VIBRATION OF TRAPEZOIDAL CANTILEVER PLATES WITH PARTIAL ROOT CHORD SUPPORT*

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ABSTRACT

The vibration characteristics of trapezoidal cantilever plates with partial root chord support, typical of some fins used on rockets and missiles, are presented. Natural frequencies and modes are calculated by lumped mass method using measured flexibility influence coefficients. Also the frequencies and nodal patterns are obtained by experiment. The variation of frequencies and nodal patterns with varying partial root chord support is studied.

Notation:

a	..	leading edge length
D	..	flexural rigidity of plate, $= Eh^3 / 12 (1-\nu^2)$
$[D]$	..	Dynamical Matrix $= [F] [M]$
E	..	Young's modulus of the material of the plate
$[F]$	..	Flexibility matrix
h	..	Thickness of plate
$[M]$	..	Mass matrix, diagonal in the present case
ρ	..	Mass density
ω	..	Circular frequency
ν	..	Poisson's ratio

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The fins and control surfaces of rockets and missiles can be considered structurally as cantilever plates[1,2]. More often than not, rocket fins and control surfaces are trapezoidal in shape[3,4]. The fins and control surfaces are attached to the missile body in various ways. These vary from one of clamping all along the root chord to clamping over only a part of the root chord[5,6] (herein-after referred to as partial root chord support) for fins, and hub or hub-pin support[7,8] for control surfaces (See Fig. 1). The study of vibration and flutter characteristics of cantilever plates with partial root chord support is, therefore, of considerable practical significance.

In this note, the variation of the frequencies and nodal patterns with different configurations of partial root chord support is studied in the case of a trapezoidal geometry which is quite typical[9].

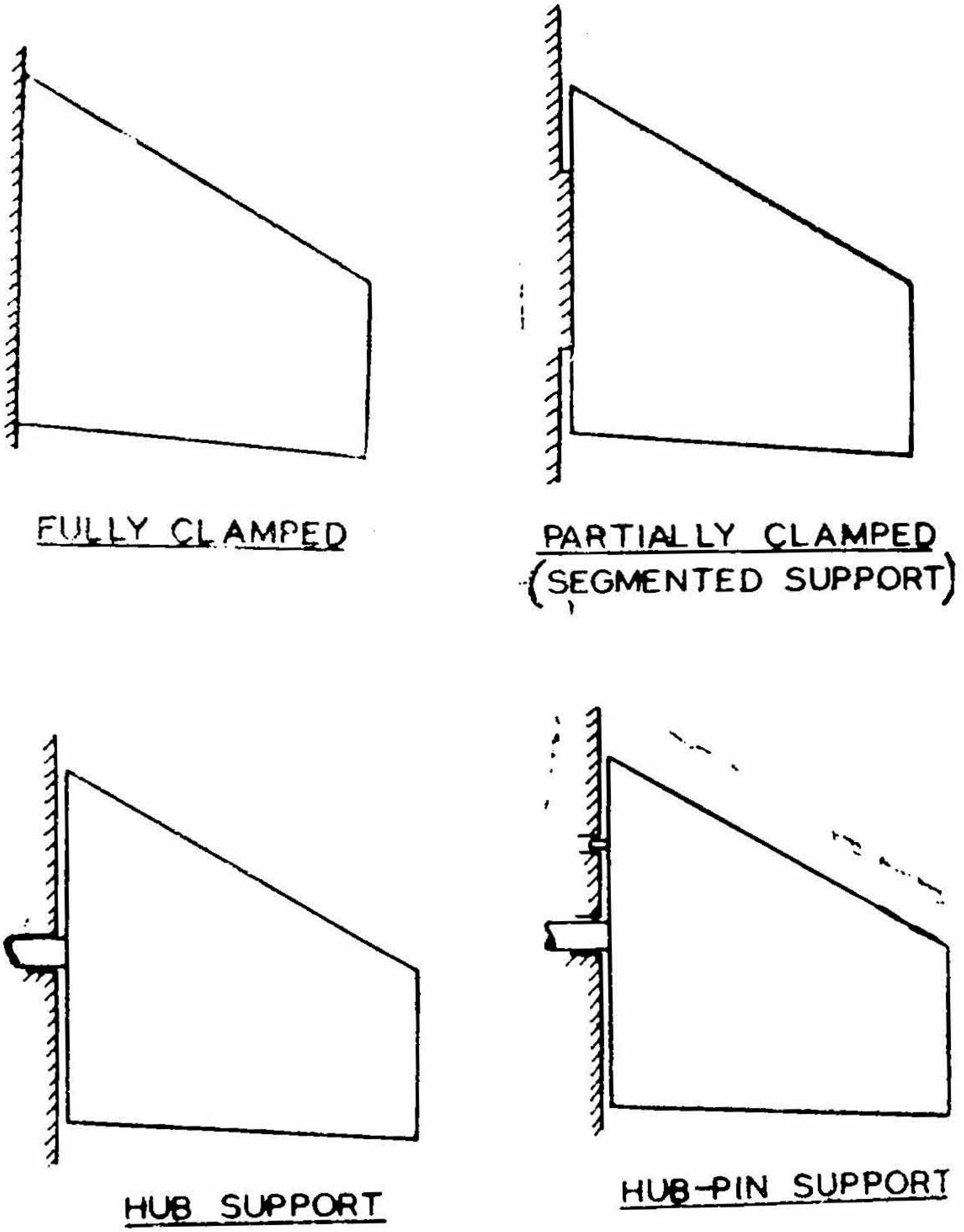
A drawing of the test plate along with the grid geometry is shown in Fig. 2. Partial root chord support was achieved by making a cut along the root chord from the leading and / or trailing edge side. The details of the different configurations so obtained are given below :

(1) Fully clamped at root[9]
(2) 25% root chord free from leading edge (LE)
(3) 50% root chord free from LE
(4) 25% root chord free from trailing edge (TE)
(5) 50% root chord free from TE
(6) 25% root chord free from LE and
25% root chord free from TE
(7) 50% root chord free from LE and
25% root chord free from TE.

Natural frequencies and nodal patterns of the different configurations of the test plates were determined by resonance tests. The natural frequencies were located by the well known "Quadrature Response" criterion[13]. The natural frequencies in *Hz.* and the nodal patterns are given in Figs. 3-9. The frequency values ω quoted in *Hz.* may be converted to non-dimensional form $\lambda = \sqrt{\frac{\rho h}{D}} \; \frac{\omega a^2}{\pi^2}$ to facilitate their use in design for other configurations of similar geometry and boundary conditions, by using the properties of the test plate. The nodal patterns were obtained as usual by sprinkling fine grains of dry sand.

For calculation of frequencies and mode shapes, the mass matrix $[M]$ was obtained by lumping the mass of appropriate areas at the grid points and the flexibility matrix $[F]$ by measuring the influence coefficients. The eigen values and eigenvectors of the dynamical matrix $[D] = [F][M]$ were obtained using a library routine based on the QR – Transformation method[12]. The frequencies and some of the nodal patterns obtained from this calculation are given in Figs. 3 to 9.

The agreement between the calculated and measured frequencies may be considered to be satisfactory on the whole. The discrepancies are believed to be mainly due to discretization to only 15 grid points.



FULLY CLAMPED

PARTIALLY CLAMPED (SEGMENTED SUPPORT)

HUB SUPPORT

HUB-PIN SUPPORT

FIG. 1

Method of Root Support for Fins and Control Surfaces

The effect of partial root chord support on frequencies and nodal patterns is quite interesting. The influence of a 25% cut at root from the leading edge side (Configuration 2) is found to be negligible except in the case of higher frequencies. It may also be noted that the frequency values decrease by increasing the cut from 25% to 50% from leading edge side. (See Figs. 4 and 5). The influence of a cut from the trailing edge side appears to be far more significant in lowering the frequencies. For example, the frequencies of Configuration 4, with 25% cut from the trailing edge side are

$a = 8\sqrt{3}$ in.
$h = 1/16$ in.

Material : MILD STEEL
$E = 30 \times 10^6$ lbs./in.^{2}
$\rho = 0.000725$ lb. sec.^{2} / in.^{4}

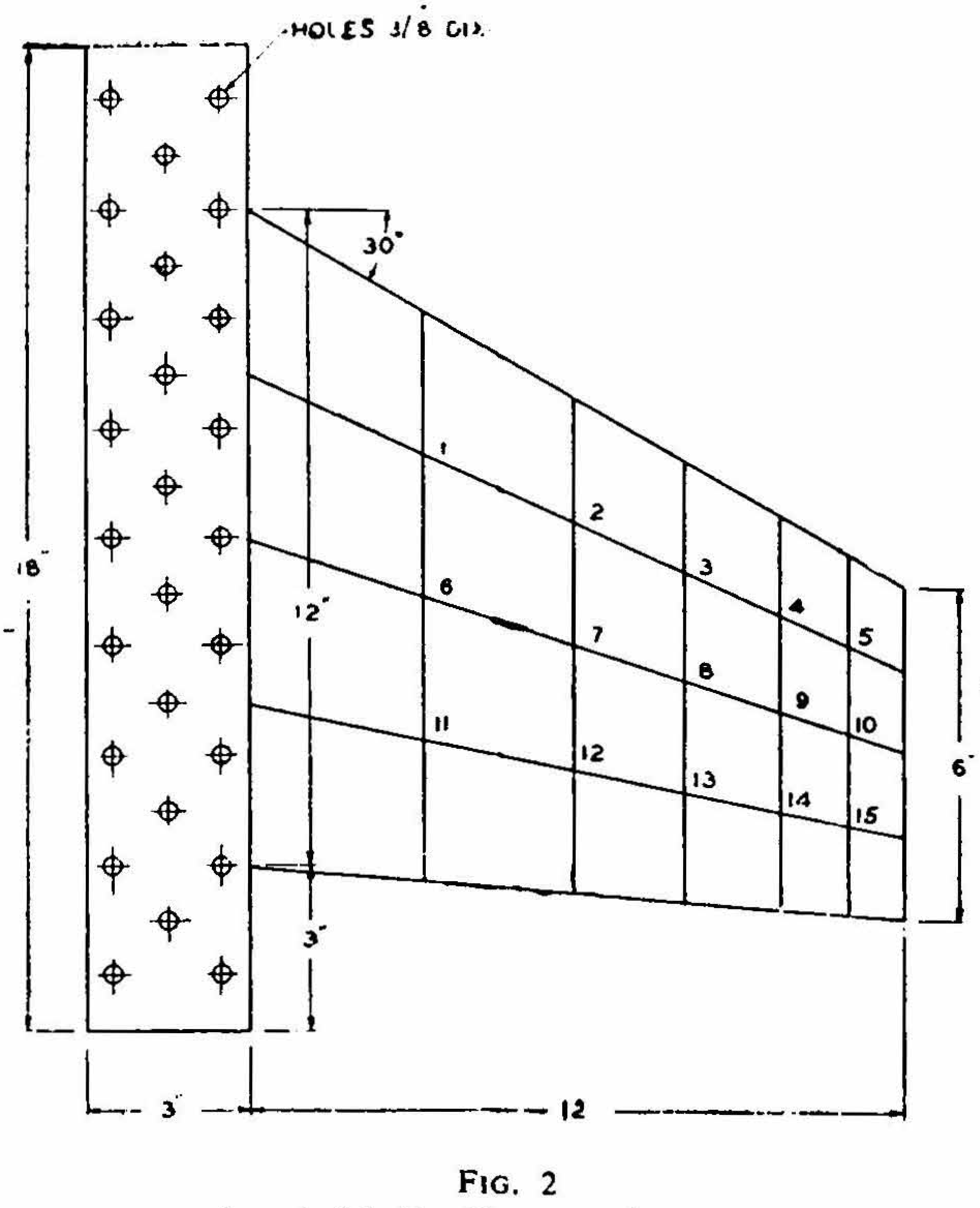


FIG. 2
Layout of the Test-Plate and Grid Geometry

lower than the frequencies of Configurations 1 and 2. The decrease in frequencies is very pronounced when the cut from the trailing edge side is increased from 25% to 50%.

FUNDAMENTAL :

E. 18 *Hz.* *L.* 15 *Hz.*

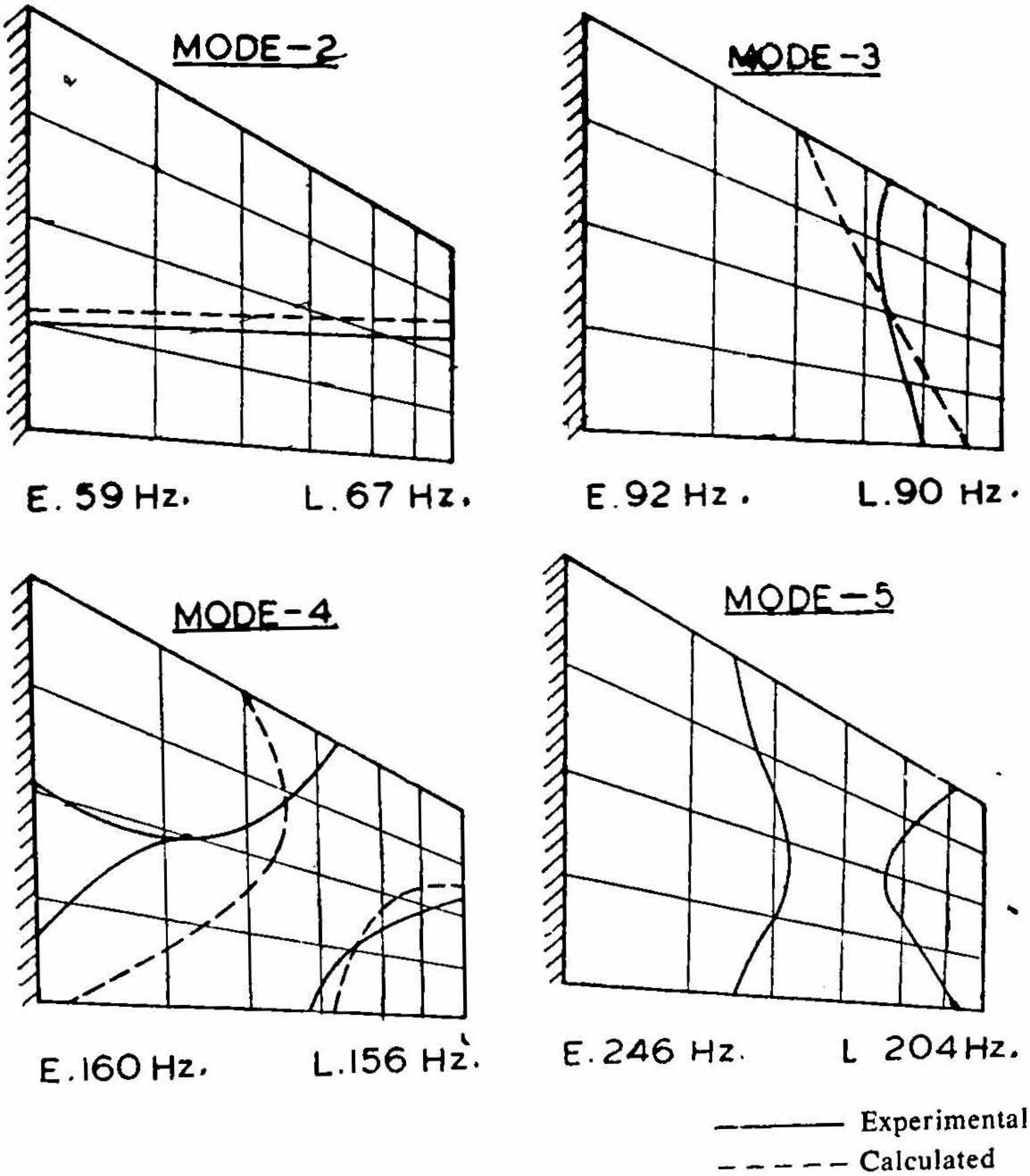


FIG. 3

Nodal Patterns of the Plate - Fully Clamped Root Chord (Configuration 1)

FUNDAMENTAL :

E. 18 *Hz*. *L*. 15 *Hz*.

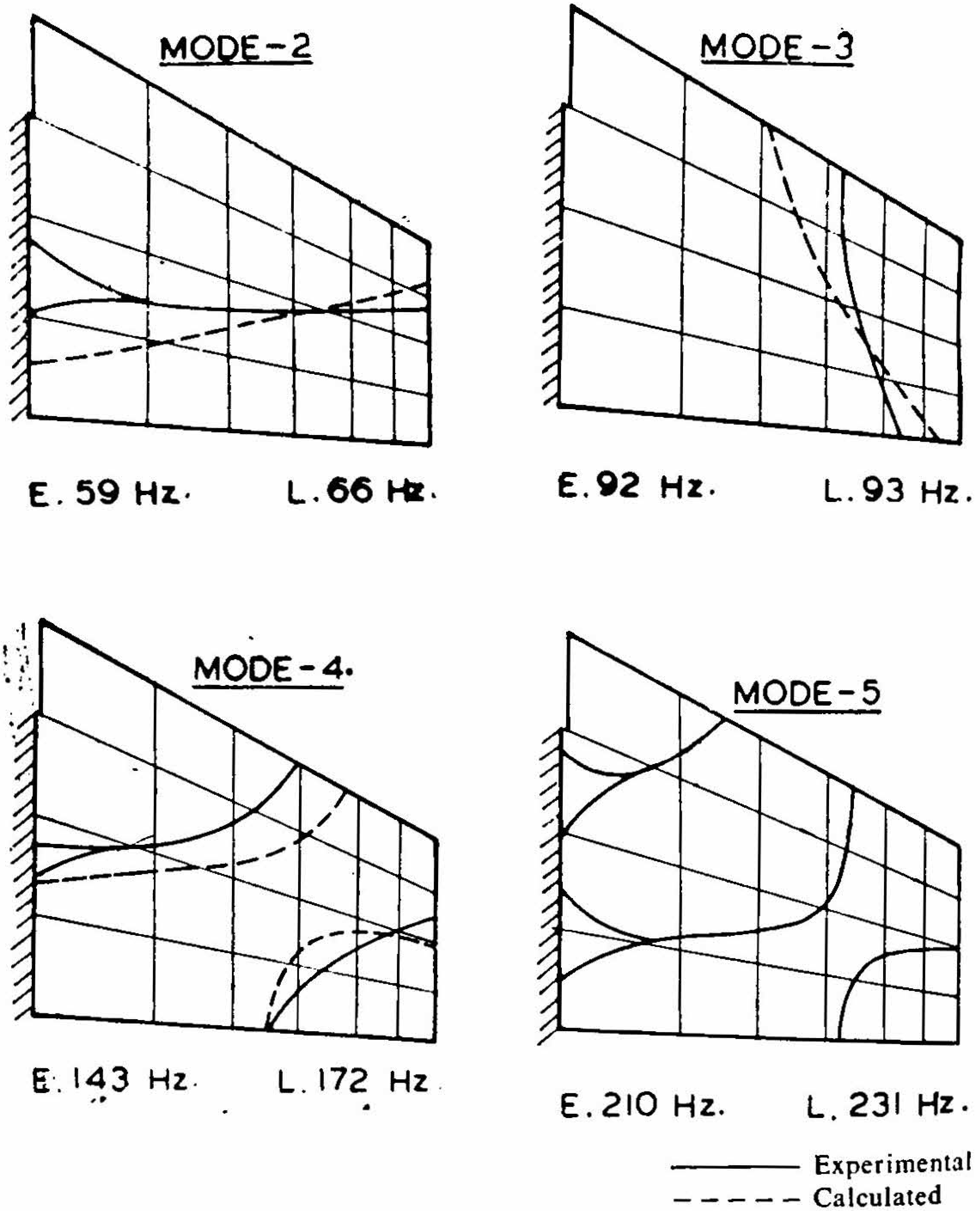


FIG. 4

Nodal Patterns of the Plate - 25% free at the Leading Edge (Configuration 2)

FUNDAMENTAL :

E. 17 *Hz.* *L.* 14.5 *Hz.*

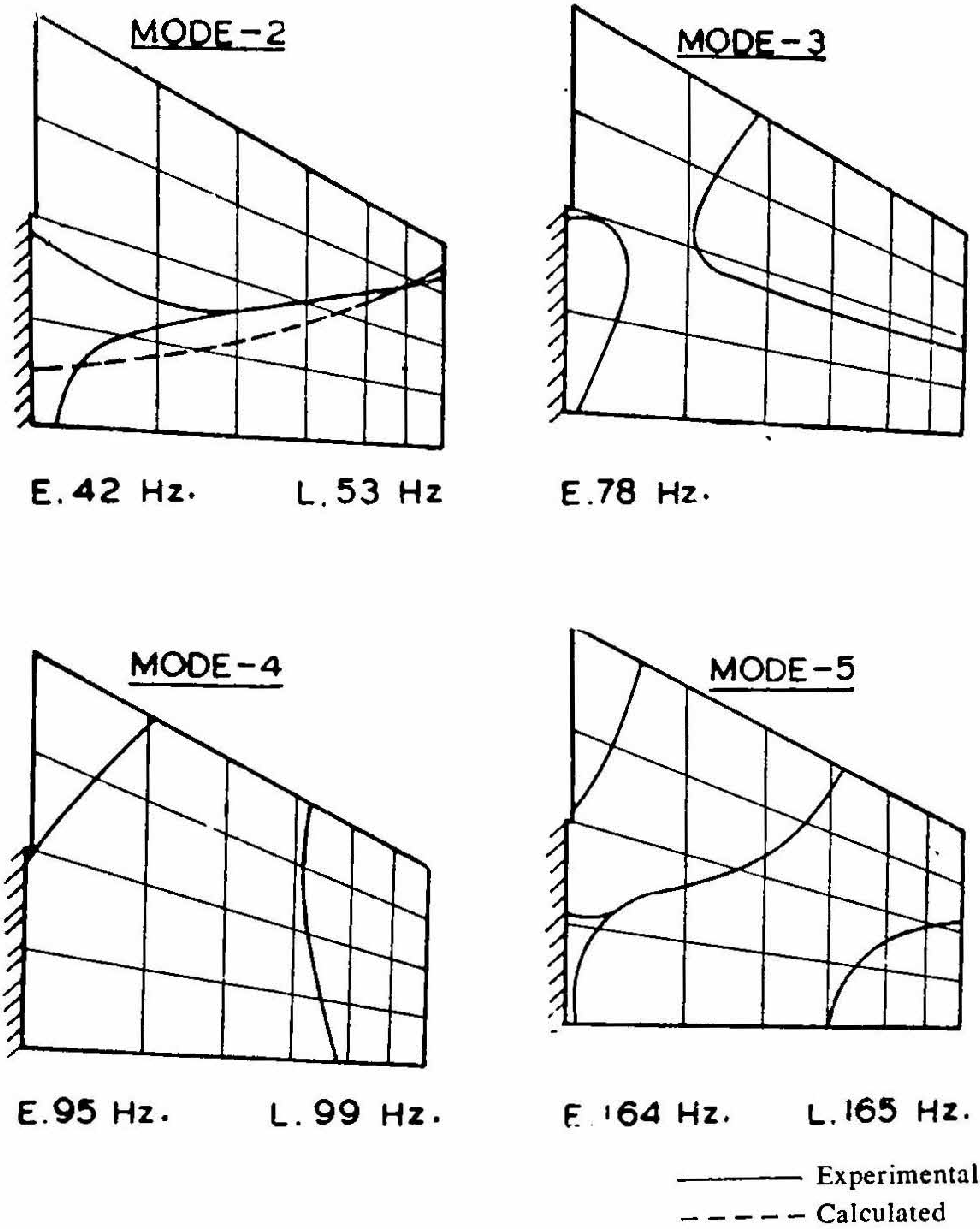


FIG. 5

Nodal Patterns of the Plate - 50% free at the Leading Edge (Configuration 3)

FUNDAMENTAL :

E. 16 *Hz.* *L.* 13.5 *Hz.*

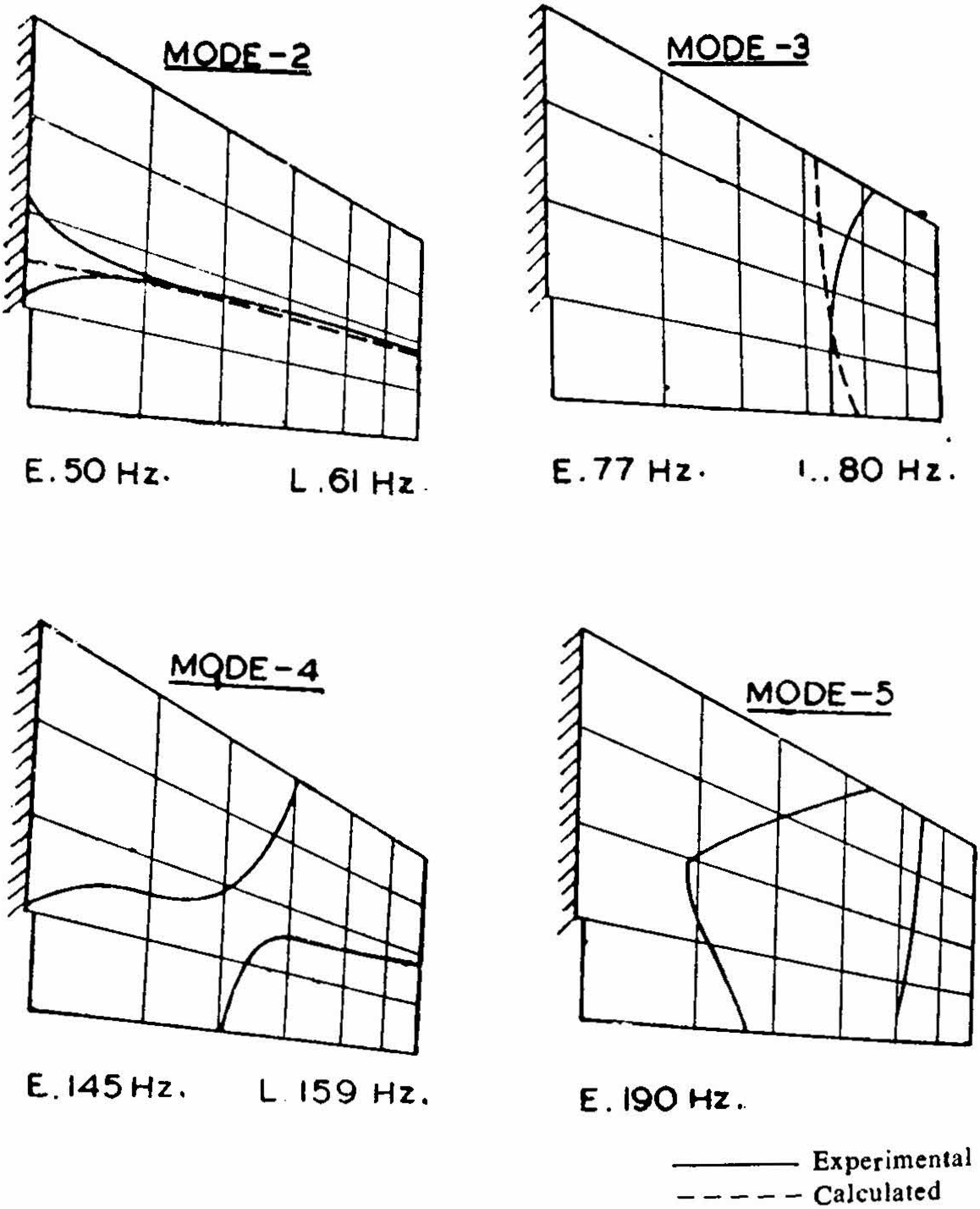


FIG. 6

Nodal Patterns of the Plate - 25% free at the Trailing Edge (Configuration 4)

FUNDAMENTAL :

E. 16 *Hz.* *L.* 13.5 *Hz.*

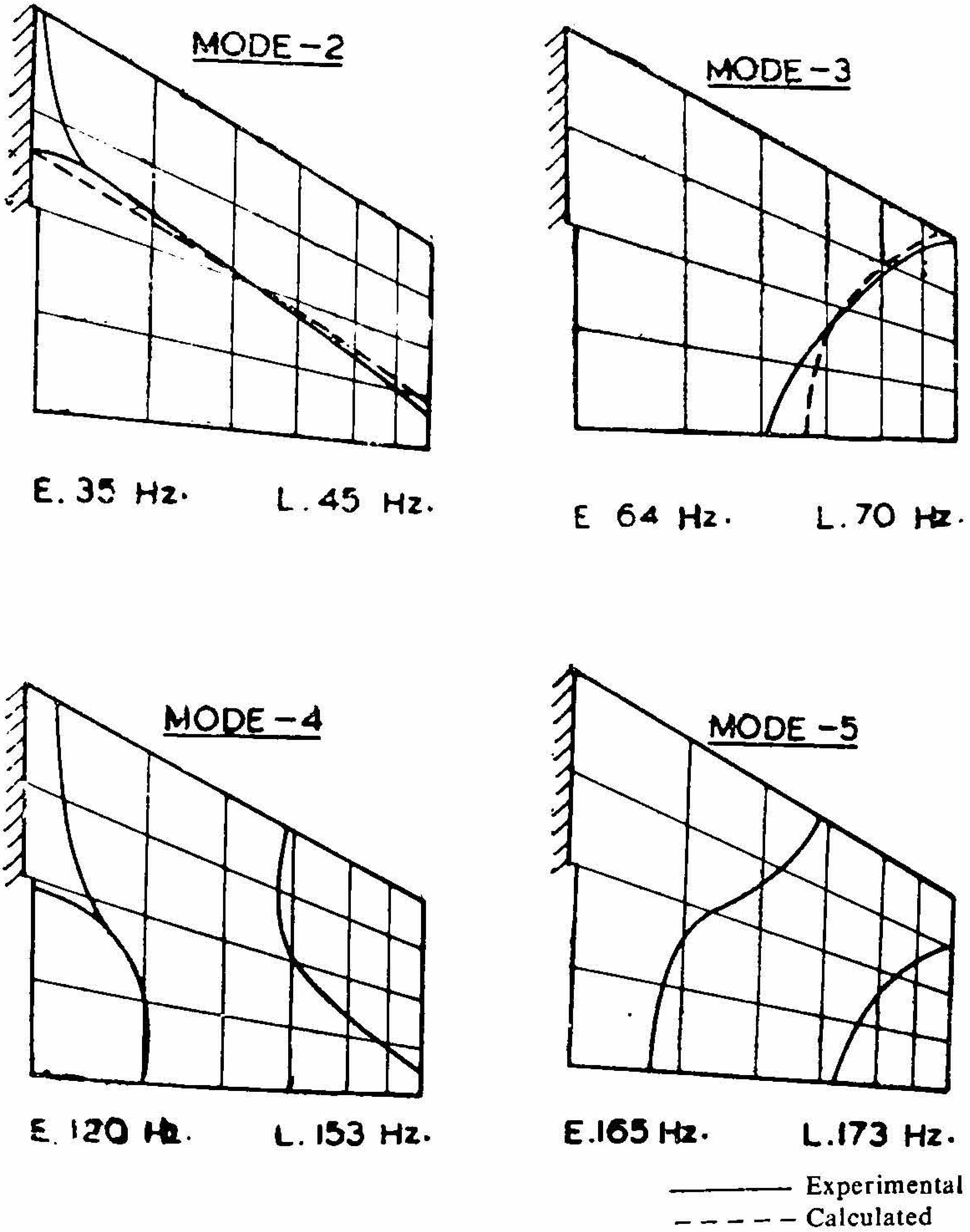


FIG. 7

Nodal Patterns of the Plate - 50% free at the Trailing Edge (Configuration 5)

FUNDAMENTAL :

E. 14.5 *Hz.* *L.* 12 *Hz.*

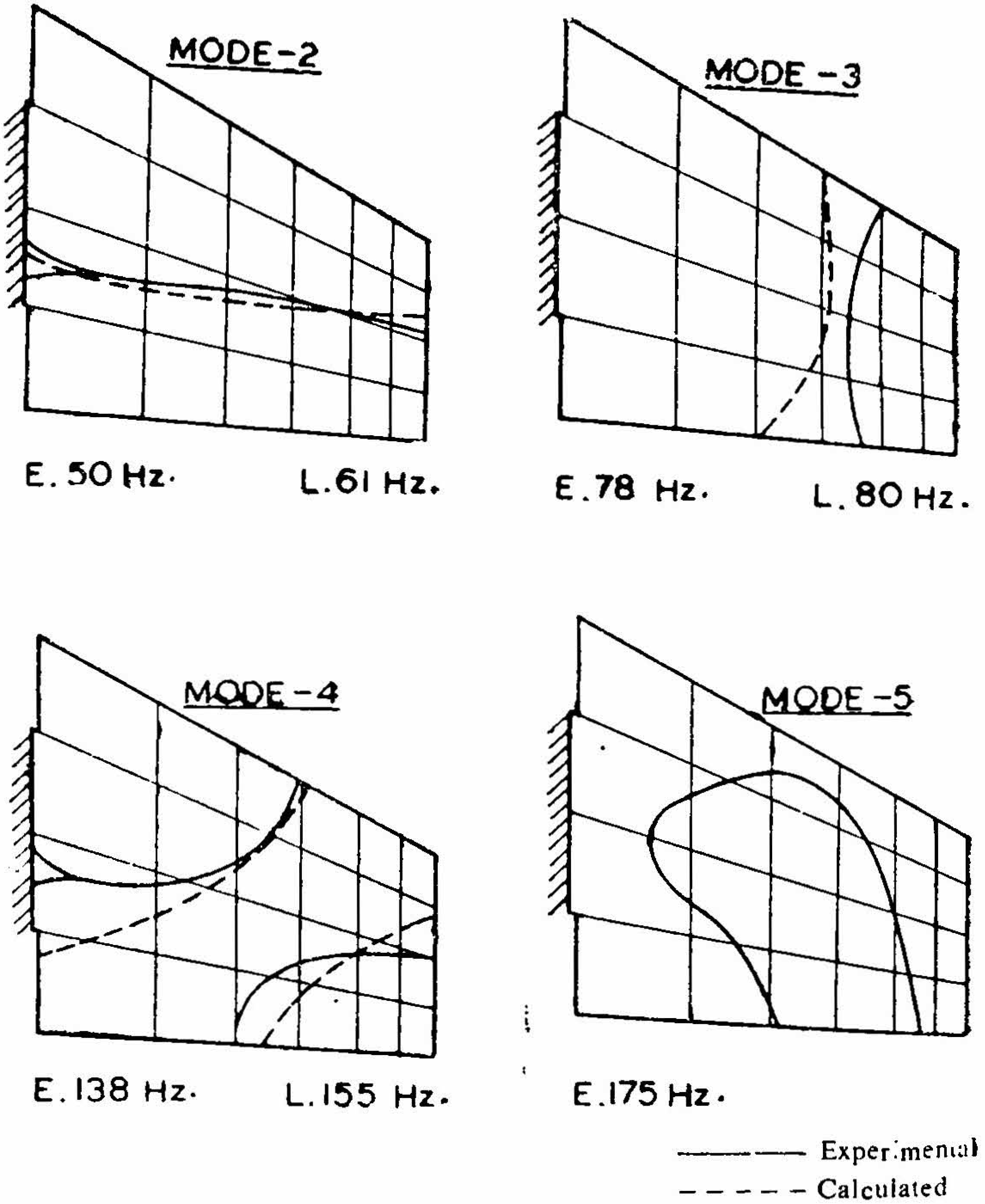


FIG. 8

Nodal Patterns of the Plate - 25% free at the Leading Edge and 25% free at the Trailing Edge (Configuration 6)

FUNDAMENTAL :

E. 12 *Hz.* *L.* 10 *Hz.*

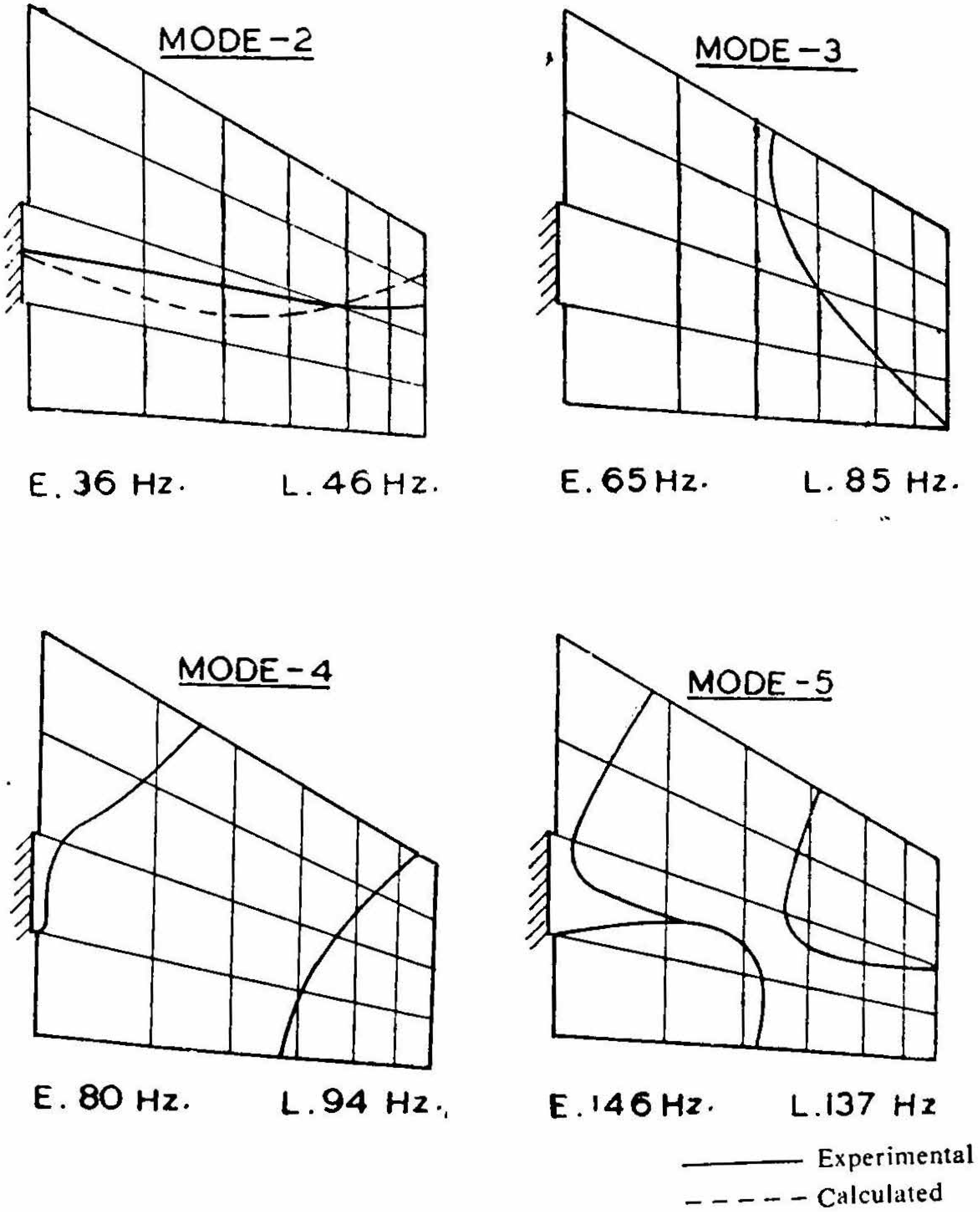


FIG. 9

Nodal Patterns of the Plate - 50% free at the Leading Edge and 25% free at the Trailing Edge (Configuration 7)

The significant changes in the nodal patterns of the different plate configurations are readily seen by an examination of Figs. 3--9 in succession.

One important feature to notice is that although the cut along the root chord from the trailing edge side lowers the frequencies quite significantly, Configuration 3 with 50% cut from leading edge side has the ratio of the torsional frequency to fundamental as the least, namely 2.46. This suggests a stronger susceptibility of this configuration to coupled bending-torsion flutter. The Configurations 7 and 5 come next (the ratio being 2.48 and 2.92 respectively) in the order of their susceptibility to flutter on this basis. These features are noteworthy in the design applications of rocket fins with partial root chord support.

REFERENCES

1. Gustafson, P. N., Stokey, W. F., and Zorowski, C. F., .. *J. Aero. Sci.*, 1953, **20**, 331.
2. ————, ... *J. Aero. Sci.*, 1954 **21**, 621.
3. Cadambe, V., Kumaraswamy, M. P., and Kaul, R. K., ... *J. Inst. Engrs. (India)*, 1956, **36**, 1429.
4. Foersching, H., ... DLR FB 66—11, Aerodynamische Versuchsanstalt, Gottingen, 1966.
5. Ohman, L. H., and Dixon, R. C., Aeronautical Report. LR—377, Nat. Res. Council of Canada. Ottawa. 1963.
6. Austin, R. N., Caughfield, D. A., and Plass, Jr. H. J., .. Developments in Theoretical and Applied Mechanics, Vol. 1, Plenum Press, 1963, 1-24
7. Craig, R. R., Plass, Jr. H. J., and Caughfield, D. A., ... Report DRL—518, Defence Research Lab., Univ. of Texas, 1964.
8. Craig, R. R., and Plass, Jr. H. J., AIAA Journal, 1965. **3**, 1177.
9. Kalyanaraman, K. .. .. M. E. Project Report, *Dept. of Aero. Engg.*, *I. I. Sc.*, 1969.
10. Sepaha, S. P. .. .. M. E. Project Report, *Dept. of Aero. Engg.*, *I. I. Sc.*, 1970.
11. Meirovitch, L. .. .. Analytical Methods in Vibrations, Macmillan and Co., 1967.
12. Wilkinson, J. H., ... Algebraic Eigenvalue Problem, Oxford University Press, 1965.
13. Hawkins, F. J. .. .. Tech. Rep. No. 65142, Royal Aircraft Establishment, Farnborough, 1965.