. The equation, therefore, takes the form

$$\frac{dv}{d\delta} = \frac{T - \cdot 053\,(1 - \cdot 67 \cos 2\delta)\, v - \sin 2\delta - \sin \delta}{v}$$

The singularities of the above equation are $(\delta, 0)$, where δ is given by $T = \sin \delta + \cdot 25 \sin 2\delta$. This equation is plotted in Fig. 5 from which we find that

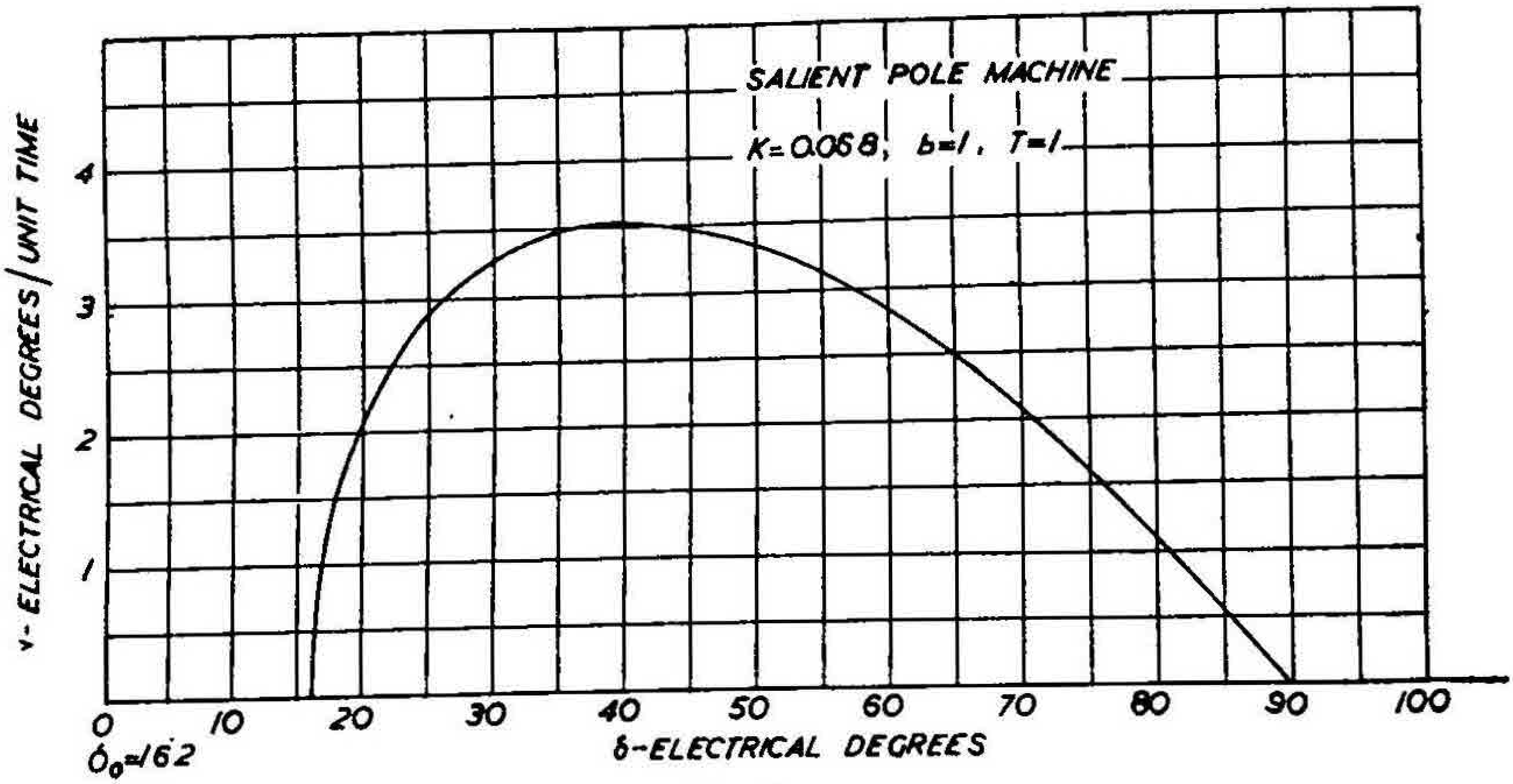

FIG. 7

the maximum steady power is $1 \cdot 1 \times 16 \cdot 73$ K.W. If $T > 1 \cdot 1$ there are no singularities and consequently no positions of equilibrium.

For various values of T, the procedure is to plot the separaticee. The following values of T are tried. $T = \cdot 95, 1, 1 \cdot 04, 1 \cdot 07, 1 \cdot 09, 1 \cdot 1$. The corresponding values of δ_s where $\sin \delta_s + \cdot 25 \sin 2\delta_s = T$, are $95^\circ \cdot 5$, 90°, 85°, 80°, 75° and $68^\circ \cdot 5$. The value of δ_s is the higher of the two roots since it corresponds to the saddle singularity.

The increment in δ, namely h, is taken as -3° and the computation continued somewhat beyond the point, where the slope becomes zero. Thereafter, the roles of δ and v are interchanged to avoid the infinite slope at $(\delta_0, 0)$. The curves are plotted from the numerical values and the results are shown from Figs. 8–13.

The values of δ_0 for various cases, and the corresponding loads T_0, as obtained from these curves, are given on page 9.

It is interesting to plot the above results with T_0 as abscissa and T_1, T as ordinates. These graphs are given in Fig. 14. The results obtained from the actual experiment on the machine to determine the stability limits are also plotted there, which check reasonably well with the calculated data. The graphs $(T_0 - T)$ are nearly straight lines up to a point and thereafter becoming drooping and merge with the horizontal line through the point $(0, 1 \cdot 1)$ shown dotted in the figure. For $T_0 - \cdot 85$, T is $1 \cdot 1$ and for all values of $T_0 > \cdot 85$, T is just equal to $1 \cdot 1$, which corresponds to the maximum steady power. The machine is said to be critically damped for this particular load T_0. Because, any further increase of K beyond the value $\cdot 053$ will not increase the value of T. For values of $T_0 < \cdot 85$, on the other

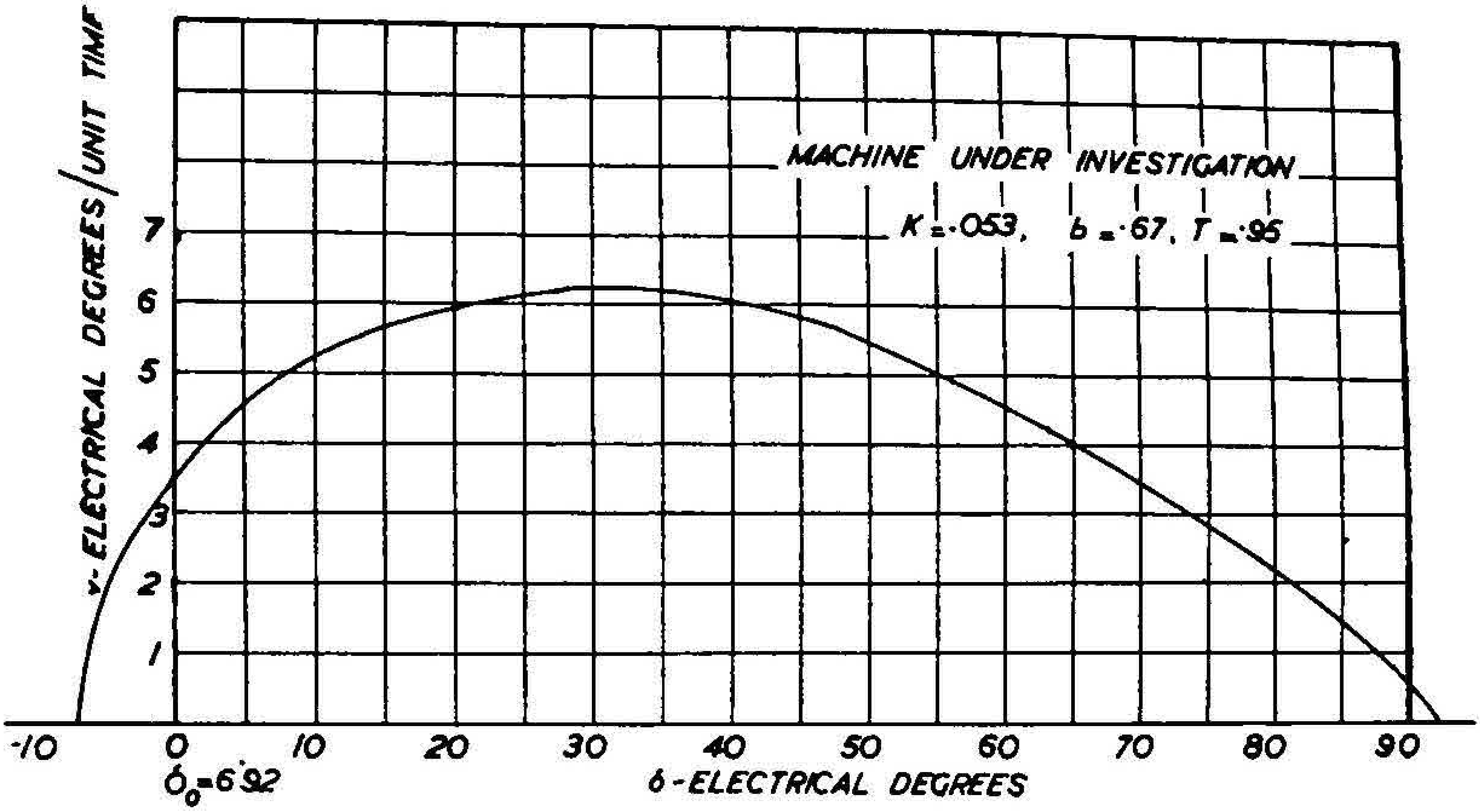
MACHINE UNDER INVESTIGATION
K = ·053, b = ·67, T = ·95
v-ELECTRICAL DEGREES/UNIT TIME
δ-ELECTRICAL DEGREES
δ₀=6·92
-10
0
10
20
30
40
50
60
70
80
90
1
2
3
4
5
6
7

FIG. 8

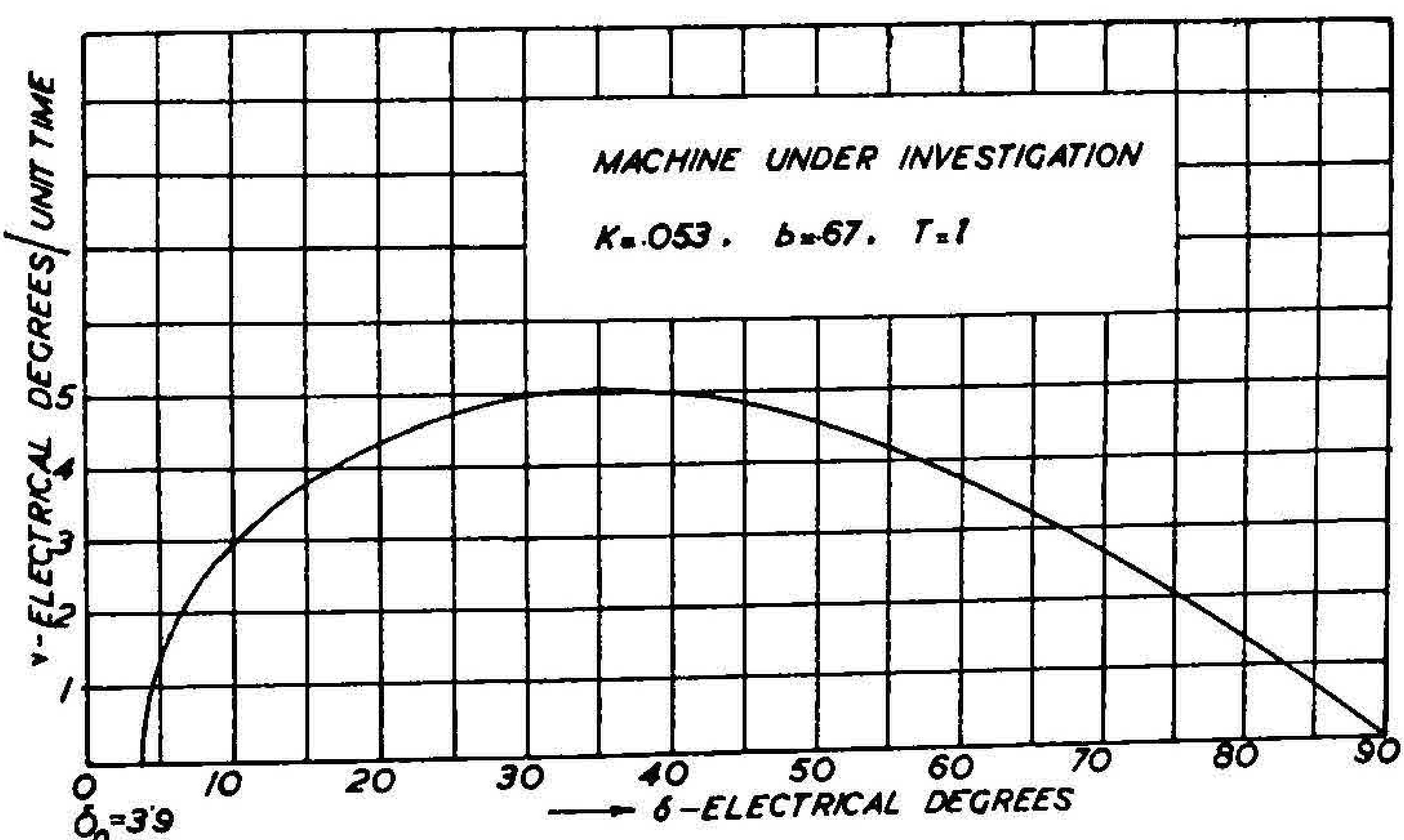
MACHINE UNDER INVESTIGATION
K=·053, b=·67, T=1
v-ELECTRICAL DEGREES/UNIT TIME
δ-ELECTRICAL DEGREES
δ₀=3·9
0
10
20
30
40
50
60
70
80
90
1
2
3
4
5

FIG. 9

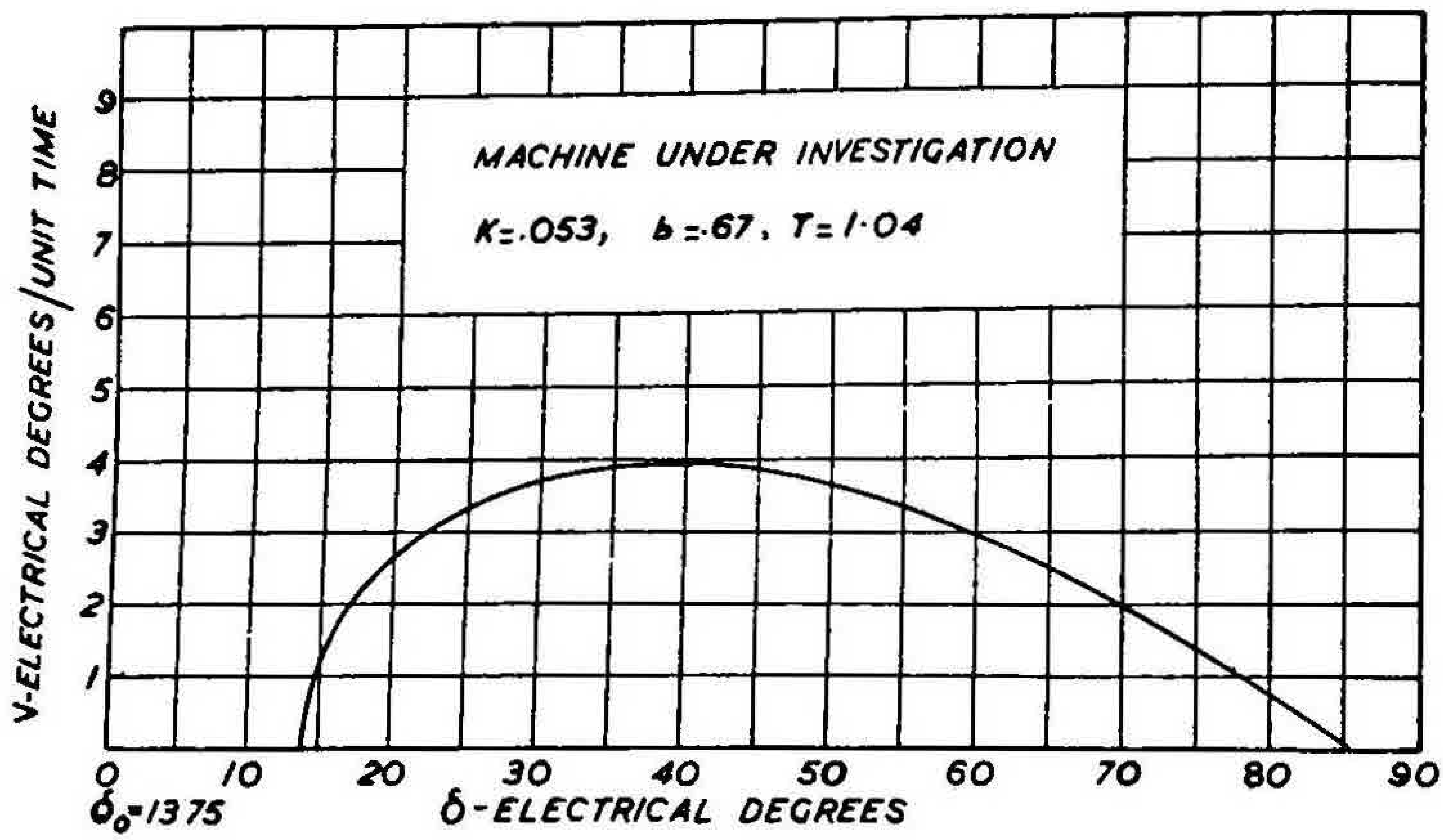
MACHINE UNDER INVESTIGATION
K=.053, b=.67, T=1·04
V-ELECTRICAL DEGREES/UNIT TIME
δ-ELECTRICAL DEGREES
δ₀=13·75
0
10
20
30
40
50
60
70
80
90
1
2
3
4
5
6
7
8
9

FIG. 10

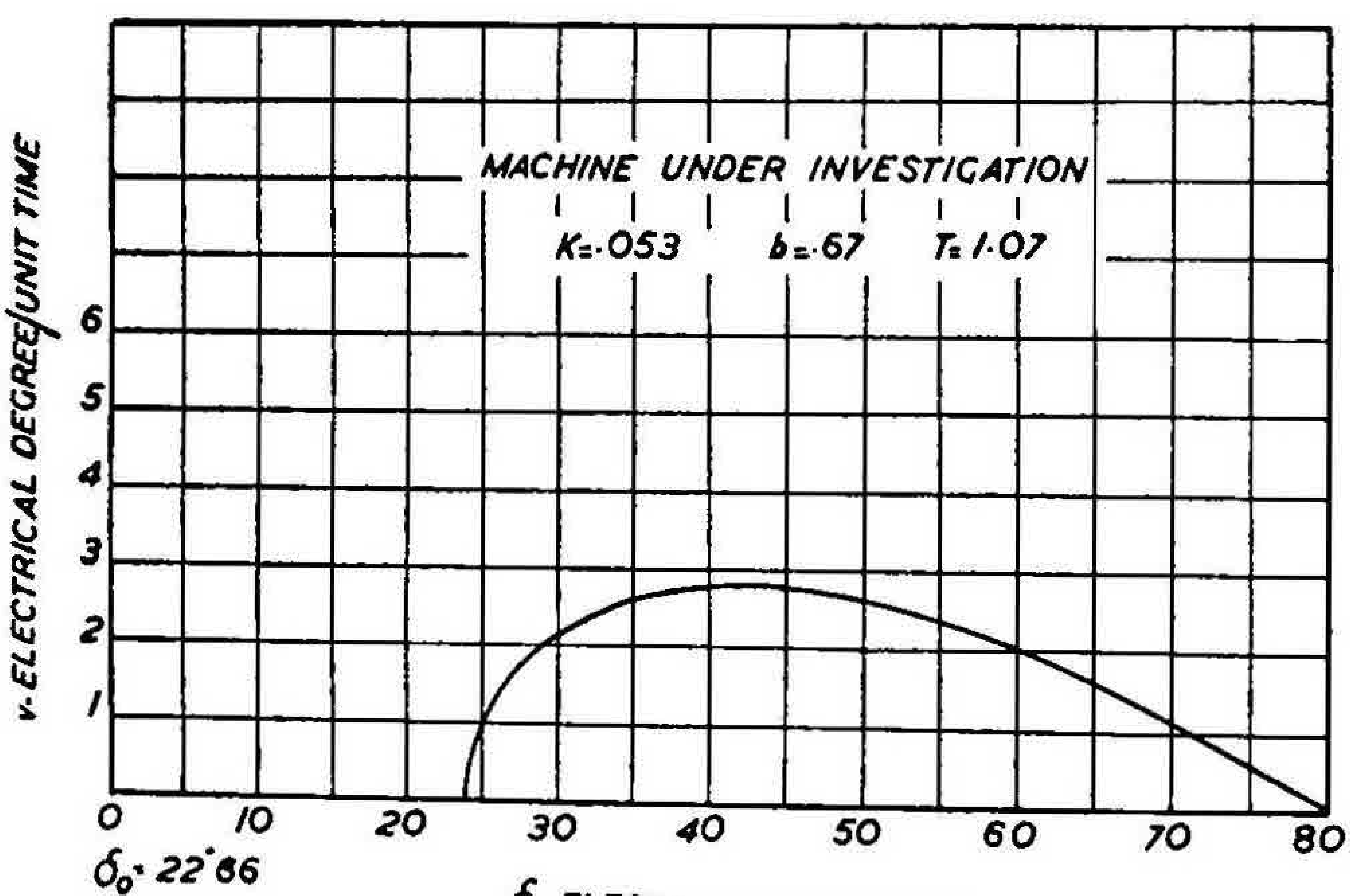
MACHINE UNDER INVESTIGATION
K=.053 b=.67 T=1·07
v-ELECTRICAL DEGREE/UNIT TIME
δ-ELECTRICAL DEGREES
δ₀=22°66
0
10
20
30
40
50
60
70
80
1
2
3
4
5
6

FIG. 11

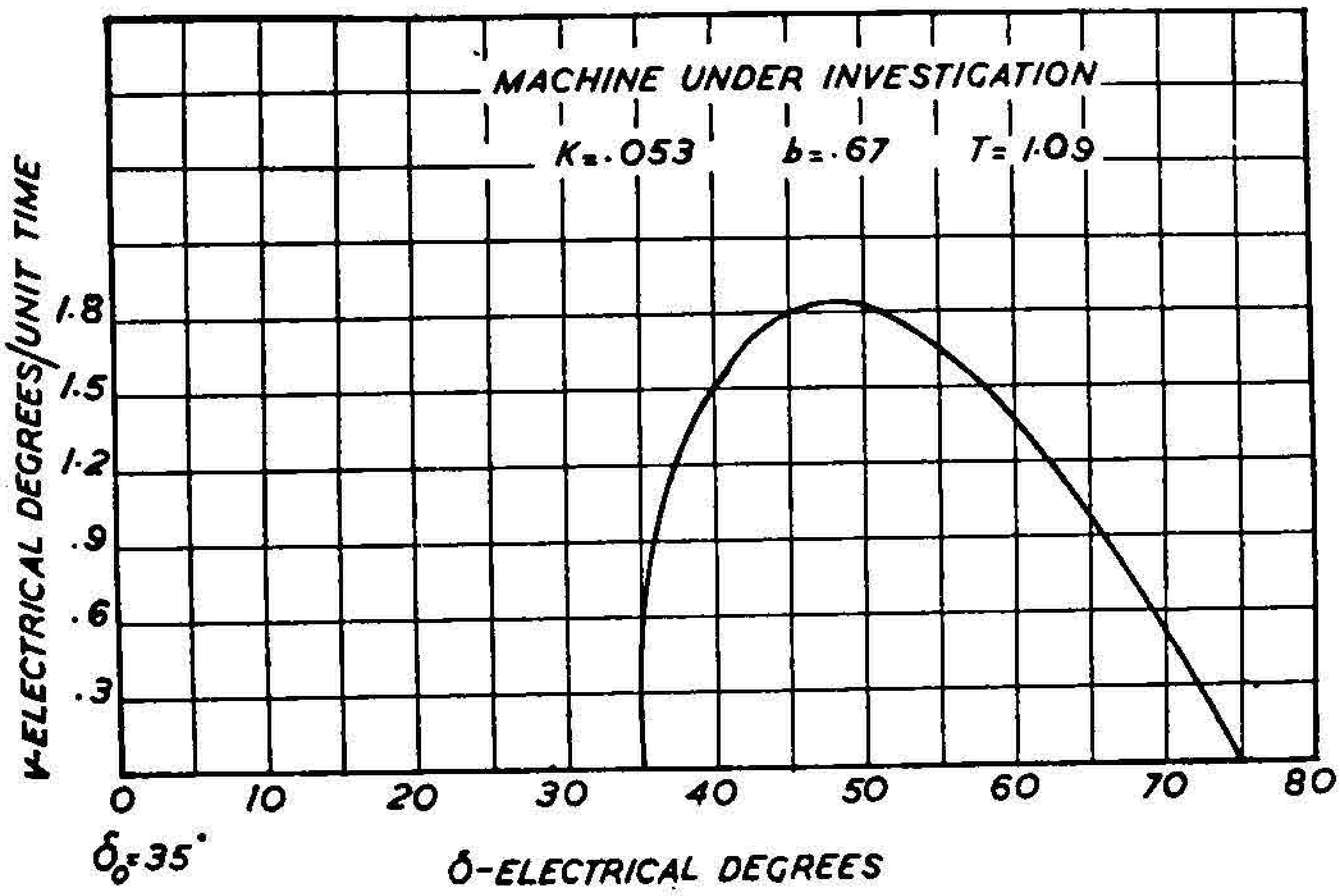
MACHINE UNDER INVESTIGATION
K=.053 b=.67 T=1.09
V-ELECTRICAL DEGREES/UNIT TIME
1.8
1.5
1.2
.9
.6
.3
0
10
20
30
40
50
60
70
80
δ_0=35°
δ-ELECTRICAL DEGREES

FIG. 12

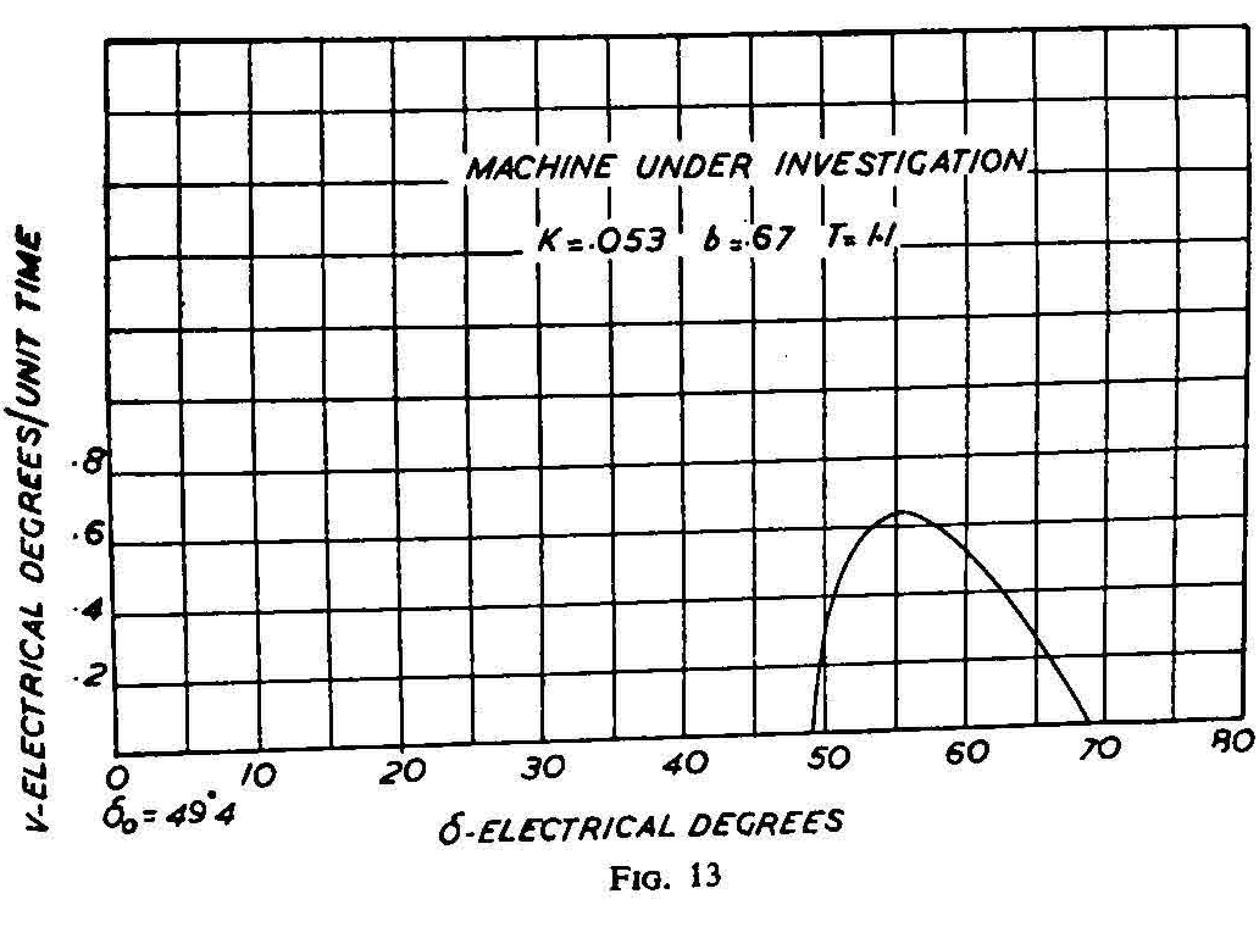
MACHINE UNDER INVESTIGATION
K=.053 b=.67 T=1.1
V-ELECTRICAL DEGREES/UNIT TIME
.8
.6
.4
.2
0
10
20
30
40
50
60
70
80
δ_0=49.4°
δ-ELECTRICAL DEGREES

FIG. 13

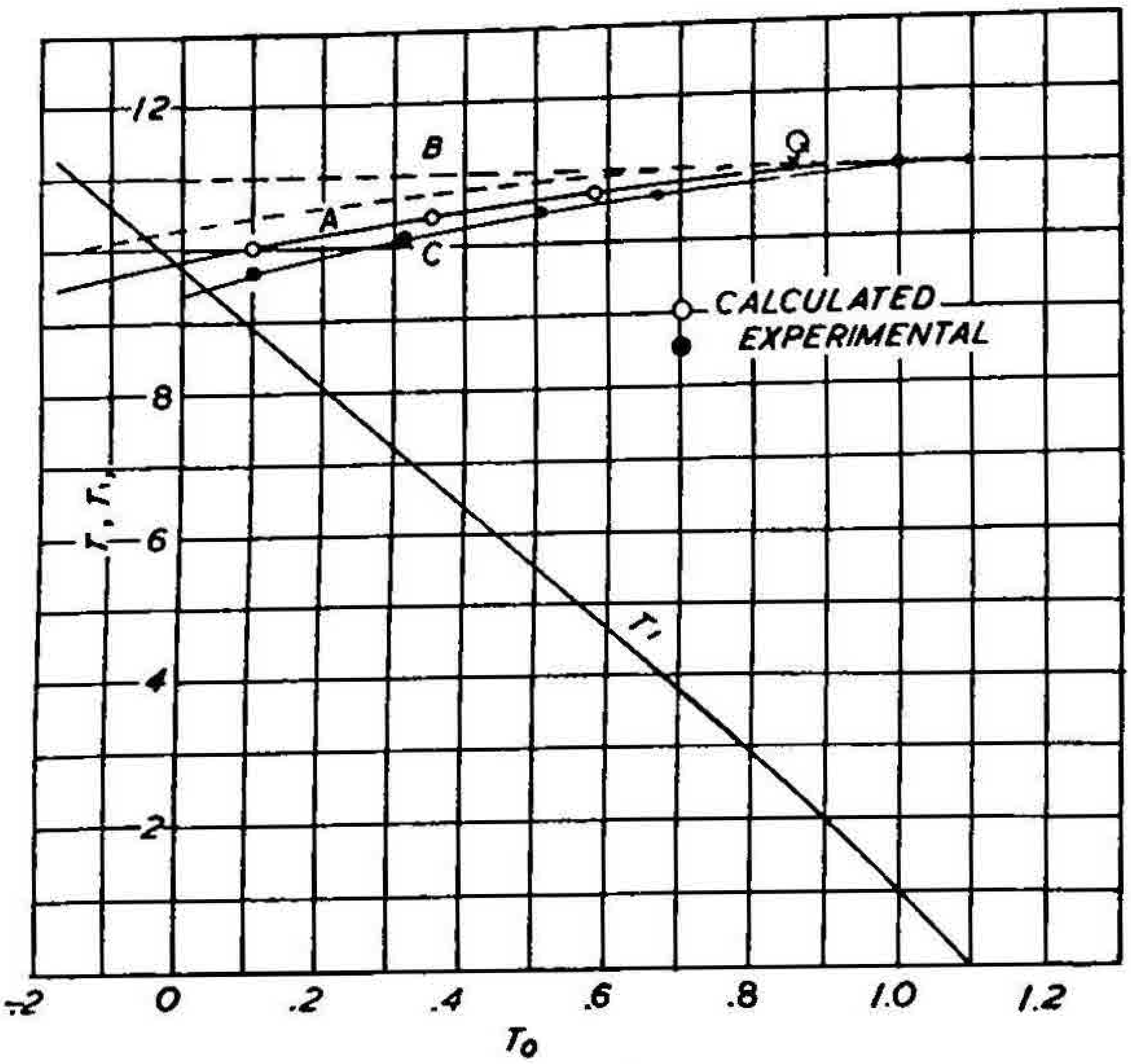
12
B
A
C
CALCULATED
EXPERIMENTAL
8
T, T_1
6
T_1
4
2
-.2
0
.2
.4
.6
.8
1.0
1.2
T_0

FIG. 14

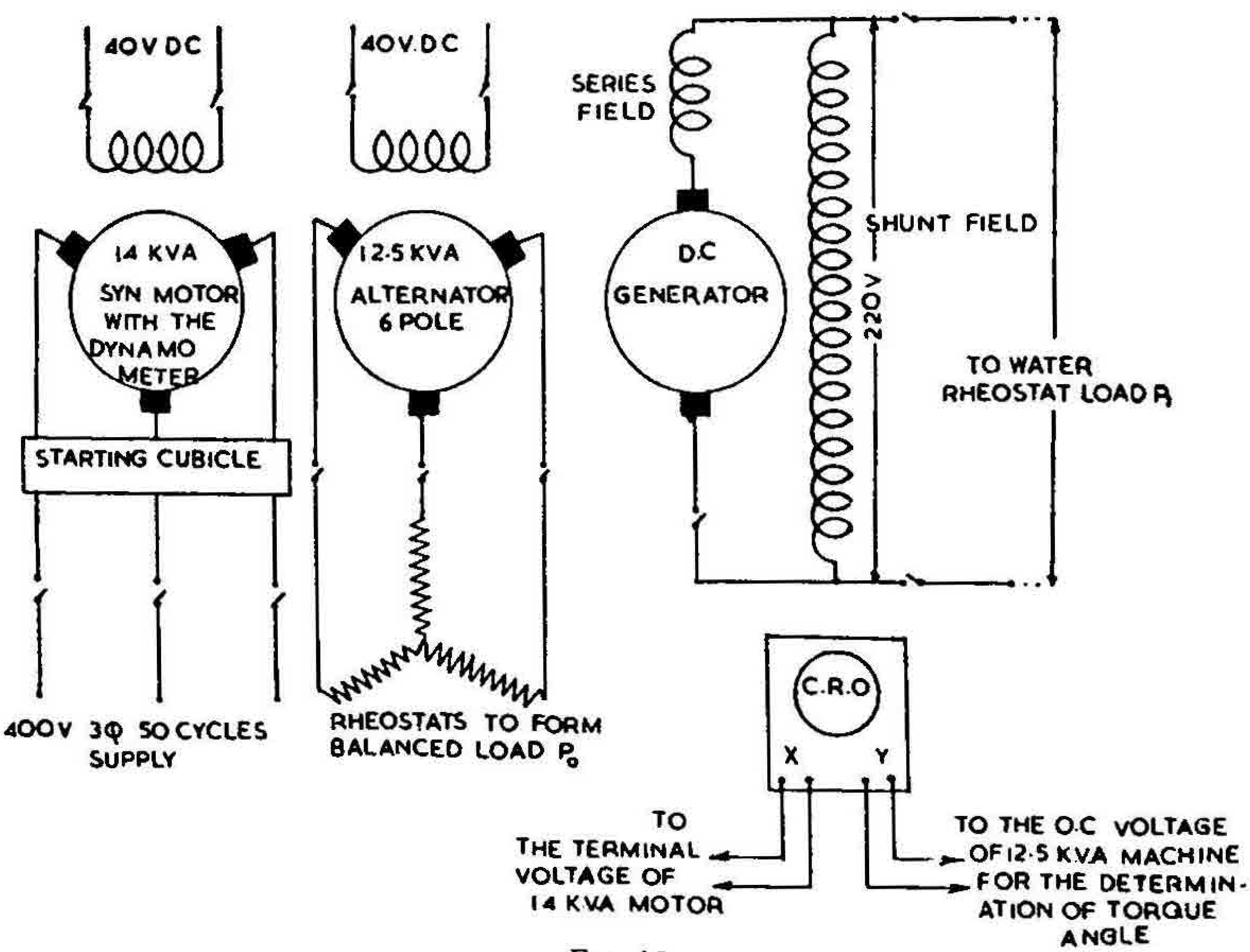
40V DC
40V.DC
SERIES FIELD
14 KVA SYN MOTOR WITH THE DYNAMO METER
12·5 KVA ALTERNATOR 6 POLE
D.C GENERATOR
SHUNT FIELD
220V
TO WATER RHEOSTAT LOAD P_1
STARTING CUBICLE
400V 3Φ 50 CYCLES SUPPLY
RHEOSTATS TO FORM BALANCED LOAD P_0
C.R.O
X
Y
TO THE TERMINAL VOLTAGE OF 14 KVA MOTOR
TO THE O.C VOLTAGE OF 12·5 KVA MACHINE FOR THE DETERMINATION OF TORQUE ANGLE

FIG. 15

T	δ_s	δ_0	T_0	$T_1 (T - T_0)$
·95	95°·5	− 6°·9	− ·18	−1·13
1·00	90°·0	3°85	·102	·893
1·04	85°·0	13°·75	·353	·687
1·07	80°·0	23°·66	·585	·485
1·09	75°·0	35°·00	·810	·28
1·1	68°·5	49°·40	1·006	·094

hand, it is possible to increase T by increasing the value of K, the damping coefficient. If A represents the $(T_0 - T)$ graph for $K = \cdot 053$ and B represents the dotted horizontal through (0, 1·1), the graphs $(T_0 - T)$ for values of $K > \cdot 053$ lie between A and B. The greater the value of the damping coefficient is, the nearer it lies to the straight line B.

In the same figure are plotted the graphs $(T_0 - T)$, *i.e.*, the initial load T_0 against the limiting load T_1. This is also nearly a straight line.

Though these points are obtained for the machine considered by accurate numerical computation involving considerable amount of labour, it is possible to arrive at these graphs, with much less labour and reasonable accuracy, and predict the values of T_1 and T for any given T_0, in the following way. The point Q in Fig. 14 corresponding to the critically damped load T_0 for the machine should be obtained very accurately in the first instance. This load T_0 is such that the corresponding T is the maximum steady power T_{max}. Let this value be called $(T_0)_c$. For values of $T < (T_0)_c$, the graph is nearly a straight line. Thus it is enough if we obtain numerically T for two or three values of T_0 less than $(T_0)_c$. For $T_0 > (T_0)_c$, the graph coincides with the line B.

In this way, it is possible to predetermine the limiting loads for various values of T_0, quickly and accurately.

IV. Experimental Verification

The machine under investigation is a salient pole synchronous machine, 14 KVA, 50 v, 6 pole, 1,000 r.p.m., coupled to another 12·5 KVA, 50 v, 6 pole synchronous machine and a D.C. generator as shown in Fig. 15. The D.C. generator is used as a load to the machine under test. The 12·5 KVA machine is provided with a stator, which can be rotated, the angle of rotation being measured on a graduated disc attached to the frame of the 14 KVA machine. The 14 KVA alternator is provided with a cradle type dynamometer, which is used to measure the torque on the shaft.

As shown earlier in Section I, the total torque acting on the shaft of the motor, when the machine is not running at steady speed, consists of the induction motor torque due to amortisseur and field windings, the synchronous motor torque given by the usual formula $\sin\delta + T_r \sin 2\delta$ and the inertial torque. The sum of these torques should be equal to the external torque at any instant of time.

The experimental investigation is divided into two parts:

(i) The measurements of constants P_j, P_d, P_m, b and P_r, to be used in the differential equation.[1]

(ii) The experimental verification of the results obtained by numerical analysis in Section III.

(i) *Measurement of constants.*—Since the machine is assumed to be running at speeds, very nearly synchronous, each one of the expressions for torque can be expressed as power in K.W.

Inductior motor power $= P_d(1 + b\cos 2\delta)\, d\delta/dt$

where P_d is expressed in K.W./electrical degree/second and b is a numerical factor where s' is the slip expressed in electrical degree per second.

If s is the fractional slip at any speed defined by where N_s is the synchronous speed and N, the actual speed,

$$s' = \frac{s \times N_s \times 360 \times p}{60 \times 2} = 360\, sf = -360\, fst$$

where f is the frequency.

$$\frac{d\delta}{dt} = -360 fs, \quad P_d = \frac{P_L}{360 fs}$$

K.W./electrical degree/second. P_L is the average output in K.W. at any slip S.

The value of P_L is obtained from the power slip diagram, experimentally plotted by running the machine as an induction motor with field winding short circuted and applying balanced three phase voltage to the stator under these conditions. In addition to the average power P_d, there is also present pulsating power given by $bP_d \times d\delta/dt \cos 2\delta + P_r \sin 2\delta$. This can be written as

$$\sqrt{P_r^2 + b^2 P_d^2\, 306^2 f^2 s^2}\, \cos(2\delta + \epsilon)$$

Thus the amplitude of the pulsating power at double the slip frequency is

$$\sqrt{P_r^2 + b^2 P_d^2\, 360^2 f^2 s^2}$$

P_r is the reluctance power and is found out from the values of X_d and X_q as

$$\frac{3V^2 (X_d - X_q)}{2X_d X_q}$$

which gives 4·18 K.W., as we shall see later. Thus, from a table of the pulsating power, obtained experimentally, and the reluctance power, the coefficient b is calculated.

The torque slip characteristics are obtained with the help of the dynamometer and checked also with the help of the D.C. generator coupled to the set. The readings are given in Table I.

The readings of the dynamometer given in kg. are converted into output in K.W. as follows. The speed is nearly the synchronous speed, *i.e.*, 1,020 r.p.m. at 50·5 cycles. The distance from the centre of the shaft to the dynamometer is 13".

If T is the reading of the dynamometer in kg.

$$\text{Torque} = \frac{T \times 1000}{453\cdot 6} \times \frac{13}{12} \text{ lb.–ft.}$$

$$\text{Output} = \frac{T \times 13{,}000 \times 6\cdot 28 \times 1{,}020}{453\cdot 6 \times 12 \times 60} \text{ ft.–lb./sec.}$$

$$= \cdot 339\, T \text{ K.W.}$$

$$P_d = \frac{3 \times \cdot 339 \times 102}{360 \times 50\cdot 5} \cdot \left(\frac{272}{172}\right)^2$$

$$= \cdot 009 \text{ K.W./elec. deg./sec.}$$

If T_a is the reading of the amplitude of the pulsating torque in kg.

$$T_a \times \cdot 339 \times \left(\frac{217}{172}\right)^2 = \sqrt{P_r^2 + b^2 P_d^2\, 360^2 f^2 s^2}$$

This is $\simeq P_r$ when $S = \cdot 0049$. This value from the tables is obtained as 3·78 K.W., which as we find later, agrees reasonably well with the value for the reluctance power obtained from the values of X_d and X_q.

Utilizing the above expression, the value of the coefficient b is calculated from the readings and is tabulated in Table I. This gives an average value of ·67 for b.

The moment of inertia is measured by the usual retardation test. The readings are given in Table II. The inertial power P_j is expressed in K.W./electrical degree/sec.², where P_j is proportional to the moment of inertia of the rotating system. If ω is the angular velocity in mechanical radians/sec. and $g = 32\cdot 2$, inertial power in ft.–lb./sec. is

$$\frac{d}{dt}\left(\tfrac{1}{2}\cdot I\,\frac{\omega^2}{g}\right) = \frac{I\omega}{g} \cdot \frac{d\omega}{dt} \cdot \frac{746}{550} \text{ K.W.}$$

$$= \frac{I}{g} \cdot \frac{2\pi N}{60} \times \frac{746}{550} \times \frac{4\pi}{360\, p} \cdot \frac{d\Omega}{dt} \text{ K.W.}$$

TABLE I

Frequency 50·5; A.C. Volts phase to Neutral 172 V

Speed r.p.m.	Slip s	Average reading of the dynomometer in Kgm. T	Amplitude of the pulsating component in Kgm. T_a	Average output in K.W. $t \times \cdot 339 \times (219/172)^2$	Amplitude of the pulsating power in K.W. $T_a \times \cdot 339 \times (217/172)^2$	Calculated value b*	D.C. volts	D.C. amps.
1015	·0049	1·50	7	·8	3·71	..	98	·5
1010	·0098	3·00	7·25	1·6	3·84	·62	98	5·0
1005	·0147	4·50	..	2·4	..	..	98	9·0
1000	·0196	6·00	8·20	3·2	4·35	·71	98	11·50
990	·0294	9·00	9·30	4·8	4·94	·68	97	16·50
980	·0392	12·50	..	6·6	..	..	97	21·50

* b is calculated by making use of the formula $T_a \times \cdot 339 \times (217/172)^2 = p_r^2 + b^2 p_d^2 360^2 f^2 s^2$

TABLE II

Retardation Test - speed 750 r.p.m. Current 4·4 A. Field Current ·26 amps. Voltage 74·5.

t in seconds	Voltage
0	121·5
4	110
8	103
12	97
16	90·5
20	85
24	79
28	74
32	68
36	63
40	58
44	53
48	49

where Ω is in electrical degrees and p is the number of poles. Since $N = 120\, f/p$ nearly, the above expression for the inertial power becomes

$$\frac{If}{p^2} \cdot 18{\cdot}48 \times 10^{-6} \times \frac{d\Omega}{dt} \text{ K.W.}$$

$$P_j = 18{\cdot}48 \times \frac{If}{p^2} \times 10^{-6} \text{ K.W./elec. deg./sec.} = {\cdot}00174$$

(The moment of inertia has the value 67·1 lb.ft.² from Table II.)

The steady state expression for power in terms of the torque angle is $P_m \sin \delta + P_r \sin 2\delta$, where

$$P_m = \frac{3EV}{X_d \times 1{,}000} \text{ K.W.}; \quad P_r = \frac{3V^2 (X_d - X_q)}{2X_d X_q \times 1{,}000} \text{ K.W.};$$

E is the induced voltage per phase; V is the terminal voltage per phase; X_d, X_q are the direct and quadrature axis reactances.

X_d and X_q are determined from the usual open and short circuit tests in the laboratory. The readings are given in Table III. X_q is obtained from the armature current oscillogram with field circuit open and reduced voltage applied to the armature, the machine being driven externally at a speed slightly less than the synchronous speed. The values obtained for X_d and X_q are $6 \cdot 6\ \Omega$ and $4 \cdot 77\ \Omega$ respectively.

TABLE III

Open and short circuit data

Excitation amps.	O.C. voltage	Excitation amps.	Short circuit current amps.
·5	39	·5	6·4
·7	56	·7	8·7
·9	73	·9	10·9
1·0	80	1·0	11·9
1·2	93	1·2	14
1·4	106	1·4	16
1·5	113		
1·8	132	1·5	17·2
2·05	150	1·6	18·4

Armature resistance per phase ·46 ohms.
Average value of X_d 6·6 ohms.

Then we obtain $P_m = 16 \cdot 5$ K.W., $P_r = 4 \cdot 11$ K.W., $P_j = \cdot 00174$, $P_d = \cdot 009$.

$$K \text{ for electrical degrees} = \frac{P_d}{\sqrt{P_j P_m}} = 0 \cdot 053$$

$$T_r = \frac{4 \cdot 11}{16 \cdot 5} = \cdot 25$$

and

$$b = \cdot 67.$$

Utilizing the values of the above constants, the differential equation is solved numerically for various values of $T = P/P_m$ and the results are given in Section III.

The stability limits are obtained and plotted in Fig. 14. The experimental verification of these results is given below.

Experimental Verification

(ii) *Experimental Varification.*—The machine under investigation is started as an induction motor and pulled into synchronism after obtaining speed nearly synchronous. The machine now runs as a synchronous motor. The field winding of the second alternator, which is coupled to this machine, is excited and the armature connected to three rheostats, which are connected in star and form a balanced load P_0 on this alternator. The D.C. machine, which is also on the same shaft, is used as an additional load P_1 on the synchronous motor. This D.C. machine is used as a compound generator and loaded by means of a water rheostat. The load P_0 on the second alternator is kept constant and the load on the D.C. side is steadily increased up to a certain value P_1. The D.C. load is then taken off by opening the switch keeping the position of the water rheostat unchanged. After steady conditions are obtained, the switch is suddenly closed so that the load P_1 comes suddenly on the shaft.

This method of loading is justified as long as the time constants due to the inductance of the machine are very small compared to the mechanical time constant of the rotating system. For a fixed value of P_0, this experiment is repeated with increasing values of P_1 until the machine pulls out of step. The results are given in Table IV and plotted in Fig. 14.

Table IV

T_0	T	P_0 K.W.	P K.W.
·1	·97	1·65	16·00
·31	1·01	5·12	16·67
·5	1·05	8·25	17·33
·66	1·06	10·9	17·5

Conclusion

The non-linear problem of the determination of stability limits of a salient pole synchronous motor, under transient conditions due to sudden loads on the shaft, is numerically solved. The relationship between the initial load on the shaft and the corresponding limiting load is discussed and it is shown how it is possible to predetermine this with minimum amount of numerical computation. This fact is illustrated by applying it to a machine in the laboratory and comparing with the experimental results.

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