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MEASUREMENT OF SURFACE STRAINS IN DIAPHRAGMS.

By Professor B. C. Carter, Sir J. C. Ghosh, D.Sc., F.N.I., M. V. C. Sastri, Ph.D., A.R.I.C., and K. V. Chinnappa, M.Sc.

In the employment of metal diaphragms in pressure gauges it is almost invariably the practice to measure the central deflection of the plate. This method has good sensitivity with moderate pressures, up to, say, 100 lb. per sq. in. As the maximum pressure is increased, however, the plate thickness needs to be greater in relation to the diameter and the maximum deflection decreases unless the diameter is increased sufficiently. The permissible diameter being limited, special methods are needed at high pressures to measure deflections with sufficient accuracy to obtain pressure measurements correct to within ± 1 per cent. On the other hand, when pressure measurements are based upon surface strain, the same range of surface strain is available in all cases for diaphragms of the same material, as they require to be designed for the same limiting strains whatever the maximum pressure may be. Moreover, there is no need to adopt a larger diameter than is required by the technique of surface-strain measurement, and this is quite moderate.

Fig. 1.

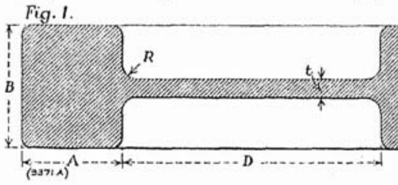
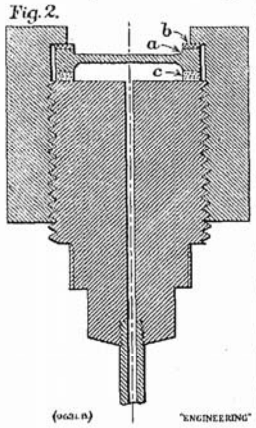


Fig. 2.



Like the central deflection, the surface strains are proportional to the pressure applied. Though various theoretical estimates of the surface strains are found in literature* for various boundary conditions, experimental data have not been available. With the aid of the bonded wire strain gauges, developed during the last war, surface strains can be measured very accurately and minutely by means of a resistance bridge. This prompted us to verify if the measurement of radial strains on circular diaphragms by this method could be rendered suitable for accurate pressure gauging. The results reported in this article, which were obtained at the India Institute of Science, Bangalore, show that the method has considerable merit.

The manner in which the diaphragms are held is important. Initial experiments with plain thin discs held tightly at the edge showed that the strains produced in such diaphragms were profoundly influenced by the conditions of clamping and were not reproducible. These difficulties, however, were completely overcome by employing diaphragms with integral holding rims of sufficient thicknesses and fillet radii, as shown in section in Fig. 1. The key to the lettering used in Fig. 1 is given in Table I, where the dimensions of the four diaphragms employed and the maximum service-pressure differences will be found; the pressure differences which would produce appreciable permanent set are about 7 per cent. higher. The diaphragms were all machined out of thick plates of spring steel, clamped at the rim in suitable unions and work-hardened *in situ* by subjecting them repeatedly to cycles of high and low pressures, with the maximum service-pressure differences as the respective upper limits.

Table I.—*Dimensions of Diaphragms and Maximum Service Pressure Differences.*

Diaphragm No.	I.	II.	III.	IV.
Thickness over flat portion (t in.)	0·0945	0·0735	0·064	0·0420
Diameter of diaphragm (D in.)	1·250	1·226	1·250	1·25
Fillet radius (R in.) ..	0·04	0·05	0·06	0·15
Radius of flat portion $\left(\frac{D}{2} - R\right)$	0·585	0·563	0·505	0·475
Width of rim (A in.) ..	0·21	0·22	0·21	0·21
Thickness of rim (B in.)	0·20	0·19	0·18	0·25
Maximum service pressure difference (lb. per sq. in.)	1,300	900	700	450

Two well-matched strain gauges, made by Messrs. H. Tinsley and Company, Limited, South Norwood, each of about 200 ohms resistance and measuring $\frac{5}{8}$ in. by $\frac{5}{64}$ in. over the active portion of the gauge, were then mounted with Durofix (cellulose cement), one centrally on the diaphragm to measure the strain and the other on the union nut to compensate for variations in ambient temperature. Fig. 2 shows the high-pressure union with the diaphragm a held in place between a copper washer b on top and a Klingerit washer c beneath. Since the measurements were made under static conditions, a direct-current bridge of the Callendar and Griffiths type (used in platinum-resistance thermometry) was employed, using as null indicator a Moll reflecting galvanometer with sensitivity of 500 mm. per microampere at 1 metre distance.

The resistance-measurements were translated into strain values by means of the usual formula

$$\frac{\Delta\,\Omega}{\Omega} = F\frac{\Delta\,L}{L} = F\,e,$$

where

Ω = initial resistance of the unstrained gauge;

$\Delta\,\Omega$ = change in resistance of the same gauge as a result of the strain;

$e = \frac{\Delta L}{L}$ = strain to be measured;

and

F = the so-called gauge factor, fixed by the makers as 1·95 for the gauges employed in the present work.

With the electrical arrangements given above, resistances could be compared with an error of less than 400 micro-ohms; the strain measurements were therefore correct to one micro-inch per inch. For maximum accuracy it is necessary to ensure that the measuring and compensating gauges are matched to within 0·1 per cent. and that they are both at the same temperature. The latter precaution is of particular importance in the measurement of gas pressures as pressure changes in gases may not be isothermal and the temperature differences may well be sufficient to vitiate the accuracy of the measurement. Unless maximum thermal conductance between the diaphragm and the external nut is ensured by having a copper washer here, it is necessary to allow about five minutes between each alteration of pressure and the measurement of resistance in order that temperature equilibrium may be re-established. With the copper washer inserted, the time lag was reduced to about a minute. As reference gauge, a Budenberg standard test gauge with a 10-in. dial and reading to 2,000 lb. per square inch was employed. The pointer of this gauge was set by the makers to read the absolute pressures directly on the dial. The gauge calibrations were checked with a deadweight tester and found to be correct to ± 1 lb. per square inch up to 1,500 lb. per square inch. The experimental arrangement shown in Fig. 3, page 582, was found to be convenient for the calibration of strain against pressure. By means of the combined inlet and bleeder valve shown, the pressure in the gauge system could be increased to any desired value by allowing compressed air to enter from a cylinder, or could be reduced by allowing some of the gas to escape.

The results of the pressure against strain calibrations are shown graphically in Fig. 4, from which it can be seen that the relationship is perfectly linear. Within the limits of maximum operating pressures given in Table I, the diaphragms were thus found to be perfectly elastic, responding to rising and falling pressures without hysteresis and without creep. The strain measurements were reproducible even after the diaphragms had been dismantled and re-assembled, thus showing the absence of end effects or clamping effects. It should be mentioned, however, that, due probably to an inherent weakness of the cement used for mounting the strain gauges, the basic setting of the resistance bridge, i.e., the reading for atmospheric pressure, showed a slight variation when left over a period of several days, and after three months the strain gauge must be replaced if the original standard of reproducibility is to be maintained.

Table II shows a typical set of values obtained in four experiments with diaphragm III. It is evident from this table that the largest average deviation from the mean values is 1·25, i.e., $\frac{1}{6}$ per cent. of the full range value of 706 micro-inches per inch. Even at the lower end of the scale (100 lb. per square inch) the average error is less than 0·5 per cent. of the mean value for that pressure, but less than $\frac{1}{11}$ per cent. of the full-scale value.

Apart from providing a means of accurate measurement of pressure, the method possesses all the advantages inherent in electrical methods, such as wide choice of accessory apparatus, interchangeability of components, etc. If a strain-gauge bridge of conventional design is used (such as those manufactured by the Baldwin Locomotive Works, Philadelphia, U.S.A., or Messrs. H. Tinsley and Company, Limited, South Norwood, London, S.E.25), the method will be found to be particularly suitable where it is desired to read, record and control the static pressures at a number of distant points with a high degree of precision and reliability. This would be analogous to remote control of temperature with a potentiometric indicator, recorder and controller.

Table II.—Results Obtained with Diaphragm III in Four Experiments.

Absolute Pressure, Lb. per sq. in.	Strain in Micro-in. per in. 1.	2.	3.	4.	Mean.	Average Deviation from Mean.	Percentage of $\frac{\text{Av. Devn.}}{\text{Av. Strain.}}$
13·4*	0	0	0	0	0	—	—
100	86	86	85	86	85·7	0·4	0·46
200	189	189	187·5	187·5	188·3	0·7	0·37
300	291	290	290	290	290·3	0·4	0·14
400	394	395	394	395	394·5	0·5	0·13
500	499	499	496·5	496·5	497·8	1·25	0·25
600	601·5	603	600·5	601·5	601·6	0·68	0·11
700	706	706	706	706	706	0·00	0·00

* Atmospheric pressure (Bangalore).

The radial and tangential stresses at any point on the surface of a circular diaphragm clamped and held encastré at the edge and subjected to a uniform fluid pressure are given by the following equations, which apply strictly for the case where the ratio of thickness to radius is about 0·1

$$p_r = \frac{3}{8}\,P\,\frac{r^2}{t^2}\left\{\left(1+\frac{1}{m}\right) - \left(3+\frac{1}{m}\right)\frac{x^2}{r^2}\right\}. \quad (1)$$

$$p_t = \frac{3}{8}\,P\,\frac{r^2}{t^2}\left\{\left(1+\frac{1}{m}\right) - \left(1+\frac{3}{m}\right)\frac{x^2}{r^2}\right\}. \quad (2)$$

* Love, *A Treatise on the Mathematical Theory of Elasticity*, Cambridge Univ. Press (1927), Chap. XXII; Timoshenko and Lessels, *Applied Elasticity*, Constable and Company (1928), pages 260-295; H. Hencky, "Uber den Spannungszustand in kreisrunden Platten," *Zeit. f. Math. u. Phys.*, 1915, vol. 63, pages 311-317; Clemmow. *Proc. Roy. Soc.*, *A*, 1926, vol. 112, page 559.

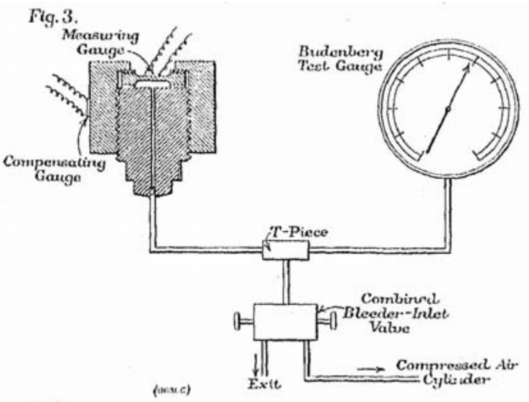


Fig. 3.

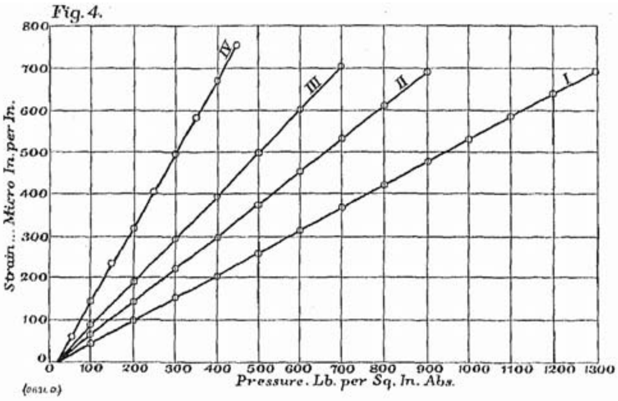


Fig. 4.

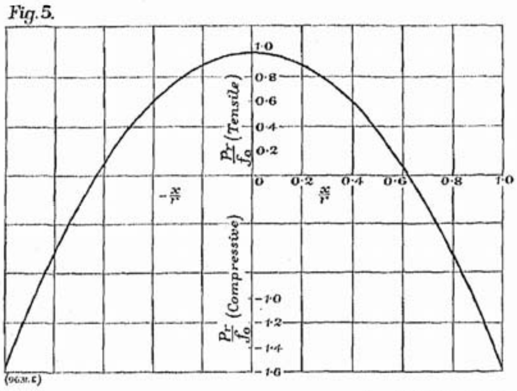


Fig. 5.

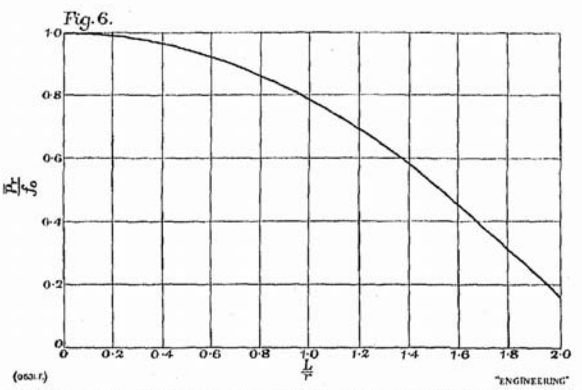


Fig. 6.

where

p_r = radial stress;
p_t = tangential stress;
$\frac{1}{m}$ = Poisson's Ratio;
r = radius of the plate;
x = distance of the point in question from the centre of the diaphragm; and
P = the applied pressure difference.

The radial and tangential stresses and strain thus vary from point to point on the surface of the plate. Denoting the term $\frac{3}{8} P \frac{r^2}{t^2}\left(1 + \frac{1}{m}\right)$ by f_0, as usual, and taking $m = \frac{10}{3}$ for steel, the values of $\frac{p_r}{f_0}$ corresponding to various values of $\frac{x}{r}$ have been worked out from equation (1) and the plot of $\frac{p_r}{f_0}$ over $\frac{x}{r}$ is given in Fig. 5. This graph shows at a glance the distribution of stresses along any given diameter of the circular plate. Hence it follows that a strain-gauge mounted along the diameter of a diaphragm subjected to a uniform pressure will experience a gradation of strains along its length and the strain value obtained experimentally will therefore be the weighted average of these values. The transverse strain sensitivity of the gauge is only about 2 to 3 per cent. of the longitudinal strain sensitivity and may be disregarded. The average radial strain over the length of the gauge can be estimated as follows.

Let L be the length of the active portion of the strain gauge, and a and b the parts of this length on either side of the centre. Since from (1),

$$p_r = f_0\left\{1 - \frac{3m+1}{m+1}\frac{x^2}{r^2}\right\} \quad . \quad . \quad (3)$$

the average value of p_r over the length $+a$ to $-b$ is given by

$$r = \frac{f_0}{a+b}\int_{-b}^{a}\left(1 - \frac{3m+1}{m+1}\frac{x^2}{r^2}\right)dx$$
$$= f_0\left\{1 - \frac{3m+1}{3(m+1)}\frac{L^2 - 3ab}{r^2}\right\}. \quad . \quad (4)$$

In the special case where the strain gauge is mounted symmetrically above the centre, $a = b$ and equation (4) reduces to

$$\bar{p}_r = f_0\left\{1 - \frac{3m+1}{12(m+1)}\frac{L^2}{r^2}\right\}. \quad . \quad (5)$$

whence the average strain

$$\bar{e}_r = \frac{f_0}{E}\left\{1 - \frac{3m+1}{12(m+1)}\frac{L^2}{r^2}\right\}. \quad . \quad (6)$$

E being the Young's Modulus.

At the centre $\bar{p}_r = f_0$ and the value of $\frac{\bar{p}_r}{f_r}$ therefore represents the ratio of the average stress to the stress at the centre. The values of $\frac{p_r}{f_0}$ for various values of $\frac{L}{r}$ have been calculated from equation (5), taking $m = \frac{10}{3}$, and are shown in Fig. 6. This graph shows directly what fraction of the maximum central strain is represented by the experimental strain value measured with a strain gauge of a given length in relation to the diameter of the plate and mounted symmetrically about the centre. Thus, with a gauge which has an active length equal to half the diameter of the diaphragm (i.e., $\frac{L}{r} = 1$), the observed strain will be 79 per cent. of the maximum strain at the centre.

From equation (6) we can obtain a theoretical estimate of the slope of observed strain plotted against pressure, i.e., of $\frac{d\bar{e}_r}{dP}$. Substituting for f_0 and taking $m = \frac{10}{3}$ and $E = 30 \times 10^6$ for steel, we get

$$\frac{d\bar{e}_r}{dP} = \frac{13}{800}\frac{r^2}{t^2}\left\{1 - \frac{11}{52}\frac{L^2}{r^2}\right\} \times 10^{-6}_{n} \quad . \quad (7)$$

In the case of the diaphragms used in the present experiments (Fig. 1), the value of r, the effective radius, cannot be taken as equal to the radius of the flat portion only, but should extend a little beyond into the region of the fillet radius also, because it would be wrong to assume that the latter region is completely insensitive to pressure changes. How much of the fillet radius should be included in the effective radius can be deduced arbitrarily by trial and error and it has thus been found that the simple assumption $r = \frac{D}{2} - \frac{2R}{3}$, (R being the fillet radius), i.e., the inclusion of one-third of the fillet radius in the effective radius, gives sufficiently consistent results. The experimental values and the theoretical estimates based on these assumptions are given in Table III.

It is also possible to deduce the limit of proportionality of the material of the diaphragm from the data given in this article. From the observed values of the maximum service pressure, the effective radius r, and the thickness t of the diaphragm, the

TABLE III.—COMPARISON OF THEORETICAL AND EXPERIMENTAL VALUES.

Diaphragm No.	Thickness, t in.	Total Diameter, D in.	Fillet Radius, R in.	Effective Radius, r in.	$\frac{d\bar{e}_r}{dP}$ in $\frac{\text{micro-in. per in.}}{\text{lb. per sq. in.}}$		Percentage Deviation of Experiment from Theory, $\frac{\text{Expt.} - \text{Theory}}{\text{Theory}} \times 100$
					Theoretical Calculated from Eq. (7).	Experimental.	
I	0·0945	1·245	0·04	0·596	0·500	0·542	+8·0
II	0·0735	1·226	0·05	0·580	0·763	0·786	+3·0
III	0·0640	1·250	0·06	0·585	1·030	1·030	0
IV	0·0420	1·250	0·15	0·525	1·778	1·743	−2·0

value of f_0, the maximum permissible stress at the centre, can be calculated, since

$$f_0 = \frac{3}{8} P \frac{r^2}{t^2}\left(1 + \frac{1}{m}\right).$$

As can be seen from Fig. 5, the maximum stress that the plate is subjected to under any given load is at the periphery and this is a compressive stress on the gauge side of magnitude equal to $\frac{20}{13}$ times the maximum tensile stress at the centre; there is a corresponding tensile stress on the other side of the rim. These values obtained from the present experiments are collected in Table IV.

TABLE IV.

Diaphragm No.	Effective Radius, *r* in.	Thickness, *t* in.	Maximum Service Pressure-Difference, Lb. per sq. in.	Corresponding Stress at Centre, Lb. per sq. in.	Maximum Radial Stress at Periphery, Lb. per sq. in.
I	0·598	0·0945	1,390	27,200	41,900
II	0·580	0·0735	960	29,100	44,800
III	0·585	0·0640	750	30,600	47,100
IV	0·525	0·0420	480	36,600	56,400

The reason for the lower values in the last column of Table IV for the thicker diaphragms is not apparent,. as the same material has been used throughout. There are differences in stress-concentration factors and there may also have been differences in work hardening during machining. When designing diaphragms, therefore, a conservative value should be taken for the limit of proportionality of the material concerned.