

# THE DISPERSION OF THE PIEZO-OPTIC CONSTANTS OF VITREOUS SILICA

BY E. S. JOG

(*Department of Physics, Indian Institute of Science, Bangalore-3*)

Received February 26, 1957

## ABSTRACT

The piezo-optic constants  $q_{11}$  and  $q_{12}$  of fused silica have been determined for various wavelengths in the range from 220 to 570  $m\mu$ . Over this range the value of  $q_{11}$  increases from  $-0.24 \times 10^{-13}$  to  $+0.11 \times 10^{-13}$ , while that of  $q_{12}$  increases from  $2.35 \times 10^{-13}$  to  $2.49 \times 10^{-13}$  C.G.S. units. The dispersion of these constants with wavelength has been established. The values of  $(q_{12} - q_{11})$  calculated from the observed values of the individual constants agree very well with those calculated from the measured values of the stress optical coefficient  $C$  (described in the preceding paper) throughout the wavelength range examined. The elasto-optic constants  $p_{11}$  and  $p_{12}$  and the change of refractive index with density have been evaluated.

## 1. INTRODUCTION

In the preceding paper the author has described the results obtained from a study of the stress optical coefficient of fused silica and its variation with wavelength. These investigations have now been extended to the piezo-optic constants of the same sample.

## 2. EXPERIMENTAL ARRANGEMENT

A method based on the production of localised interference fringes first developed by G. N. Ramachandran (1947) was employed by the author. The experimental arrangement used is illustrated in Fig. 1.  $S_1$  was the source of illumination.  $P_1$  was an ordinary fused quartz plate by which the light was reflected on to the fused quartz specimen  $S_2$  which was under stress. The localised interference fringes produced by the light coming from the two surfaces of the specimen were passed through the double image prism  $P_2$  and were focussed on the slit  $S_3$  of the Hilger small quartz spectrograph. The slit  $S_3$  was kept fairly wide. The optic axis of the quartz lens  $L_2$  was parallel to the direction of the rays. The principal axes of  $P_2$  were oriented such that the two images  $I_1$  and  $I_2$  of the fringe system falling on the slit have their vibration directions parallel and perpendicular to the direction of pressure respectively. As  $L_2$  was not achromatic its position had to be altered for each wavelength. In order to get well-defined fringes it was very necessary to restrict the aperture of  $L_2$ .



The stressing apparatus was similar to that used by Ramachandran (1947). For making the pressure distribution uniform lead sheets were used for padding, and the position of the specimen was adjusted several times till a uniform pressure distribution was obtained as seen through the crossed polaroids. Care was also taken to see that the full load applied to the stressing beam was effective on the specimen.

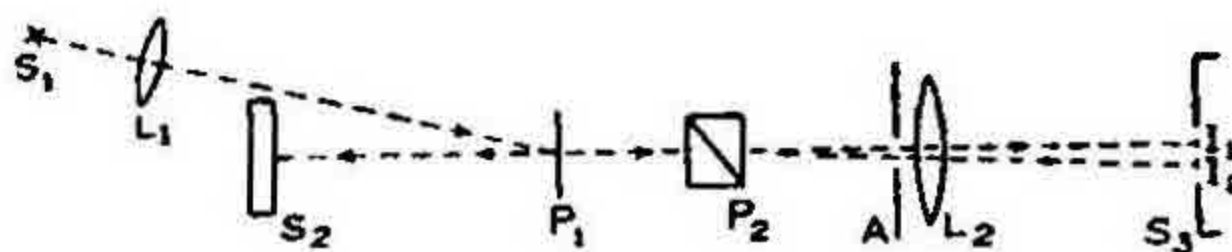


FIG. 1. Experimental Arrangement.

For photographing the fringe system a point source mercury arc and a copper arc were used as sources of illumination. The actual experiment was carried out as follows. For a particular wavelength the position of  $L_2$  was adjusted. Photographs first with the dead load of about 1 kg. on each of the hangers and with the addition load of 8684 gm. and lastly again with the previous dead load were taken. The maximum pressure used was about  $1.6 \text{ kg./mm.}^2$ . A scratch made on the specimen very nearly at the centre of the specimen appeared on the negatives as a bright sharp line and served as a reference line. The diameters of the rings were measured in a direction parallel to the scratch and through a fixed point. These measurements were done with the aid of Hilger cross slide comparator. There was slight variation in the room temperature during the time of exposure which varied from 1 to 15 minutes for a single photograph depending upon the intensity of radiation. The error arising from this change in the temperature of the room between the two photographs with and without load was eliminated by taking the average diameters for the two photographs of the dead load before and after loading into the calculation of the fraction of the fringe width. In addition, the experiment was repeated twice for each wavelength.

In the above experimental arrangement the stress distribution was essentially different for each wavelength. Therefore the whole experiment was repeated with a slightly different optical arrangement which enabled the author to obtain more well-defined fringe system and the same stress distribution for each wavelength. In this type of arrangement the central one-third portion of the surfaces of the specimen were partly aluminised. This enabled to photograph the transmission fringe system and it was also possible to study the uniformity of the stress distribution through the crossed polaroids. The light was incident directly on the specimen  $S_2$  and the transmission fringe system was focussed on to the slit of the spectrograph with the aid of an off-axis concave mirror. The double image prism was kept at a suitable point whereas the semi-reflecting fused quartz plate was dispensed with. Here also a small aperture restricting the reflecting surface of the mirror to a very narrow region was essential. The source used for this experiment was a



copper arc run at 4 amp. current. The spectrograph was mounted in a vertical plane.

In this arrangement the photographs for each component were taken separately. The scratch was not photographed instead care was taken to photograph the same section for the different loads. The fringes were very well defined and hence it was possible to verify the proportionality law for the change in the refractive index with load for the horizontal component (*i.e.*, perpendicular to the direction of pressure) by measuring the retardation for two different additional loads namely 8883 and 6415 gm. respectively. The retardations for different wavelengths were found to be directly proportional to the load within the limits of experimental error. For the vertical component as the 95% or more contribution to the retardation was due to the change in the thickness, the proportionality law was not tested. Therefore the experiment was repeated with the same additional load 8883 gm. The average values of the retardation were used for the calculation of the piezo-optic constants.

Two different plates A ( $20 \times 6.80 \times 1.285$ ) mm. and B ( $18.5 \times 5.95 \times 1.29$ ) mm. were cut from the same sample of fused quartz supplied by the Thermal Syndicate Ltd., London. They were suitably polished and the final dimensions are given above. The Plate A was actually used for determining the stress optical coefficient by the author (Jog, 1957). Both plates were used for the piezo-optic measurements. The direction of pressure was along the length and the light direction was perpendicular to the faces ( $6.8 \times 20$ ) mm. and ( $5.95 \times 18.5$ ) mm. respectively.

### 3. RESULTS AND DISCUSSIONS

A typical photograph of the fringe system for the horizontal component obtained in the second arrangement is reproduced in Fig. 2 on Plate II. From the measured diameters of the rings appearing with and without load the fringe shift  $f$  as a fraction of fringe width was calculated by using relation

$$f = 2 \frac{d_{m'}^2 - d_m^2}{d_{m+1}^2 - d_m^2 + d_m^2 - d_{m'-1}^2} \quad (1)$$

where  $d_m$  and  $d_{m+1}$  are the diameters of the  $m$ th and  $m + 1$ th fringes respectively. The dashed quantities refer to the corresponding diameters with the additional load. This relation was found to be most satisfactory since the differences between the squares of the successive diameters of the rings were almost constant and particularly for the specimen B. The average values of  $f$  from the measurement of at least four diameters were used for calculating the retardation  $R_x$  which is given by

$$R_x = \frac{f_x \cdot b \cdot \lambda}{P'g} \quad (2)$$

where  $f_x$  is the fraction for the additional load  $P'$  in gm.,  $b$  is the breadth of the specimen and  $x = 1, 2$  for components parallel and perpendicular to the direction of pressure respectively. The values of  $R_x$  calculated by using equation (2)



are given in Table I for both the experimental set-up. The path retardation  $R_r$  is related to the piezo-optic constants  $q_{12}$  and  $q_{11}$  as given in the following equation.

$$R_r = n^3 q_{12} - 2nS_{12} \quad (3)$$

The values of  $q_{11}$  and  $q_{12}$  calculated by using (3) are also given in Table I. Results obtained by the first experimental set-up are given in Table I a and those obtained by the second arrangement are included in Table I b. The refractive indices for the various wavelengths were taken from *International Critical Tables*, and the elastic constant  $S_{12} = -2.46 \times 10^{-13}$  C.G.S. units was calculated from the most probable values of the modulus of compression  $K$  and modulus of rigidity  $R$  given by Sosman (1923).

As already indicated any error arising from the non-uniform pressure distribution, the effective load being less than the applied load and the variation of the room temperature were reduced to minimum by taking proper experimental

TABLE I

Obs. No.	$\lambda$ in $m\mu$	Specimen	$n$	$R_r \times 10^{13}$ $= (n^3 q_{12} - 2nS_{12}) \times 10^{13}$		$q_{12} \times 10^{13}$	$q_{11} \times 10^{13}$
				$x=2$	$x=1$		
				..... C.G.S. Units .....			
I a							
1	546.1	B	1.4602	14.85	7.26	2.46	.03
2	435.8	"	1.4667	15.13	7.49	2.51	.09
3	404.7	"	1.4697	15.16	7.49	2.50	.08
4	334.2	"	1.4797	15.26	7.26	2.46	-.01
5	227.6	A	1.5222	15.34	6.89	2.23	-.17
I b							
1	570.0	B	1.4591	14.71	7.68	2.42	.16
2	510.6	"	1.4618	15.03	7.35	2.51	.05
3	465.1	"	1.4646	14.77	7.57	2.41	.11
4	353.4	"	1.4765	15.36	7.29	2.52	.01
5	333.8	"	1.4797	15.02	7.46	2.39	.04
6	303.6	"	1.4859	15.28	7.15	2.43	-.05
7	282.4	"	1.4932	15.38	6.91	2.41	-.17

precautions. In the values given in Table I a the errors arising from the stress distribution and the effective load would be different for different wavelengths, and are of random character. On the other hand in the second experimental arrangement these errors appear as systematic. Hence they do not affect the dispersion of these quantities obtained from the second experiment.

A serious source of error was in the measurement of the fringe shift. It was due to the uncertainty in the determination of the position of the fringe and in the use of the somewhat approximate relation (1). Measurements showed that the

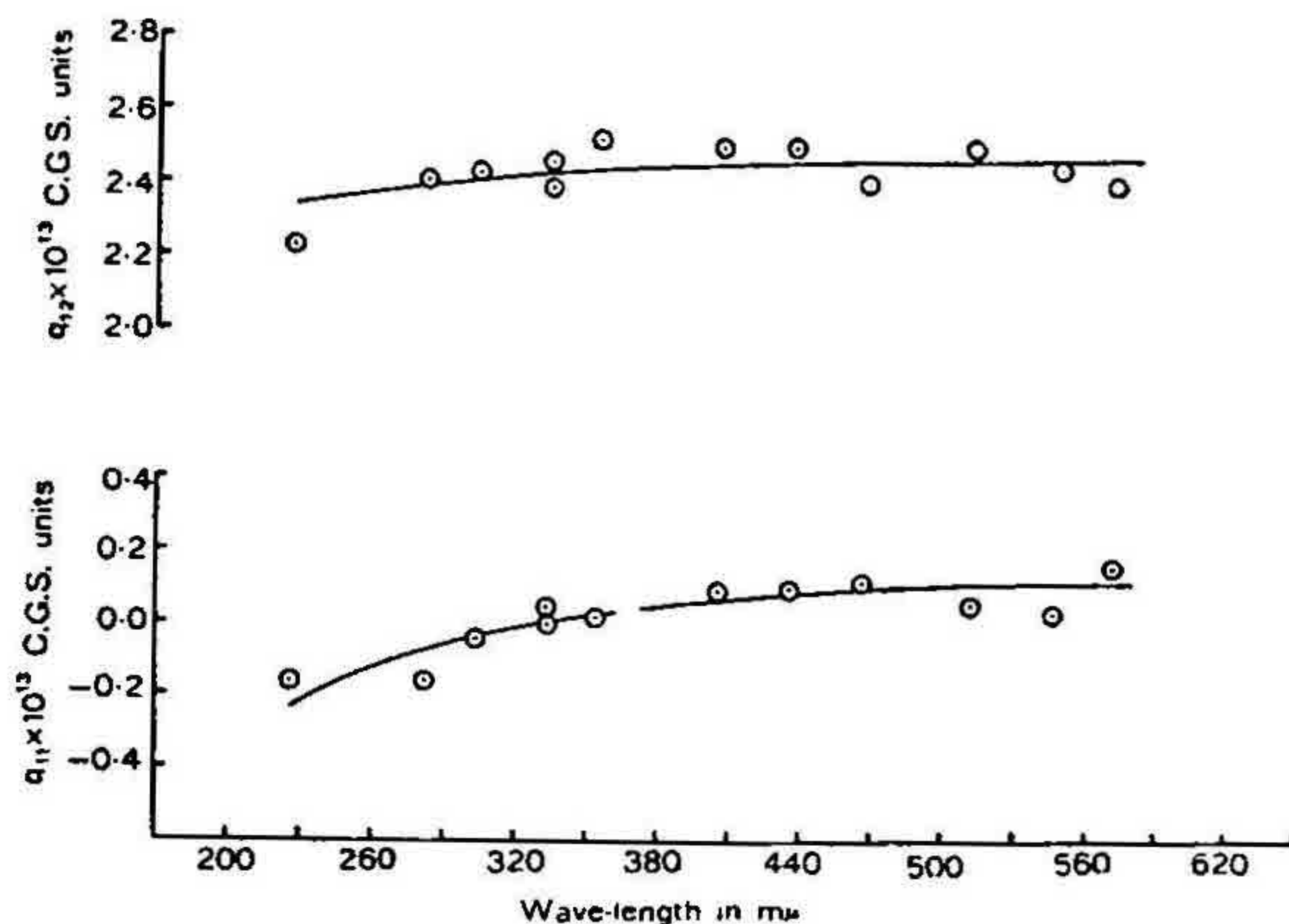


FIG. 3. The piezo-optic constants of vitreous silica.

uncertainty in the fringe shift was of the order of 0.5% of the fringe width. The corresponding errors in the observed values of  $q_{12}$  and  $q_{11}$  could be estimated as  $\pm 0.05 \times 10^{-13}$  and  $\pm 0.04 \times 10^{-13}$  respectively. The values given in Table I are therefore accurate only to this extent.

Any variation in the values of  $S_{12}$  taken for calculation in Equation (3) will affect the absolute magnitudes but not the dispersion of the quantities  $q_{12}$  and  $q_{11}$ , the effect on the absolute values of  $q_{11}$  being relatively much greater than that of  $q_{12}$ .

The observed values of  $q_{12}$  and  $q_{11}$  are plotted in Fig. 3, and smooth curves were drawn. These curves could be represented by the Equations (4) and (5) given below:

$$C_2 = \frac{n^3}{2} q_{12} = \frac{1}{2n} \left[ 11.15 + \frac{\lambda^2 m_1 2\lambda_1}{(\lambda^2 - \lambda_1^2)^2} (0.28) \right] \quad (4)$$

$$C_1 = \frac{n^3}{2} q_{11} = \frac{1}{2n} \left[ 0.74 + \frac{\lambda^2 m_1 2\lambda_1}{(\lambda^2 - \lambda_1^2)^2} (-0.41) \right] \quad (5)$$

where  $m_1 = .74655$ ,  $\lambda_1 = .107044 \mu$ . These values are the same as those used by Martens (Jog, 1957),  $\lambda$  is the wavelength in  $\mu$  and  $C$  in brewsters. It is observed that  $q_{11}$  exhibits a dispersion which is relatively greater than that exhibited by  $q_{12}$ . The values of  $q_{12}$  and  $q_{11}$  have been calculated using Equations (4) and (5) and these are entered in Table II. The values of  $q_{12} - q_{11}$  are also given in the fourth column. In the fifth column of Table II are given the corresponding values of



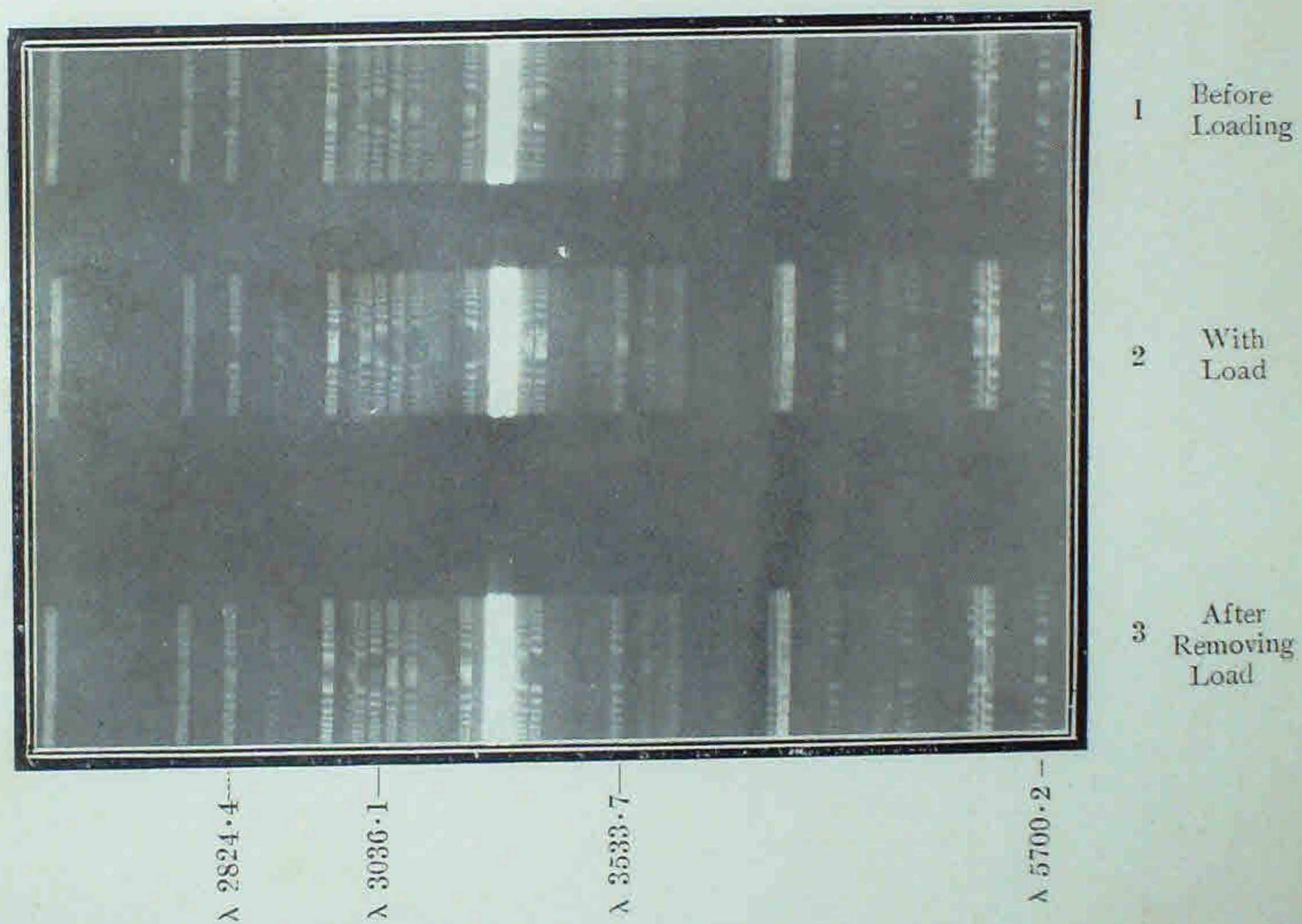


FIG. 2

$q_{12} - q_{11}$  calculated from the values of  $C$  obtained from the smooth curve given in the preceding paper (E. S. Jog, 1957). The agreement between the values given in columns four and five which are obtained from independent methods is very satisfactory.

The elasto-optic constants  $p_{11}$  and  $p_{12}$  and the change in the refractive index with density  $dn/d\rho$  are calculated by using the following relations and are also included in Table II.

$$p_{11} = C_{11}q_{11} + 2C_{12}q_{12} \quad (6)$$

$$p_{12} = C_{11}q_{12} + C_{12}(q_{11} + q_{12}) \quad (7)$$

$$\frac{dn}{d\rho} = \frac{n^3}{6\rho}(p_{11} + 2p_{12}) \quad (8)$$

where the constants  $C_{11}$  and  $C_{12}$  have been calculated from the most probable values of  $K$  and  $R$  given by Sosman (1923).

TABLE II

$\lambda$ in $m\mu$	$q_{12}$ $\times 10^{13}$	$q_{11}$ $\times 10^{13}$	$(q_{12} - q_{11})$ $\times 10^{13}$	$(q_{12} - q_{11})$ $\times 10^{13}$ $= 2C/n^3$	$p_{12}$	$p_{11}$	$p_{12} - p_{11}$	$dn/d\rho$
230	2.35	-0.24	2.59	2.58	0.218	0.060	0.158	0.132
250	2.37	-0.16	2.53	2.54	0.221	0.067	0.154	0.132
300	2.41	-0.04	2.45	2.48	0.227	0.077	0.150	0.132
350	2.44	+0.02	2.42	2.44	0.231	0.083	0.148	0.133
400	2.46	+0.06	2.40	2.41	0.233	0.087	0.146	0.133
450	2.47	+0.08	2.39	2.39	0.235	0.089	0.146	0.133
500	2.48	+0.10	2.38	2.37	0.236	0.090	0.146	0.133
550	2.49	+0.11	2.38	2.35	0.237	0.092	0.145	0.133

From Table II, it is seen that:—

(i) The piezo-optic constants  $q_{11}$  and  $q_{12}$  and the elasto-optic constants  $p_{11}$  and  $p_{12}$  increase with wavelength. The total dispersion throughout the range  $\lambda = 230$  to  $\lambda 550 m\mu$  for  $q_{12}$  and  $p_{12}$  is very small being less than 10%, whereas for  $p_{11}$  it is 50%. In the case of  $q_{11}$  one cannot speak of any percentage increase as the sign of the quantity changes. However, this change of the sign may not be



significant, since the magnitude of  $q_{11}$  is much dependent on the actual value of  $S_{12}$  used in Equation (3).

(ii) The quantities  $(q_{12} - q_{11})$  and  $(p_{12} - p_{11})$  decrease with wavelength, the total dispersion being only about 10%.

(iii) The quantity  $dn/d\rho$  is practically constant for all the wavelengths.

In conclusion, the author is grateful to Professor R. S. Krishnan for his kind interest and guidance in the work. His thanks are also due to Dr. K. Vedam and to Dr. S. Ramaseshan for useful discussions and to the Deccan Education Society, Poona, for the financial assistance.

#### REFERENCES

1. Jog, E. S. .. *Jour. Ind. Inst. Sci.*, 1957, 39A, 93.
2. .. *International Critical Tables*, 1929, 6, 341.
3. Ramachandran, G. N. .. *Proc. Ind. Acad. Sci.*, 1947, 25 A, 208.
4. Sosman .. *The Properties of Silica*, 1927.