THEORY OF OPEN RESONATOR WITH AN AXIAL DIELECTRICAL ROD *

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1. ABSTRACT

The Q factor of a microwave resonator consisting of an uniform circular dielectric rod excited by HE_{11} and E_{01} modes and terminated at both ends by circular metal plates has been derived. The constant percentage power contour for both the modes show that most of the energy is located within the resonator and very little is lost by radiation outside the resonator.

2. INTRODUCTION

Theoretical and experimental studies of microwave resonator, closed and open types and resonator consisting of an axial uniform and corrugated metal rod have been made and reported elsewhere, by Chatterjee, *et.al*[1-11]. The object of the present paper is to derive an expression for the Q-factor of a resonator consisting of an uniform circular dielectric rod and excited in HE_{11} and E_{01} modes and terminated at both ends by circular metal plates of diameter much larger than the diameter of the dielectric rod. All the other sides of the resonator are open. It is also the object to determine the constant percentage power contours round the rod and show that the loss of the energy by radiation is insignificantly small when the diameter of the end-plates are much larger compared to the wavelength of excitation.

3. FIELD COMPONENTS

The field components inside and outside the dielectric rod when it is excited by HE_{11} mode are as follows (See Fig. 1).

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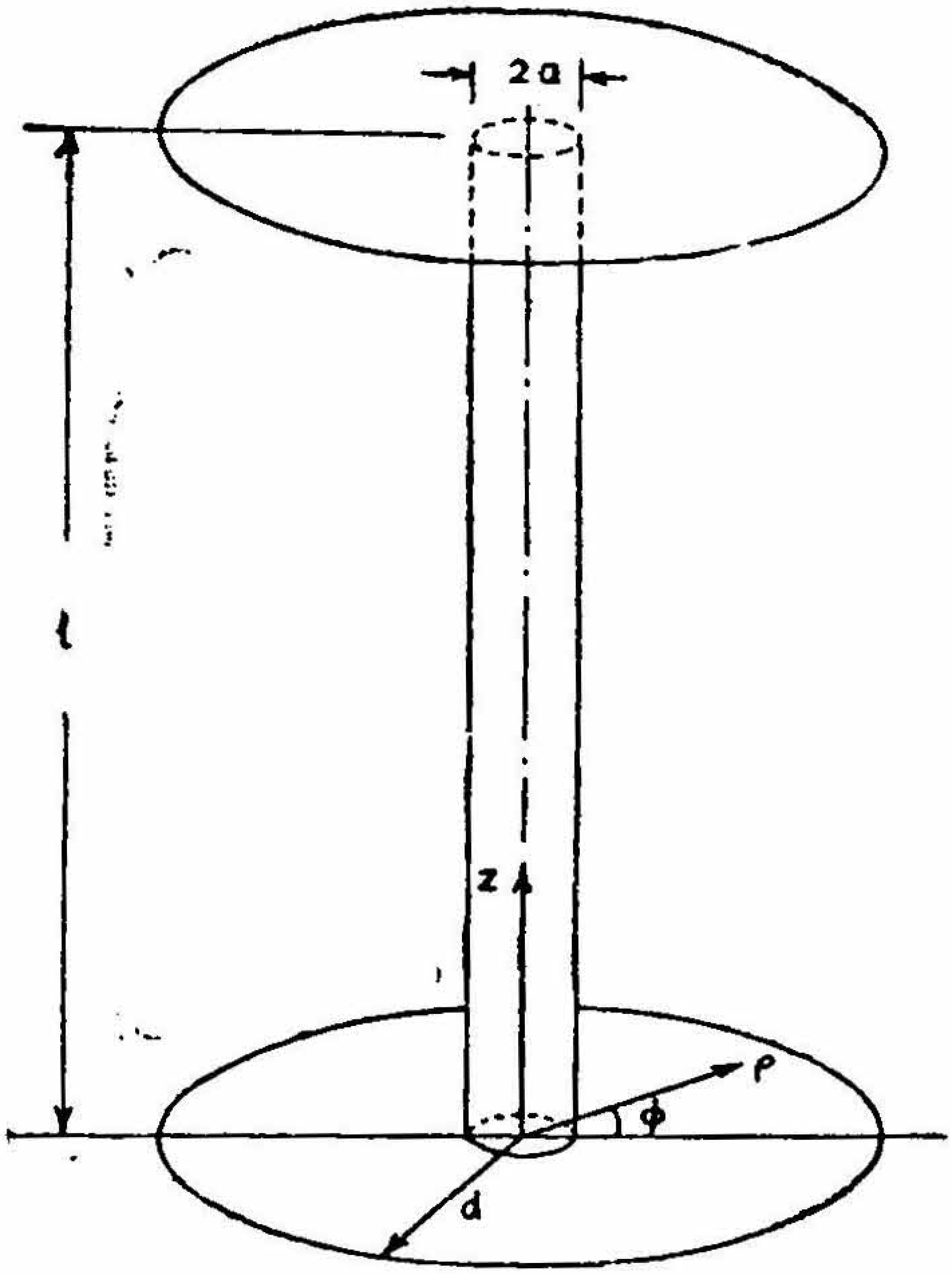


FIG. 1

For $\rho \leq a$ Medium 1 (Inside the rod):

$$E_{\rho 1} = -B\left[1/\rho\, J_1(k_1\rho) + \frac{b\beta k_1}{B\omega\epsilon_1} J_1'(k_1\rho)\right] \sin\phi\, e^{-j\beta z}$$

$$E_{\phi 1} = -B\left[k_1 J_1'(k_1\rho) + \frac{b}{B}\,\frac{\beta}{\omega\epsilon_1}\,\frac{1}{\rho} J_1(k_1\rho)\right] \cos\phi\, e^{-j\beta z}$$

$$E_{z1} = B\left[\frac{b}{B}\,\frac{k_1^2}{j\omega\epsilon_1} J_1(k_1\rho)\right] \sin\phi\, e^{-j\beta z}$$

$$H_{\rho 1} = B\left[\frac{\beta k_1}{\omega\mu_0} J_1'(k_1\rho) + \frac{b}{B}\,\frac{1}{\rho} J_1(k_1\rho)\right] \cos\phi\, e^{-j\beta z}$$

$$H_{\phi 1} = -B\left[\frac{1}{\rho}\,\frac{\beta}{\omega\mu_0} J_1(k_1\rho) + \frac{b}{B} k_1 J_1'(k_1\rho)\right] \sin\phi\, e^{-j\beta z}$$

$$H_{z1} = -B\left[(k_1^2/j\omega\mu_0)\, J_1(k_1\rho)\right]\cos\phi\; e^{-j\beta z} \qquad [1]$$

For $\rho \geq a$ Medium 2 (outside the rod):

$$E_{\rho 2} = -C\left[\frac{1}{\rho} H_1^{(1)}(k_2\rho) + \frac{c}{C}\frac{\beta k_2}{\omega\epsilon_0} H_1^{(1)\prime}(k_2\rho)\right]\sin\phi\; e^{-j\beta z}$$

$$E_{\phi 2} = -C\left[k_2 H_1^{(1)\prime}(k_2\rho) + \frac{c}{C}\frac{1}{\rho}\frac{\beta}{\omega\epsilon_0} H_1^{(1)}(k_2\rho)\right]\cos\phi\; e^{-j\beta z}$$

$$E_{z2} = C\left[\frac{c}{C}\frac{k_2^2}{j\omega\epsilon_0} H_1^{(1)}(k_2\rho)\right]\sin\phi\; e^{-j\beta z}$$

$$H_{\rho 2} = C\left[\frac{\beta k_2}{\omega\mu_0} H_1^{(1)\prime}(k_2\rho) + \frac{c}{C}\frac{1}{\rho} H_1^{(1)}(k_2\rho)\right]\cos\phi\; e^{-j\beta z}$$

$$H_{\phi 2} = -C\left[\frac{1}{\rho}\frac{\beta}{\omega\mu_0} H_1^{(1)}(k_2\rho) + \frac{c}{C} k_2 H^{(1)\prime}(k_2\rho)\right]\sin\phi\; e^{-j\beta z}$$

$$H_{z2} = -C\left[(k_2^2/j\omega\mu_0)\, H_1^{(1)}(k_2\rho)\right]\cos\phi\; e^{-j\beta z} \qquad [2]$$

4. Boundary Conditions

In order to derive the characteristic equation and determine the propagation constant, the following boundary conditions are applied at the interface $(\rho = a)$ between the dielectric and air.

$$E_{z1} = E_{z2}; \qquad H_{z1} = H_{z2}$$
$$E_{\phi 1} = E_{\phi 2}; \qquad H_{\phi 1} = H_{\phi 2} \qquad [3]$$

5. Characteristic Equation

By using the proper field components and appropriate boundary conditions, the following characteristic equation is obtained.

$$\left[\frac{1}{x_1}\frac{J_1'(x_1)}{J_1(x_1)} - \frac{1}{x_2}\frac{H_1^{(1)\prime}(x_2)}{H_1^{(1)}(x_2)}\right]\left[\frac{\epsilon_{r1}}{x_1}\frac{J_1'(x_1)}{J_1(x_1)} - \frac{1}{x_2}\frac{H_1^{(1)\prime}(x_2)}{H_1^{(1)}(x_2)}\right]$$

$$= \frac{(x_1^2 - x_2^2)(x_1^2 - \epsilon_{r1} x_2^2)}{x_1^4 x_2^4} \qquad [4]$$

Where, $x_1 = k_1 a, \quad x_2 = k_2 a$

$$\epsilon_{r1} = \epsilon_1 / \epsilon_0$$

$$x_1^2 + (x_2/j)^2 = \left(\frac{2\pi a}{\lambda_0}\right)^2 \quad (\epsilon_{r1} = 1) \qquad [5]$$

6. Excitation Constants

The excitation contants B, C, b and c are related as follows.

$$\frac{C}{B} = \frac{x_1^2}{x_2^2} \frac{J_1(x_1)}{H_1^{(1)}(x_2)}$$

$$\frac{C}{b} = \frac{x_1^2}{x_2^2} \frac{1}{\epsilon_{r1}} \frac{J_1(x_1)}{H_1^{(1)}(x_2)}$$

$$\frac{b}{B} = \frac{\beta \epsilon_1}{\omega \mu_0} \frac{x_1^2 - x_2^2}{x_1^2 x_2^2} \left[\frac{\epsilon_1}{x_1} \frac{J_1'(x_1)}{J_1(x_1)} - \frac{\epsilon_0}{x_2} \frac{H_1^{(1)\prime}(x_2)}{H_1^{(1)}(x_2)} \right]^{-1}$$

$$\frac{c}{C} = \frac{\beta \epsilon_0}{\omega \mu_0} \frac{x_1^2 - {}^2_2}{x_1^2 x_2^2} \left[\frac{\epsilon_1}{x_1} \frac{J_1'(x_1)}{J_1(x_1)} - \frac{\epsilon_0}{x_2} \frac{H_1^{(1)\prime}(x_2)}{H_1^{(1)}(x_2)} \right] \qquad [6]$$

The above relations are obtained from the field components and using boundary conditions.

7. Standing Waves

The standing waves formed along the rod due to reflections taking place at both ends of the rod are represented on a vector basis as follows.

$$E_{zs} = E_{z+} + E_{z-} \qquad H_{zs} = H_{z+} + H_{z-}$$

$$E_{\rho s} = E_{\rho +} + E_{\rho +} \qquad H_{\rho s} = H_{\rho +} + H_{\rho -}$$

$$E_{\phi s} = E_{\phi +} + E_{\phi -} \qquad H_{\phi s} = H_{\phi +} + H_{\phi -} \qquad [7]$$

where,

$$E_{z+} = + E_{z-} \qquad H_{z+} = - H_{z-}$$

$$E_{\rho +} = - E_{\rho -} \qquad H_{\rho +} = + H_{\rho -}$$

$$E_{\phi +} = - E_{\phi -} \qquad H_{\phi +} = + H_{\phi -} \qquad [8]$$

The field components of the standing waves are therefore, for $\rho \leq a$.

$$E_{\rho 1s} = -2B\left[\frac{1}{\rho} J_1(k_1 \rho) + \frac{b}{B} \frac{\beta k_1}{\omega \epsilon_1} J_1'(k_1 \rho)\right] \sin\phi \sin\beta z$$

$$E_{\phi 1s} = -2B\left[k_1 J_1'(k_1 \rho) + \frac{b}{B} \frac{\beta}{\omega \epsilon_1} J_1(k_1 \rho)\right] \cos\phi \sin\beta z$$

$$E_{z1s} = 2B\left[\frac{b}{B} \frac{k_1^2}{j\omega \epsilon_1} J_1(k_1 \rho)\right] \sin\phi \cos\beta z$$

$$H_{\rho 1s} = 2B\left[\frac{k_1 \beta}{\omega \mu_0} J_1'(k_1 \rho) + \frac{b}{B} \frac{1}{\rho} J_1(k_1 \rho)\right] \cos\phi \cos\beta z$$

$$H_{\phi 1s} = -2B\left[\frac{1}{\rho} \frac{\beta}{\omega \mu_0} J_1(k_1 \rho) + \frac{b}{B} k_1 J_1'(k_1 \rho)\right] \sin\phi \cos\beta z$$

$$H_{z1s} = -2B\left[\frac{k_1^2}{j\omega \mu_0} J_1(k_1 \rho)\right] \cos\phi \sin\beta z \qquad [9]$$

The components in the region outside the rod $\rho \geq a$ can be written similarly.

8. RESONANT WAVE

The field componets E_ρ and E_ϕ at $z=0$ and $z=1$ satisfy the conditions $E_\phi=0$ and $E_\rho=0$ which lead to $\sin\beta z=0$ which requires $\beta l=m\pi$ where, m is a positive integer (1, 2, 3 . . .) indicating the number of half cycle variation of the field components in the z-direction. The components of the resonant waves are obtained substituting $\beta=(m\pi/l)$ in the equations for slanding waves.

For $\rho \leq a$

$$E_{\rho 1r} = -2B\left[\frac{1}{\rho} J_1(k_1 \rho) + \frac{b}{B} \frac{\beta k_1}{\omega \epsilon_1} J'_1(k_1 \rho)\right] \sin\phi \sin\left(\frac{m\pi}{l}\right) z$$

$$E_{\phi 1r} = -2B\left[k_1 J_1'(k_1 \rho) + \frac{b}{B} \frac{1}{\rho} \frac{\beta}{\omega \epsilon_1} J_1(k_1 \rho)\right] \cos\phi \sin\left(\frac{m\pi}{1}\right) z$$

$$E_{z1r} = -2B\left[\frac{b}{B} \frac{k_1^2}{j\omega \epsilon_1} J_1(k_1 \rho)\right] \sin\phi \cos\left(\frac{m\pi}{1}\right) z$$

$$H_{\rho 1r} = 2B\left[\frac{k_1 \beta}{\omega \mu_0} J_1'(k_1\rho) + \frac{b}{B}\frac{1}{\rho} J_1(k_1\rho)\right] \cos\phi \cos\left(\frac{m\pi}{l}\right) z$$

$$H_{\phi 1r} = -2B\left[\frac{1}{\rho}\frac{\beta}{\omega\mu_0} J_1(k_1\rho) + \frac{b}{B} k_1 J_1'(k_1\rho)\right] \sin\phi \cos\left(\frac{m\pi}{l}\right) z$$

$$H_{z1r} = -2B\left[\frac{k_1^2}{j\omega\mu_0} J_1(k_1\rho)\right] \cos\phi \sin\left(\frac{m\pi}{l}\right) z \qquad [10]$$

$\rho \geq a$;

$$E_{\rho 2r} = -2C\left[\frac{1}{\rho} H_1^{(1)}(k_2\rho) + \frac{c}{C}\frac{\beta k_2}{\omega\epsilon_0} H_1^{(1)\prime}(k_2\rho)\right] \sin\phi \sin\left(\frac{m\pi}{l}\right) z$$

$$E_{\phi 2r} = -2C\left[k_2 H_1^{(1)\prime}(k_1\rho) + \frac{c}{C}\frac{\beta}{\omega\epsilon_0}\frac{1}{\rho} H_1^{(1)}(k_2\rho)\right] \cos\phi \sin\left(\frac{m\pi}{l}\right) z$$

$$E_{z2r} = 2C\left[\frac{c}{C}\frac{k_2^2}{j\omega\epsilon_0} H_1^{(1)}(k_2\rho)\right] \sin\phi \cos\left(\frac{m\pi}{l}\right) z$$

$$H_{\sigma 2r} = 2C\left[\frac{\beta k_2}{\omega\mu_0} H_1^{(1)\prime}(k_2\rho) + \frac{c}{C}\frac{1}{\rho} H_1^{(1)}(k_2\rho)\right] \cos\phi \cos\left(\frac{m\pi}{l}\right) z$$

$$H_{\phi 2s} = -2C\left[\frac{1}{\rho}\frac{\beta}{\omega\mu_0} H_1^{(1)}(k_2\rho) + \frac{c}{C} k_2 H_1^{(1)\prime}(k_2\rho)\right] \sin\phi \cos\left(\frac{m\pi}{l}\right) z$$

$$H_{z2r} = -2C\left[\frac{k_2^2}{j\omega\mu_0} H_1^{(1)}(k_2\rho)\right] \cos\phi \sin\left(\frac{m\pi}{l}\right) z \qquad [11]$$

If the field inside the resonator is completely described by equations [10] and [11], the mode of oscillation is pure HE_{11}. If the frequency of excitation or the distance between the end plates is so adjusted that the above conditions are satisfied, the cavity formed by the structure with the two end plates is said to be in resonance

9. Magnetic Energy Stored

The energy stored in the magnetic field consists of two parts W_{M1}, in the field inside the rod and W_{M2} in the field outside the rod (W_{M2}). The total enery W_M stored in the magnetic field is therefore.

$$W_M = W_{M1} + W_{M2}$$

$$= \frac{\mu_0}{2}\int_{\rho=0}^{a}\int_{\phi=0}^{2\pi}\int_{z=0}^{l} |H_1|^2 \rho\, d\rho\, d\phi\, dz$$

$$+ \frac{\mu_0}{2}\int_{\rho=a}^{d}\int_{\phi=0}^{2\pi}\int_{z=0}^{l} |H_2|^2 \rho\, d\rho\, d\phi\, dz \qquad [12]$$

where,

$$|H_1|^2 = |H_{\rho 1r}|^2 + |H_{\phi 1r}|^2 + |H_{z1r}|^2$$
$$|H_2|^2 = |H_{\rho 2r}|^2 + |H_{\phi 2r}|^2 + |H_{z2r}|^2 \quad [13]$$

The energy stored in the two media are :

$$WM1 = \frac{\mu_0}{2}\pi \frac{1}{2} 4B^2 \Bigg\{ \int_{\rho=0}^{a} \left(\frac{k_1 \beta}{\omega \mu_c}\right)^2 \rho [J_1(k_1 \rho)]^2 d\rho$$
$$+ \int_{\rho=0}^{a} \left(\frac{b}{B}\right)^2 \frac{1}{\rho} [J_1(k_1 \rho)]^2 d\rho + \int_{\rho=0}^{a} \frac{2b\beta k_1}{B\omega\mu_0} J_1(k_1 \rho) J_1'(k_1 \rho) d\rho$$
$$+ \int_{\rho=0}^{a} \left(\frac{\beta}{\omega\mu_0}\right)^2 \frac{1}{\rho} [J_1(k_1 \rho)]^2 d\rho + \int_{\rho=0}^{a} \left(\frac{b}{B}\right)^2 k_1^2 \rho [J_1'(k_1 \rho)]^2 d\rho$$
$$+ \int_{\rho=0}^{a} \frac{2b\beta k_1}{B\omega\mu_0} J_1'(k_1 \rho) J_1(k_1 \rho) d\rho - \left(\frac{k_1^2}{\omega\mu_0}\right)^2 \int_{\rho=0}^{a} \rho [J_1(k_1 \rho)]^2 d\rho$$

which results in

$$WM1 = \pi\mu_0 B l^2 \Bigg[\left\{ \left(\frac{\beta}{\omega\mu_0}\right)^2 + \left(\frac{b}{B}\right)^2 - \left(\frac{k_1}{\omega\mu_0}\right)^2 \right\} \left\{ \frac{(k_1 a)^2}{2} \langle [J_0(k_1 a)]^2$$
$$+ [J_1(k_1 a)]^2 \rangle \right\} - \left\{ \left(\frac{\beta}{\omega\mu_0} - \frac{b}{B}\right)^2 \right\} [J_1(k_1 a)]^2$$
$$+ \left(\frac{k_1}{\omega\mu_0}\right)^2 \left\{ k_1 a \, J_0(k_1 a) \, J_1(k_1 a) \right\} \Bigg] \quad [14]$$

Similarly,

$$WM_2 = \pi\mu_0 C l^2 \Bigg[\left\{ \left(\frac{\beta}{\omega\mu_0}\right)^2 + \left(\frac{c}{C}\right)^2 - \left(\frac{k_2}{\omega\mu_0}\right)^2 \right\} \Bigg(\left\{ \frac{(k_2 d)^2}{2} [H_0^{(1)}(k_2 d)]^2$$
$$+ [H_1^{(1)}(k_2 d)]^2 \right\} - \frac{(k_2 a)^2}{2} \{ [H_0^{(1)}(k_2 a)]^2 - [H_1^{(1)}(k_2 a)]^2 \} \Bigg)$$
$$- \left(\frac{\beta}{\omega\mu_0} - \frac{c}{C}\right)^2 \{ [H_1^{(1)}(k_2 d)]^2 - [H_1^{(1)}(k_2 a)]^2 \}$$
$$+ \left(\frac{k_2}{\omega\mu_0}\right)^2 \{ (k_2 d) H_0^{(1)}(k_2 d) H_1^{(1)}(k_2 d) - (k_2 a) H_0^{(1)}(k_2 a) H_1^{(1)}(k_2 a) \} \Bigg] \quad [15]$$

10. Power Loss

The total power lost P is composed of the power lost P_d in the dielectric rod, power lost in the end plates P_e and power lost by radiation P_R outside the resonatar, *i.e.*,

$$P = P_d + P_e + P_R \qquad [16]$$

If the diameters 'd' of the end plates is much larger than the diameter $(2a)$ of the rod, the loss by radiation P_R outside the resonator can be ignored compared to the other two terms.

The power lost in the dielectric rod is given by

$$P = \frac{1}{2} \int_{\phi=0}^{2\pi} \int_{\rho=0}^{a} \sigma_1 \, |E|^2 \, \rho \, d\rho \, d\phi \qquad [17]$$

where $\sigma_1 = \omega \, \epsilon_1 \tan \delta$ and

$$|E|^2 = |E_{\rho 1}|^2 + |E_{\phi 1}|^2 + |E_{z1}|^2$$

Substituting the values of the field components, equation [17] becomes

$$P_d = \sigma_1 \frac{1}{2} \pi B^2 \Bigg[\int_{\rho=0}^{a} \frac{1}{\rho} [J_1(k_1 \rho)]^2 \, d\rho + \int_{\rho=0}^{a} \left(\frac{b \beta k_1}{B \omega \epsilon_1} \right)^2 [J_1'(k_1 \rho)]^2 \rho \, d\rho$$

$$+ \int_{\rho=0}^{a} \frac{2b \beta k_1}{B \omega \epsilon_1} J_1'(k_1 \rho) J_1(k_1 \rho) \, d\rho + \int_{\rho=0}^{a} k_1^2 \rho \, [J_1'(k_1 \rho)]^2 \, d\rho \quad .$$

$$+ \int_{\rho=0}^{a} \left(\frac{b \beta}{B \omega \epsilon_1} \right)^2 \frac{1}{\rho} [J_1(k_1 \rho)]^2 \, d\rho + \int_{\rho=0}^{a} \frac{2b \beta k_1}{B \omega \epsilon_1} J_1'(k_1 \rho) J_1(k_1 \rho) \, d\rho$$

$$+ \int_{\rho=0}^{a} \left(\frac{b k_1^2}{B \omega \epsilon_1} \right)^2 \rho \, [J_1(k_1 \rho)]^2 \, d\rho) \Bigg]$$

$$= \frac{\pi l \sigma_1 B^2}{2} \Bigg\langle \left\{ \frac{(k_1 a)^2}{2} \left[1 + \left(\frac{b \beta}{B \omega \epsilon_1} \right)^2 \right] - \frac{1}{2} \left(\frac{b k_1^2 a}{B \omega \epsilon_1} \right)^2 \right\} \{J_0(k_1 a)\}^2$$

$$+ \{J_1(k_1 a)\}^2 \left[- \left(1 - \frac{b \beta}{B \omega \epsilon_1} \right)^2 + \frac{(k_1 a)^2}{2} \left(\left\{ 1 + \frac{b \beta}{B \omega \epsilon_1} \right\}^2 \right) - \frac{1}{2} \left(\frac{b k_1^2 a}{B \omega \epsilon_1} \right)^2 \right]$$

$$+ \{J_0(k_1 a) \, J_1(k_1 a)\} \left\{ \frac{1}{k_1 a} \left(\frac{b k_1^2 a}{B \omega \epsilon_1} \right) \right\} \Bigg\rangle \qquad [18]$$

The power lost in the two end plates is

$$P = 2 \times \frac{1}{2\sqrt{2}} \left(\frac{\omega \mu_e}{\sigma_e}\right)^{1/2} \int_{\rho=a}^{d} \int_{\phi=0}^{2\pi} |H_{tan}|^2 \rho \, d\rho \cdot d\phi \qquad [19]$$

where,

σ_e = conductivity of the end plates

$|H_{tan}|^2 = |H_{\rho 2}|^2 + |H_{\phi 2}|^2$

Substituting the value of the field components, equation [19] after integration yields.

$$P = \pi C^2 \left(\frac{\omega \mu_e}{2 \sigma_c}\right)^{1/2} \left[\left\{ \left(\frac{\beta k_2}{\omega \mu_0}\right)^2 + \left(\frac{c k_2}{C}\right)^2 \right\} \left\{ \frac{d^2}{2} [H_0^{(1)}(k_2 d)]^2 - \frac{a^2}{2} [H_0^{(1)}(k_2 a)]^2 \right\} \right.$$

$$\left. + \left\{ \left(\frac{\beta k_2}{\omega \mu_0}\right)^2 + \left(\frac{c k_2}{C}\right)^2 + \left(\frac{\beta}{\omega \mu_0} - \frac{c}{C}\right)^2 \right\} \left\{ \frac{d^2}{2} [H_1^{(1)}(k_2 d)]^2 - \frac{a^2}{2} [H_1^{(1)}(k_2 a)]^2 \right\} \right] \qquad [20]$$

11. 'Q' FACTOR

The quality factor Q of the resonator is defined as

$$Q = \omega \cdot \frac{W_{M1} + W_{M2}}{P_{total}} = \omega \frac{W_{M1} + W_{M2}}{P_e + P_d} \qquad [21]$$

Substituting appropriate expressions for the maximum energy stored in the magnetic field and the total power lost, the expression for Q becomes

$$Q = \frac{2\pi f (\mu_0/\epsilon_0) [X + (C^2/B^2) Y]}{(\frac{1}{2} \omega \epsilon_{r_1} \tan \delta)(U + V) + (C^2/B^2)(1/\epsilon_0 l)(\omega \mu_0 / 2\sigma_e)^{1/2} W} \qquad [22]$$

where

$$X = \left\{ \left(\frac{\beta}{\omega \mu_0}\right)^2 + \left(\frac{b}{B}\right)^2 - \left(\frac{k_1}{\omega \mu_0}\right)^2 \right\} \frac{(k_1 a)^2}{2} [\{J_0(k_1 a)\}^2 + \{J_1(k_1 a)\}^2]$$

$$- \left\{ \left(\frac{\beta}{\omega \mu_0} - \frac{b}{B}\right)^2 \right\} \{[J_1(k_1 a)]^2\} + \left\{ \left(\frac{k_1}{\omega \mu_0}\right)^2 \right\} \{(k_1 a) J_0(k_1 a) J_1(k_1 a)\}$$

$$Y = \left\langle \left\{ \left(\frac{\beta}{\omega\mu_0}\right)^2 + \left(\frac{c}{C}\right)^2 - \left(\frac{k_2}{\omega\mu_0}\right)^2 \right\} \left[\left\{ \frac{(k_2 d)^2}{2} [H_0^{(1)}(k_2 d)]^2 + [H_1^{(1)}(k_2 d)]^2 \right\} - \frac{(k_2 a)^2}{2} \left\{ [H_0^{(1)}(k_2 a)]^2 + H_1^{(1)}(k_2 a)]^2 \right\} \right] - \left\{ \frac{\beta}{\omega\mu_0} - \left(\frac{c}{C}\right) \right\}^2 \{ [H_1^{(1)}(k_2 d]^2 - [H_1^{(1)}(k_2 a)]^2 \} + \left(\frac{k_2}{\omega\mu_0}\right)^2 \{ (k_2 d)\, H_0^{(1)}(k_2 d)\, H_1^{(1)}(k_2 d) - (k_2 a)\, H_0^{(1)}(k_2 a)\, H_1^{(1)}(k_2 a) \} \right\rangle$$

$$U = \left\{ \frac{(k_1 a)^2}{2} \left[1 + \left(\frac{b\beta}{B\omega\epsilon_1}\right)^2 \right] - \frac{1}{2}\left(\frac{bk_1^2 a}{B\omega\epsilon_1}\right)^2 \right\} [J_0(k_1 a)]^2$$

$$V = \left\langle \{[J_1(k_1 a)]^2\} \left\{ -\left(1 - \frac{b\beta}{B\omega\epsilon_1}\right)^2 + \frac{(k_1 a)^2}{2} \right\} \left[1 + \left(\frac{b\beta}{B\omega\epsilon_1}\right)^2 \right] - \frac{1}{2}\left(\frac{bk_1^2 a}{B\omega\epsilon_1}\right)^2 \right\} + \{J_0(k_1 a)\, J_1(k_1 a)\} \left\{ \frac{1}{k_1 a} \left(\frac{bk_1^2 a}{B\omega\epsilon_1}\right)^2 \right\} \right\rangle$$

$$W = \left\{ \left(\frac{\beta k_2}{\omega\mu_0}\right)^2 + \left(\frac{ck_2}{C}\right)^2 \right\} \left\{ \frac{d^2}{2} [H_0^{(1)}(k_2 d)]^2 - \frac{a^2}{2} [H_1^{(1)}(k_2 a)]^2 + \left\{ \left(\frac{Bk_2}{\omega\mu_0}\right)^2 + \left(\frac{c}{C} k_2\right)^2 + \left(\frac{\beta}{\omega\mu_0} - \frac{c}{C}\right)^2 \right\} \left\{ \frac{d^2}{2} [H_1^{(1)}(k_2 d)]^2 - \frac{a^2}{2} [H_1^{(1)}(k_2 a)]^2 \right\} \right.$$

12. Evaluation of k_1 and k_2

The values of radial propagation constants k_1 and k_2 are found from the values of x_1 and x_2 which are determined by solving the characteristic equation (4) with the aid of equation (5). The variation of radial propagation constants with $(2a/\lambda_0)$ is shown in Fig. 2. Fig. 3 shows the variation of axial phase constant with $(2a/\lambda_0)$ which is determined from the relation between k and β.

13. Evaluation of 'Q'

The values of Q of the resonator as a function of $(2a/\lambda)$, L, $\tan\delta$, σ_c and λ calculated from equation (22) with proper values of k_1, k_2 and β are ploted in figures 4 - 8 respectively.

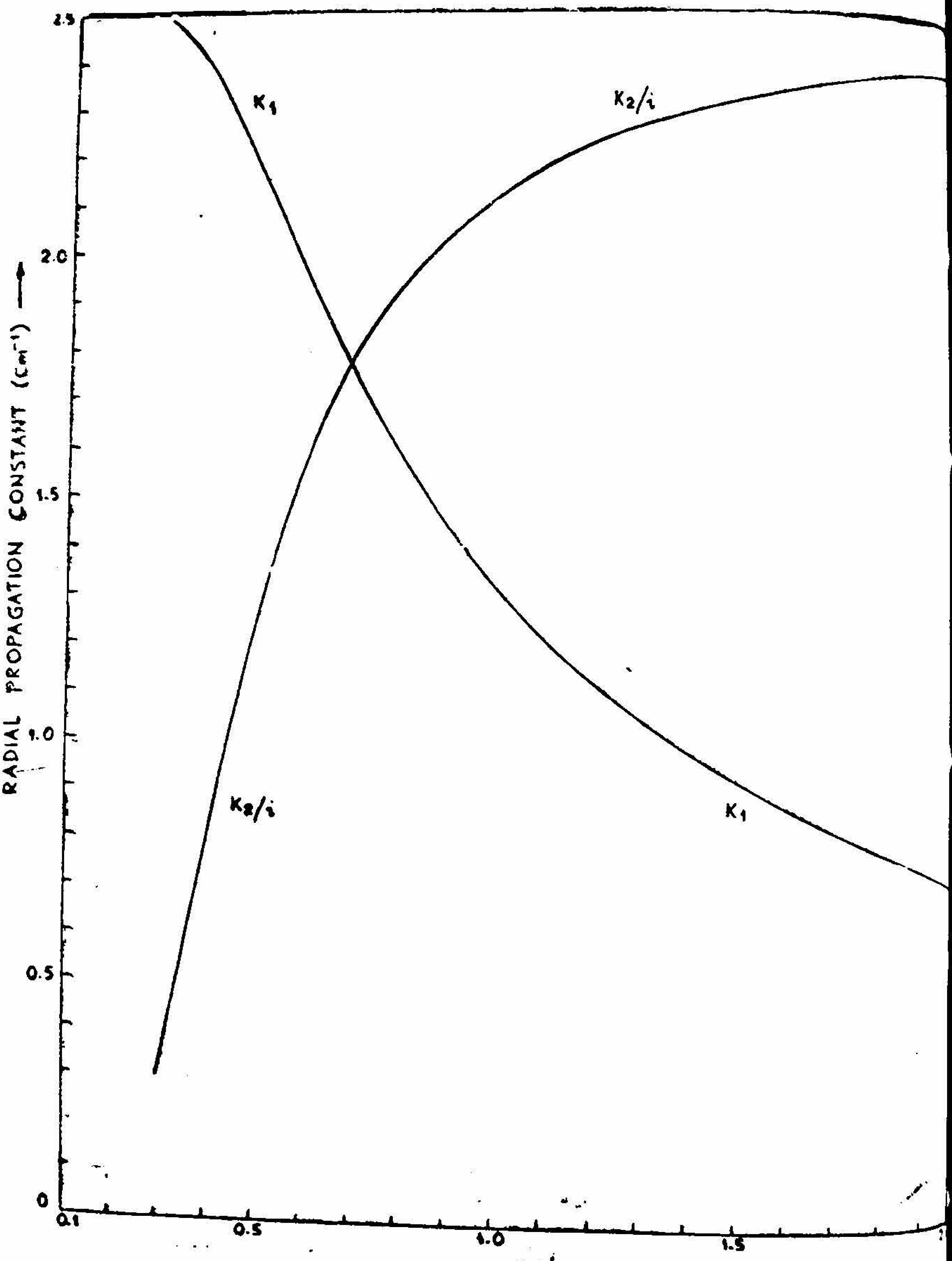

RADIAL PROPAGATION CONSTANT (cm⁻¹)
2.5
2.0
1.5
1.0
0.5
0
0.1
0.5
1.0
1.5
K1
K2/i
K2/i
K1
2a/λ

FIG. 2

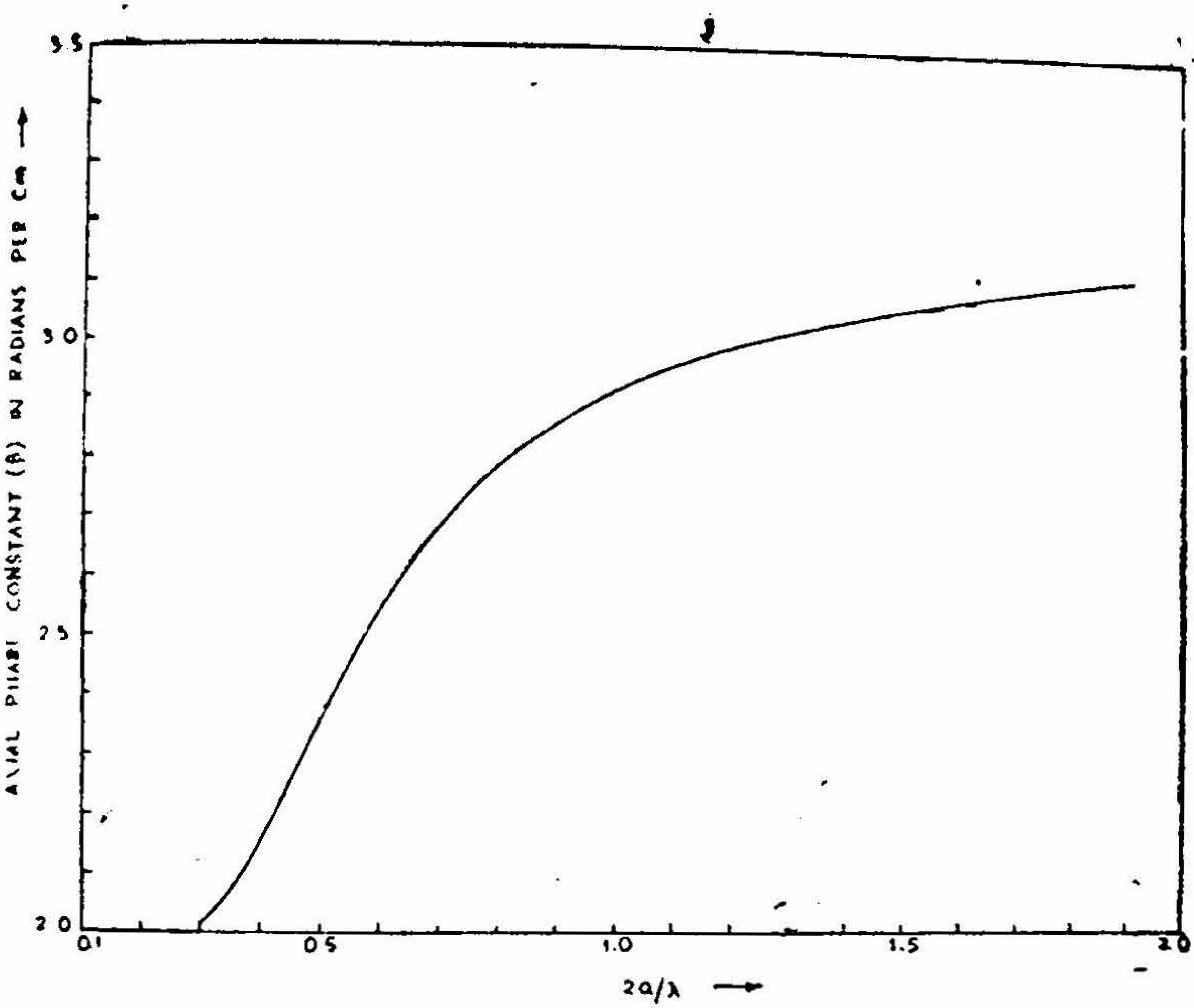


FIG. 3

14. FIELD COMPONENTS E_{01} – MODE

The field components inside ($\rho \leq a$) and outside the rod ($\rho \geq a$) are respectively

Medium 1 : $\rho \leq a$

$$E_{\rho 1} = \frac{Bk_1\beta}{\omega\epsilon_1} J_1(k_1\rho) \exp(-j\beta z)$$

$$E_{z1} = \frac{Bk_1^2}{j\omega\epsilon_1} J_0(k_1\rho) \exp(-j\beta z)$$

$$H_{\phi 1} = Bk_1 J_1(k_1\rho) \exp(-j\beta z) \quad [24]$$

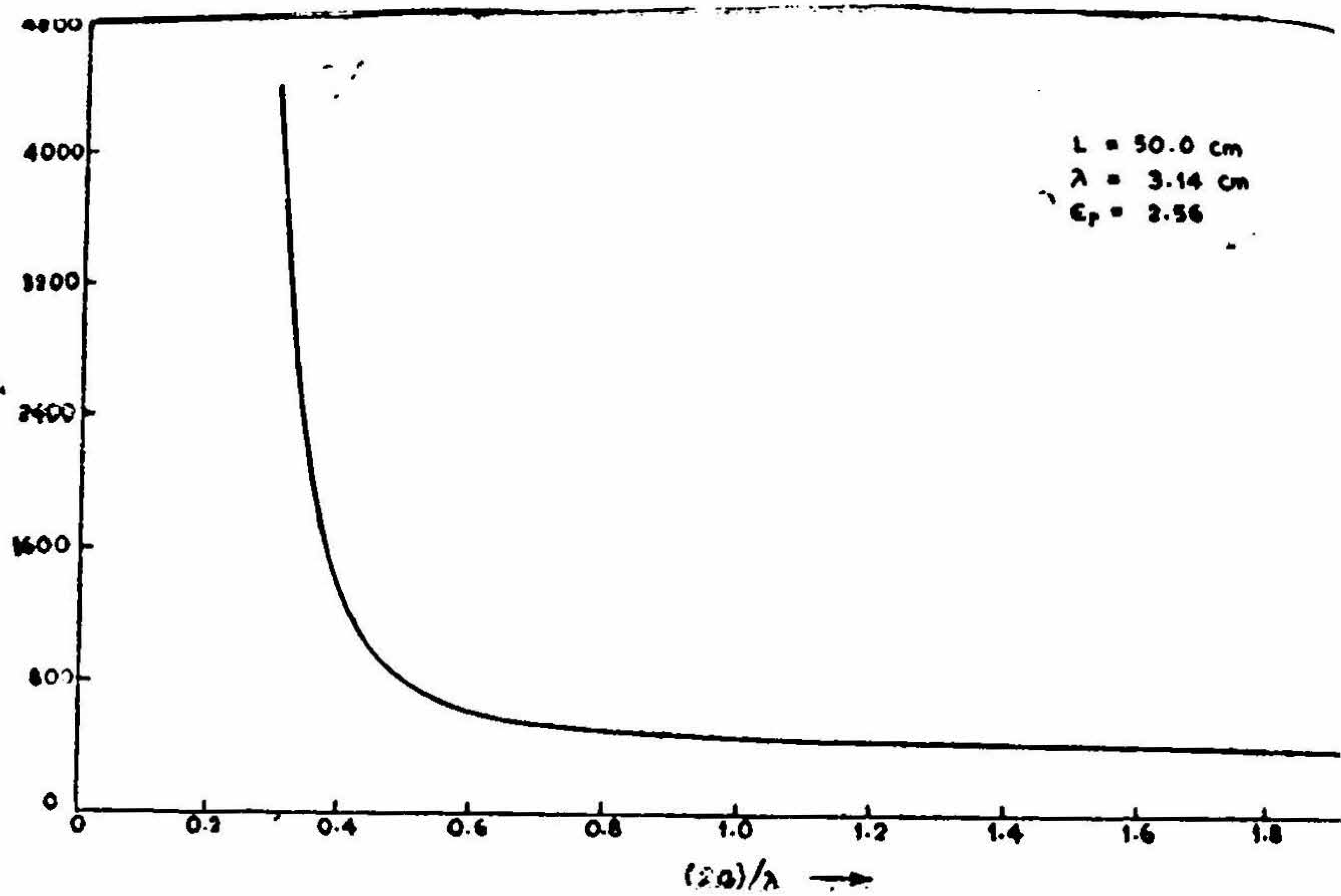

4000
3200
2400
1600
800
0
0
0.2
0.4
0.6
0.8
1.0
1.2
1.4
1.6
1.8
L = 50.0 cm
λ = 3.14 cm
ε_r = 2.56
(2a)/λ →

FIG. 4

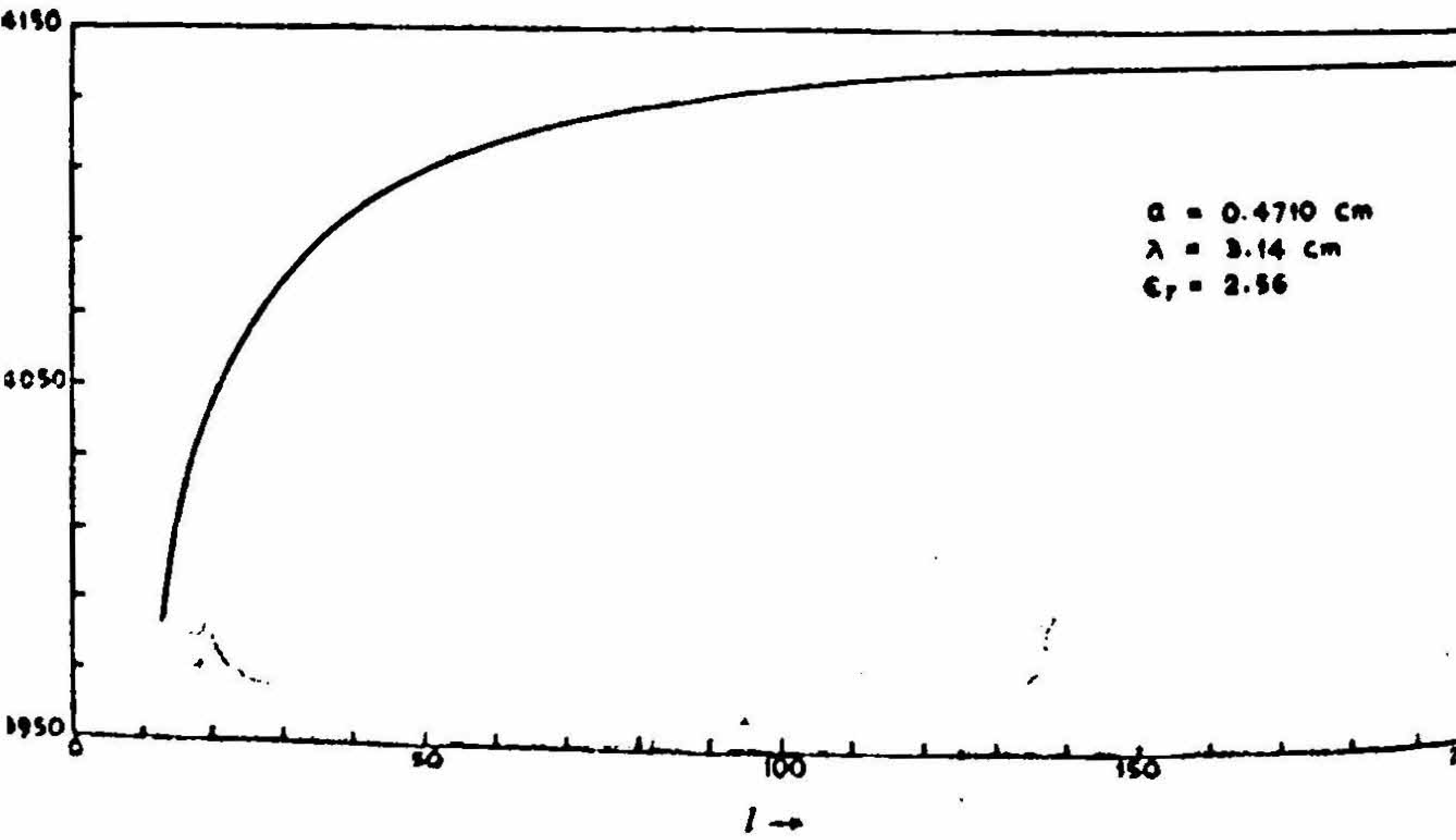

4150
4050
3950
0
50
100
150
a = 0.4710 cm
λ = 3.14 cm
ε_r = 2.56
l →

FIG. 5

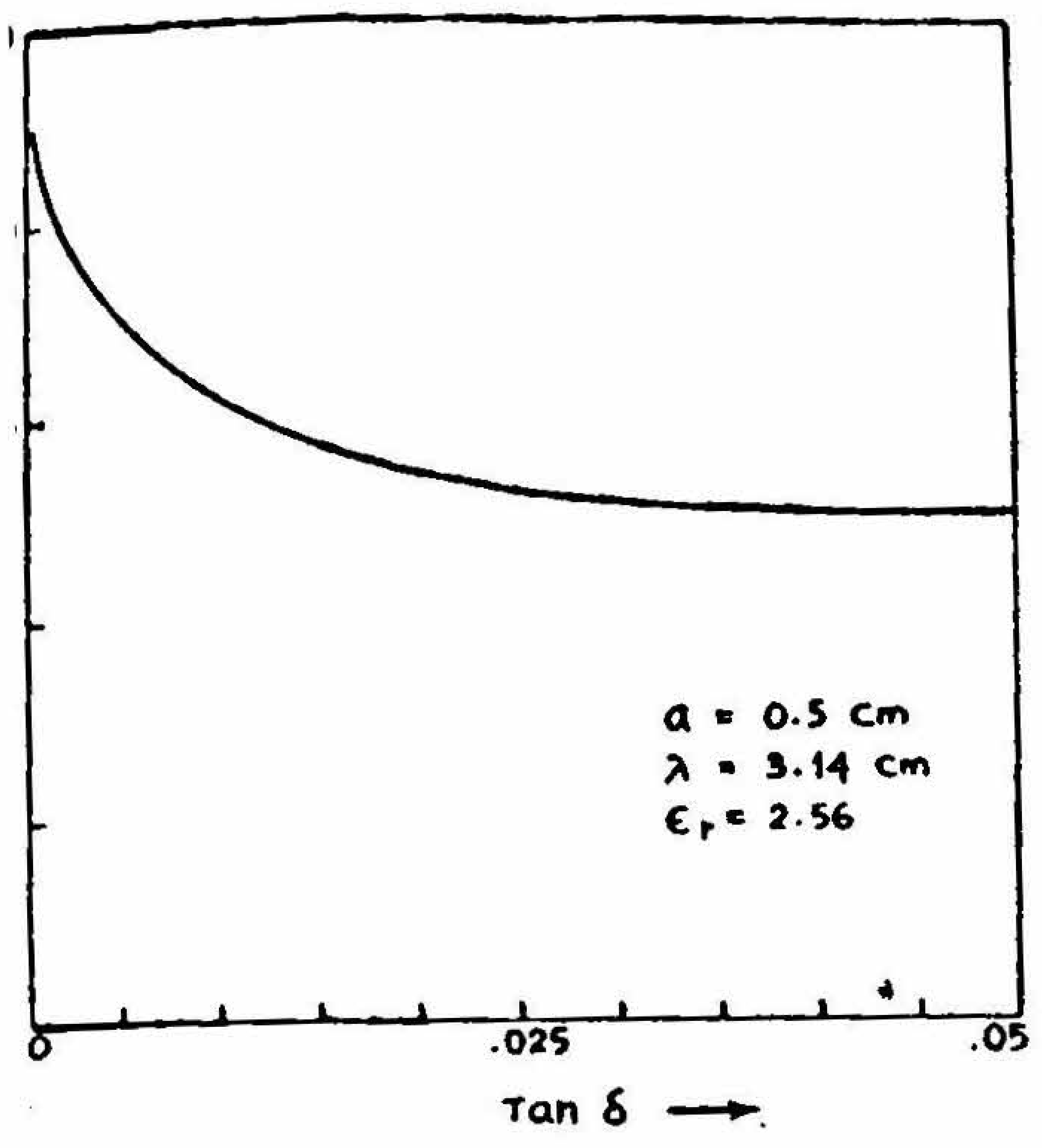


FIG. 6

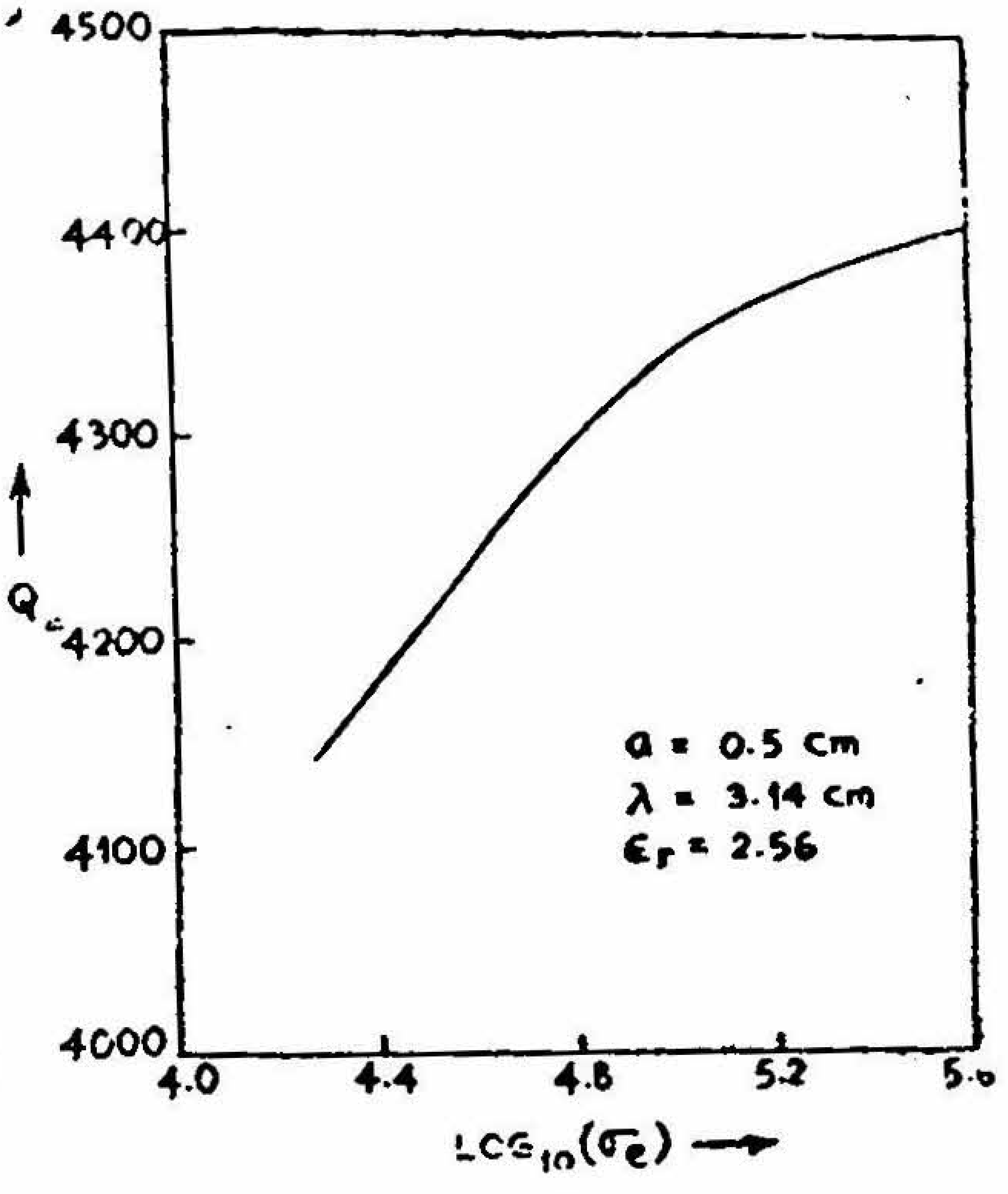


FIG. 7

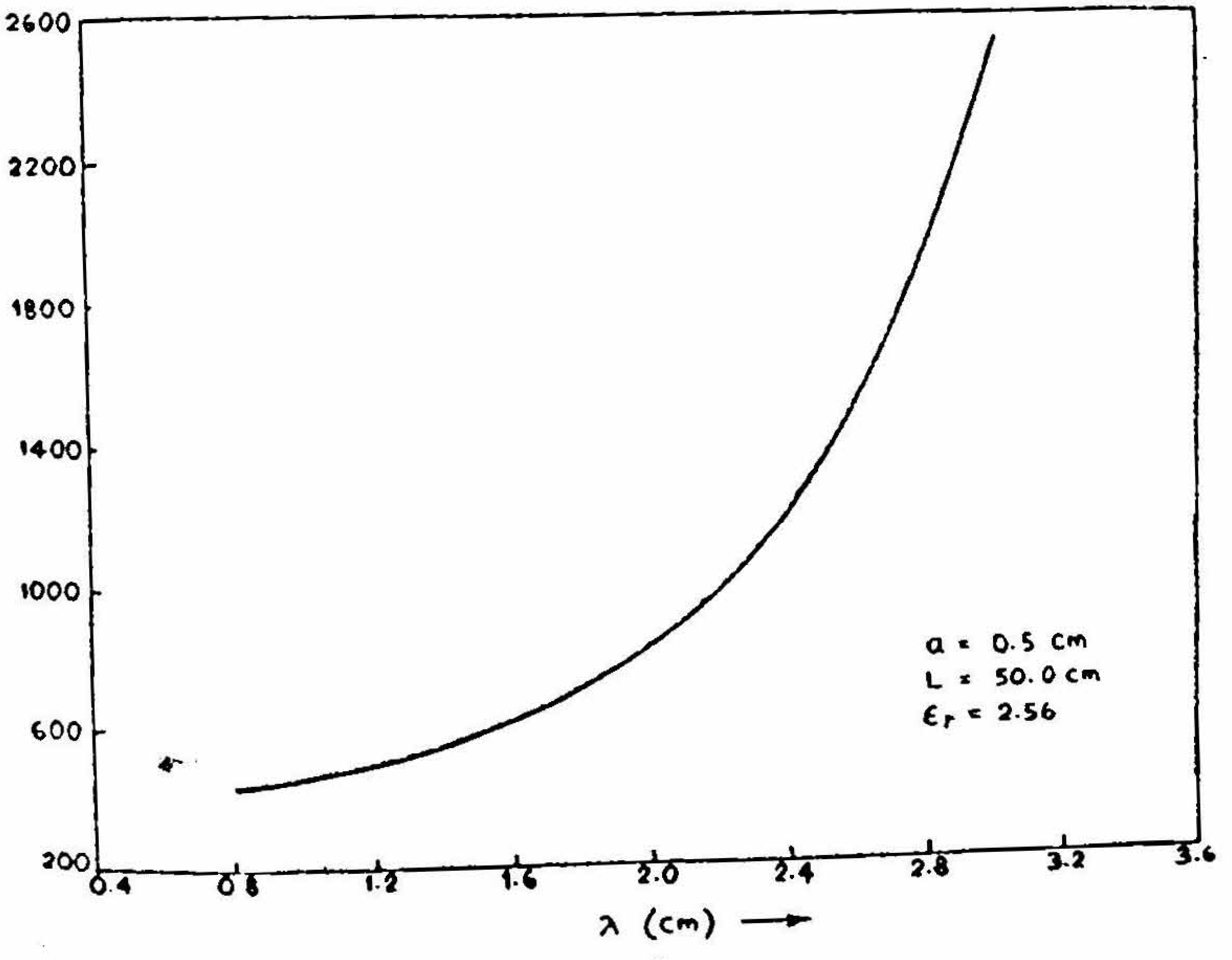


FIG. 8

Medium 2 : $\rho \geq a$

$$E_{\rho 2} = \frac{C k_2 \beta}{\omega \epsilon_0} H_0^{(1)} (k_2 \rho) \exp(-j \beta z)$$

$$E_{z2} = \frac{C k_2^2}{j \omega \epsilon_0} H_0^{(1)} (k_2 \rho) \exp(-j \beta z)$$

$$H_{\phi 2} = C k_2 H_1^{(1)} (k_2 \rho) \exp(-j \beta z) \qquad [25]$$

15. STANDING WAVES

In this case, the standing waves are represented by

$$E_{zs} = E_{z+} + E_{z-}, \qquad E_{\rho s} = E_{\rho +} + E_{\rho -}, \qquad H_{\phi s} = H_{\phi +} + H_{\phi -} \qquad [26]$$

where,

$$E_{z+} = \quad E_{z-} \qquad E_{\rho +} = \quad E_{\rho -} \qquad H_{\phi +} = +H_{\phi -} \qquad [27]$$

16. RESONANT WAVES

Due to the vanishing of the tangential components of the electric field at the two end plates ($z=0$ and $z=1$), $\beta = n\pi/1$, where n is a positive integer. The field components of the resonant waves are

Medium 1 : $\rho \leq a$

$$E_{\rho 1r} = 2j B \frac{k_1 \beta}{\omega \epsilon_1} J_1 (k_1 \rho) \sin\left(\frac{n \pi z}{l}\right)$$

$$E_{z1r} = B \frac{k_1^2}{j \omega \epsilon_1} J_0 (k_1 \rho) \cos\left(\frac{n \pi z}{l}\right)$$

$$H_{\phi 1r} = 2 B k_1 J_1 (k_1 \rho) \cos\left(\frac{n \pi z}{l}\right) \qquad [28]$$

Medium 2 : $\rho \geq a$

$$E_{\rho 2r} = 2 C \frac{k_2 \beta}{\omega \epsilon_0} H_1^{(1)} (k_2 \rho) \sin\left(\frac{n \pi z}{l}\right)$$

$$E_{z2r} = 2 C \frac{k_2^2}{j \omega \epsilon_0} H_0^{(1)} (k_2 \rho) \sin\left(\frac{n \pi z}{l}\right)$$

$$H_{\phi 2r} = 2 C k_2 H_1^{(1)} (k_2 \rho) \cos\left(\frac{n \pi z}{l}\right) \qquad [29]$$

17. Maximum Energy Stored in Magnetic Field

The total maximum energy stored in the magnetic field is

$$W_M = W_{M1} + W_{M2}$$

where, the magnetic energy stored in medium 1 (W_{M1}) and in medium 2 (W_{M2}) are respectively

$$W_{M1} = \frac{\mu_0}{2} \int_V |\mathrm{H}_{max}|^2 \, d\,v$$

$$W_{M1} = \frac{\mu_0}{2} \int_{\rho=0}^{a} \int_{\phi=0}^{2\pi} \int_{z=0}^{l} 4\, B^2 k_1^2 \,[\,\mathrm{J}_1 (k_1 \rho)\,]^2 \cos^2\left(\frac{n\,\pi\, z}{l}\right) \rho \, d\rho \, d\phi \, dz$$

$$= \pi\, \mu_0\, B^2 k_1^2 \, l \, a^2 \left[\mathrm{J}_0 (k_1 a)^2 + \mathrm{J}_1 (k_1 a)^2 - \frac{2}{k_1 a} \mathrm{J}_0 (k_1 a)\, \mathrm{J}_1 a) \right] \quad [30]$$

$$W_{M2} = \pi\, \mu_0\, C^2 k_2^2 \, l \Bigg[d^2 \Big\{ [\,\mathrm{H}_0^{(1)} (k_2 d)\,]^2 + [\,\mathrm{H}_1^{(1)} (k_2 d\,)^2 - \frac{2}{k_2 d} \mathrm{H}_0^{(1)} (k_2 d)\, \mathrm{H}_1^{(1)} (k_2 d) \Big\}$$

$$- a^2 \Big\{ [\,\mathrm{H}_0^{(1)} (k_2 a)\,]^2 + [\,\mathrm{H}_1^{(1)} (k_2 a)\,]^2 - \frac{2}{k_2 a} \mathrm{H}_0^{(\,)} (k_2 a) \mathrm{H}_1^{(1)} (k_2 a) \Big\} \Bigg] \quad [31]$$

Therefore, the total maximum energy stored in the magnetic field is

$$W_M = \pi\, \mu_0\, l\, a^2\, B^2 \Bigg[k_1^2 \Big\{ \{\mathrm{J}_0(k_1 a)\}^2 + \{\mathrm{J}_1(k_1 a)\}^2 - \frac{2}{k_1 a} \mathrm{J}_0 (k_1 a)\, \mathrm{J}_1 (k_1 a) \Big\}$$

$$- \frac{C^2}{B^2} k_2^2 \Bigg(\frac{d^2}{a^2} \Big\{ [\mathrm{H}_0^{(1)} (k_2 d)]^2 + [\mathrm{H}_1^{(1)} (k_2 d)]^2 - \frac{2}{k_1 a} \mathrm{H}_0^{(1)}(k_2 d) \mathrm{H}_1^{(1)} (k_2 d) \Big\}$$

$$- \Big\{ [\mathrm{H}_0^{(1)} (k_2 a)]^2 + [\mathrm{H}_1^{(1)} (k_2 a)]^2 - \frac{2}{k_2 a} \mathrm{H}_0^{(1)} (k_2 a)\, \mathrm{H}_1^{(1)} (k_2 a) \Big\} \Bigg) \Bigg] \quad [32]$$

18. Power Loss in the Resonator F_{01} – Mode

The total power loss in the resonator is

$$P = P_d + P_e \quad [33]$$

neglecting the loss by radiation outside the resonator. Where, the power loss in the two end plates is

$$P_e = 2 \times \frac{1}{2\sqrt{2}} \left(\frac{\omega \mu e}{\sigma e} \right)^{1/2} \int_{\rho=a}^{d} \int_{\phi=0}^{2\pi} | \mathrm{H}_{\tan} |^2 \rho \, d\rho \quad d\phi \qquad [34]$$

where $|\mathrm{H}_{\tan}| = |\mathrm{H}_{\phi 2}|$, σ_e and μ_e are the conductivity and permeability respectively of the end plates and d is the radius of the end plate

Substituting $\mathrm{H}_{\phi 2}$ in the integrand, eqation [34] becomes

$$P_e = \pi k_2^2 C^2 \left(\frac{\omega \mu_e}{2 \sigma_e} \right)^{1/2} \Bigg[d^2 \Big([\mathrm{H}_0^{(1)} (k_2 d)]^2 + \mathrm{H}_1^{(1)} (k_2 d)^2$$

$$- \frac{2}{k_2 d} \mathrm{H}_0^{(1)} (k_2 d) \, \mathrm{H}_1^{(1)} (k_2 d) \Big)$$

$$- a^2 \Big(\{\mathrm{H}_0^{(1)} (k_2 a)\}^2 + \{\mathrm{H}_1^{(1)} (k_2 a)\}^2 - \frac{2}{k_2 a} \mathrm{H}_0^{(1)} (k_2 a) \, \mathrm{H}_1^{(1)} (k_2 a) \Big) \Bigg] \qquad [35]$$

The power loss in the dielectric rod is

$$P_d = \frac{1}{2} \int_{\phi=0}^{2\pi} \int_{\rho=0}^{a} \sigma_1 |E|^2 \rho \, d\rho \; d\phi \qquad [36]$$

where, $\sigma_1 = \omega \epsilon_1 \tan \delta$ and $|E|^2 = |E_{\rho 1}|^2 + |E_{z1}|^2$

Substituting $E_{\tan}$ in the integrand, equation [36] reduces to

$$P_d = \frac{\pi l \mathrm{B}^2 k_1^2 a^2 \sigma_1}{2 \omega^2 \epsilon_1^2} \Bigg[[\{\mathrm{J}_0 (k_1 a)\}^2 + \{\mathrm{J}_1 (k_1 a)\}^2] (\beta^2 - k_1^2)$$

$$- 2 \frac{\beta^2}{k_1 a} \mathrm{J}_0 (k_1 a) \, \mathrm{J}_1 (k_1 a) \Bigg] \qquad [36a]$$

19. 'Q' FACTOR : E_{01} – MODE

$Q = \omega (W_M/P)$ gives

$$Q = \frac{(\mu_0/\epsilon_0) [R_1 + (C/B)^2 R_2]}{[(\omega \epsilon_{r1} \tan \delta)/2] [R_3] + (C/B)^2 (1/l\epsilon_0)(\omega \mu_e/2\sigma_e)^{1/2} [R_4]} \qquad [37]$$

where, R_1, R_2, R_3 and R_4 are given by the following expressions.

$$R_1 = a^2 k_1^2 \left\{ [\mathrm{J}_0 (k_1 a)]^2 + [\mathrm{J}_1 (k_1 a)]^2 - \frac{2}{k_1 a} \mathrm{J}_0 (k_1 a) \, J_1 (k_1 a) \right\}$$

$$R_2 = k_2^2 \left[d^2 \left\{ [H_0^{(1)}(k_2 d)]^2 + [H_1^{(1)}(k_2 d)]^2 - \frac{2}{k_2 d} H_0^{(1)}(k_2 d)\, H_1^{(1)}(k_2 d) \right\} \right.$$
$$\left. - a^2 \left\{ [H_0^{(1)}(k_2 a)]^2 + [H_1^{(1)}(k_2 a)]^2 - \frac{2}{k_2 a} H_0^{(1)}(k_2 a)\, H_1^{(1)}(k_2 a) \right\} \right]$$
$$R_3 = \left(\frac{k_1}{\omega \epsilon_1} \right)^2 \left[\{ [J_0(k_1 a)]^2 + [J_1(k_1 a)] \} + \{ (\beta^2 - k_1^2) \} - \frac{2}{k_1 a} J_0(k_1 a)\, J_1(k_1 a) \right]$$
$$R_4 = k_2^2 \left[d^2 \left\{ [H_0^{(1)}(k_2 d)]^2 + [H_1^{(1)}(k_2 d)]^2 - \frac{2}{k_2 d} H_0^{(1)}(k_2 d)\, H_1^{(1)}(k_2 d) \right\} \right.$$
$$\left. - a^2 \left\{ [H_0^{(1)}(k_2 a)]^2 + [H_1^{(1)}(k_2 a)]^2 - \frac{2}{k_2 a} H_0^{(1)}(k_2 a)\, H_1^{(1)}(k_2 a) \right\} \right]$$

The value of C/B is given by the following expression

$$(C/B)^2 = \left[\frac{k_1^2}{k_2^2} \frac{1}{\epsilon_{r1}} \frac{J_0(k_1 a)}{H_0^{(1)}(k_2 a)} \right]^2$$

The variation of Q for E_0 mode as a function of $2a/\lambda_0$, L, $\tan\delta$ and σ, are shown in Fig. 9—13.

20. Constant Percentage Power Contours

In calculating the Q-factor of the resonator, radiation loss outside the resonator has been assumed to be negligible. This is justified if most of the energy is located inside the resonator, *i.e.*, within a radius 'd' from the axis of the resonator where 'd' is the radius of the end plates. This can be determined by calculating the constant percentage power contour.

The amount of relative power flow outside the guide can be represented from the constant percentage power contours round the guide which are determined as follows.

If $\rho = r_1, r_2, r_3 \cdots r_n$ represent the radii of the circles representing the contours inside which constant powers $P_{z1}, P_{z2}, P_{z3} \cdots P_{zn}$ flowing along the rod are located, then the ratio of powers with respect to the total power P_{zn} is

$$P_{z1}/P_{zn} = W_1 \quad \text{at} \quad \rho = r_1$$
$$P_{z2}/P_{zn} = W_2 \quad \text{at} \quad \rho = r_2$$
$$P_{z3}/P_{zn} = W_3 \quad \text{at} \quad \rho = r_3$$
$$\vdots$$
$$P_{zn}/P_{zn} = W_n \quad \text{at} \quad \rho = r_n$$

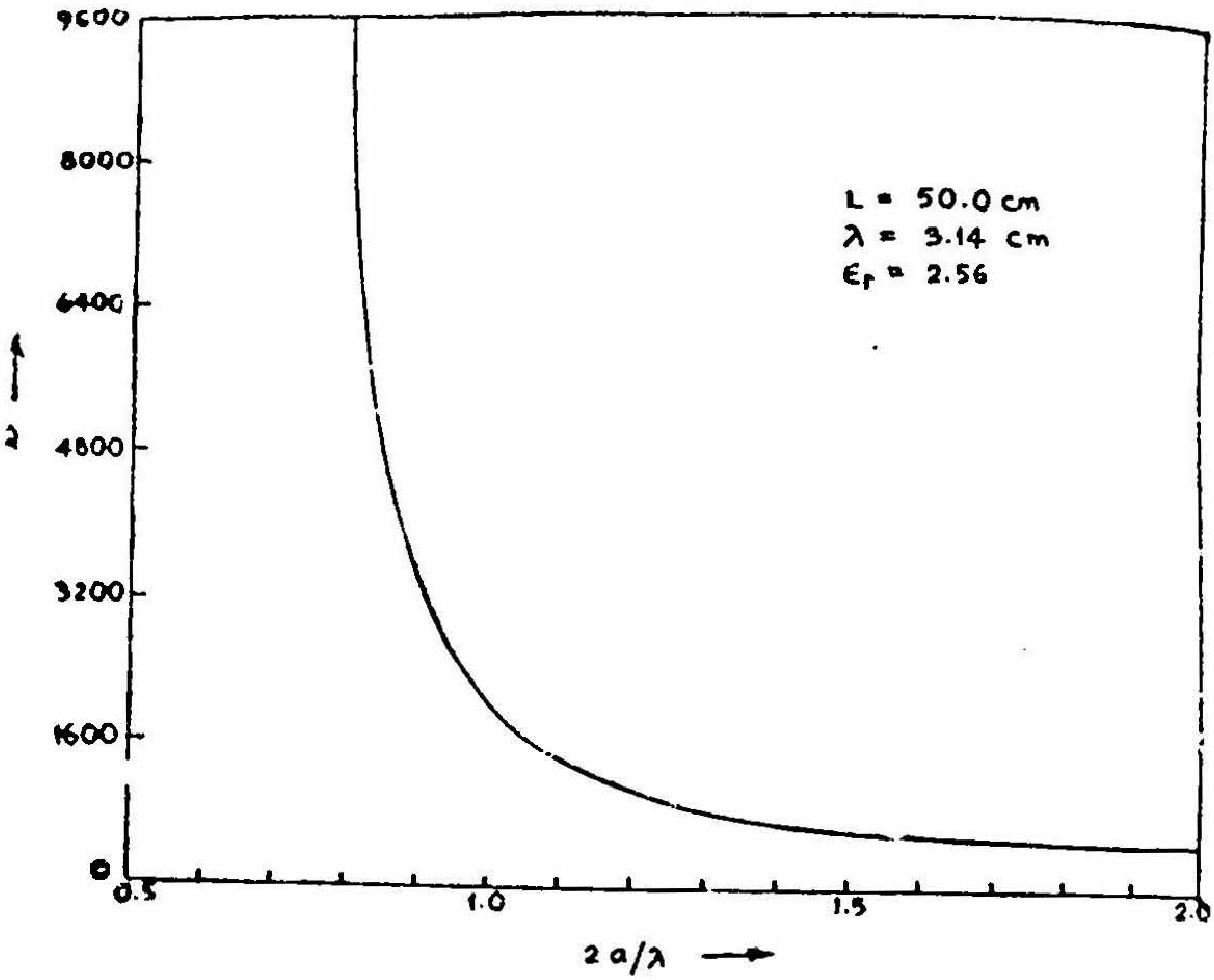

9600
8000
6400
4800
3200
1600
0
0.5
1.0
1.5
2.0
2 a/λ
L = 50.0 cm
λ = 3.14 cm
εr = 2.56

FIG. 9

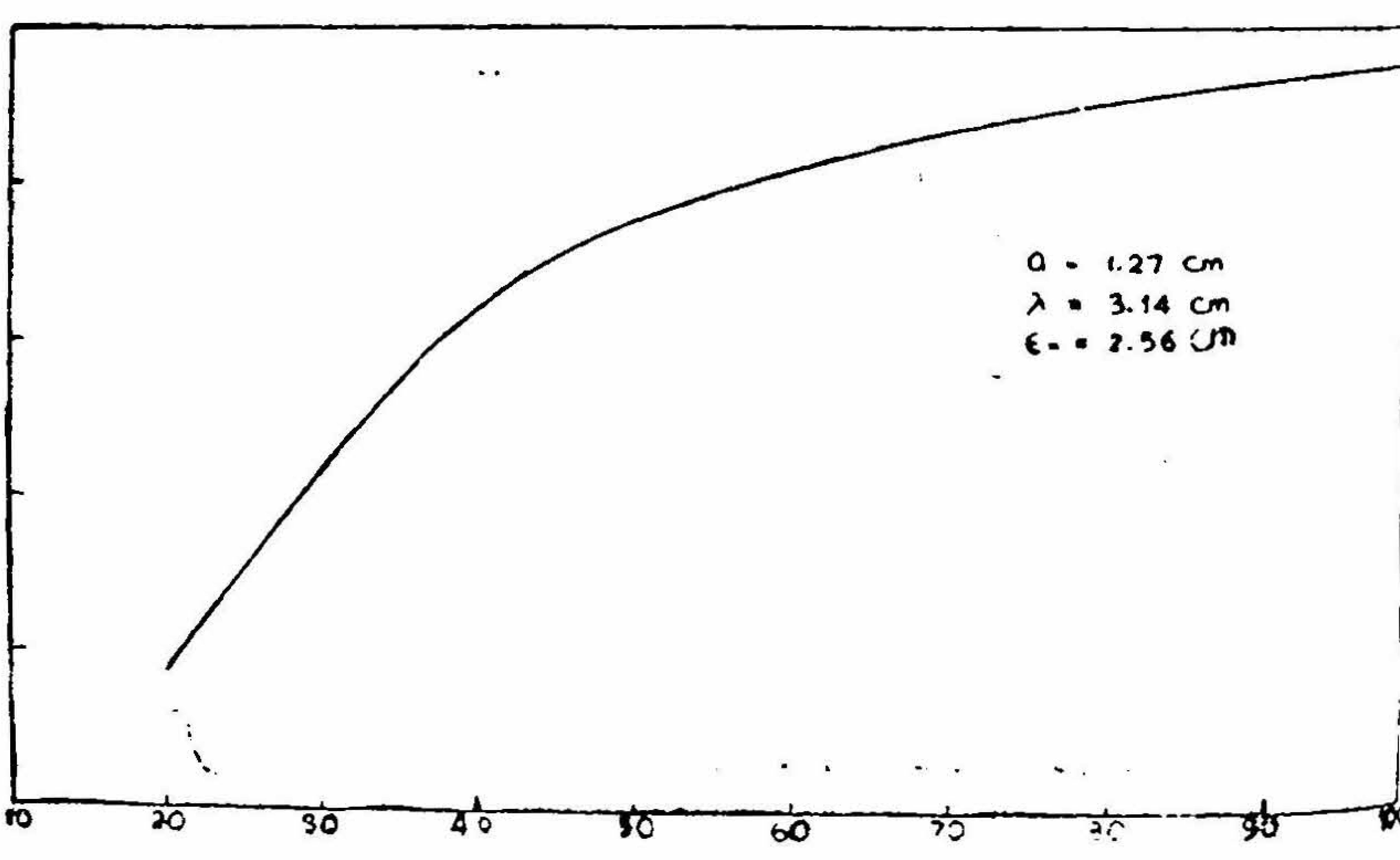

a = 1.27 cm
λ = 3.14 cm
10
20
30
40
50
60
90

FIG. 10

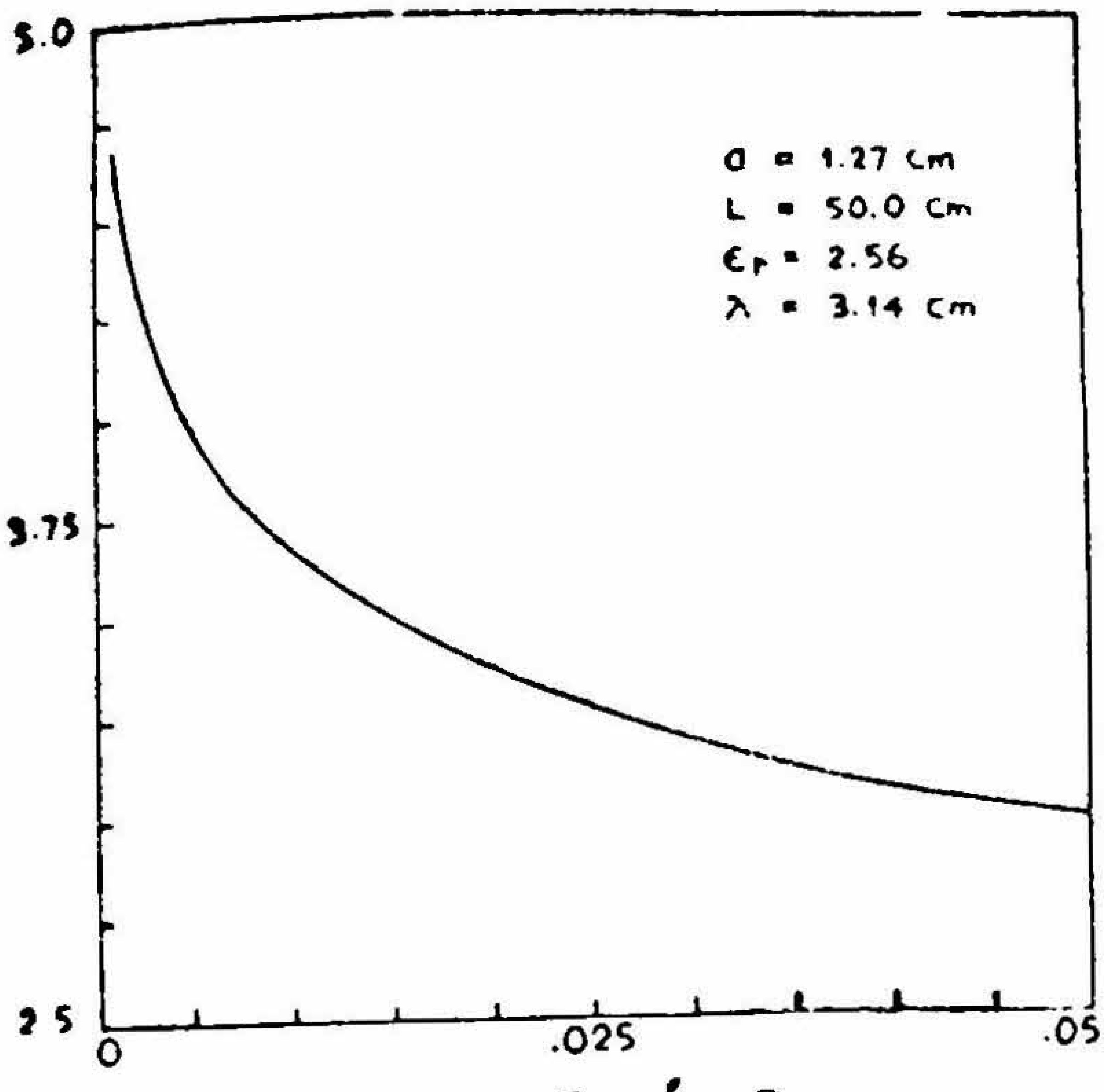


FIG. 11

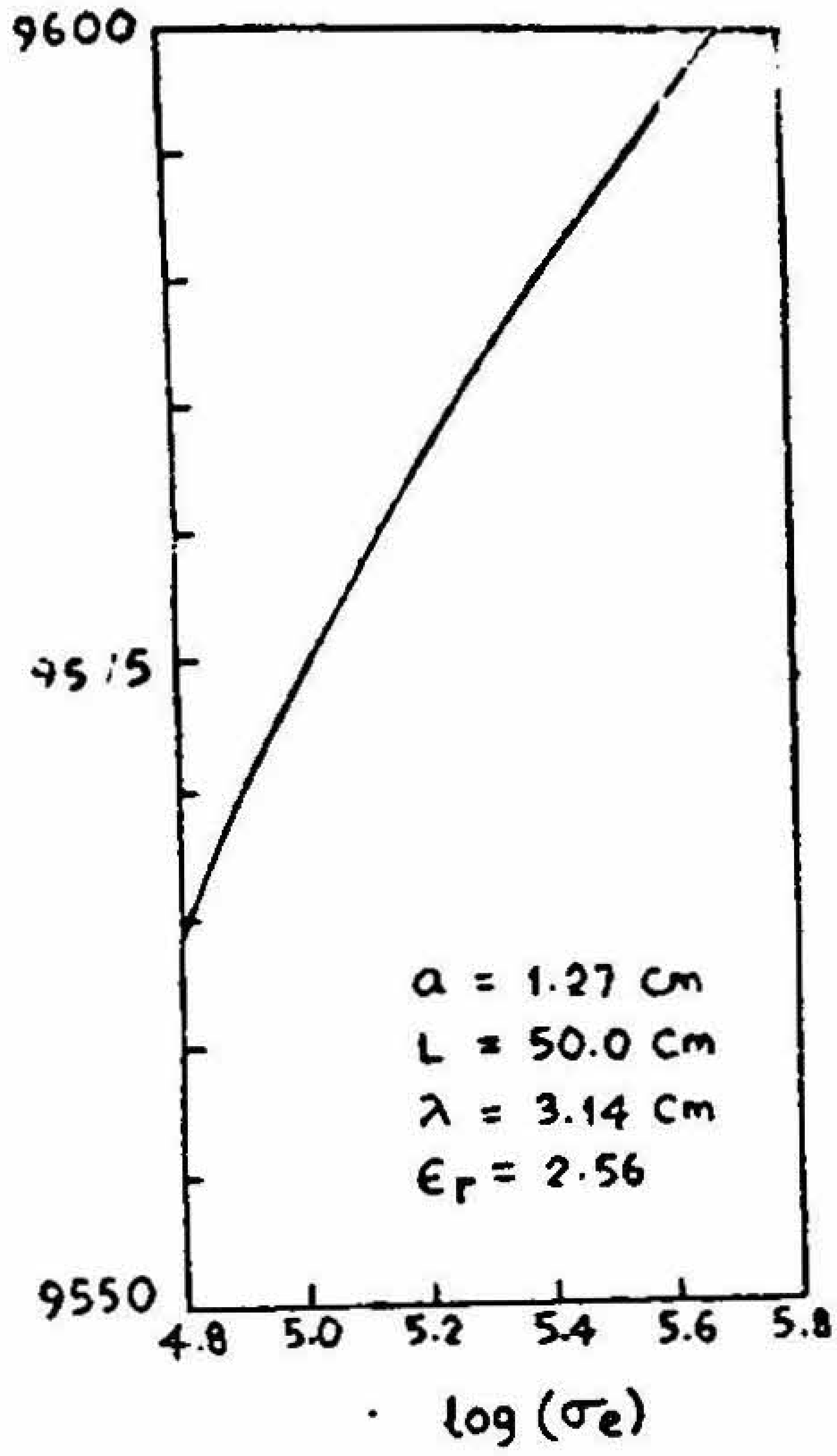


FIG. 12

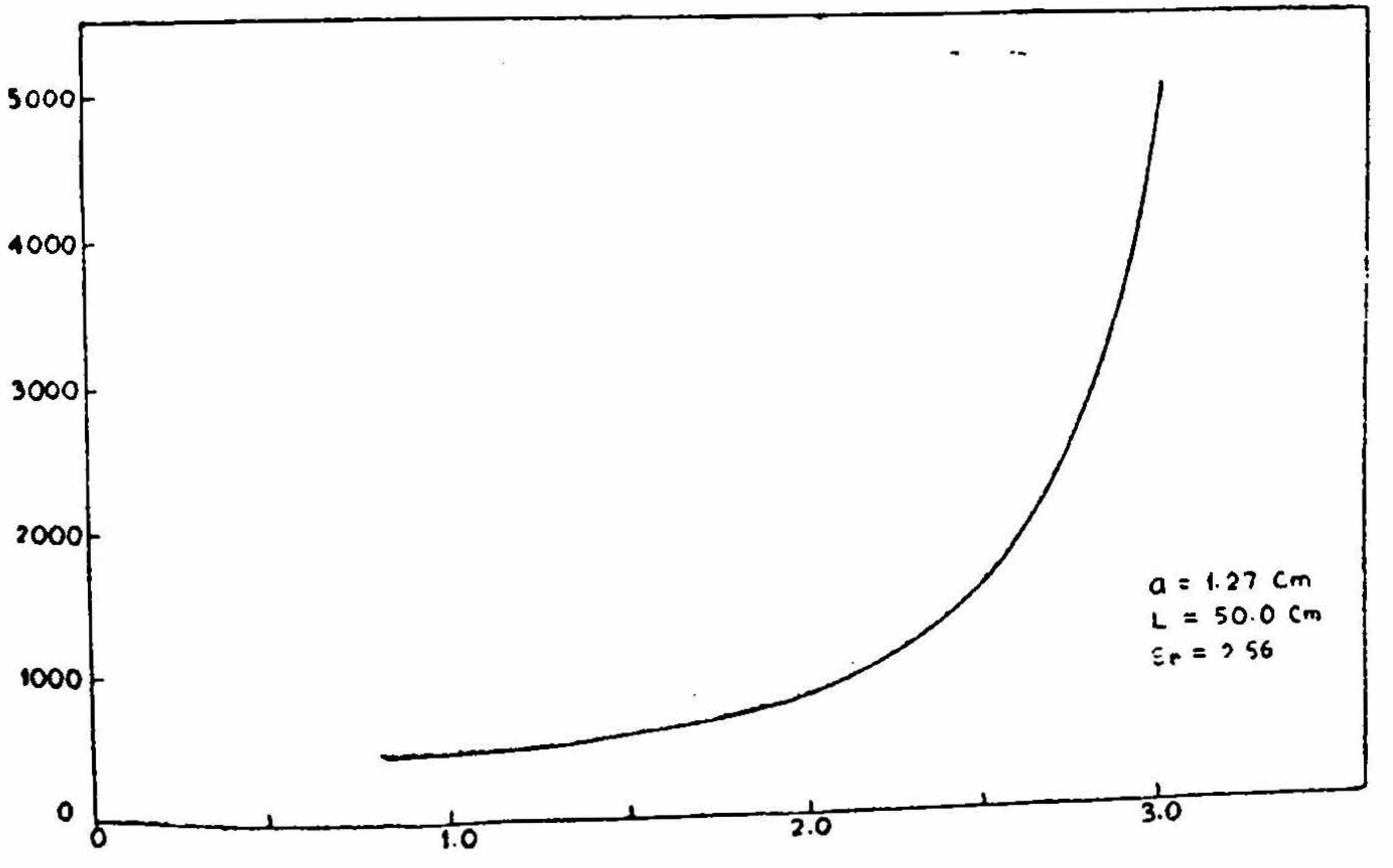


FIG. 13

where,

$P_{z1} = P_{z1}^{i} + P_{z1}^{0}$ contained within a radius $\rho = r_1$.

$P_{z2} = P_{z2}^{i} + P_{z1}^{0}$ contained within a radius $\rho = r_2$.

$P_{z3} = P_{z3}^{i} + P_{z3}^{0}$ contained within a radius $\rho = r_3$.

$\vdots$

$P_{zn} = P_{zn}^{i} + P_{zn}^{0}$ contained within a radius $\rho = r_n$.

P_{zn} represents the total power contained within a contour of radius r_n and $r_1 < r_2 < r_3 \cdots < r_n$. The values of P_{z1}, P_{z2}, $P_{z3} \cdots P_{zn}$, are determined from equation [46] for H E mode and for the E_0-mode by replacing the integrals

$$\int_{\rho=a}^{\infty} \text{ by } \int_{\rho=a}^{\rho=r_1}, \int_{\rho=a}^{\rho=r_2} \cdots \int_{\rho=a}^{\rho=r_n}$$

respectively.

20.1 *Power Contour for* H E *Mode*

The power flow in the radial, azimuthal and longitudinal direetions are

$$P_\rho = \frac{1}{2} \, Re \iint_S [\, E_\phi \, H_z^* - E_z \, H_\phi^* \,] \, \rho d\phi \, dz \tag{39a}$$

$$P_\phi = \frac{1}{2} \, Re \iint_S [\, E_z \, H_\rho^* - E_\rho \, H_z^* \,] \, d\rho \, dz \tag{39b}$$

$$P_z = \frac{1}{2} \, Re \iint_S [E_\rho \, H_\phi^* - E_\phi \, H_\rho^* \,] \, \rho \, d\rho \, d\phi \tag{39c}$$

The power launched in the dielectric rod will be transmitted in the longitudinal direction, where there is no radiation. But if some power is lost by radiation, the power will not only be transmitted in the longitudinal direction but also in the radial and azimuthal directions. In order to determine how much power is lost by radiation, the power flow P_ρ and P_ϕ in the ρ and ϕ directions respectively are also calculated.

The power flow inside p^i and P^0 in the three directions are

$$p_z^i = B\,B^*\Bigg[\frac{b}{B}\pi k_1\left(1-\frac{\gamma_1^2}{\omega^2\mu_0\epsilon_1}\right)\int_{\rho=0}^{a} J_0(k_1\rho)\,J_1(k_1\rho)\,d\rho$$

$$-\frac{\pi\gamma_1 k_1}{j\omega}\left(\frac{1}{\mu_0}+\frac{b^2}{B^2}\frac{1}{\epsilon_1}\right)\int_{\rho=0}^{a} J_0(k_1\rho)\,J_1(k_1\rho)\,d\rho$$

$$+\frac{\pi\gamma_1}{j\omega}\left(\frac{1}{\mu_0}+\frac{b^2}{B^2}\frac{1}{\epsilon_1}\right)\int_{\rho=0}^{a}\frac{1}{\rho}\{J_1(k_1\rho)\}^2\,d\rho$$

$$-\frac{b}{B}\pi\left(1-\frac{\gamma_1^2}{\omega^2\mu_0\epsilon_1}\right)\int_{\rho=0}^{a}\frac{1}{\rho}\{J_1(k_1\rho)\}^2\,d\rho$$

$$+\frac{\pi\gamma_1 k_1^2}{2j\omega}\left(\frac{1}{\mu_0}+\frac{b^2}{B^2}\frac{1}{\epsilon_1}\right)\int_{\rho=0}^{a}\rho\{J_0(k_1\rho)\}^2\,a\rho\Bigg] \qquad [40]$$

$$P_z^0 = C\,C^*\Bigg[\frac{c}{C}\pi k_2\left(1-\frac{\gamma_2^2}{\omega^2\mu_0\epsilon_0}\right)\int_{p=a}^{\infty} H_0^{(1)}(k_2\rho)\,H_1^{(1)}(k_2\rho)\,d\rho$$

$$-\frac{\pi\gamma_2 k_2}{j\omega}\left(\frac{1}{\mu_0}+\frac{c^2}{C^2}\frac{1}{\epsilon_0}\right)\int_{\rho=a}^{\infty} H_0^{(1)}(k_2\rho)\,H_1^{(1)}(k_2\rho)\,d\rho$$

$$+\frac{\pi\gamma_2}{j\omega}\left(\frac{1}{\mu_0}+\frac{c^2}{C^2}\frac{1}{\epsilon_0}\right)\int_{\rho=a}^{\infty}\frac{1}{\rho}[H_1^{(1)}(k_2\rho)]^2\,d\rho$$

$$-\frac{c}{C}\left(1-\frac{\gamma_2^2}{\omega^2\mu_0\epsilon_0}\right)\int_{\rho=a}^{\infty}\frac{1}{\rho}\{H_1^{(1)}(k_2\rho)\}^2\,d\rho$$

$$+\frac{\pi\gamma_2 k_2^2}{2j\omega}\left(\frac{1}{\mu_0}+\frac{c^2}{C^2}\frac{1}{\epsilon_0}\int_{\rho=a}^{\infty}\rho\{H_0^{(1)}(k_2\rho)\}^2\,d\rho \qquad [41]$$

$$p^i_\phi = j\frac{BB^*}{4}\left[\frac{b}{B}\frac{2l\beta_1 k_1^2}{\omega^2\mu_0\epsilon_1}\int_{\rho=0}^{a}\frac{1}{\rho}\{J_1(k_1\rho)\}^2 d\rho\right.$$

$$-\frac{b}{B}\frac{2l\beta_1 k_1^3}{\omega^2\mu_0\epsilon_1}\int_{\rho=0}^{a} J_0(k_1\rho)\,J_1(k_1\rho)\,d\rho$$

$$\left.-\frac{lk_1^2}{\omega}\left(\frac{1}{\mu_0}+\frac{b^2}{B^2}\frac{1}{B^2}\right)\int_{\rho=0}^{a}\frac{1}{\rho}\{J_1(k_1\rho)\}^2 d\rho\right] \quad [42]$$

$$P^0_\phi = j\frac{CC^*}{4}\left[\frac{c}{C}\frac{2l\beta_2 k_2^2}{\omega^2\mu_0\epsilon_0}\int_{\rho=a}^{\infty}\frac{1}{\rho}\{H_1^{(1)}(k_2\rho)\}^2 d\rho\right.$$

$$-\frac{c}{C}\frac{2l\beta_2 k_2^3}{\omega^2\mu_0\epsilon_0}\int_{\rho=a}^{\infty} H_1^{(1)}(k_2\rho)\,H_1^{(1)}(k_2\rho)\,d\rho$$

$$\left.-\frac{lk_2^2}{\omega}\left(\frac{1}{\mu_0}+\frac{c^2}{C^2}\frac{1}{\epsilon_0}\right)\int_{\rho=a}^{\infty}\frac{1}{\rho}\{H_1^{(1)}(k_2\rho)\}^2 d\rho\right] \quad [43]$$

In calculating p^i_ϕ and P^0_ϕ, the limit of z has been taken from $z=0$ to $z=l$ where 'l' is the length of the dielectric rod p^i_ϕ and P^0_ϕ has been evaluated for $\phi=\pi/4$ as $P_\phi=0$ for $\phi=0$, $\pi/2$, π, $3\pi/2$, and 2π.

$$p^i = j\,BB^*\left[\frac{\pi}{2}\frac{l\rho k_1^3}{\omega}\left(\frac{1}{\mu_0}-\frac{b^2}{B^2}\frac{1}{\epsilon_1}\right)J_0(k_1\rho)\,J_1(k_1\rho)\right.$$

$$\left.-\frac{\pi}{2}\frac{lk_1^2}{\omega}\left(\frac{1}{\mu_0}-\frac{b^2}{B^2}\frac{1}{\epsilon_1}\right)\{J_1(k_1\rho)\}^2\right] \quad [44]$$

$$P^0_\phi = j\,CC^*\left[\frac{\pi}{2}\frac{l\rho k_2^3}{\omega}\left(\frac{1}{\mu_0}-\frac{c^2}{C^2}\frac{1}{\epsilon_0}\right)H_0^{(1)}(k_2\rho)\,H_1^{(1)}(k_2\rho)\right.$$

$$\left.-\frac{\pi}{2}\frac{lk_2^2}{\omega}\left(\frac{1}{\mu_0}-\frac{c^2}{C^2}\frac{1}{\epsilon_0}\right)\{H_1^{(1)}(k_2\rho)\}^2\right] \quad [45]$$

In calculating p^l and P^0 the limit of ϕ has been taken from $\phi=0$ to $\phi=2\pi$ and the limit of z is from $z=0$ to $z=l$. p^l and P^0 have been evaluated for

$\rho = a$, The axial propagation constant $\gamma = j\beta$, and $(\beta_1 = \beta_2 = \beta)$ has been obtained from the solution of the characteristic equation which gives k_1 and k_2 and from equation relating β to the radial eonstants k.

The power flow p_z^i and P_z^0 for different values of $2a/\lambda$. have been ealculated for perspex rod ($\epsilon_{r1} = 2.56$) and the results are tabulated below.

TABLE 1

Power flow inside and outside the rod in the z-direction

$2a/\lambda_0$	P_z^i (Watts/Sq. em.)	P_z^0 (Watts/Sq. cm.)	$P_z = p_z^i + P_z^0$ (Watts/Sq. cm.)
0.8	$0.01363 \; \lvert B \rvert^2$	$0.0^{2}1308 \; \lvert B \rvert^2$	$0.0149 \; \lvert B \rvert^2$
0.6	$0.01349 \; \lvert B \rvert^2$	$0.01698 \; \lvert B \rvert^2$	$0.0305 \; \lvert B \rvert^2$
0.4	$0.0^{2}9965 \; \lvert B \rvert^2$	$0.0448 \; \lvert B \rvert^2$	$0.0548 \; \lvert B \rvert^2$
0.3	$0.0^{2}7806 \; \lvert B \rvert^2$	$0.0490 \; \lvert B \rvert^2$	$0.0569 \; \lvert B \rvert^2$

TABLE 2

Power flow inside and outside the rod in the radial (ρ) direction

$2a/\lambda_0$	p^i Watts/Sq. cm.	P^0 Watts/Sq. cm.	$P = p^i + P^0$ Watts per sq. cm.
0.8	$j\ 0.0^{4}9086\ 1 \lvert B \rvert^2$	$j\ 0.0^{3}6389\ 1 \lvert B \rvert^2$	$j\ 0.0^{3}7298\ 1 \lvert B \rvert^2$
0.6	$-j\ 0.0^{4}6965\ 1 \lvert B \rvert^2$	$j\ 0.0^{3}2418\ 1 \lvert B \rvert^2$	$j\ 0.0^{3}1722\ 1 \lvert B \rvert^2$
0.4	$-j\ 0.0^{2}2031\ 1 \lvert B \rvert^2$	$j\ 0.0^{2}1387\ 1 \lvert B \rvert^2$	$-j\ 0.0^{3}6440\ 1 \lvert B \rvert^2$
0.3	$-j\ 0.0^{2}3466\ 1 \lvert B \rvert^2$	$-j\ 0.02500\ 1 \lvert B \rvert^2$	$-j\ 0.02847\ 1 \lvert B \rvert^2$

TABLE 3

Power flower inside and outside the guide in the azimuthal direction

$2a/\lambda_0$	$p^i{}_\phi$ Watts/cm²	$P^0{}_\phi$ Watts/cm²	$P_\phi = p^i{}_\phi + P^0{}_\phi$ Watts/cm²
0.8	$-j\ 0.0^{2}4620\ 1 \lvert B \rvert^2$	$-j\ 0.0^{3}7518\ 1 \lvert B \rvert^2$	$-j\ 0.0^{2}5371\ 1 \lvert B \rvert^2$
0.6	$-j\ 0.0^{2}7099\ 1 \lvert B \rvert^2$	$-j\ 0.0^{2}2197\ 1 \lvert B \rvert^2$	$-j\ 0.0^{2}9296\ 1 \lvert B \rvert^2$
0.4	$-j\ 0.0^{2}8581\ 1 \lvert B \rvert^2$	$-j\ 0.02134\ 1 \lvert B \rvert^2$	$-j\ 0.0299\ 1 \lvert B \rvert^2$
0.3	$-j\ 0.0^{2}6279\ 1 \lvert B \rvert^2$	$-j\ 0.0107\ 1 \lvert B \rvert^2$	$-j\ 0.0169\ 1 \lvert B \rvert^2$

TABLE 4

Values of k_1, k_2 and for the rod excited in HE_{11} mode=3.2 cms.

$2a/\lambda_0$	k_1	$-jk_2$	β
0.30	2.46	0 43	2.00
0.40	2.42	0.54	2.04
0.50	2.15	1.20	2.32
0.60	1.92	1.58	2.52
0.70	1.71	1.80	2.66
0.80	1.56	1.93	2.75
0 90	1.41	2.04	2.83
1.00	1.28	2.12	2.89
1.50	0.90	2.31	3.04
2.00	0.69	2.38	3.08

The power flow P_ρ and P_ϕ are reactive and hence the relative power flowing inside the guide is expressed as a percentage as follows

$$W = \frac{p_z^i}{p_z^i + P_z^0} = \frac{p_z^i / P_z^0}{1 + p_z^i / P_z^0} \; 100 \qquad [46]$$

The rusults of computation of p_z^i / P_z as a function of $2a/\lambda_0$ for $\epsilon_{r1} = 2\ 56$ and as a function of $2a/\lambda_0 = 0.8$ at $\lambda = 3.2$ cm. for the HE_{11} mode are shown in Figures 14 and 15, respectively. Figure 16 represents graphically the relation between $W\%$ and $\rho = r_1, r_2, \ldots r_n$ for different values of $(2a/\lambda_0)$ and $\epsilon_{rs} = 2.56$. From Fig. 17, the radii of the constant percentage power contours for different values of $(2a/\lambda_0)$ are ploted.

20.2 *Power contour for E_0 mode*

The power flow inside and outside the dieletric rod excited in E_0 mode has been calculated following the above method,

$$p_z^i = \frac{\pi B^2 \beta a^2 k_1^2}{2\,\omega\epsilon_0\,\epsilon_{r1}} \left([J_0(k_1 a)]^2 + [J_1(k_1 a)]^2 - \frac{2\,J_0(k_1 a)\,J_1(k_1 a)}{k_1 a} \right) \qquad [47]$$

$$P_z^0 = Re\,\frac{\pi D^2 k_2^2 \beta}{2\,\omega\epsilon_0} \left(d^2 \{ [H_0^{(1)}(k_2 d)]^2 + [H_1^{(1)}(k_2 d)]^2 - \frac{2H_0^{(1)}(k_2 d)\,H_1^{(1)}(k_2 d)}{k_2 d} \right)$$

$$- a^2 \left([H_0^{(1)}(k_2 a)]^2 + [H_1^{(1)}(k_2 a)]^2 - \frac{2H_0^{(1)}(k_2 a)\,H_1^{(1)}(k^2 a)}{k_2 a} \right)$$

where B and D are related by the following relation

$$D=\frac{B}{\epsilon_{r1}}\left(\frac{k_1}{k_2}\right)^2\left[\frac{J_0(k_1a)}{H_0^{(1)}(k_2a)}\right]$$

which is obtained by using the appropriate field components and boundary condition. The total power flow is therefore,

$$P=\frac{\pi B^2\beta k_1^2}{2\omega\epsilon_0\epsilon_{r1}}\left[a^2\left([J_0(k_1a)]^2+[J_1(k_1a)]^2-\frac{2J_0(k_1a)J_1(k_1a)}{k_1a}\right)\right.$$
$$+\left\{\frac{1}{\epsilon_{r1}}\left(\frac{k_1^2}{k_2^2}\right)\frac{J_0(k_1a)}{H_0^{(1)}(k_2a)}\right\}d^2\left([H_0^{(1)}(k_2d)]^2+[H_1^{(1)}(k_2d)]^2\right.$$
$$\left.-\frac{2H_0^{(1)}(k_2d)H_1^{(1)}(k_2d)}{k_2d}\right)-a^2\left([H_0^{(1)}(k_2a)]^2+[H_1^{(1)}(k_2a)]^2\right.$$
$$\left.\left.-\frac{2H_0^{(1)}(k_2a)H_1^{(1)}(k_2a)}{k_2a}\right)\right]$$

Figures 18 and 19 respectively represent $W\%$ versus ρ and constant percentage power contours.

21. Conclusions

The following interesting points emerge as a result of the above theoretical investigations.

(i) For the same value of $2a/\lambda_0$, and ϵ_{r1}, more power is concentrated near the rod in the case of HE mode compared to the E_0 mode excitation.

(ii) The power flow inside the rod for HE mode excitation is more compared to that in the case of E_0 mode (12) excitation.

(iii) For the same value of L, $2a/\lambda_0$, ϵ_{r1}, σe and $\tan\delta$ the Q factor of the resonator is greater in the case of E_0-mode than in the case of HE mode excitation. This is justified due to the fact that in the case of HE mode though the loss in the end plates due to finite σe is less than E_0 mode due to less radial field spread, but the loss due to $\tan\delta$ in the case of HE-mode is comparatively much more than in the case of E_0-mode due to higher concentration of power inside the rod.

(iv) The nature of variation of the Q-factor with wavelength of excitation is practically the same in both the cases of excitation.

It may therefore be concluded that a dielectric rod excited in HE mode will act as a better surface wave guide than when excited in E_0 mode provided the dielectric rod is made of material having low value of loss tangent.

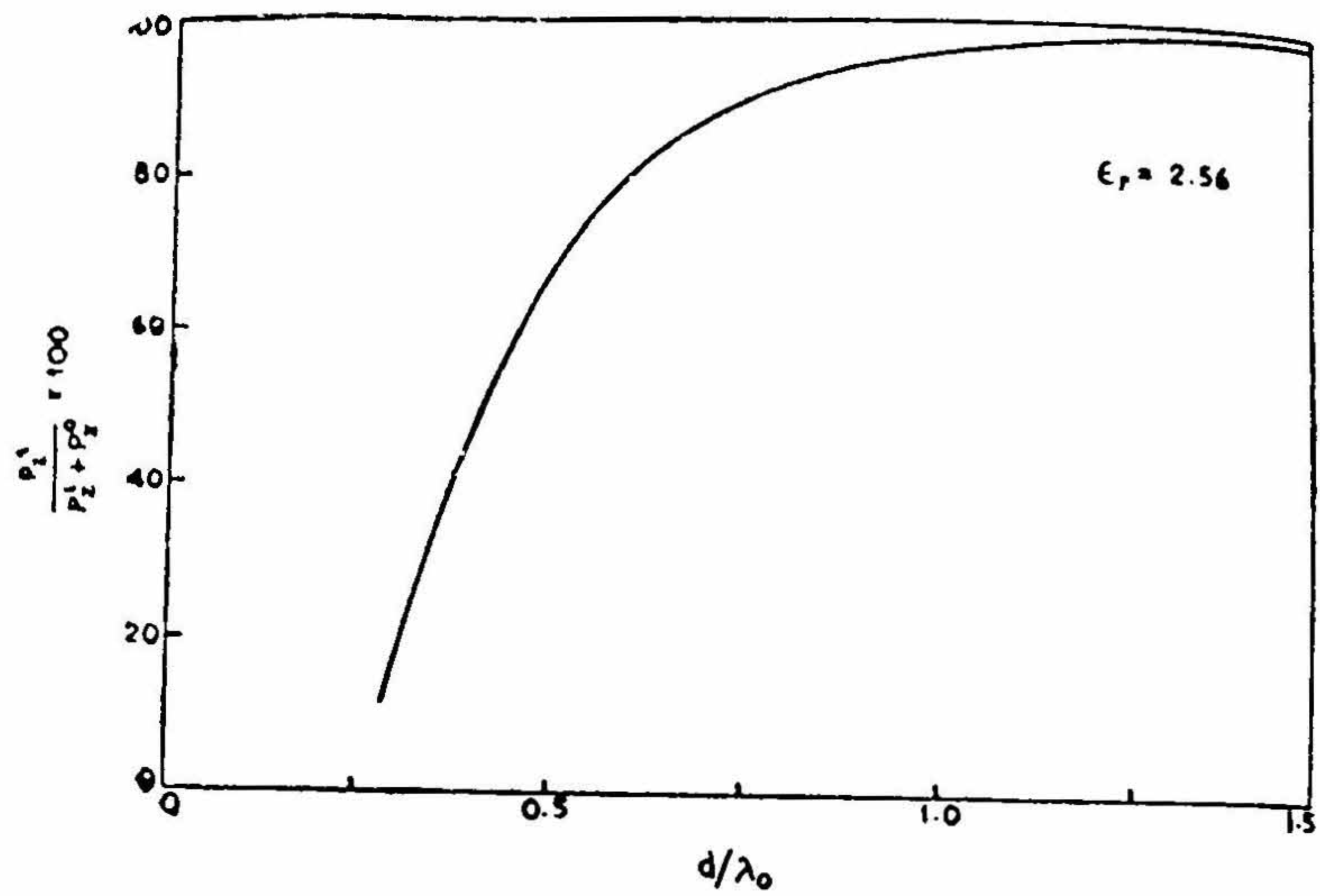

100
80
60
40
20
0
$\frac{P_z^1}{P_z^1 + P_z^0} \times 100$
$\epsilon_r = 2.56$
0
0.5
1.0
1.5
d/λ_0

FIG. 14

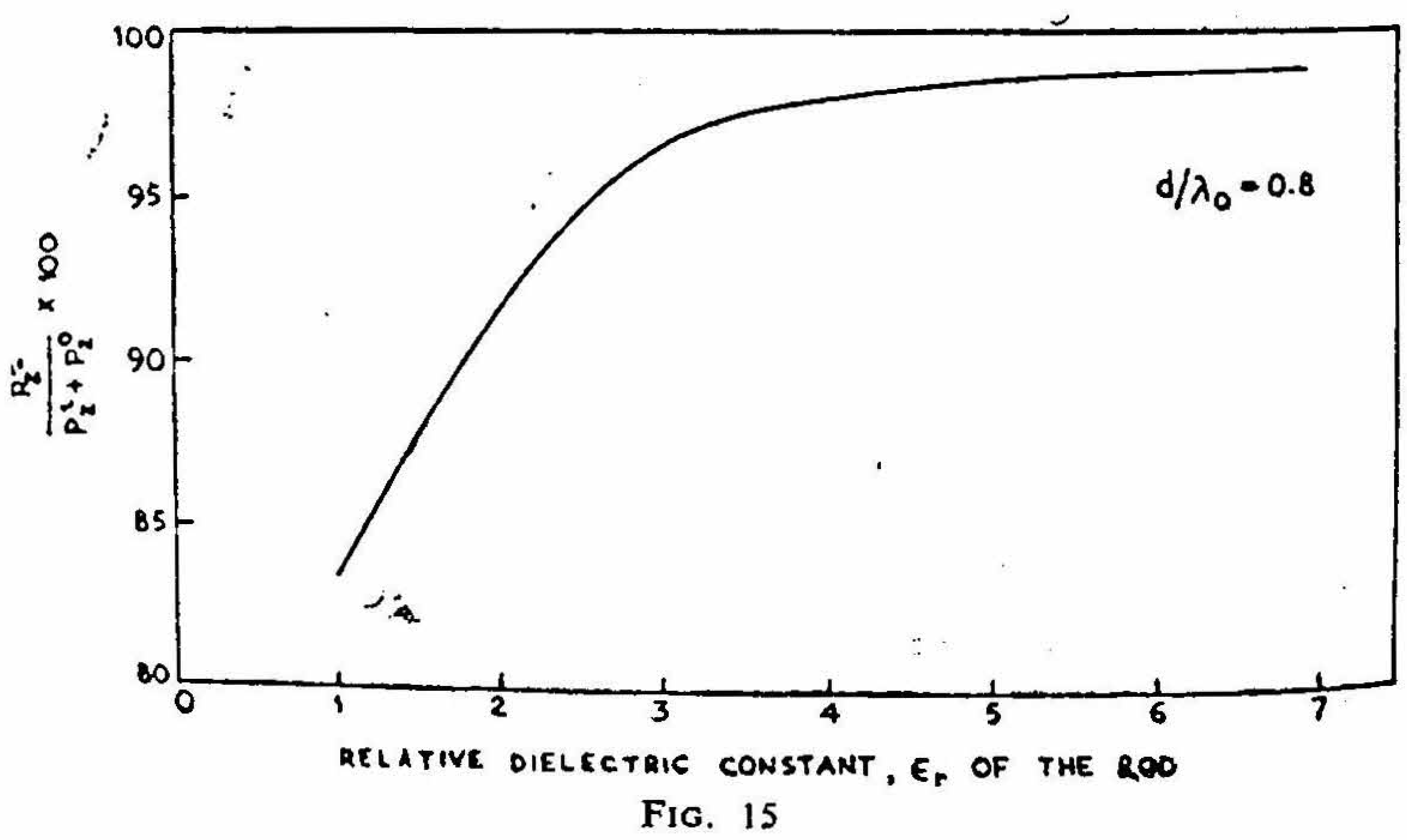

100
95
90
85
80
$\frac{P_z^-}{P_z^1 + P_z^0} \times 100$
$d/\lambda_0 = 0.8$
0
1
2
3
4
5
6
7
RELATIVE DIELECTRIC CONSTANT, ϵ_r OF THE ROD

FIG. 15

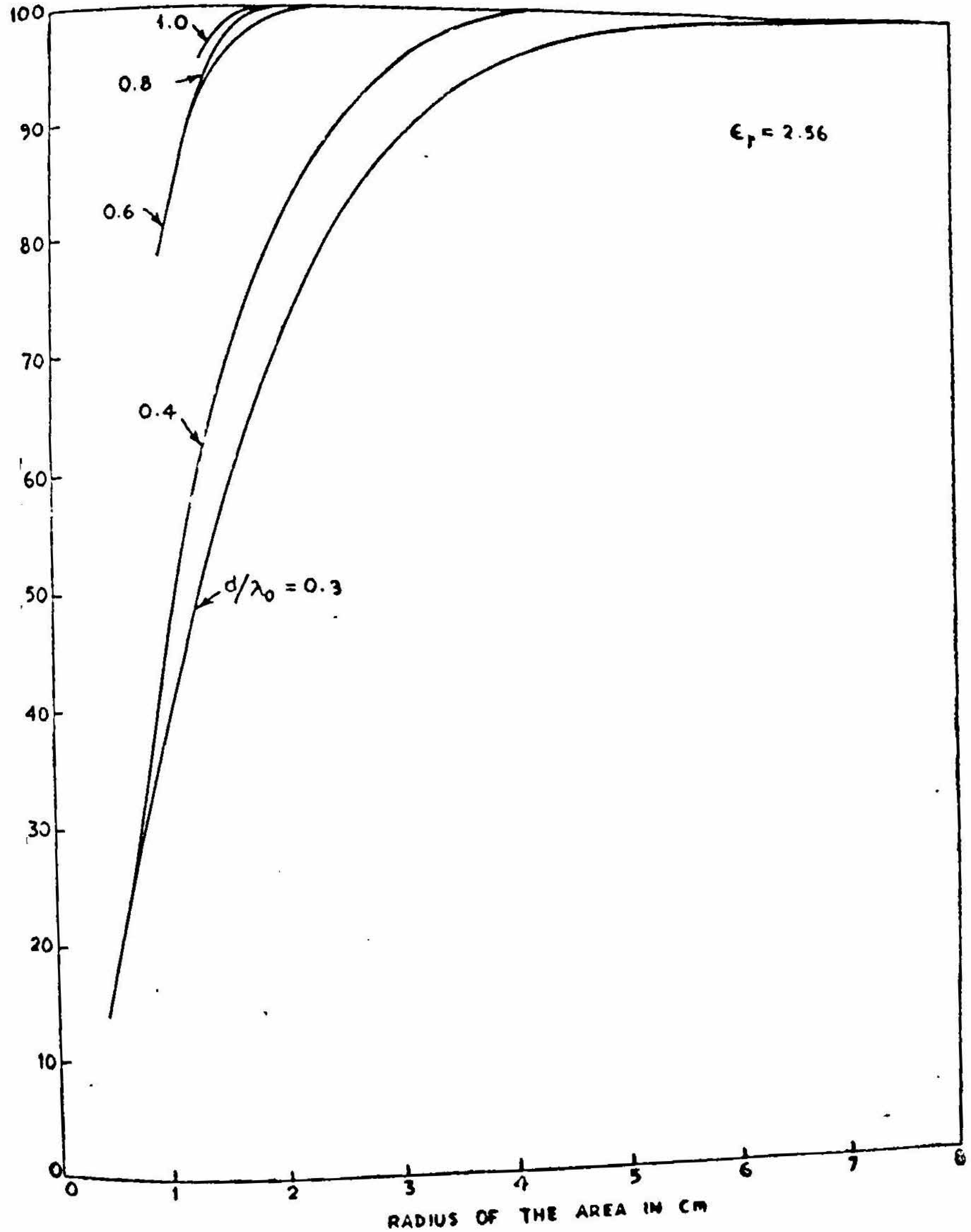

100
90
80
70
60
50
40
30
20
10
0
1.0
0.8
0.6
0.4
d/λ₀ = 0.3
ε_r = 2.56
0
1
2
3
4
5
6
7
8
RADIUS OF THE AREA IN CM

FIG. 16

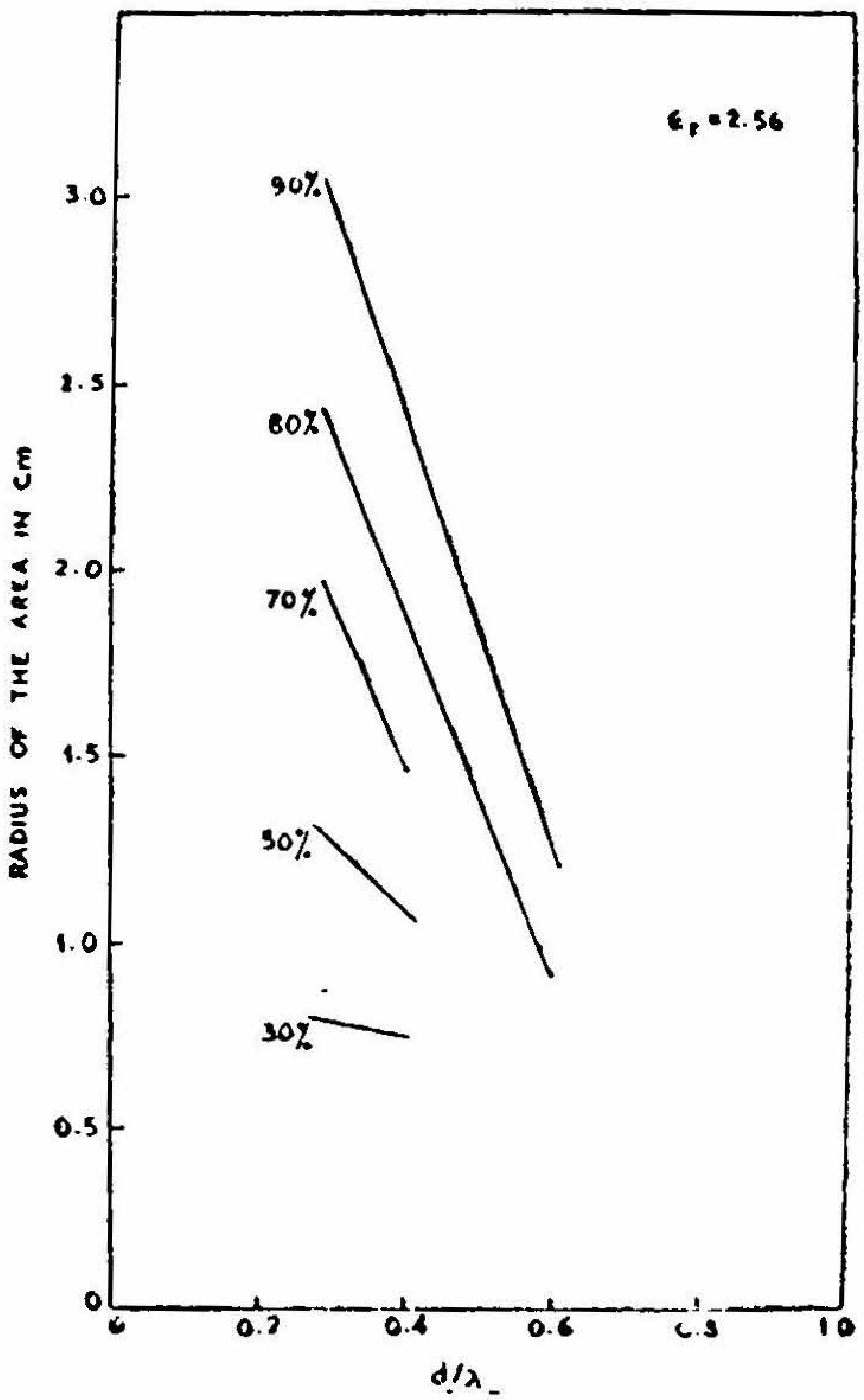

ε_r = 2.56
90%
80%
70%
50%
30%
3.0
2.5
2.0
1.5
1.0
0.5
0
0.2
0.4
0.6
0.8
1.0
RADIUS OF THE AREA IN Cm
d/λ

FIG. 17

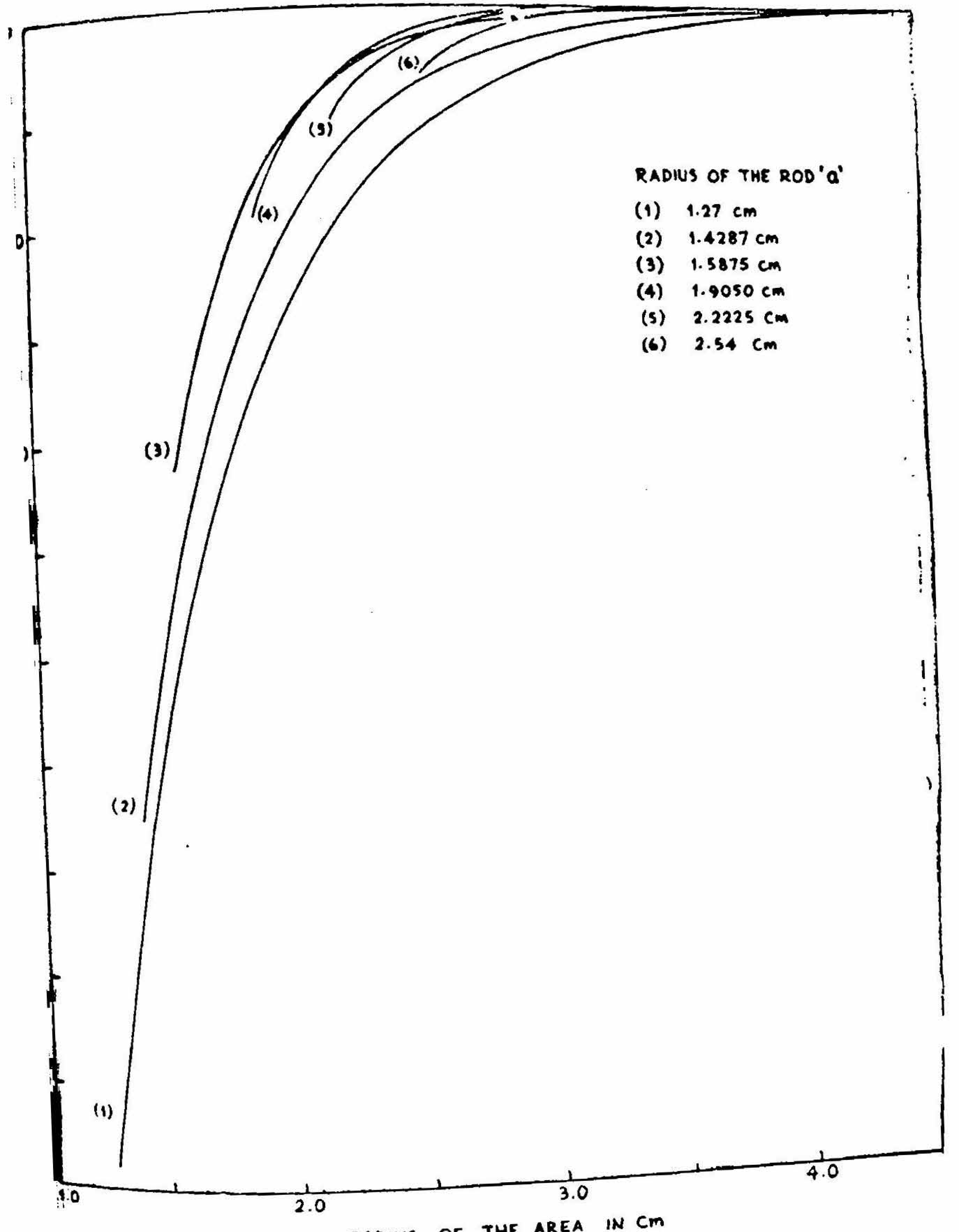
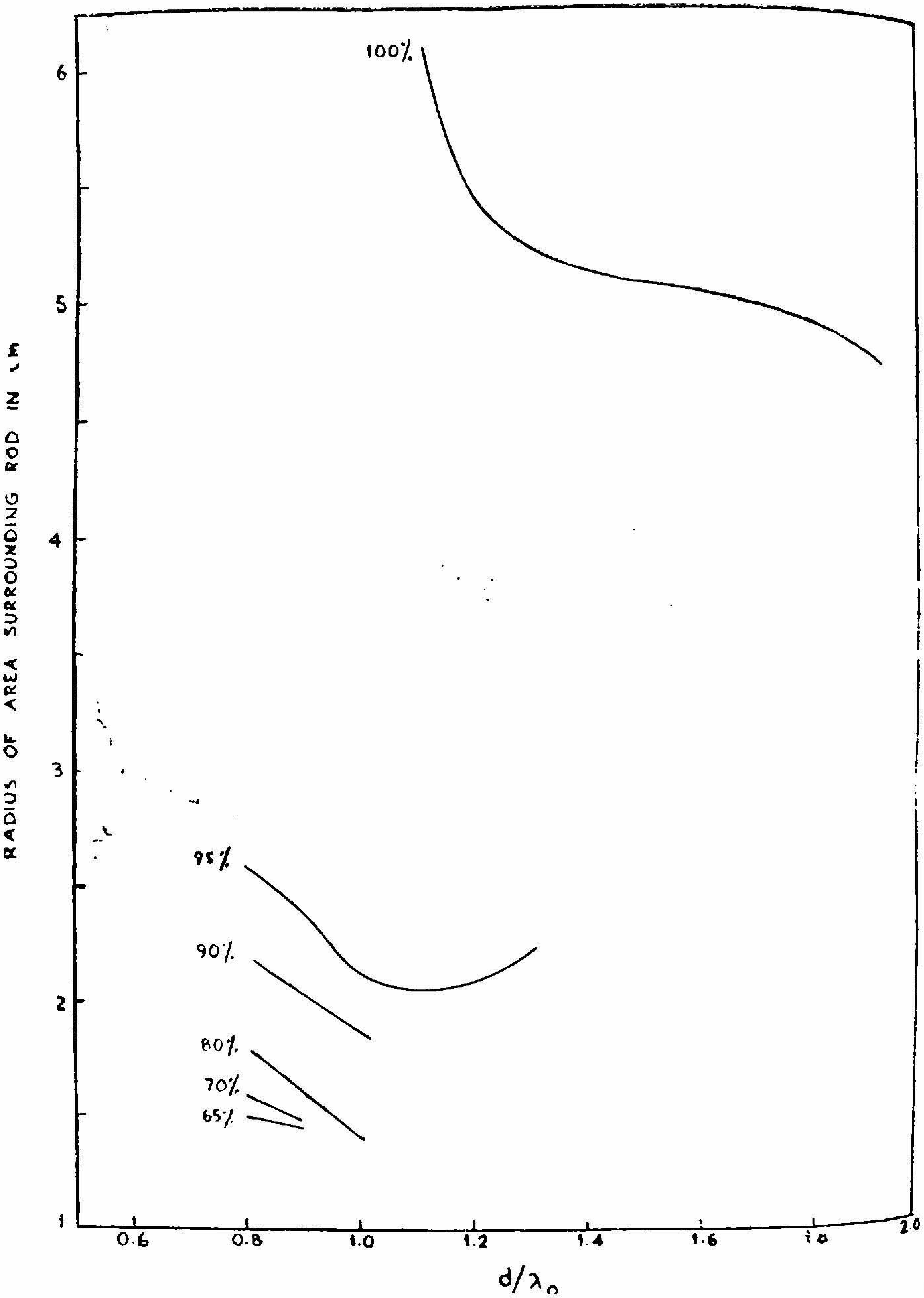

RADIUS OF THE ROD 'a'
(1) 1.27 cm
(2) 1.4287 cm
(3) 1.5875 cm
(4) 1.9050 cm
(5) 2.2225 cm
(6) 2.54 cm
(1)
(2)
(3)
(4)
(5)
(6)
1.0
2.0
3.0
4.0
RADIUS OF THE AREA IN Cm

FIG. 18



RADIUS OF AREA SURROUNDING ROD IN cm
6
5
4
3
2
1
100%
95%
90%
80%
70%
65%
0.6
0.8
1.0
1.2
1.4
1.6
2.0
d/λ_0

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