SOME INVESTIGATIONS ON DIELECTRIC AERIALS—PART II

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Abstract

An expression for the electric field intensity at a distant point from a circular dielectric rod aerial excited in the HE_{11} mode has been derived by utilising Huyghen's principle. The rod has been considered as an assemblage of a large number of Huyghen's radiators distributed all over the surface. It is found from the expression of radiation pattern that there is very little difference in the structure of the radiation pattern and beam width of the major lobe in both the $\Phi = 0°$ and $\Phi = 90°$ planes.

Introduction

Halliday and Kiely (1947) have derived an expression for the radiation pattern of a circular dielectric rod aerial excited in the HE_{11} mode by considering the rod to consist of two sets of end-fire arrays arranged in a broadside fashion in one axial plane only (Fig. 1). The sources of the array are Huyghen's radiators. The object of the present paper is to generalise the above theory with a different mathematical approach. The rod is considered as consisting of a very large number of Huyghen's radiators, distributed in all possible planes (Fig. 2) instead of one
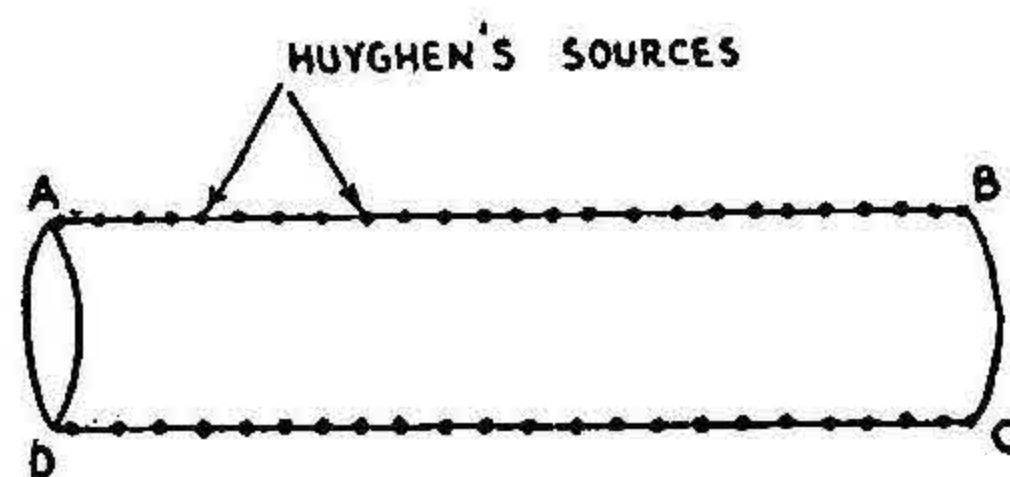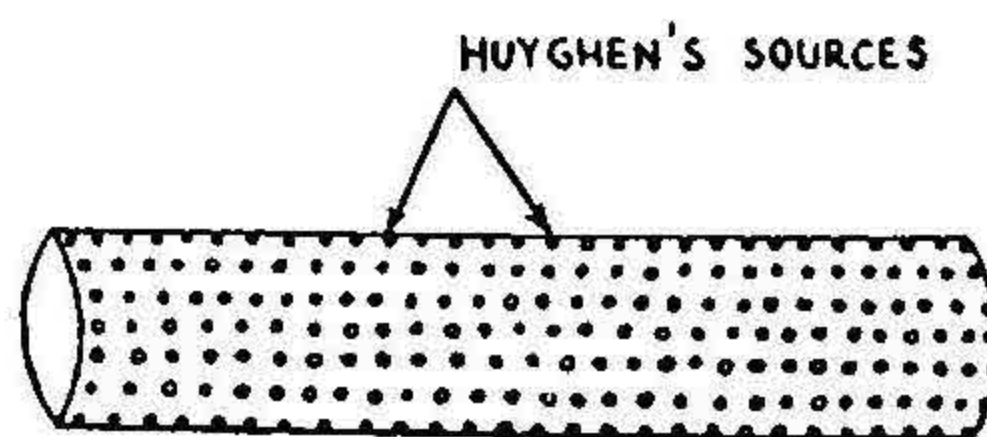

Fig. 1

Fig. 2

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plane only. It may be mentioned that both the theories are scalar as compared to the vector treatment made by the authors (1956) in a recent paper.

FIELD AT A DISTANT POINT

To calculate the field at a distant point P due to this infinite number of sets of radiating elements, it is necessary to calculate the field at P due to an infinite number of radiating elements distributed on the circumference of a cross-section TT′ (Fig. 3) of the cylinder and then to multiply this field by the field due to one set of elements AB (Fig. 1).
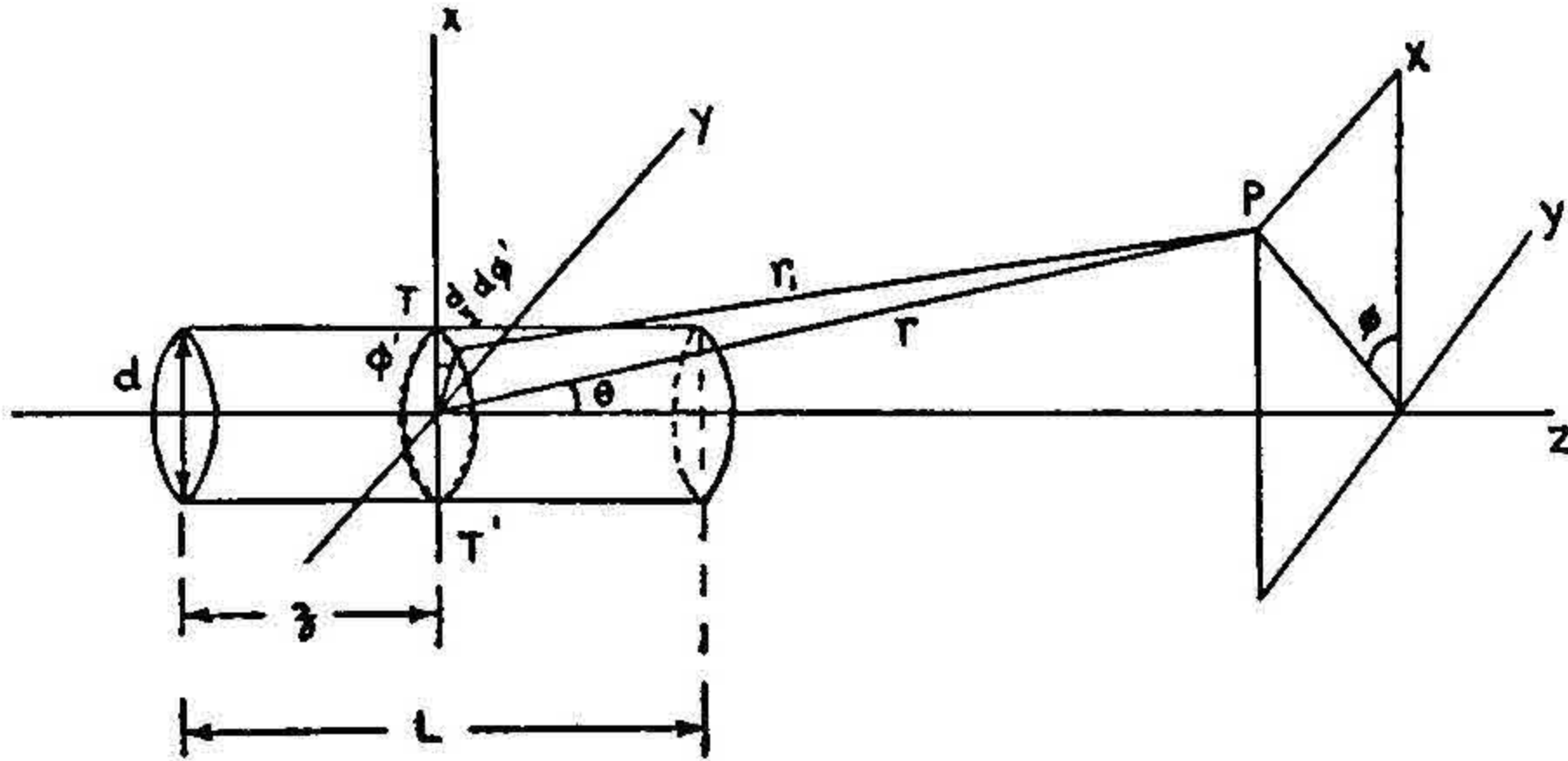

FIG. 3

The distant point P is designated by the spherical polar co-ordinates (r, θ, Φ) and the location of the element on the circumference of the cross-section TT′ is specified by the cylindrical co-ordinates (ρ, Φ', z). Since we consider only one cross-section, $z = $ constant and $\rho = d/2$. When $r \gg L$, the length of the rod,

$$r_1 = \text{distance of the element } \frac{d}{2} d\Phi' \text{ from } P$$

$$\cong r - \frac{d}{2} \sin \phi' \sin \phi \sin \theta - \frac{d}{2} \cos \phi' \cos \phi \sin \theta$$

where $r = $ distance of the centre of the cross-section from P. The field dE_2 at P due to a point source in the infinitesimal element $\frac{d}{2} \delta\phi'$

$$= \frac{A}{r_1} e^{j\omega(t - r_1/v_0)}$$

$$= \frac{A}{r_1} e^{j(\omega t - k r_1)} \tag{1}$$

where

$v_0 = $ velocity of the electromagnetic waves in free space.

$k = $ phase constant $2\pi/\lambda_0$ in free space.

$A = $ constant involving magnitude of excitation.

We assume the strength of the point sources on the circumference of the cross-section TT' to vary as the electromagnetic power which is the same as $\vec{E} \times \vec{H}$. The latter varies as $\cos^2 \phi'$ or $\sin^2 \phi'$ (vide Part I).

Therefore, the total field E_2 at P due to all the point sources on the circumference of the cross-section is given by

$$E_2 = \int_{\phi'=0}^{2\pi} \frac{A}{r_1} \sin^2 \phi' \, e^{j(\omega t - kr_1)} \frac{d}{2} \, d\phi'$$

$$\cong \frac{Ad}{2r} e^{j(\omega t - kr)} \int_{\phi'=0}^{2\pi} \sin^2 \phi' \, e^{jk \sin \theta} \cos(\phi - \phi') \frac{d}{2} \, d\phi'$$

$$= -\frac{Ad}{2r} e^{j(\omega t - kr)} \left[\pi \left\{ J_0 \left(\frac{kd}{2} \sin \theta \right) + \cos 2\phi \, J_2 \left(\frac{kd}{2} \sin \theta \right) \right\} \right.$$

$$\left. + 4 \sqrt{\frac{\pi}{2}} \, \Gamma(2) \sin 2\phi \, \frac{J_{3/2} \left(\frac{kd}{2} \sin \theta \right)}{\left(\frac{kd}{2} \sin \theta \right)^{\frac{3}{2}}} \right] \tag{2}$$

The field E_1 due to one set of elements AB is (Appendix)

$$E_t = E_0 \frac{\sin \left\{ \frac{L}{2} (\beta - k \cos \theta) \right\}}{\frac{L}{2} (\beta - k \cos \theta)} \tag{3}$$

Therefore, the total field at P due to the whole rod is

$$E_p = E_0 \frac{Ad}{2r} e^{j(\omega t - kr)} \frac{\sin \left\{ \frac{L}{2} (\beta - k \cos \theta) \right\}}{\frac{L}{2} (\beta - k \cos \theta)}$$

$$\times \left[\pi \left\{ J_0 \left(\frac{kd}{2} \sin \theta \right) + \cos 2\phi \, J_2 \left(\frac{kd}{2} \sin \theta \right) \right\} \right.$$

$$\left. + 4 \sqrt{\frac{\pi}{2}} \, \Gamma(2) \sin 2\Phi \frac{J_{3/2} \left(\frac{kd}{2} \sin \theta \right)}{\left(\frac{kd}{2} \sin \theta \right)^{\frac{3}{2}}} \right] \tag{4}$$

The gamma function $\Gamma(2) = 1$.

In the $\Phi = 0°$ plane

$$E_\rho \propto \left\{ J_0\left(\frac{kd}{2}\sin\theta\right) + J_2\left(\frac{kd}{2}\sin\theta\right) \right\} \frac{\sin\left\{\frac{L}{2}(\beta - k\cos\theta)\right\}}{\frac{L}{2}(\beta - k\cos\theta)} \qquad (5)$$

In the $\Phi = 90°$ plane

$$E_\rho \propto \left\{ J_0\left(\frac{kd}{2}\sin\theta\right) - J_2\left(\frac{kd}{2}\sin\theta\right) \right\} \frac{\sin\left\{\frac{L}{2}(\beta - k\cos\theta)\right\}}{\frac{L}{2}(\beta - k\cos\theta)} \qquad (6)$$

DISCUSSION

Fig. 4 shows the radiation pattern (E_ρ/E_{max} vs. θ) in the $\Phi = 0°$ plane for a polystyrene rod of length $3\lambda_0$ and diameter $0.46\lambda_0$. The radiation pattern plotted in the $\Phi = 90°$ plane shows almost the same structure and has therefore not been reported.
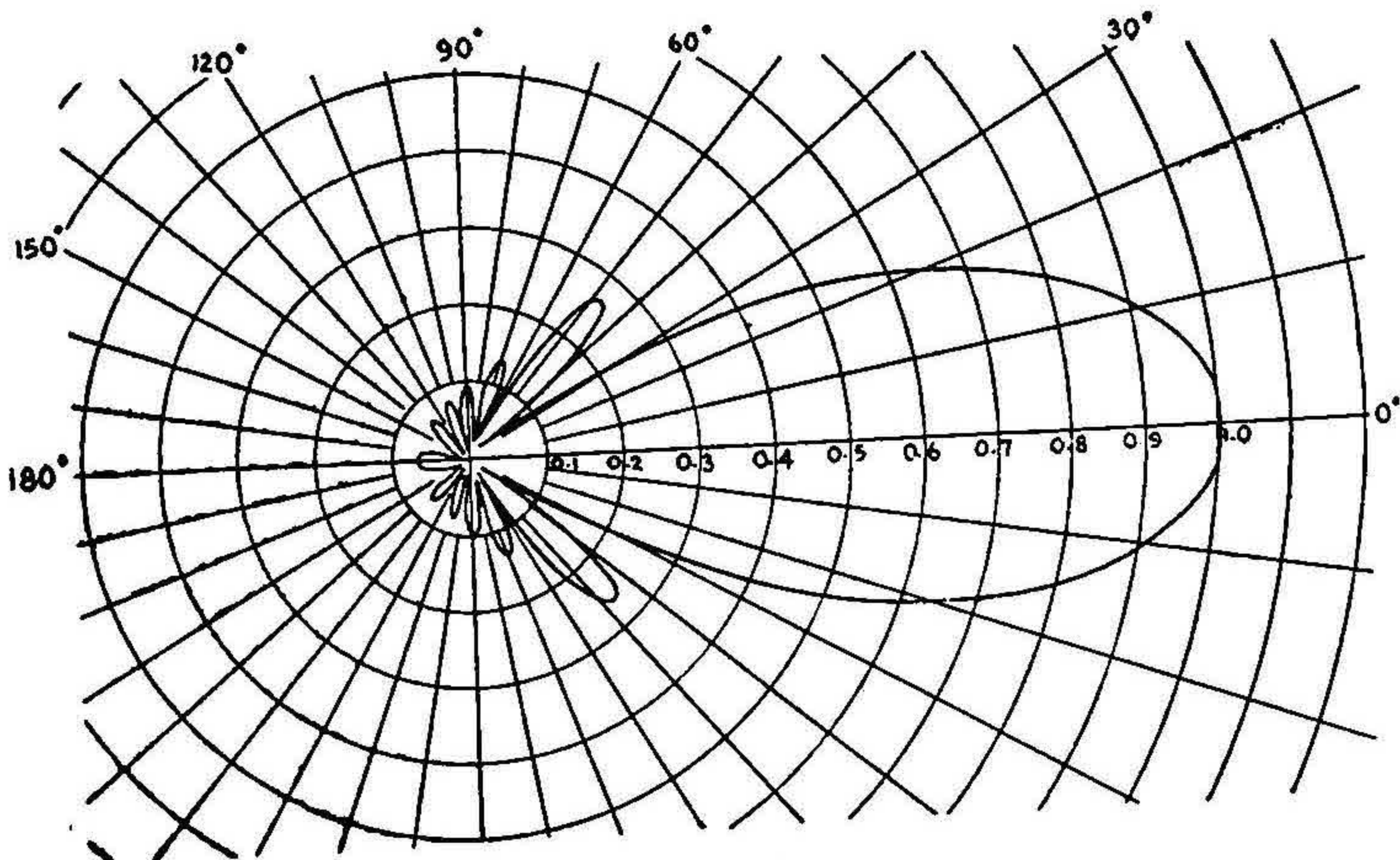

FIG. 4

The expression for the radiation pattern derived by Halliday and Kiely (*loc. cit.*) is (*Appendix*)

$$\frac{E_p}{E_{max}} = \frac{(\beta - k)\sin\left\{\frac{L}{2}(\beta - k\cos\theta)\right\}}{(\beta - k\cos\theta)\sin\left\{\frac{L}{2}(\beta - k)\right\}} \tag{7}$$

The following Tables I and II show a comparative study of the radiation pattern obtained from equations (5), (6) and (7).

TABLE I

Positions of maxima and relative strengths of lobes

Extended theory of Halliday and Kiely Equations (5) and (6)				Halliday and Kiely's theory, Equation (7)	
$\Phi = 0°$ plane		$\Phi = 90°$ plane			
Angle of maxima in degrees	Relative strengths	Angle of maxima in degrees	Relative strengths	Angle of maxima in degrees	Relative strengths
0	1·00	0	1·00	0	1·00
47	0·27	47	0·19	47	0·30
				55	0·20
70	0·14	70	0·07	70	0·17
90	0·10	90	0·04	90	0·13
110	0·08	110	0·04	110	0·10
135	0·07	135	0·05	135	0·08
180	0·07	180	0·07	180	0·07

The beam width of the major lobes at half power points in both the $\Phi = 0°$ and $\Phi = 90°$ planes in the case of the extended theory as well as in the case of the Halliday and Kiely's theory is found to be 36°. Whereas the beam widths obtained from the theory (Chatterjee *et al.*, *loc. cit.*) based on Schelkunoff's Equivalence principle are 36° and 42° in the $\Phi = 0°$ and $\Phi = 90°$ planes respectively. It is also found in this case that the structure of the radiation pattern differs in both the planes for the same rod.

The reasons for the difference in the results obtained in Part 1 of this paper and the present one may be explained as follows:

Table II

Positions of null

Extended theory of Halliday and Kiely Equations (5) and (6)		Halliday and Kiely's theory, Equation (7)
$\Phi = 0°$	$\Phi = 90°$	
35	35	35
..	..	52
60	60	60
80	80	80
100	100	100
120	120	120
150	150	150

(*i*) The theory based on Schelkunoff's Equivalence principle is a vector formulation, whereas the present theory including that of Halliday and Kiely based on the original Huyghen's principle is essentially a scalar formulation. The former approach is regarded as more correct as it gives the direction of the field at a distant point as well as different beam widths in different planes. This is justified due to the asymmetric nature of the mode of excitation. Whereas the theory based on Huyghen's principle does not indicate the direction of the field at a distant point nor does it show any variation of the beam widths in different planes, even though the mode of excitation is asymmetric.

(*ii*) Huyghen's ray theory being essentially a salar treatment does not satisfy Maxwell's equations. Hence such a representation of the electromagnetic field with the help of the Huyghen's principle is not rigorously justified, even though there is fair agreement in the case of major lobes obtained by different methods.

If however, the Huyghen's radiators are to represent correctly the electromagnetic field satisfying Maxwell's equations with proper boundary conditions, the boundary surface of the aerial should be considered as carrying surface and line charges which account for the electric and magnetic current sheets. In the case, when the surface is a closed one, the effects of the line charges cancel out. This revised concept of the Huyghen's principle first proposed by Larmor (1903) and later modified by Kottler (1923), if suitably applied, will lead to the same result as obtained by the application of Schelkunoff's principle as the latter principle is nothing but a revived form of Kottler's vector formulation of the original Huyghen's principle.

APPENDIX

The field due to one set of radiators AB (Fig. 1) is (Halliday and Kiely, *loc. cit.*)

$$E_1 = E_0 \frac{\sin\left\{\frac{\pi L}{\lambda_0}(K - \cos\theta)\right\}}{\frac{\pi L}{\lambda_0}(K - \cos\theta)} \tag{8}$$

We have

$$\frac{1}{K^2} = \left(\frac{\lambda}{\lambda_0}\right)^2 = \frac{1}{\bar{\epsilon} - x_1^2 \left(\frac{\lambda_0}{\pi d}\right)^2}$$

where, $\bar{\epsilon} = \epsilon/\epsilon_0$ relative dielectric constant of the rod

$$x_1 = k_1 \frac{d}{2}$$

Hence,

$$K = \sqrt{\bar{\epsilon} - \frac{k_1^2 \lambda_0^2}{4\pi^2}}$$

Let

$$k = \frac{2\pi}{\lambda_0} \quad k_1^2 = \omega^2 \mu_1 \epsilon_1 + \gamma^2 = \beta_1^2 - \beta^2$$

$$\therefore \quad K = \sqrt{\bar{\epsilon} - \frac{\beta_1^2 - \beta^2}{k^2}}$$

But

$$\beta_1 = \frac{\omega}{v_1} = k\sqrt{\bar{\epsilon}}$$

$$\therefore \quad K = \frac{\beta}{k} \tag{9}$$

From (8) and (9) the field at a distant point due to one set of array elements is

$$E_1 = E_0 \frac{\sin\left\{\frac{L}{2}(\beta - k\cos\theta)\right\}}{\frac{L}{2}(\beta - k\cos\theta)}$$

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