SWITCHING TRANSIENTS IN THREE-PHASE INDUCTION MOTORS RUN ON SINGLE PHASE SUPPLY

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ABSTRACT

The subject of switching transients in polyphase induction motors and synchronous machines has been studied in very great detail by several investigators in the past, but no published factual data exist dealing with the analysis of the problem of switching transients in a three-phase induction motor run as a Capacitor Motor on single phase supply. This particular problem has been studied in this paper in detail by applying the Laplace Transform in a slightly modified form.

It is shown that the developed electrical torque and the voltage across the condenser do not exhibit any abnormal values during the first few cycles after the closing of the switch, whether switching is done at the maximum value or zero value of the applied voltage. This result is in striking contrast with the results obtained earlier with a plain general purpose Capacitor Motor.

INTRODUCTION

Single phase operation of three-phase motors has been a subject of very great interest in the past to both the design and operating engineers. Quite a large amount of literature exists dealing with the analysis of this problem by the method of Symmetrical Components. In a recent publication[7] the method of Dyadic Circuit Analysis, has been applied to this problem and analytical expressions have been obtained with the aid of which it is possible to determine the performance characteristics of the motor from standstill to synchronous speed.

It is a well-known fact that auxiliary impedances could be used to shift the phase currents and terminal voltages of a polyphase motor in order to operate it on single phase supply. The two standard methods are the Series Impedance Method and the Shunt Impedance Method. By a judicious choice of the circuit elements it is possible to have reasonably good balancing both as regards phase and magnitude of the currents and voltages. A three-phase motor run on single phase supply as a Capacitor Motor is by far the most important case of the Shunt Impedance Method of operation. In the paper referred to above, expressions for the line current and torques have been derived for this case as well.

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In a very recent paper, Habermann[10] has made a comprehensive study of this problem by applying the method of symmetrical components and come to the conclusion that it would be far more economical to use a plain single phase motor, rather than a combination of a three-phase motor and a phase converter. He has shown that a three-phase motor in conjunction with a phase converter has an appreciably lower breakdown torque than the motor operating singly, supplied with balanced three-phase power. The performance results and the formulation from which they have been derived by Habermann, indicate clearly that for the motor to give normal performance when operated on single phase supply with a phase converter, it would be very necessary to use an oversized three-phase motor.

This paper deals with the problem of Switching Transients in a three-phase motor operated on single phase supply with a phase converter. This is an extension of the work described in a paper by the author entitled ' Switching Transients in Single Phase Induction Motors,' published in the 1956 *Transactions of the A.I.E.E.*† The analysis of this problem has been carried out with the help of the Laplace Transform in a slightly modified form. This facilitates the easily determined machine constants to be used directly in the formulæ, for quick computation of the results.

ASSUMPTIONS

In accordance with the well-known limitations of the performance equations for induction motors the following simplifying assumptions are made in this study.

(1) The electrical switching is accomplished in zero time.

(2) Both the rotor and stator have symmetrical windings.

(3) The rotor is perfectly smooth and the self-inductances of the windings are independent of the rotor position.

(4) The effect of magnetic saturation, hysteresis and eddy current losses are completely disregarded.

(5) The resistances and inductances are considered to be unaffected by the absolute frequencies, of the currents in the stator and rotor.

(6) In the low frequency transients encountered in induction motor studies, the effects of inter-turn capacitances of the windings are neglected.

THEORY

Fig. 1 shows the phase converter with a motor having delta connected stator winding. An auto-transformer could also be used to obtain more satisfactory operating characteristics. The converter consists of two capacitors of unequal

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ratings, with a voltage sensitive relay that cuts out the larger condenser and connects the smaller one to the circuit at the correct speed. The running capacitor is assumed to have a resistance 5 per cent. of its reactance magnitude. The equivalent circuit at standstill developed by Habermann in the paper referred to earlier is given in Fig. 2.

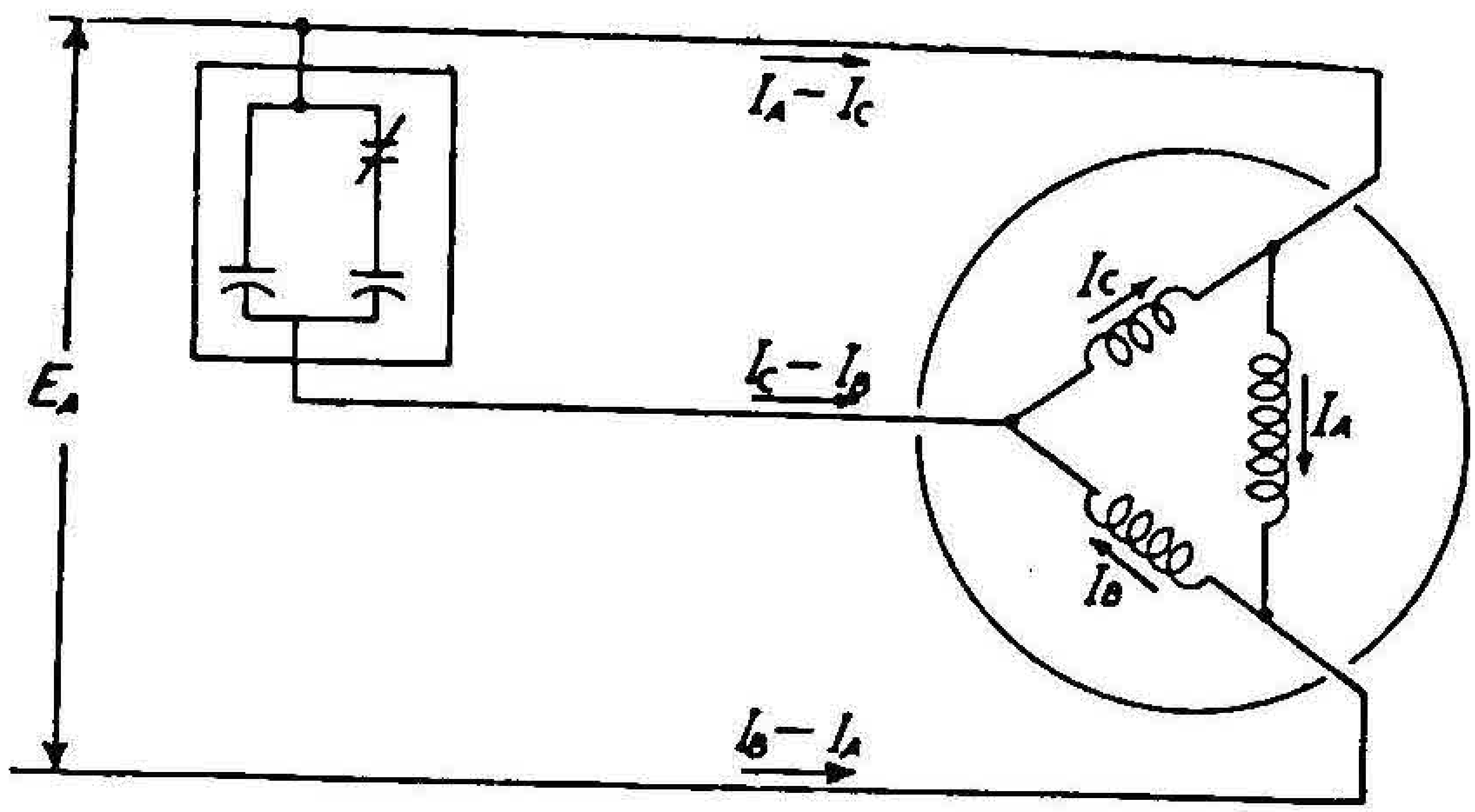

FIG. 1. Converter and three-phase motor on single phase supply.

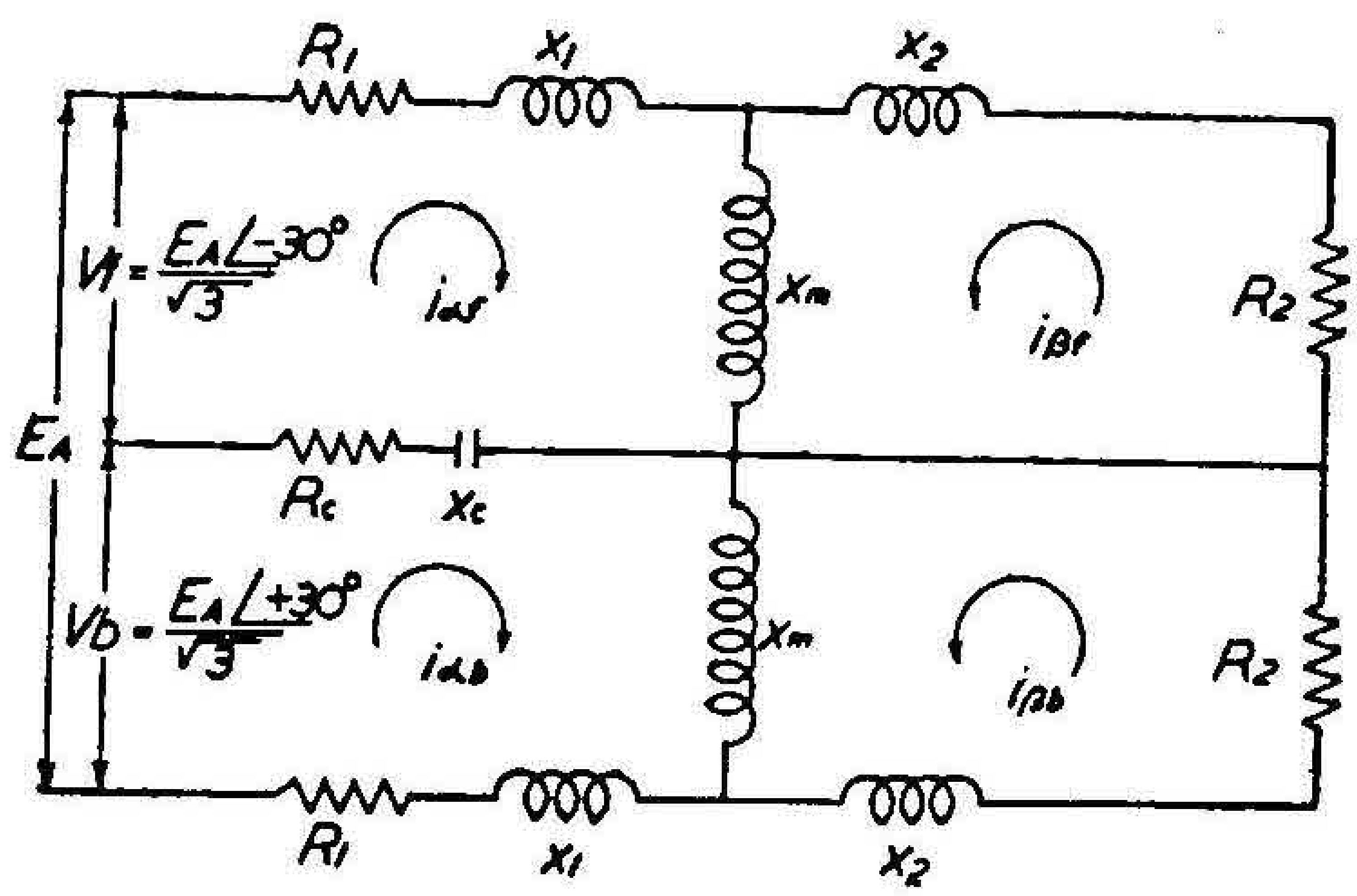

FIG. 2. Equivalent circuit of three-phase motor with converter on a single phase supply.

The mesh equations of Fig. 2 can be written down by inspection, as follows:

$$R_1 i_{\alpha f} + X_a \frac{di_{\alpha f}}{dt} + X_m \frac{di_{\beta f}}{dt} + R_c (i_{\alpha f} - i_{\alpha b}) + X_c \int_0^t (i_{\alpha f} - i_{\alpha b})\, dt = V_f(t) \tag{1}$$

$$R_1 i_{\alpha b} + X_a \frac{di_{\alpha b}}{dt} + X_m \frac{di_{\beta b}}{dt} + R_c (i_{\alpha b} - i_{\alpha f}) + X_c \int_0^t (i_{\alpha b} - i_{\alpha f})\, dt = V_b(t) \tag{2}$$

$$X_m \frac{di_{af}}{dt} + R_2 i_{\beta f} + X_\beta \frac{di_{\beta f}}{dt} = O \tag{3}$$

$$X_m \frac{di_{ab}}{dt} + R_2 i_{\beta b} + X_\beta \frac{di_{\beta b}}{dt} = O \tag{4}$$

By taking the Laplace Transform, on both sides, and rearranging terms suitably, the following four equations are obtained.

$$\begin{aligned} I_{af} \left\{ R_1 + R_c + sX_a + \frac{X_c}{s} \right\} + I_{ab} \left\{ - R_c - \frac{X_c}{s} \right\} \\ + I_{\beta f}\, sX_m + I_{\beta b} \,.\, O = V_f(s) \end{aligned} \tag{5}$$

$$\begin{aligned} I_{af} \left\{ - R_c - \frac{X_c}{s} \right\} + I_{ab} \left\{ R_1 + R_c + sX_a + \frac{X_c}{s} \right\} \\ + I_{\beta f} \,.\, O + I_{\beta b} \,.\, sX_m = V_b(s) \end{aligned} \tag{6}$$

$$I_{af} \,.\, sX_m + I_{ab} \,.\, O + I_{\beta f} (R_2 + sX_\beta) + I_{\beta b} \,.\, O = O \tag{7}$$

$$I_{af} \,.\, O + I_{ab} \,.\, sX_m + I_{\beta f} \,.\, O + I_{\beta b} (R_2 + sX_\beta) = O \tag{8}$$

In all the equations given above, the independent variable is taken as (t) = time in radians or $\omega = 2\pi f$ times the time in seconds $= \omega t'$. This facilitates the inductances being replaced by the better known values of reactances, measured at the power supply frequency. The differential operator d/dt in the integro-differential equations, 1 to 4, is thus rendered dimensionless in character, since $\omega t'$ is a pure ratio of two quantities having the same dimensional value. The roots of the characteristic equation are also dimensionless and are usually expressed as complex fractions of the angular frequency at which the reactances are given. An additional advantage accruing from this particular choice of the independent variable is, that, on actual oscillograms of voltages and currents obtained experimentally the time scale can usually be determined by a proper timing wave.

The equations, (5) to (8), can be rewritten in the following way by making the simplifying substitutions indicated here.

Let

$$A = R_1 + R_c + sX_a + \frac{X_c}{s}$$

$$B = - \left\{ R_c + \frac{X_c}{s} \right\}$$

$$C = sX_m$$

$$D = R_2 + sX_\beta$$

Therefore,

$$V_f = AI_{af} + BI_{ab} + CI_{\beta f} + O \,.\, I_{\beta b} \tag{9}$$

$$V_b = BI_{af} + AI_{ab} + O \,.\, I_{\beta f} + CI_{\beta b} \tag{10}$$

$$O = CI_{af} + O \,.\, I_{ab} + DI_{\beta f} + O \,.\, I_{\beta b} \tag{11}$$

$$O = O \,.\, I_{af} + CI_{ab} + O \,.\, I_{\beta f} + DI_{\beta b} \tag{12}$$

In matrix notation:

$$\begin{vmatrix} V_f \\ V_b \\ O \\ O \end{vmatrix} = \begin{vmatrix} A & B & C & O \\ B & A & O & C \\ C & O & D & O \\ O & C & O & D \end{vmatrix} \cdot \begin{vmatrix} I_{af} \\ I_{ab} \\ I_{\beta f} \\ I_{\beta b} \end{vmatrix}$$

$$\overline{V} = \overline{\triangle} \cdot \overline{I} \tag{13}$$

$$\text{Det}\,\triangle = \begin{vmatrix} A & B & C & O \\ B & A & O & C \\ C & O & D & O \\ O & C & O & D \end{vmatrix} = \{C^2 - D(A - B)\}\{C^2 - D(A + B)\}$$

$$I_{af} = \frac{\begin{vmatrix} V_f & B & C & O \\ V_b & A & O & C \\ O & O & D & O \\ O & C & O & D \end{vmatrix}}{\text{Det}\,\triangle} = \frac{V_f\,(AD^2 - C^2D) - V_bBD^2}{\text{Det}\,\triangle} \tag{14}$$

$$I_{ab} = \frac{\begin{vmatrix} A & V_f & C & O \\ B & V_b & O & C \\ C & O & D & O \\ O & O & O & D \end{vmatrix}}{\text{Det}\,\triangle} = \frac{-V_fBD^2 + V_b\,(AD^2 - C^2D)}{\text{Det}\,\triangle} \tag{15}$$

$$I_{\beta f} = \frac{\begin{vmatrix} A & B & V_f & O \\ B & A & V_b & C \\ C & O & O & O \\ O & C & O & D \end{vmatrix}}{\text{Det } \Delta} = \frac{V_f\,(C^2 - ACD) + V_b BCD}{\text{Det } \Delta} \tag{16}$$

$$I_{\beta b} = \frac{\begin{vmatrix} A & B & C & V_f \\ B & A & O & V_b \\ C & O & D & O \\ O & C & O & O \end{vmatrix}}{\text{Det } \Delta} = \frac{V_f BCD + V_b\,(C^2 - ACD)}{\text{Det } \Delta} \tag{17}$$

In the equivalent circuit shown in Fig. 2, the two voltages V_f and V_b can be written down as follows, on the assumption that the applied voltage E_A is passing through its zero value at the time of closing the switch.

$$V_f\,(t) = \frac{E_A}{\sqrt{3}} \sin\,(t - 30)$$

$$V_b\,(t) = \frac{E_A}{\sqrt{3}} \sin\,(t + 30)$$

The Laplace Transform of the above functions can be written down as:

$$\mathcal{L}\, V_f\,(t) = V_f\,(s) = \frac{E_A}{\sqrt{3}} \left[\frac{\sqrt{3}}{2} \frac{1}{(1 + s^2)} - \frac{s}{2\,(1 + s^2)} \right] \tag{18}$$

$$\mathcal{L}\, V_b\,(t) = V_b\,(s) = \frac{E_A}{\sqrt{3}} \left[\frac{\sqrt{3}}{2} \frac{1}{(1 + s^2)} + \frac{s}{2\,(1 + s^2)} \right] \tag{19}$$

Substituting these expressions in equation (14) and rearranging terms suitably,

$$I_{af} = \frac{E_A}{2\,(1 + s^2)} \left[\frac{D}{\{D\,(A + B) - C^2\}} + \frac{sD}{\sqrt{3}\,\{C^2 - D\,(A - B)\}} \right] \tag{20}$$

Substitution of the values of A, B, C and D gives the following expression for I_{af}

$$\frac{E_A}{2(1+s^2)}\left[\frac{R_2+sX\beta}{(R_2+sX_\beta)(R_1+sX_a)-s^2X_m^2}\right.$$

$$\left.+\frac{1}{\sqrt{3}}\frac{s(R_2+sX_\beta)}{s^2X_m^2-(R_2+sX_\beta)\left(R_1+2R_c+sX_a+\frac{2X_c}{s}\right)}\right]$$

This can be further simplified to the following form:

$$\frac{E_A}{2\sigma X_a}\left[\frac{s+K_\beta}{(1+s^2)\left\{s^2+s\frac{K_a+K_\beta)}{\sigma}+\frac{K_aK_\beta}{\sigma}\right\}}\right.$$

$$\left.-\frac{s^2(s+K_\beta)}{\sqrt{3}(1+s^2)\left\{s^3+s^2\frac{(K_{ac}+K_\beta)}{\sigma}+s\frac{(X_c'+K_\beta K_{ac})}{\sigma}+\frac{K_\beta X_c'}{\sigma}\right\}}\right] \tag{21}$$

Similarly it can be shown that the expression for I_{ab} is given by:

$$\frac{E_A}{2\sigma X_a}\left[\frac{s+K_\beta}{(1+s^2)\left\{s^2+s\frac{(K_a+K_\beta)}{\sigma}+\frac{K_aK_\beta}{\sigma}\right\}}\right.$$

$$\left.+\frac{s^2(s+K_\beta)}{\sqrt{3}(1+s^2)\left\{s^3+s^2\frac{(K_\beta+K_{ac})}{\sigma}+s\frac{(X_c'+K_\beta K_{ac})}{\sigma}+\frac{K_\beta X_c'}{\sigma}\right\}}\right] \tag{22}$$

The only difference between the above two equations (21) and (22) is in the sign of the second term.

The current in phase A, being the sum of I_{af} and I_{ab}, can be expressed in the following form:

$$I_A(s)=\frac{E_A}{\sigma X_a}\left[\frac{(s+K_\beta)}{(1+s^2)\left\{s^2+s\frac{(K_a+K_\beta)}{\sigma}+\frac{K_aK_\beta}{\sigma}\right\}}\right]$$

$$=\frac{E_AK_\beta}{\sigma X_a p_1 p_2}\frac{1+as}{(1+s^2)(1+T_1s)(1+T_2s)} \tag{23}$$

where $a=1/K_\beta$, T_1 and T_2 are the reciprocals of the two roots of the characteristic equation. By taking the inverse Laplace Transform the following general expression is obtained for the current in phase A as a function of 't'.

$$i_A(t) = \frac{E_A}{\sigma X_a}\left[\frac{\sqrt{1+K_\beta^2}}{\sqrt{(1+p_1^2)(1+p_2^2)}} \cdot \sin(t-\theta) + \frac{K_\beta - p_1}{(p_2-p_1)(1+p_1^2)} \cdot e^{-p_1 t} + \frac{K_\beta - p_2}{(p_1-p_2)(1+p_2^2)} \cdot e^{-p_2 t}\right]$$

where

$$\theta = \tan^{-1}\frac{1}{p_1} + \tan^{-1}\frac{1}{p_2} - \tan^{-1}\frac{1}{K_\beta}.$$

For the usual values of the constants of three-phase induction motors, $K_\beta \ll 1$ and p_2 is negligible compared with p_1 and 1. The above expression can therefore be simplified to the following form:

$$i_A(t) = \frac{E_A}{\sigma X_a}\left[\frac{\sin(t-\theta)}{\sqrt{1+p_1^2}} + \frac{\bar{e}^{p_1 t}}{(1+p_1^2)} - \frac{p_2}{p_1} \cdot e^{-p_2 t}\right] \tag{24}$$

Detailed computation of $i_A(t)$ is given in Appendix A. The current in phase A consists of two decaying D.C. terms in addition to the steady state A.C. term. Out of the two transient D.C. terms one is extremely weak in magnitude but has a large time constant.

Rotor Currents.—The currents in the forward and backward fields of the rotor are given below.

$$I_{\beta f} = \frac{E_A}{2(1+s^2)}\left[\frac{C}{\{C^2 - D(A+B)\}} - \frac{1}{\sqrt{3}}\frac{sC}{\{C^2 - D(A-B)\}}\right] \tag{25}$$

$$I_{\beta b} = \frac{E_A}{2(1+s^2)}\left[\frac{C}{\{C^2 - D(A+B)\}} + \frac{1}{\sqrt{3}}\frac{sC}{\{C^2 - D(A-B)\}}\right] \tag{26}$$

Substituting the values of A, B, C and D in the above equations and simplifying the expressions, in the manner indicated earlier, the following equations are obtained for the rotor currents:

$$I_{\beta f}(s) = \frac{E_A X_m}{2\sigma X_a X_\beta}\left[\frac{-s}{(1+s^2)\left\{s^2 + s\frac{(K_a+K_\beta)}{\sigma} + \frac{K_a K_\beta}{\sigma}\right\}} + \frac{1}{\sqrt{3}} \times \frac{s^3}{(1+s^2)\left\{s^3 + s^2\frac{(K_{ac}+K_\beta)}{\sigma} + s\frac{(K_\beta K_{ac} + X_c')}{\sigma} + \frac{K_\beta X_c'}{\sigma}\right\}}\right] \tag{27}$$

$$I_{\beta b}(s) = \frac{E_A X_m}{2\sigma X_a X_\beta}\left[\frac{-s}{(1+s^2)\left\{s^2 + s\frac{(K_a+K_\beta)}{\sigma} + \frac{K_a K_\beta}{\sigma}\right\}} - \frac{1}{\sqrt{3}} \times \frac{s^3}{(1+s^2)\left\{s^3 + s^2\frac{(K_{ac}+K_\beta)}{\sigma} + s\frac{(K_\beta K_{ac} + X_c')}{\sigma} + \frac{K_\beta X_c'}{\sigma}\right\}}\right] \tag{28}$$

The first term in the bracket gives rise to two decaying D.C. components in addition to the steady state value. The second term in the bracket gives rise to an extremely weak D.C. component, a damped oscillatory term and a steady state A.C. term. The frequency of the oscillatory term is, in general, higher than the base frequency. The general expression for the rotor current will, therefore, be of the following form:

$$i_{\beta f}(t) = \mathrm{E_A}\,[\mathrm{A}_1 \cos(t - \theta_1) + \mathrm{A}_2 \cos(t - \theta_2) + \mathrm{B}_1 e^{-a_1 t} + \mathrm{B}_2 e^{-a_2 t}$$
$$+ \mathrm{B}_3 e^{-a_3 t} + \mathrm{C}_1 e^{-a_4 t} \sin(b\,t - \theta_3)]$$

where A_1, A_2, etc., are constants, the values of which depend on the machine parameters. A detailed computation of $i_{\beta f}$ is given in Appendix B.

Condenser Voltage.—The voltage across the condenser is given in the following form, as a function of 's', the Laplacean parameter.

$$\mathrm{V}_c = \left(\mathrm{R}_c + \frac{\mathrm{X}_c}{s}\right)(\mathrm{I}_{af} - \mathrm{I}_{ab}) \tag{29}$$

For the purpose of estimating $V_c(t)$ it is necessary to multiply R_c by the values of $(i_{af} - i_{ab})$ at various instants of time and this gives the first part of the above expression. To obtain the second part of the above expression the expression $(i_{af} - i_{ab})$ is integrated with respect to t and then multiplied by X_c.

The final expression for V_c is given in Appendix A and the variation of V_c during the first few cycles is shown in Fig. 5.

Developed Electrical Torque.—From the Double Revolving Field theory of the induction motor, the developed electrical torque is given as:

$$\mathrm{T}_e = \mathrm{K}\,[i_{\beta f} \cdot i_{\beta f} - i_{\beta b} \cdot i_{\beta b}] \tag{30}$$

It is a well-known fact that transient asymmetrical fluxes and currents are produced whenever a highly inductive circuit is switched on to a source of A.C. voltage. The magnitude and phase of these transient terms would obviously depend on the value of the voltage at which electrical switching is accomplished. Detailed computation of the developed electrical torque is given in Appendix C. It should be clearly noted that all the expressions given earlier are for standstill condition; the actual case of the motor rapidly coming up to speed becomes extremely complicated. In general, however, the peak value of the shaft torque occurs within the first few cycles during which time the rotor will not have attained any measurable speed. There is, thus, perfect justification for disregarding the effect of rotation on the switching transients.

Equation (30) gives the instantaneous developed electrical torque which acts on the rotor. The actual magnitude of the shaft torque, however, depends on this, as well as the flexibility of the shaft, the damping and moment of inertia. If the motor is connected to a mechanical load with a gear ratio of 'n', the reflected values of moment of inertia, etc., have to be added to the corresponding quantities of the rotor. If the natural frequency of the mechanical system happens to be very close, to the supply frequency, the peak value of the torque may exceed the normal value of the developed electrical torgque, due to resonance effects. In the event of the enertia of the directly driven mechanical load, being appreciably smaller than the inertia of the rotor, then a pronounced reduction in shaft torque could be expected. If the coupling has a backlash or non-linear torque-angle characteristics, sometimes, deliberately adjusted, an appreciable increase in shaft torque, due to impacts, will be produced.

In the large majority of cases of practical interest, the motor is mechanically coupled to a heavy inertia load by a flexible coupling; in all such cases the equivalent mechanical system could be represented by the scheme shown in Fig. 3. In this mass represents rotor inertia, etc., spring represents the effect of flexible coupling and B represents friction.

$$I_m\phi''_m + B_m\phi'_m + K_m\phi_m = T_e(t)$$

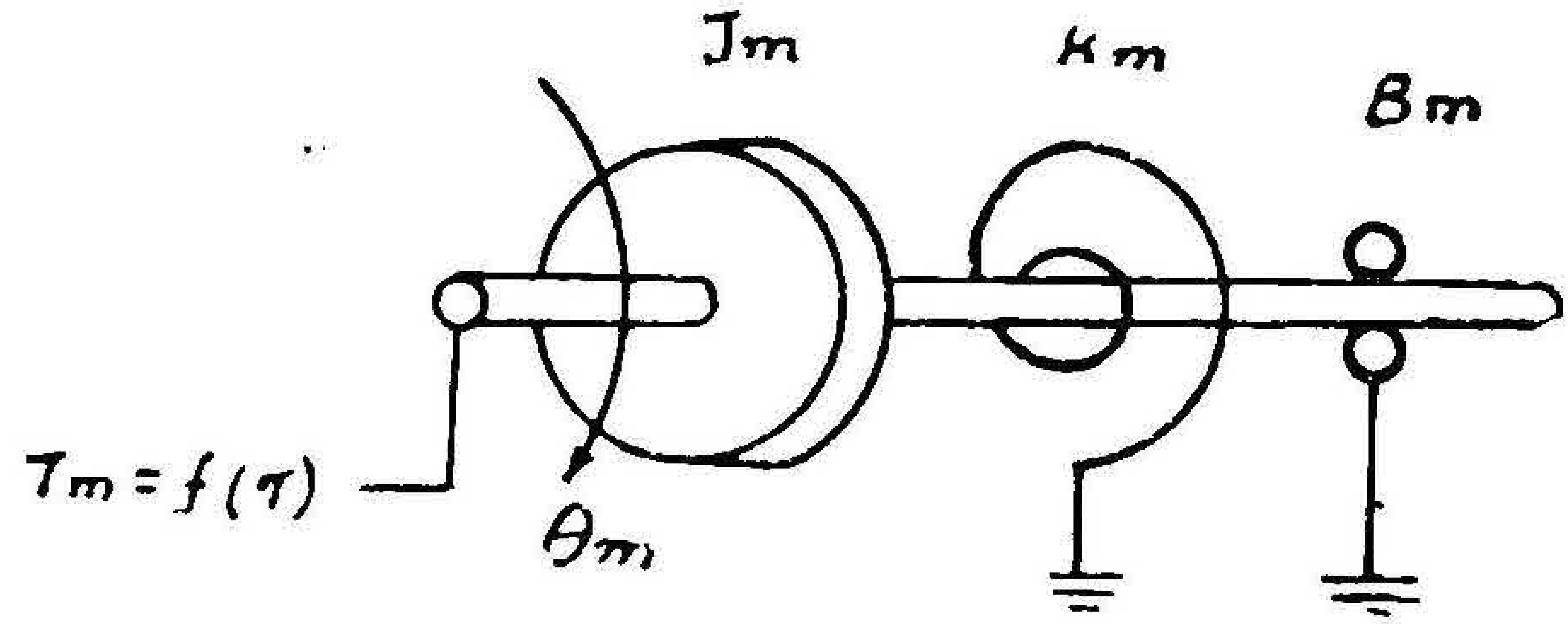

FIG. 3. Mechanical Rotational System.

With an extremely long expression for $T_e(t)$ it is not possible to solve the above equation by the simple and well-known methods; it is however possible to solve this equation with the aid of an Electronic Differential Analyzer.

The effect of closing the switch when the applied voltage passes through its maximum value is discussed in Appendix D.

CONCLUSIONS

From the detailed computation carried out, for a typical three-phase motor it is shown that the expression for torque contains, in addition to the steady state component, a very large number of transient oscillatory terms. As is to be expected in single phase operation there is a double frequency torque pulsation the like of which does not appear in the case of a Capacitor Motor, since in the latter case the operation at standstill corresponds to a fair degree of approximation to that of a two-phase motor. The time constants of the transient terms are large with the result that the developed electrical torque attains its final steady state value after about one cycle. It is interesting to note that, whether the electrical switching is performed at the maximum value or zero value of the applied A.C. potential all the transient terms in the expression for torque vanish by the end of the very first cycle. This is in sharp contrast with the results obtained in the case of a well designed, general purpose Capacitor Motor. It can, therefore, be concluded that even if starts and stops are unusually frequent fatigue due to repeated switching will not normally occur.

It is well known that single phase motors have a fairly large double frequency pulsating torque at no load as well as under normal loading conditions, so that it is very necessary to design the shaft coupling and rotor supports to handle these torques safely. It has therefore become the normal practice to make the natural frequency in torsion of the rotor and connected load well above twice the line frequency and also to make the shaft adequately strong to carry high pulsating torques. As the three-phase motor run as a Capacitor Motor on single phase supply is, usually under-rated, trouble due to mechanical failure caused by transient and double frequency torques are very rare.

The effect of varying the capacitive reactance, on the transient terms in the expression for torque is not very pronounced, as shown at the end of Appendix D.

SYMBOLS

E_A = Single phase supply voltage.

V_f = Voltage applied to the forward field in the equivalent circuit.

V_b = Voltage applied to the backward field in the equivalent circuit.

V_c = Voltage across the starting condenser.

i_{af} = Stator current in the forward field as a function of t.

i_{ab} = Stator current, in the backward field as a function of t.

I_{af} = Stator current in the forward field as a function of 's'.

I_{ab} = Stator current in the backward field as a function of 's'.

$i_{\beta f}$ = Rotor current in the forward field as a function of t.

$i_{\beta b}$ = Rotor current in the backward field as a function of t.

$I_{\beta f}$ = Rotor current in the forward field as a function of 's'.

$I_{\beta b}$ = Rotor current in the backward field as a function of 's'.

s = Laplacian parameter of complex character.

R_1 = Resistance of the stator winding (ohms/phase Δ).

R_2 = Resistance of the rotor winding (ohms/phase Δ).

R_c = Equivalent series resistance of the starting condenser.

X_1 = Leakage reactance of a stator winding (ohms/phase Δ).

X_2 = Leakage reactance of rotor winding (ohms/phase Δ).

X_m = Magnetising Reactance (ohms/phase Δ).

X_c = Capacitive reactance of starting condenser.

$X_\alpha = X_1 + X_m$. $\quad K_\alpha = \dfrac{R_1}{X_\alpha}$ $\quad \sigma = 1 - \dfrac{X_m^2}{X_\alpha \cdot X_\beta}$

$X_\beta = X_2 + X_m$. $\quad K_\beta = \dfrac{R_2}{X_\beta}$

$X_c' = \dfrac{2X_c}{X_\alpha}$ $\quad K_{\alpha c} = \dfrac{R_1 + 2R_c}{X_\alpha}$

t' = time in seconds.

t = $2\pi f$ times time in seconds = $\omega t'$.

J_m = Moment of interia of rotor, etc., and connected load.

B_m = Friction constant.

K_m = Spring constant of the coupling system.

θ_m = Angular twist of shaft coupling system between motor and load.

f = power-supply frequency at which the motor is designed to operate.

T_e = developed electrical torque.

A_1, A_2, B_1, B_2, etc., are arbitrary constants.

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APPENDIX A

The value of the current in phase A has been computed for a typical three-phase motor, the constants of which are taken from Ref. 10.

5 H.P., 3-Phase, 220 Volt Motor.

$R_1 = 1\cdot65$ $\quad X_m = 75$ $\quad$ All the values are given in ohms

$R_2 = 1\cdot35$ $\quad R_e = 0\cdot15$ ohm per phase $\triangle$

$X_1 = X_2 = 1\cdot71$ $\quad X_e = 3$ ohm

By estimation we get

$K_\alpha = 2\cdot151 \times 10^{-2}$ $\quad K_{\alpha e} = 2\cdot542 \times 10^{-2}$

$K_\beta = 1\cdot759 \times 10^{-2}$ $\quad X_e' = 7\cdot821 \times 10^{-2}$

$\sigma = 4\cdot41 \times 10^{-2}$

The roots of the characteristic equation

$$s^2 + 0\cdot8862\, s + 10^{-2} \times 0\cdot8579 = 0$$

are

$$p_1 = 0\cdot8763$$

$$p_2 = 0\cdot0098$$

The final expression for current in phase A is given below.

$$i_A(t) = E_A\,[0\cdot22256 \sin(t-\theta) + 0\cdot1658\, e^{-0\cdot8763t} + 10^{-3}\times 2\cdot659\, e^{-0\cdot0098\,t}]$$

where

$$\phi = \tan^{-1} 1\cdot1411 + \tan^{-1} 102\cdot09 - \tan^{-1} 56\cdot85 = 49^\circ\ 18'.$$

Fig. 4 shows the variation of $i_A(t)$ during the first few cycles.

The expression for i_{af} and i_{ab} as a function of time are given below.

$$\begin{aligned} i_{af}(t) = E_A\,[&0\cdot1112 \sin(t - 49^\circ\ 18') + 0\cdot0693 \sin(t - 51^\circ\ 2') \\ &+ 0\cdot0829\, e^{-0\cdot8763\,t} + 10^{-8} \times 0\cdot147\, e^{-\cdot0177\,t} + 10^{-4} \\ &\times 13\cdot297 e^{-0\cdot0098\,t} - e^{-0\cdot47875\,t}\{0\cdot00131 \sin(1\cdot24\,t - 143^\circ\ 23') \\ &- 0\cdot09873 \sin(1\cdot24\,t - 212^\circ\ 18')\}] \end{aligned}$$

$$\begin{aligned} i_{ab}(t) = E_A\,[&0\cdot1112 \sin(t - 49^\circ\ 18') - 0\cdot0693 \sin(t - 51^\circ\ 2') \\ &+ 0\cdot0829\, e^{-0\cdot8763\,t} + 10^{-4} \times 13\cdot297\, e^{-0\cdot0098t} - 10^{-8} \\ &\times 0\cdot147\, e^{-0\cdot0177\,t} + e^{-0\cdot47875\,t}\{0\cdot00131 \sin(1\cdot24\,t - 143^\circ\ 23') \\ &- 0\cdot098 \sin(1\cdot24\,t - 212^\circ\ 18')\}] \end{aligned}$$

Voltage across the Condenser.—From the values of i_{af} and i_{ab} it is possible to determine the voltage across the condenser, which is given by the following equation.

$$R_c (i_{af} - i_{ab}) + X_c \int_0^t (i_{af} - i_{ab})\, dt = V_c$$

By substituting the values of R_c, X_c, i_{af} and i_{ab} the following equation is derived for V_c:

$$V_c = E_A [0{\cdot}416 \sin (t - 138°7') - e^{-0{\cdot}478\, t} (0{\cdot}446) \sin (1{\cdot}24\, t - 143°\, 23')$$
$$- 10^{-7} \times 4{\cdot}98\, e^{-{\cdot}0177\, t} + 10^{-2} \cdot e^{-0{\cdot}478\, t} \{0{\cdot}586 \sin (1{\cdot}24\, t$$
$$- 74°\, 28') + 2{\cdot}96 \sin (1{\cdot}24\, t - 212°\, 18')\}]$$

Fig. 5 shows the condenser voltage as a function of 't' during the first cycle.

APPENDIX B

For the machine for which the constants are given in Appendix A, the characteristic cubic equation is given below.

$$s^3 + 0{\cdot}9752\, s^2 \times 1{\cdot}7836\, s + 10^{-2} \times 3{\cdot}1195 = 0$$

The roots of the above equations as determined by synthetic division are:

$$p_1' = 0{\cdot}478 + j\,1{\cdot}24$$

$$p_2' = 0{\cdot}478 - j\,1{\cdot}24$$

$$p_3' = 0{\cdot}0177$$

The final expressions for $i_{\beta f}(t)$ and $i_{\beta b}(t)$ are as follows:

$$\begin{aligned} i_{\beta f}(t) = -\,E_A\,[&+ 0{\cdot}1085 \cos(t - 138^\circ\, 16') + 0{\cdot}0677 \cos(t - 140^\circ) \\ &+ 0{\cdot}0825\, e^{-0{\cdot}8763\, t} - 10^{-2} \times 0{\cdot}163\, e^{-0{\cdot}0098\, t} + 0{\cdot}0965\, e^{-0{\cdot}47871} \\ &\times \sin(1{\cdot}24\, t - 212^\circ\, 18') + 10^{-6} \times 0{\cdot}2611\, e^{-0{\cdot}0177\, t}] \end{aligned}$$

$$\begin{aligned} i_{\beta b}(t) = -\,E_A\,[&0{\cdot}1085 \cos(t - 138^\circ 16') - 0{\cdot}677 \cos(t - 140^\circ) \\ &+ 0{\cdot}08256\, e^{-0{\cdot}8763\, t} - 10^{-2} \times 0{\cdot}16324\, e^{-0{\cdot}0098\, t} - 0{\cdot}0965\, e^{-0{\cdot}478751} \\ &\times \sin(1{\cdot}24\, t - 212^\circ\, 18') - 10^{-6} \times 0{\cdot}2611\, e^{-0{\cdot}0177\, t}] \end{aligned}$$

The transients in the rotor current consists of three decaying D.C. terms and one damped oscillatory term of frequency 1·24 times the base frequency. One of the three D.C. terms is of extremely small magnitude.

APPENDIX C

The developed electrical torque is given in symbolic form by the following expression.

$$T_e = KR_2\,[i_{\beta f} \cdot i_{\beta f} - i_{\beta b} \cdot i_{\beta b}]$$

For the machine under study, the torques due to the forward and backward fields as calculated by the above formula, are given below:

$$\begin{aligned}
T_{\text{forward}} = \frac{10^{-2}E_A^2}{1\cdot 35}\,[&5\cdot 69\cos^2(t - 139^\circ) + 5\cdot 448\, e^{-0\cdot 8763\,t}\cos(t - 139^\circ)\\
&+ 6\cdot 198\, e^{-0\cdot 4787\,t}\cos(t - 139^\circ)\sin(1\cdot 24\,t - 212^\circ\ 18')\\
&- 0\cdot 1048\, e^{-0\cdot 0098\,t}\cos(t - 139^\circ) + 1\cdot 311\, e^{-1\cdot 7526\,t}\\
&+ 1\cdot 697\, e^{-0\cdot 9575t}\sin^2(1\cdot 24\,t - 212^\circ\ 18') + 10^{-4}\\
&\times 4\cdot 857\, e^{-0\cdot 0196\,t} + 2\cdot 984\, e^{-1\cdot 355\,t}\sin(1\cdot 24\,t - 212^\circ\ 18') - 10^{-}\\
&\times 0\cdot 504\, e^{-\cdot 8861\,t} - 10^{-1}\times \cdot 5742\, e^{-0\cdot 4885\,t}\sin(1\cdot 24\,t - 212^\circ\ 18')]
\end{aligned}$$

$$\begin{aligned}
T_{\text{backward}} = \frac{10^{-2}E_A^2}{1\cdot 35}\,[&0\cdot 3036\cos^2(t - 139^\circ) + 1\cdot 26\, e^{-0\cdot 8763\,t}\cos(t - 139^\circ)\\
&- 1\cdot 4358\, e^{-0\cdot 4787t}\cos(t - 139^\circ)\sin(1\cdot 24\,t - 212^\circ\ 18') - 10^{-1}\\
&\times 0\cdot 242\, e^{-0\cdot 0098t}\cos(t - 139^\circ) + 1\cdot 311\, e^{-1\cdot 7562t} + 1\cdot 697\, e^{-0\cdot 9575t}\\
&\times \sin(1\cdot 24\,t - 212^\circ 18') + 10^{-4}\times 4\cdot 885\, e^{-\cdot 0196t} - 2\cdot 9838\, e^{-1\cdot 355t}\\
&\times \sin(1\cdot 24\,t - 212^\circ 18') - 10^{-1}\times 0\cdot 5046\, e^{-0\cdot 8861t} + 10^{-1}\\
&\times 0\cdot 5744\, e^{-0\cdot 4885t}\sin(1\cdot 24\,t - 212^\circ 18')].
\end{aligned}$$

The nett torque in the direction of rotation is therefore equal to

$$\begin{aligned}
T_e = E_A^2\,[7\cdot 932\times 10^{-2}]\,[1 + \cos 2\theta] &+ 1\cdot 564\, e^{-0\cdot 87631t}\cos\theta\\
&+ 2\cdot 86\, e^{-0\cdot 478\,t}\cos\theta\sin(1\cdot 24\,t - 212^\circ\ 18')\\
&+ 2\cdot 228\, e^{-1\cdot 355\,t}\sin(1\cdot 24\,t - 212^\circ 18') - 0\cdot 03\, e^{-0\cdot 0098t}\\
&\times \cos\theta - 0\cdot 0428\, e^{-0\cdot 4885\,t}\sin(1\cdot 24\,t - 212^\circ\ 18')]
\end{aligned}$$

where

$$\theta = (t - 139^\circ).$$

The last two items are extremely small in magnitude and could therefore be disregarded in the computation of T_e as a function of t. There are thus three transient terms all of which vanish by the end of the very first cycle. The frequency of some of the components is 1·24 times the base frequency.

APPENDIX D

The effect of closing the switch, when the applied potential passes through its maximum positive value has been investigated by taking

$$V_f \text{ as } \frac{E_A}{\sqrt{3}} \{\cos (t - 30°)\}$$

$$V_b \text{ as } \frac{E_A}{\sqrt{3}} \{\cos (t + 30°)\}$$

The approximate values of $i_A(t)$, $V_c(t)$, etc., computed for this particular case, are as follows:

$$i_A(t) = E_A\,[0{\cdot}222 \cos (t - 49°\,18') - 0{\cdot}145\, e^{-0{\cdot}8763\, t} - 10^{-4} \times 0{\cdot}2606\, e^{-0{\cdot}098\, t}]$$

$$V_c(t) = E_A\,[0{\cdot}416 \sin (t - 48°) + e^{-{\cdot}47875\, t}\{0{\cdot}023 \sin (1{\cdot}24\, t - 143°\,23') - 0{\cdot}335 \sin (1{\cdot}24\, t - 212°\,18')\}]$$

$$T_e = \frac{-E_A{}^2 10^{-2}}{1{\cdot}35}\,[5{\cdot}356 \sin^2 (t - 139°) + 3{\cdot}57\, e^{-0{\cdot}8763\, t} \sin (t - 139°) - 5{\cdot}76\, e^{-0{\cdot}47875\, t} \sin (t - 139°) \sin (1{\cdot}24\, t - 143°\,23') - 3{\cdot}832\, e^{-1{\cdot}353t} \times \sin (1{\cdot}24\, t - 143°\,23')]$$

Variation of i_A, V_c and T_e as functions of t is shown in Figs. 4, 5, and 6 respectively.

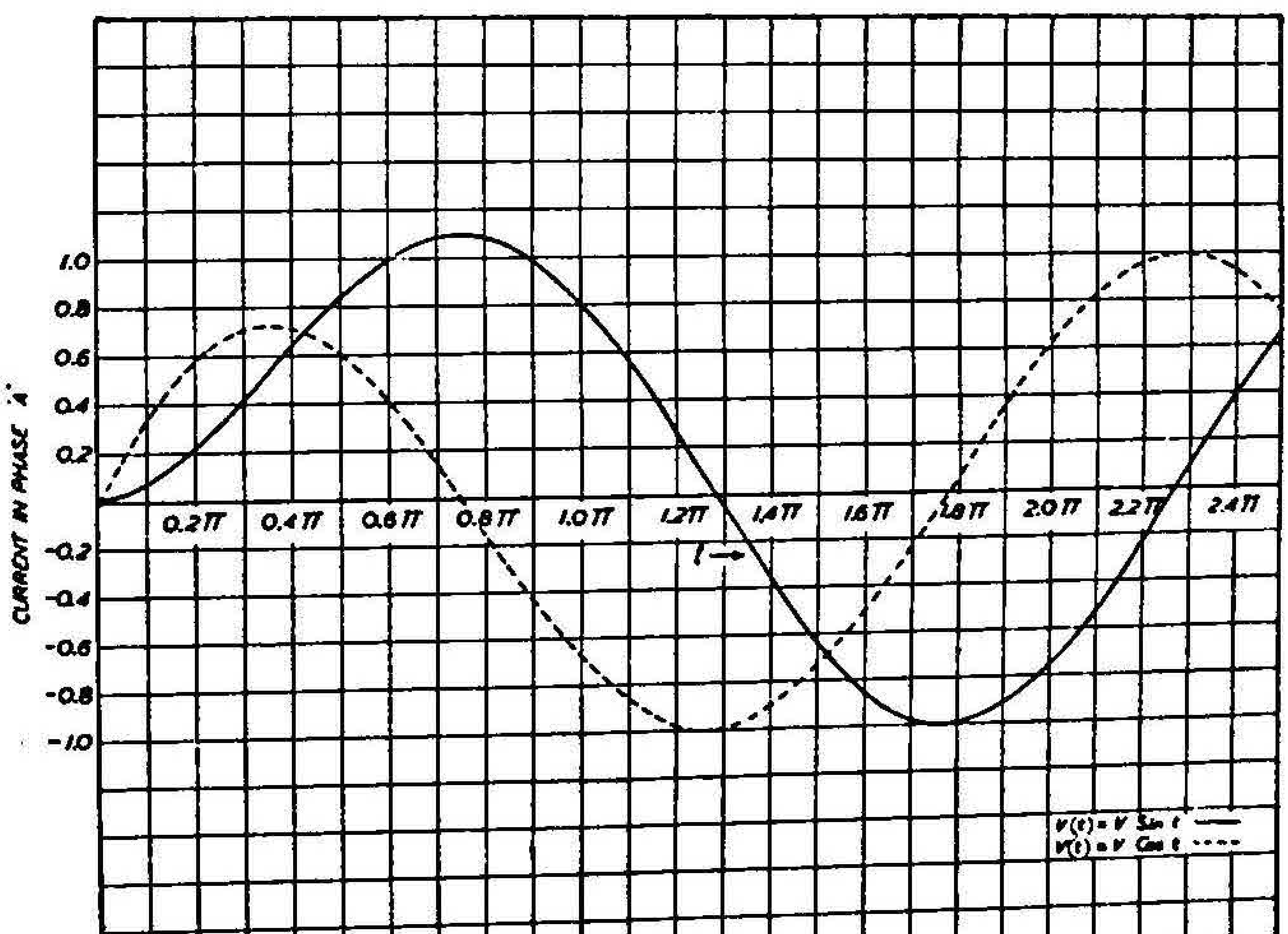

Fig. 4. Current in Phase A during the First Cycle.

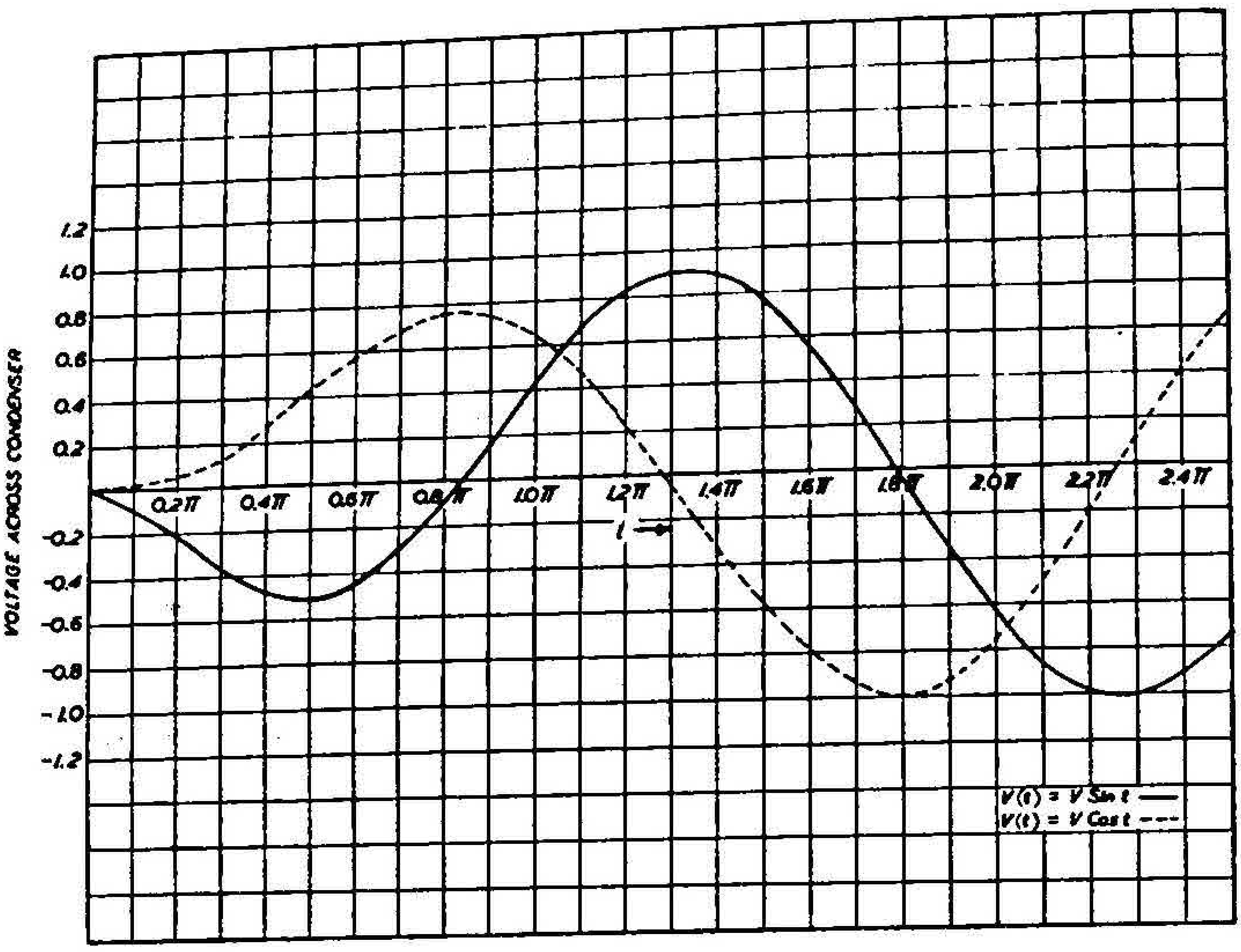

FIG. 5. Voltage across the Starting Condenser during the First Cycle.

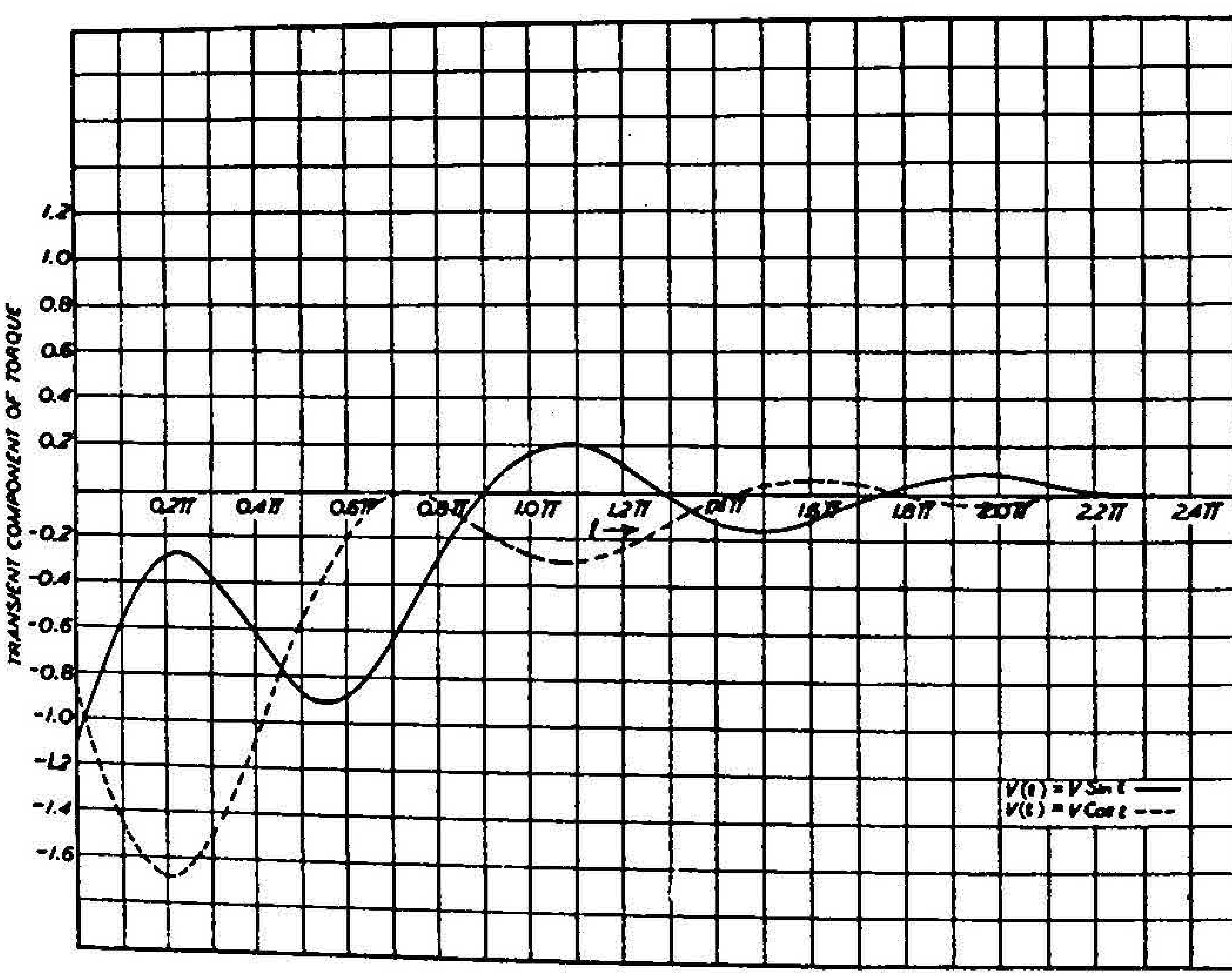

FIG. 6. Transient Part of Developed Electrical Torque of a Motor.

Variation of X_c on the transient terms:

It is obvious that the life of the transient terms depends on the roots of the characteristic cubic equation, referred to in Appendix B.

The following table gives the characteristic equation and its roots for various values of X_c.

X_c	R_c	Characteristic equation	Roots
2·0	0·100	$s^3+0{\cdot}944\,s^2+1{\cdot}19\,s+10^{-2}\times 2{\cdot}077$	$-0{\cdot}017$; $-0{\cdot}463\pm j\,{\cdot}979$
2·5	0·125	$s^3+0{\cdot}958\,s^2+1{\cdot}485\,s+10^{-2}\times 2{\cdot}596$	$-0{\cdot}017$; $-0{\cdot}471\pm j\,1{\cdot}117$
3·0	0·150	$s^3+0{\cdot}974\,s^2+1{\cdot}782\,s+10^{-2}\times 3{\cdot}11$	$-0{\cdot}017$; $-0{\cdot}478\pm j\,1{\cdot}240$
3·5	0·175	$s^3+0{\cdot}987\,s^2+2{\cdot}078\,s+10^{-2}\times 3{\cdot}637$	$-0{\cdot}017$; $-0{\cdot}485\pm j\,1{\cdot}355$
4·0	0·200	$s^3+1{\cdot}00\,s^2+2{\cdot}374\,s+10^{-2}\times 4{\cdot}155$	$-0{\cdot}017$; $-0{\cdot}493\pm j\,1{\cdot}454$
4·5	0·225	$s^3+1{\cdot}017\,s^2+2{\cdot}671\,s+10^{-2}\times 4{\cdot}67$	$-0{\cdot}017$; $-0{\cdot}504\pm j\,1{\cdot}554$
5·0	0·250	$s^3+1{\cdot}033\,s^2+2{\cdot}963\,s+10^{-2}\times 5{\cdot}192$	$-0{\cdot}017$; $-0{\cdot}508\pm j\,1{\cdot}653$

From the roots of the characteristic equation, it is easily seen that the damping factors for both the D.C. term and the oscillatory term remain practically constant over a small and permissible range of variation of X_c. The variation of the frequency, however, of the oscillatory term, changes appreciably with X_c.