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PROPAGATION OF MICROWAVES ALONG A SINGLE CONDUCTOR EMBEDDED IN THREE CO-AXIAL DIELECTRICS—PART II

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Abstract

The characteristic equation for the EH wave has been derived. It is shown, as a special case, that the asymmetric wave EH_1 cannot be propagated along a solid conductor embedded in free space due to very high attenuation. Field components in terms of the axial power flow have been derived.

Field Components of EH Wave

Replacing k by jk in Part I of this paper, for convenience of computation the following wave equations for the z components of $\vec{E}$ and $\vec{H}$ are obtained:

$$\nabla_t^2 E_z - k^2 E_z = 0$$

$$\nabla_t^2 H_z - k^2 H_z = 0 \tag{1}$$

The solutions of the above equations yield E_z and H_z from which other components are found. The field components of the EH wave are obtained as follows by combining the field components of the E and H wave.

$$E_z^{e,h} = [AI_n(kr) + B\,K_n(kr)] \cos n\theta$$

$$E_r^{e,h} = \frac{\gamma}{k^2} [AkI_n'(kr) + BkK_n'(kr)] \cos n\theta$$

$$- \frac{j\omega\mu}{k^2 r} [A'I_n(kr) + B'K_n(kr)]\, n \sin n\theta$$

$$E_\theta^{e,h} = -\frac{\gamma}{k^2 r}[AI_n(kr) + BK_n(kr)]\, n \sin n\theta$$

$$-\frac{j\omega\mu}{k^2}[A'kI_n'(kr) + B'kK_n'(kr)] \cos n\theta$$

$$H_z^{e,h} = [A'I_n(kr) + B'K_n(kr)] \cos n\theta$$

$$H_r^{e,h} = \frac{j\omega\epsilon}{k^2 r}[AI_n(kr) + BK_n(kr)]\, n \sin n\theta$$

$$+\frac{\gamma}{k^2}[A'kI_n'(kr) + B'kK_n'(kr)] \cos n\theta$$

$$H_\theta^{e,h} = \frac{j\omega\epsilon}{k^2}[AkI_n'(kr) + BkK_n'(kr)] \cos n\theta$$

$$-\frac{\gamma}{k^2 r}[A'I_n(kr) + B'K_n(kr)]\, n \sin n\theta$$

The field components in the different media can be obtained from the above equation by using the same constants for μ, ϵ, k, r, as in the previous paper

CONDITIONAL EQUATIONS

The following conditional equations are obtained by using proper boundary conditions and the field components in the different media as obtained from equation (2).

$$A_2I_n(k_2r_1) + B_2K_n(k_2r_1) = 0 \quad (a)$$

$$A_2'I_n'(k_2r_1) + B_2'K_n'(k_2r_1) = 0. \quad (b)$$

$$\frac{j\omega}{r_2}\left(\frac{\epsilon_3}{k_3^2} - \frac{\epsilon_2}{k_2^2}\right)[A_2I_n(k_2r_2) + B_2K_n(k_2r_2)]\, n \sin n\theta$$

$$-\frac{\gamma}{k_2^2}[A_2'k_2I_n'(k_2r_2) + B_2'k_2K_n'(k_2r_2)] \cos n\theta$$

$$+\frac{\gamma}{k_3^2}[A_3'k_3I_n'(k_3r_2) + B_3'k_3K_n'(k_3r_2)] \cos n\theta = 0 \quad (c)$$

$$\frac{j\omega\mu_0}{r_2}\left(\frac{\epsilon_3}{k_3^2} - \frac{\epsilon_2}{k_2^2}\right)[A_2'I_n(k_2r_2) + B_2'K_n(k_2r_2)]\, n \sin n\theta$$

$$-\frac{\gamma\epsilon_3}{k_3^2}[A_3k_3I_n'(k_3r_2) + B_3k_3K_n'(k_3r_2)] \cos n\theta \quad (3)$$

$$+\frac{\gamma\epsilon_2}{k_2^2}[A_2k_2I_n'(k_2r_2) + B_2k_2K_n'(k_2r_2)] \cos n\theta = 0 \quad (d)$$

$$A_2 I_n (k_2 r_2) + B_2 K_n (k_2 r_2) - A_3 I_n (k_3 r_2) - B_3 K_n (k_3 r_2) = 0 \qquad (e)$$

$$A_2' I_n (k_2 r_2) + B_2' K_n (k_2 r_2) - A_3' I_n (k_3 r_2) - B_3' K_n (k_3 r_2) = 0 \qquad (f)$$

$$\frac{j\omega}{r_3}\left(\frac{\epsilon_0}{k_4^2} - \frac{\epsilon_3}{k_3^2}\right)[A_3 I_n (k_3 r_3) + B_3 K_n (k_3 r_3)]\, n \sin n\theta$$

$$- \frac{\gamma}{k_3^2}[A_3' k_3 I_n' (k_3 r_3) + B_3' k_3 K_n' (k_3 r_3)] \cos n\theta$$

$$+ \frac{\gamma}{k_4^2}[A_4' k_4 H_n^{(1)\prime} (jk_4 r_3)] \cos n\theta = 0. \qquad (g)$$

$$\frac{j\omega\mu_0}{r_3}\left(\frac{\epsilon_0}{k_4^2} - \frac{\epsilon_3}{k_3^2}\right)[A_3' I_n (k_3 r_3) + B_3' K_n (k_3 r_3)]\, n \sin n\theta$$

$$- \frac{\gamma\epsilon_0}{k_4^2}[A_4 k_4 H_n^{(1)\prime} (jk_4 r_3)] \cos n\theta$$

$$+ \frac{\gamma\epsilon_3}{k_3^2}[A_3 k_3 I_n' (k_3 r_3) + B_3 k_3 K_n' (k_3 r_3)] \cos n\theta = 0 \qquad (h)$$

$$A_3 I_n (k_3 r_3) + B_3 K_n (k_3 r_3) - A_4 H_n^{(1)} (jk_4 r_3) = 0 \qquad (i)$$

$$A_3' I_n (k_3 r_3) + B_3' K_n (k_3 r_3) - A_4' H_n^{(1)} (jk_4 r_3) = 0 \qquad (j)$$

CHARACTERISTIC EQUATION FOR THE EH WAVE

In order that A's and B's in the above equations are not zero, the following relation must hold good.

$$\begin{vmatrix}
{}^{(a)}A_2 & {}^{(a)}B_2 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & {}^{(b)}A_2' & {}^{(b)}B_2' & 0 & 0 & 0 & 0 & 0 & 0 \\
{}^{(c)}A_2 & {}^{(c)}B_2 & -{}^{(c)}A_2' & -{}^{(c)}B_2' & 0 & 0 & {}^{(c)}A_3' & {}^{(c)}B_3' & 0 & 0 \\
{}^{(d)}A_2 & {}^{(d)}B_2 & {}^{(d)}A_2' & {}^{(d)}B_2' & -{}^{(d)}A_3 & -{}^{(d)}B_3 & 0 & 0 & 0 & 0 \\
{}^{(e)}A_2 & {}^{(e)}B_2 & 0 & 0 & -{}^{(e)}A_3 & -{}^{(e)}B_3 & 0 & 0 & 0 & 0 \\
0 & 0 & {}^{(f)}A_2' & {}^{(f)}B_2' & 0 & 0 & -{}^{(f)}A_3' & -{}^{(f)}B_3' & 0 & 0 \\
0 & 0 & 0 & 0 & {}^{(g)}A_3 & {}^{(g)}B_3 & -{}^{(g)}A_3' & -{}^{(g)}B_3' & 0 & {}^{(g)}A_4' \\
0 & 0 & 0 & 0 & {}^{(h)}A_3 & {}^{(h)}B_3 & {}^{(h)}A_3' & {}^{(h)}B_3' & -{}^{(h)}A_4 & 0 \\
0 & 0 & 0 & 0 & {}^{(i)}A_3 & {}^{(i)}B_3 & 0 & 0 & -{}^{(i)}A_4 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & {}^{(j)}A_3' & {}^{(j)}B_3' & 0 & -{}^{(j)}A_4'
\end{vmatrix} = 0 \quad (4)$$

where ${}^{(a)}A_2$ represents the coefficient of A_2 in equation (3 *a*) and so on. The general-characteristic equation for the EH wave obtained by solving the above determinant is as follows:

$$\Delta_1\Gamma_1 + \Delta_2\Gamma_2 + \Delta_3\Gamma_3 + \Delta_4\Gamma_4 + \Delta_5\Gamma_5 + \Delta_6\Gamma_6 + \Delta_7\Gamma_7 + \Delta_8\Gamma_8 + \Delta_9\Gamma_9 + \Delta_{10}\Gamma_{10} + \Delta_{11}\Gamma_{11} + \Delta_{12}\Gamma_{12} = 0 \quad (5)$$

where

$$\Delta_1 = \begin{vmatrix}
\frac{\gamma}{k_3} K_n'(k_3r_2)\cos n\theta & 0 & 0 \\
-\frac{\gamma}{k_3} K_n'(k_3r_3)\cos n\theta & 0 & \frac{\gamma}{k_4} H_n^{(1)'}(jk_4r_3)\cos n\theta \\
0 & -H_n^{(1)}(jk_4r_3) & 0
\end{vmatrix} \quad (6\,a)$$

$$\Delta_2 = \begin{vmatrix}
\frac{\gamma}{k_3} K_n'(k_3r_2)\cos n\theta & 0 & 0 \\
-\frac{\gamma}{k_3} K_n'(k_3r_3)\cos n\theta & 0 & \frac{\gamma}{k_4} H_n^{(1)'}(jk_4r_3)\cos n\theta \\
\frac{j\omega\mu_0}{r_3}\left(\frac{\epsilon_0}{k_4^2} - \frac{\epsilon_3}{k_3^2}\right) K_n(k_3r_3)\, n\sin n\theta & -\frac{\gamma\epsilon_0}{k_4} H_n^{(1)'}(jk_4r_3)\cos n\theta & 0
\end{vmatrix} \quad (6\,b)$$

$$\Delta_3 = \begin{vmatrix}
\frac{\gamma}{k_3} K_n'(k_3r_2)\cos n\theta & 0 & 0 \\
0 & -H_n^{(1)}(jk_4r_3) & 0 \\
K_n(k_3r_3) & 0 & -H_n^{(1)}(jk_4r_3)
\end{vmatrix} \quad (6\,c)$$

and so on for other Δ's.

$$\Gamma_1 = -\delta_{1,1} + \delta_{1,2} - \delta_{1,3} - \delta_{1,4} + \delta_{1,5} + \delta_{1,6} - \delta_{1,7} - \delta_{1,8} - \delta_{1,9} - \delta_{1,10} + \delta_{1,11} + \delta_{1,12} - \delta_{1,13} + \delta_{1,14} \quad (6\,d)$$

$$\Gamma_2 = \delta_{2,1} - \delta_{2,2} - \delta_{2,3} + \delta_{2,4} + \delta_{2,5} + \delta_{2,6} + \delta_{2,7} - \delta_{2,8} - \delta_{2,9} + \delta_{2,10} + \delta_{2,11} - \delta_{2,12} + \delta_{2,13} - \delta_{2,14} \quad (6\,e)$$

$$\Gamma_3 = +\delta_{3,1} - \delta_{3,2} - \delta_{3,3} + \delta_{3,4} + \delta_{3,5} - \delta_{3,6} - \delta_{3,7} + \delta_{3,8} + \delta_{3,9} - \delta_{3,10} - \delta_{3,11} + \delta_{3,12} - \delta_{3,13} + \delta_{3,14} + \delta_{3,15} - \delta_{3,16} + \delta_{3,17} - \delta_{3,18} - \delta_{3,19} + \delta_{3,20} - \delta_{3,21} + \delta_{3,22} + \delta_{3,23} - \delta_{3,24} - \delta_{3,25} + \delta_{3,26} + \delta_{3,27} - \delta_{3,28} - \delta_{3,29} + \delta_{3,30} + \delta_{3,31} + \delta_{3,32} + \delta_{3,33} - \delta_{3,34} - \delta_{3,35} + \delta_{3,36} \quad (6\,f)$$

and so on for other Γ's.

$$\delta_{1,1} = \frac{\gamma^2 \epsilon_2 \epsilon_3}{k_2 k_3} \mathrm{I}_n(k_3 r_3)\, \mathrm{K}_n(k_3 r_2)\, \mathrm{K}_n'(k_2 r_2)\, \mathrm{I}_n(k_2 r_1)\, \mathrm{I}_n(k_2 r_2)\, \mathrm{K}_n'(k_2 r_1) \times \mathrm{I}_n'(k_3 r_3) \cos^2 n\theta \quad (6\,g)$$

$$\delta_{2,1} = \frac{\gamma \epsilon_3}{k_3} [\mathrm{I}_n(k_3 r_3)]^2\, \mathrm{K}_n'(k_3 r_2)\, \mathrm{K}_n(k_2 r_1)\, [\mathrm{I}_n(k_2 r_2)]^2 \times \mathrm{K}_n'(k_2 r_1) \cos n\theta \quad (6\,h)$$

$$\delta_{3,1} = \frac{\gamma^3 \epsilon_3^2}{k_3^3} [\mathrm{I}_n'(k_3 r_3)]^2\, \mathrm{K}_n'(k_3 r_2)\, \mathrm{K}_n(k_2 r_1)\, [\mathrm{I}_n(k_2 r_2)]^2 \times \mathrm{K}_n'(k_2 r_1) \cos^3 n\theta \quad (6\,i)$$

and so on for other δ's.

A Special Case

A conductor of infinite conductivity embedded in free space:

In this case

$$\epsilon_2 = \epsilon_3 = \epsilon_4 = \epsilon_0$$
$$k_2 = k_3 = k_4 = k$$
$$r_2 = r_3 = r_1 \quad (7\,a)$$

This leads to

$$\Delta_{10} = \Delta_{11} = 0$$
$$\Gamma_4 = \Gamma_9 = \Gamma_{12} = 0 \quad (7\,b)$$

$$\Gamma_1 = \frac{\gamma^3\epsilon_0^3}{k^3}\,[I_n^4(K_n')^3 - 2I_n^3(K_n')^2 I_n'K_n - I_nK_n^3(I_n')^3]\cos^2 n\theta$$

$$\Gamma_2 = 2\,\frac{\gamma\epsilon_0}{k}\,[I_n^3K_n^2K_n'I_n']\cos n\theta$$

$$\Gamma_3 = \frac{\gamma^3\epsilon_0^2}{k^3}\,[(I_n')^4K_n^3 - 2(I_n')^3K_n'K_n^2I_n + 2(I_n')^2K_n(K_n')^2I_n^2 - I_n'(K_n')^3I_n^3]\cos^3 n\theta$$

$$\Gamma_5 = -2\,\frac{\gamma^2\epsilon_0}{k^2}\,[I_n^3(K_n')^2K_nI_n']\cos^2 n\theta$$

$$\Gamma_6 = 2\,\frac{\gamma^2\epsilon_0}{k^2}\,[I_n^2K_n^3(I_n')^2 - I_n^2K_n^2(I_n')^2K_n' - I_n^3(K_n')^2K_nI_n']\cos^2 n\theta$$

$$\Gamma_7 = 2\,\frac{\gamma^3\epsilon_0}{k^3}\,[(I_n')^2K_n(K_n')^2I_n^2)]\cos^3 n\theta$$

$$\Gamma_8 = \frac{\gamma^3\epsilon_0^2}{k^3}\,[(I_n')^4K_n^3 - 2I_nK_n^2(I_n')^3K_n' + 2(I_n')^2K_n(K_n')^2I_n^2 + I_n^3(K_n')^3I_n']\cos^3 n\theta \qquad (7\,c)$$

Therefore the characteristic equation (5) reduces to

$$\Delta_1\Gamma_1 + \Delta_2\Gamma_2 + \Delta_3\Gamma_3 + \Delta_5\Gamma_5 + \Delta_6\Gamma_6 + \Delta_7\Gamma_7 + \Delta_8\Gamma_8 = 0 \qquad (8\,a)$$

which yields

$$\begin{aligned} &H_n^{(1)}H_n^{(1)\prime}[(K_n')^4I_n^4 + K_n^4(I_n')^4] \\ &+ H_n^{(1)}H_n^{(1)\prime}K_nK_n'I_nI_n'[I_n^2(K_n')^2 - 3K_n^2(I_n')^2] \\ &+ 2(K_n')^2I_n^3I_n'[(H_n^{(1)\prime})^2K_n^2 - (K_n')^2(H_n^{(1)})^2] \\ &+ 2K_n^3(H_n^{(1)\prime})^2I_n^2(I_n')^2[K_n' - K_n] = 0 \end{aligned} \qquad (8\,b)$$

If $n = 1$, then using recurrence relations (Watson, 1922), equation (8 b) reduces to

$$\begin{aligned} &I_1^4\left(\frac{K_1}{kr_1} + K_0\right)^4 + K_1^4\left(I_0 - \frac{I_1}{kr_1}\right)^4 - K_1I_1^3\left(I_0 - \frac{I_1}{kr_1}\right)\left(K_0 + \frac{K_1}{kr_1}\right)^3 \\ &+ 3K_1^3I_1\left(K_0 + \frac{K_1}{kr_1}\right)\left(I_0 - \frac{I_1}{kr_1}\right)^3 \\ &\div K_1^4\left(I_0 - \frac{I_1}{kr_1}\right)\left[jH_0^{(1)} - \frac{H_1^{(1)}}{kr_1}\right]^2 \\ &+ K_1^3\left(I_0 - \frac{I_1}{kr_1}\right)\left(K_0 + \frac{K_1}{kr_1}\right)\left[jH_0^{(1)} - \frac{H_1^{(1)}}{kr_1}\right]^2 \end{aligned}$$

$$\cdots - \left(K_0 + \frac{K_1}{kr_1}\right)^4 I_1 (H_1^{(1)})^2$$

$$- \left(K_0 + \frac{K_1}{kr_1}\right)^2 I_1 K_1^2 \left[jH_0^{(1)} - \frac{H_1^{(1)}}{kr_1}\right]^2$$

$$= -j2I_1^2\left(I_0 - \frac{I_1}{kr_1}\right) \div H_1^{(1)}\left[jH_0^{(1)} - \frac{H_1^{(1)}}{kr_1}\right] \qquad (9\,a)$$

Therefore

$$I_1^2\left(I_0 - \frac{I_1}{kr_1}\right) = 0 \qquad (9\,b)$$

and

$$I_1^4\left(\frac{K_1}{kr_1} + K_0\right)^4 + K_1^4\left(I_0 - \frac{I_1}{kr_1}\right)^4$$

$$- K_1 I_1^3\left(I_0 - \frac{I_1}{kr_1}\right)\left(K_0 + \frac{K_1}{kr_1}\right)^3$$

$$+ 3K_1^3 I_1\left(K_0 + \frac{K_1}{kr_1}\right)\left(I_0 - \frac{I_1}{kr_1}\right)^3 = 0 \qquad (9\,c)$$

In the above equations the argument kr_1 of the Bessel functions has been omitted for convenience. From (9 *b*)

either $I_1(kr_1) = 0$ which yields $k = 0$

or

$$I_0(kr_1) = \frac{I_1(kr_1)}{kr_1} \qquad (9\,d)$$

For $k = 0$, we obtain the phase velocity $\omega/\beta = c$. But at microwave frequencies, the surface resistivity of the conductor being appreciable, the phase velocity cannot be equal to the free space velocity c. Hence the root $k = 0$ is physically inadmissible.

A plot of $I_0(kr_1)$ *vs.* kr_1 and $I_1(kr_1)/kr_1$ *vs.* kr_1 in Fig. 1 shows no intersection. Hence condition (9 *d*) is not valid.

The left-hand side of the equation (9 *c*) has been plotted *vs.* kr_1 in Fig. 1, Graph 3. It is evident that the equation (9 *c*) is true for $k > 5$. Taking $k = 5$ and $r_1 = 10^{-3}$ metre, $\gamma \simeq 5 \times 10^4$. γ is real (*i.e.*) the attenuation constant $\alpha \cong 5 \times 10^4$ nepers per metre. This leads to the conclusion that the asymmetric wave EH_1, even though it is excited on a solid conductor embedded in free space, becomes rapidly attenuated and hence cannot be propagated. This result agrees fully with the prediction of Hondros (1909). Therefore, the general characteristic equation (5) for the EH wave derived above is correct.

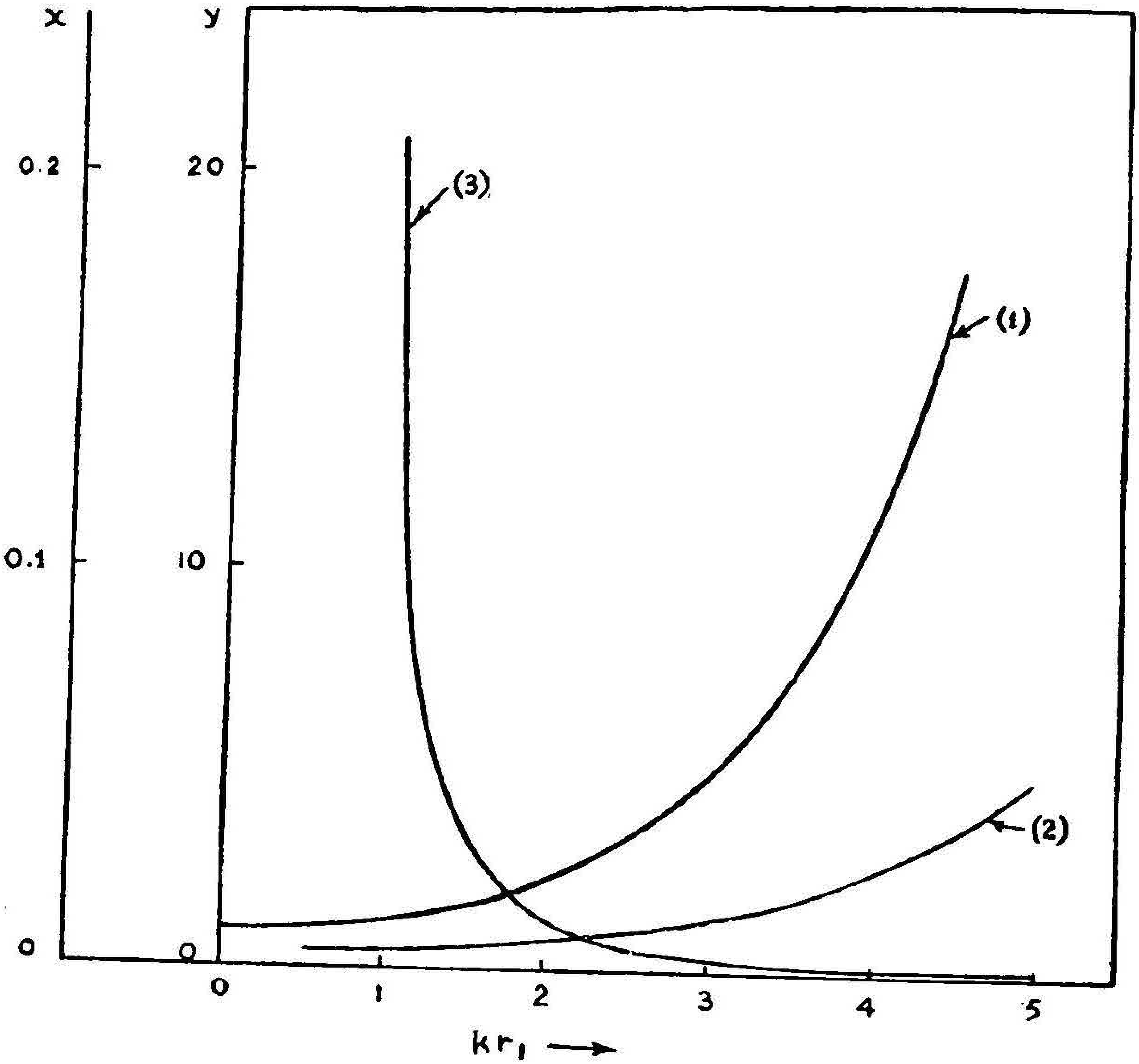

FIG. 1. (1) $I_0\ (kr_1)$ vs. kr_1. (2) $\frac{I_1\ (kr_1)}{kr_1}$ vs. kr_1. (3) L. H. S. of Equation 9*c* vs. kr_1. *x* and *y* represent graphs (3) and (1), (2) respectively.

EVALUATION OF THE EXCITATION CONSTANTS

The excitation constants A's and B's can be evaluated from equation (3) and the result is as follows:

$$
\begin{aligned}
B_2 &= A_2 a_n & A_2' &= A_2 w_n \\
A_3 &= A_2 p_n & B_2' &= A_2 v_n \\
A_4 &= A_2 q_n & B_3' &= A_2 y_n \\
& & A_3' &= A_2 y_n' \\
B_3 &= A_2 m_n & A_4' &= A_2 q_n'
\end{aligned}
\tag{10}
$$

where,

$$a_n = -\frac{I_n(k_2r_1)}{K_n(k_2r_1)} \qquad a_n' = -\frac{I_n'(k_2r_1)}{K_n'(k_2r_1)}$$

$$b_n = \frac{I_n(k_3r_2)}{I_n(k_2r_2) + a_nK_n(k_2r_2)} \qquad c_n = \frac{K_n(k_3r_2)}{I_n(k_2r_2) + a_nK_n(k_2r_2)} = \frac{K_n(k_3r_2)}{f_n}$$

$$b_n' = \frac{I_n(k_3r_2)}{I_n(k_2r_2) + a_n'K_n(k_2r_2)} \qquad c_n' = \frac{K_n(k_3r_2)}{I_n(k_2r_2)\, a_n'K_n(k_2r_2)} = \frac{K_n(k_3r_2)}{f_n'}$$

$$d_n = b_n[I_n'(k_2r_2) + a_nK_n'(k_2r_2)] \qquad d_n' = b_n'[I_n'(k_2r_2) + a_n'K_n'(k_2r_2)]$$

$$e_n = c_n[I_n'(k_2r_2) + a_nK_n'(k_2r_2)] \qquad e_n' = c_n'[I_n'(k_2r_2) + a_n'K_n'(k_2r_2)]$$

$$f_n = I_n(k_2r_2) + a_nK_n(k_2r_2) \qquad f_n' = I_n(k_2r_2) + a_n'K_n(k_2r_2)$$

$$g_n = \frac{d_n}{k_2} - \frac{I_n'(k_3r_2)}{k_3} \qquad g_n' = \frac{I_n'(k_3r_2)}{k_3} - \frac{d_n'}{k_2}$$

$$h_n = \frac{e_n}{k_3} - \frac{K_n'(k_3r_2)}{k_3} \qquad h_n' = \frac{K_n'(k_3r_2)}{k_3} - \frac{e_n'}{k_2}$$

$$i_n = \frac{I_n(k_2r_2)}{I_n(k_3r_2)} + a_n\frac{K_n(k_2r_2)}{I_n(k_3r_2)} \qquad i_n' = \frac{I_n(k_2r_2)}{I_n(k_3r_2)} + a_n'\frac{K_n(k_2r_2)}{I_n(k_3r_2)}$$

$$j_n = \frac{K_n(k_2r_2)}{I_n(k_3r_2)} \qquad l_n = I_n'(k_2r_2) + a_nK_n'(k_2r_2)$$

$$m_n = \frac{\frac{\epsilon_2}{k_2}l_n - i_ng_n - \frac{\epsilon_3}{k_3}i_nI_n'(k_3r_2)}{h_n - g_nj_n - \frac{\epsilon_3}{k_3}j_nI_n'(k_3r_2) + \frac{\epsilon_3}{k_3}K_n'(k_3r_2)}$$

$$p_n = \frac{f_n - m_nK_n(k_3r_2)}{I_n(k_3r_2)} \qquad q_n = \frac{p_nI_n(k_3r_3) + m_nK_n(k_3r_3)}{H_n^{(1)}(jk_4r_3)}$$

$$r_n = h_n - g_nj_n \qquad s_n = \left[\frac{\epsilon_2}{k_2}l_n - \frac{\epsilon_3}{k_3}i_nI_n'(k_3r_2)\right]$$

$$t_n = \left[\frac{\epsilon_3}{k_3}j_nI_n'(k_3r_2) - \frac{\epsilon_3}{k_3}K_n'(k_3r_2)\right] \qquad u_n = \frac{s_nr_n - i_ng_nt_n}{\xi f_n'(t_n - r_n)}$$

$$v_n = u_na_n' \qquad w_n = \frac{u_nI_n(k_2r_2) + v_nK_n(k_2r_2)}{I_n(k_3r_2)}$$

$$x_n = \frac{K_n(k_3r_2)}{I_n(k_3r_2)} \qquad y_n = \frac{z_n}{z_n'}$$

$$z_n = \zeta_n \left[I_n(k_2r_2) + a_nK_n(k_2r_2) - \left\{ \frac{\gamma}{k_2} u_nI_n'(k_2r_2) + v_nK_n'(k_2r_2) - \frac{\gamma}{k_3} w_n \right\} \cos n\theta \right]$$

$$z_n' = -\gamma \cos n\theta \left[\frac{K_n'(k_3r_2)}{k_3} - \frac{x_nI_n'(k_3r_2)}{k_3} \right]$$

$$\xi = \frac{j\omega\mu_0}{r_2} n \sin n\theta \left(\frac{\epsilon_3}{k_3^2} - \frac{\epsilon_2}{k_2^2} \right) \qquad \zeta_n = \frac{j\omega}{r_2} n \sin n\theta \left(\frac{\epsilon_3}{k_3^2} - \frac{\epsilon_2}{k_3^2} \right)$$

$$y_n' = \frac{w_nI_n(k_3r_2) + v_nK_n(k_3r_2)}{y_n} \qquad q_n' = \frac{y_n'I_n(k_3r_3) + y_nK_n(k_3r_3)}{H_n^{(1)}(jk_4r_3)}$$

The excitation constant A_2 can be expressed in terms of the axial power flow $P_z^{e,h}$ as follows (Chatterjeee, *loc. cit.*):

$$P_z^{e,h} = A_2^2 x_n (k_2, k_3, k_4, r_1, r_2, r_3, \omega)$$

or

$$A_2 = \tau_n \sqrt{P_z^{e,h}} = \tau_n \psi_z \tag{11}$$

FIELD COMPONENTS OF THE EH WAVE IN DIFFERENT MEDIA IN TERMS OF THE POWER FLOW

The field components of the EH wave are obtained from (2), (10) and (11) as follows:

In medium 2 $r_2 \geqslant r \geqslant r_1$

$$E_{z2}^{e,h} = \tau_n\psi_z [I_n(k_2r) + a_nK_n(k_2r)] \cos n\theta$$

$$E_{r2}^{e,h} = \frac{\gamma}{k_2} \tau_n\psi_z [I_n'(k_2r) + a_nK_n'(k_2r)] \cos n\theta - \frac{j\omega\mu_0}{k_2^2 r} \tau_n\psi_z [u_nI_n(k_2r) + v_nK_n(k_2r)] n \sin n\theta$$

$$E_{\theta 2}^{e,h} = -\frac{\gamma}{k_2^2 r} \tau_n\psi_z [I_n(k_2r) + a_nK_n(k_2r)] n \sin n\theta - \frac{j\omega\mu_0}{k_2} \tau_n\psi_z [u_nI_n'(k_2r) + v_nK_n'(k_2r)] \cos n\theta$$

$$H_{z2}^{e,h} = \tau_n\psi_z [u_nI_n(k_2r) + v_nK_n(k_2r)] \cos n\theta$$

$$H_{z2}^{e,h} = \frac{j\omega\epsilon_2}{k_2^2 r}\tau_n\psi_z\,[I_n(k_2r) + a_nK_n(k_2r)]\,n\sin n\theta$$

$$+\frac{\gamma}{k_2}\tau_n\psi_z\,[u_nI_n'(k_2r) + v_nK_n'(k_2r)]\cos n\theta$$

$$H_{\theta 2}^{e,h} = \frac{j\omega\epsilon_2}{k_2}\tau_n\psi_z\,[I_n'(k_2r) + a_nK_n'(k_2r)]\cos n\theta$$

$$-\frac{\gamma}{k_2^2 r}\tau_n\psi_z\,[u_nI_n(k_2r) + v_nK_n(k_2r)]\,n\sin n\theta$$

In medium 3 $r_3 \geqslant r \geqslant r_2$

$$E_{z3}^{e,h} = \tau_n\psi_z\,[p_nI_n(k_3r) + m_nK_n(k_3r)]\cos n\theta$$

$$E_{r3}^{e,h} = \frac{\gamma}{k_3}\tau_n\psi_z\,[p_nI_n'(k_3r) + m_nK_n'(k_3r)]\cos\theta$$

$$-\frac{j\omega\mu_0}{k_3^2 r}\tau_n\psi_z\,[y_n'I_n(k_3r) + y_nK_n(k_3r)]\,n\sin n\theta$$

$$E_{\theta 3}^{e,h} = -\frac{\gamma}{k_3^2 r}\tau_n\psi_z\,[p_nI_n(k_3r) + m_nK_n(k_3r)]\,n\sin n\theta$$

$$-\frac{j\omega\mu_0}{k_3}\tau_n\psi_z\,[y_n'I_n'(k_3r) + y_nK_n'(k_3r)]\cos n\theta$$

$$H_{z3}^{e,h} = \tau_n\psi_z\,[y_n'I_n(k_3r) + y_nK_n(k_3r)]\cos n\theta$$

$$H_{r3}^{e,h} = \frac{j\omega\epsilon_3}{k_3^2 r}\tau_n\psi_z\,[p_nI_n(k_3r) + m_nK_n(k_3r)]\,n\sin n\theta$$

$$+\frac{\gamma}{k_3}\tau_n\psi_z\,[y'_nI_n'(k_3r) + y_nK_n'(k_3r)]\cos n\theta$$

$$H_{\theta 3}^{e,h} = \frac{j\omega\epsilon_3}{k_3}\tau_n\psi_z\,[p_nI_n'(k_3r) + m_nK_n'(k_3r)]\cos n\theta$$

$$-\frac{\gamma}{k_3^2 r}\tau_n\psi_z\,[y_n'I_n(k_3r) + y_nK_n(k_3r)]\,n\sin n\theta$$

In medium 4 $r \geqslant r_3$

$$E_{z4}^{e,h} = \tau_n\psi_z q_n H_n^{(1)}(jk_4r)\cos n\theta$$

$$E_{r4}^{e,h} = \frac{\gamma}{k_4}\tau_n\psi_z q_n H_n^{(1)\prime}(jk_4r)\cos n\theta$$

$$-\frac{j\omega\mu_0}{k_4^2 r}\tau_n\psi_z q_n' H_n^{(1)}(jk_4r)\,n\sin n\theta$$

$$E_{\theta 4}^{e,h} = -\frac{\gamma}{k_4^2 r}\tau_n \psi_z q_n H_n^{(1)}(jk_4 r)\, n \sin n\theta$$

$$-\frac{j\omega\mu_0}{k_4}\tau_n \psi_z q_n' H_n^{(1)\prime}(jk_4 r)\cos n\theta$$

$$H_{z4}^{e,h} = \tau_n \psi_z q_n' H_n^{(1)}(jk_4 r)\cos n\theta$$

$$H_{r4}^{e,h} = \frac{j\omega\epsilon_0}{k_4^2 r}\tau_n \psi_z q_n H_n^{(1)}(jk_4 r)\, n \sin n\theta$$

$$+\frac{\gamma}{k_4}\tau_n \psi_z q_n' H_n^{(1)\prime}(jk_4 r)\cos n\theta$$

$$H_{\theta 4}^{e,h} = \frac{j\omega\epsilon_0}{k_4}\tau_n \psi_z q_n H_n^{(1)\prime}(jk_4 r)\cos n\theta$$

$$-\frac{\gamma}{k_4^2 r}\tau_n \psi_z q_n' H^{(1)}(jk_4 r)\, n \sin n\theta$$

REFERENCES

Chatterjee, S. K. and Chatterjee, (Mrs.), R. — *Jour. Ind. Inst. Sci.*, 1956, **38**, 157.

Watson, G. N. .. *A Treatise on the Theory of Bessel Functions*, Cambridge Univerisity Press, 1922.

Handros, D. .. *Ann. d. Phys.*, 1909, **30**, 905.