

DIELECTRIC SPHERICAL AERIALS EXCITED IN THE UNSYMMETRIC HYBRID MODE AT MICROWAVE FREQUENCIES

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ABSTRACT

Approximate expressions for the radiation field and gain of a dielectric spherical aerial excited in the unsymmetric hybrid mode have been derived. The calculated radiation patterns and gains of several aerials of varying diameter have been verified by experimental results. The input impedance of these aerials have also been measured and reported.

1. INTRODUCTION

In 1941, Stratton and Chu¹ considered the problem of forced oscillations of a conducting sphere which is excited in an infinite number of TM modes by a delta-function electric source field applied normally across the equatorial plane. Since then some work^{2,3,4} has been done on metal spherical aerials. There has also been some work on spherical arrays^{5,6,7}. Very recently Chatterjee and Crosswell⁸ have reported some experimental work on the radiation patterns of wave guide excited dielectric spheres as compared with those of metal horns.

In this paper, an attempt has been made to derive theoretically expressions for the radiation pattern and the gain of a dielectric spherical aerial excited in the unsymmetric hybrid mode by a circular waveguide. The calculated radiation patterns and gains of aerials of various diameters have been verified by experimental results. The measured input impedances of these aerials have also been reported.

2. STATEMENT OF THE PROBLEM

The geometry of the structure is given in Fig. 1.

The aerial consists of a part of a sphere ABC of radius R which subtends an angle $\alpha = \pi - \theta'$ at its centre O , and which is terminated in a

cylindrical dielectric rod $ACDE$ which is terminated in a dielectric cone DFE . The portion $FEACDF$ which consists of the cylindrical dielectric rod and the dielectric cone is fitted tightly into a cylindrical metal waveguide which is carrying the dominant TE_{11} mode. This dominant TE_{11} mode which is azimuthally unsymmetric in the metal waveguide is transformed into the dominant unsymmetric HE_{11} mode in the dielectric cylinder $ACDE$, which in its turn transforms into an unsymmetric hybrid mode in the dielectric sphere.

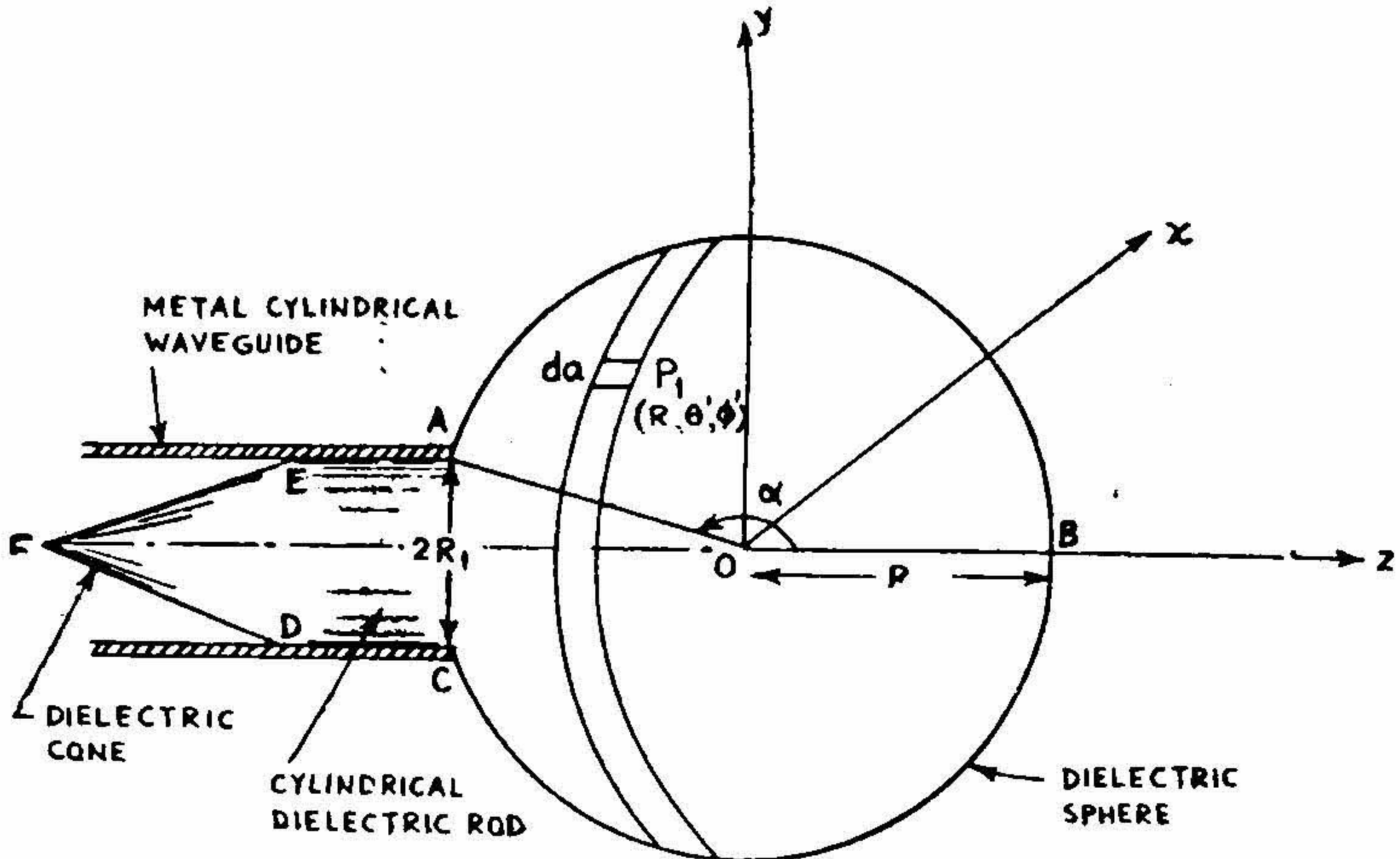


FIG. 1

Geometry of the Structure

The electric and magnetic fields in the cylindrical waveguide vary as $\cos \phi$ or $\sin \phi$ (or as Re or $\text{Im} \{e^{j\phi}\}$) for the TE_{11} mode, and hence they vary in the same manner in the dielectric cylinder for the HE_{11} mode as well as in the dielectric sphere for this hybrid mode. (r', θ', ϕ') are the spherical polar coordinates of any point in the dielectric sphere and (r, θ, ϕ) are the spherical polar coordinates of the distant point P .

In this paper, expressions for the radiation field components at a distant point as well as for the gain of the dielectric spherical aerial are derived, and compared with experimental results.

3. RADIATED FIELD AT A DISTANT POINT

The spherical aerial is approximated by a finite, but large number n of circular electric current loops of varying diameter as shown in Fig. 2.

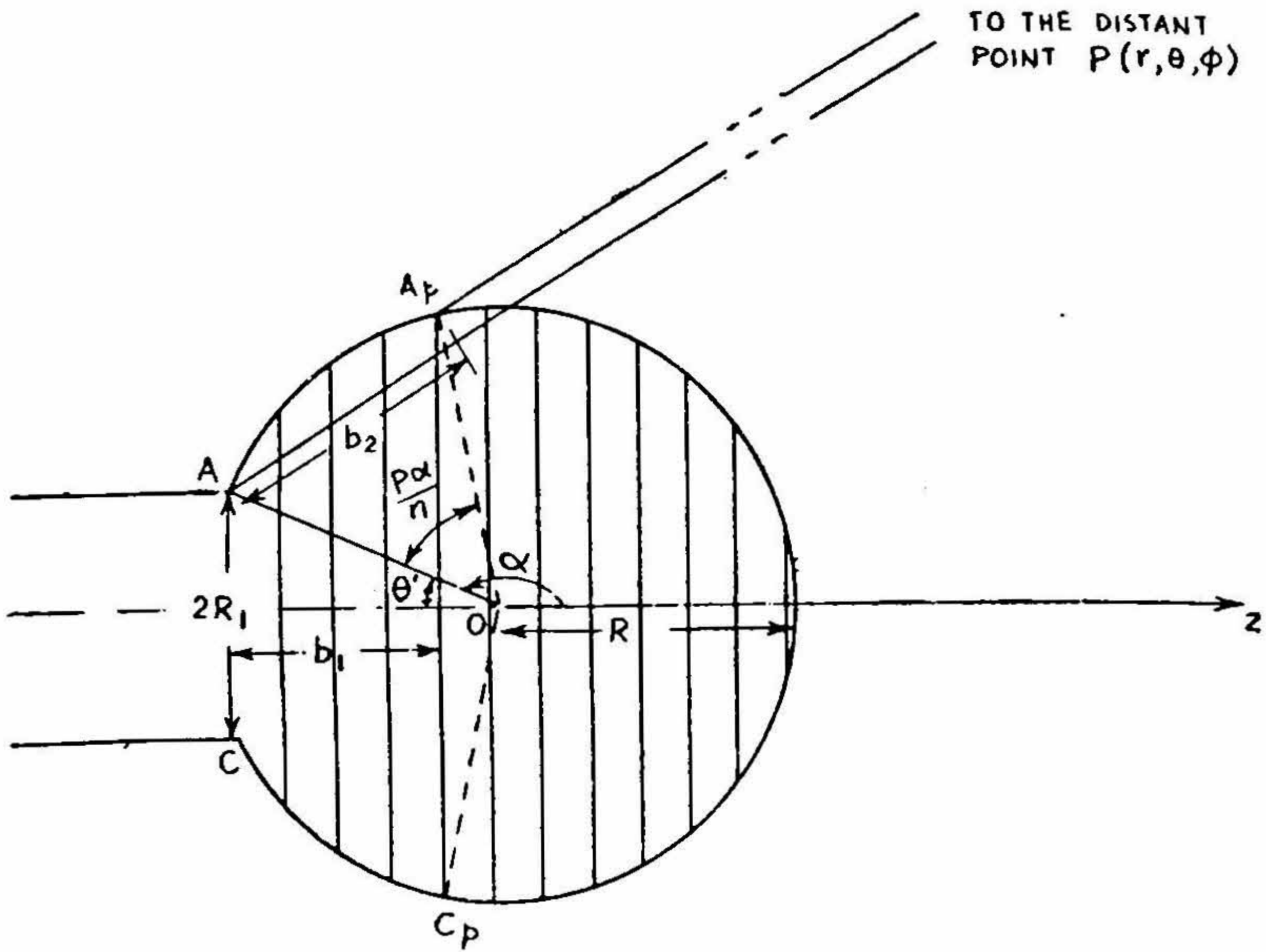


FIG. 2

Array Approximation of the Spherical Aerial

The radiation field at a distant point $P(r, \theta, \phi)$ is determined by the vectorial sum of the radiation fields of all the n current carrying circular loops.

The above approximation assumes only electric current loops, neglecting the magnetic current loops, as a first attempt to derive the radiation field, in the absence of an accurate knowledge of the electromagnetic field inside and outside the dielectric sphere excited in this manner and hence of the actual surface electric and magnetic currents on the dielectric spherical aerial.

If ' a ' is the radius of the p^{th} electric current carrying loop, assuming that the current varies as $\text{Re}(C e^{j\phi'})$ on this loop, the x and y components of the magnetic vector potential A at the distant point P are given by

$$\begin{aligned}
 A_{xp} &= \frac{C}{r} e^{j(\omega t - \beta_0 r)} \int_{\phi'=0}^{2\pi} e^{j\beta_0 a \sin \theta \cos(\theta - \theta')} e^{j\phi'} a \sin \phi' d\phi' \\
 &= jC \frac{e^{j(\omega t - \beta_0 r)}}{r} \pi a [J_0(\beta_0 a \sin \theta) + J_2(\beta_0 a \sin \theta) e^{j2\phi}] \quad [1]
 \end{aligned}$$

$$\begin{aligned}
 A_{yp} &= \frac{C}{r} e^{j(\omega t - \beta_0 r)} \int_{\phi'=0}^{2\pi} e^{j\beta_0 a \sin \theta \cos(\phi - \phi')} e^{j\phi} a \cos \phi' d\phi' \\
 &= \frac{C\pi a}{r} e^{j(\omega t - \beta_0 r)} [J_0(\beta_0 a \sin \theta) - J_2(\beta_0 a \sin \theta) e^{j2\phi}]
 \end{aligned} \tag{2}$$

[In the final expressions, the real parts of equation [1] and [2] are considered] where

$\beta_0 = \omega \sqrt{(\mu_0 \epsilon_0)}$ = wave number in free space

ω = frequency

$$a = R \sin \left(\theta' + \frac{p\alpha}{n} \right)$$

C = amplitude constant.

As there are n such current loops, the angle subtended by the p^{th} loop at the centre O of the sphere is given by

$$\delta = \theta' + \frac{p\alpha}{n} \tag{3}$$

Taking the loop for which $p=0$ as the reference loop AC , the path difference between the p^{th} loop $A_p C_p$ and the reference loop AC due to propagation in the dielectric sphere is

$$b_1 = R \left[\cos \theta' - \cos \left(\theta' + \frac{p\alpha}{n} \right) \right] \tag{4}$$

The path difference between the two loops due to propagation in free space is

$$b_2 = R \left[\cos \theta' - \cos \left(\theta' + \frac{p\alpha}{n} \right) \right] \cos \theta \tag{5}$$

where R = radius of the sphere. Hence the total phase difference between the fields at the distant point $P(r, \theta, \phi)$ due to the two loops is given by

$$\Delta \phi_p = \frac{2\pi}{\lambda_0} R \left[\cos \theta' - \cos \left(\theta' + \frac{p\alpha}{n} \right) \right] (K - \cos \theta) \tag{6}$$

where K is a constant which is analogous to λ_0/λ_g , where λ_0 is the free-space wavelength, and λ_g may be called the guide wavelength in the dielectric sphere.

The the x and y components of the total magnetic vector potential \vec{A} due to all the n loops at the distant point (r, θ, ϕ) is given by

$$\begin{aligned}
 A_x &= \sum_{p=0}^n A_{xp} \\
 &= j \frac{C}{r} e^{j(\omega t - \beta_0 r)} \pi R \sum_{p=0}^n \left[\left\{ J_0 \left[(\beta_0 R \sin \theta \sin (\theta' + p\alpha/n)) \right] \right. \right. \\
 &\quad \left. \left. + e^{j2\phi} J_2 \left[\beta_0 R \sin \theta \sin \left(\theta' + \frac{p\alpha}{n} \right) \right] \right\} \sin \left(\theta' + \frac{p\alpha}{n} \right) e^{j\Delta\phi p} \right] \quad [7]
 \end{aligned}$$

and

$$\begin{aligned}
 A_y &= \sum_{p=0}^n A_{yp} \\
 &= \frac{C}{r} e^{j(\omega t - \beta_0 r)} \pi R \sum_{p=0}^n \left[\left\{ J_0 \left[\beta_0 R \sin \theta \sin (\theta' + p\alpha/n) \right] \right. \right. \\
 &\quad \left. \left. - e^{j2\phi} J_2 \left[\beta_0 R \sin \theta \sin \left(\theta' + \frac{p\alpha}{n} \right) \right] \right\} \sin \left(\theta' + \frac{p\alpha}{n} \right) e^{j\Delta\phi p} \right] \quad [8]
 \end{aligned}$$

The θ and ϕ components of A are given by

$$A_\phi = -A_x \sin \phi + A_y \cos \phi \quad [9]$$

$$A_\theta = (A_x \cos \phi + A_y \sin \phi) \cos \theta \quad [10]$$

And the electric field E at distant point P is given by

$$\vec{E} = -j \omega \mu \vec{A} \quad [11]$$

Taking the real parts of equations [7] and [8] and substituting in equations [9], [10] and [11], the electric field at a distant point P in the $\phi = 0^\circ$ plane is given by

$$\begin{aligned}
 \vec{E} &= \vec{u}_\phi E_\phi \\
 &= -\vec{u}_\phi j \omega \mu C \pi \frac{e^{j(\omega t - \beta_0 r)}}{r} R \sum_{p=0}^n \left[\left\{ J_0 \left\langle (\beta_0 R \sin \theta) \sin \left(\theta + \frac{p\alpha}{n} \right) \right\rangle \right. \right. \\
 &\quad \left. \left. - J_2 \left\langle \beta_0 R \sin \theta \sin \left(\theta' + \frac{p\alpha}{n} \right) \right\rangle \right\} \sin \left(\theta' + \frac{p\alpha}{n} \right) e^{j\Delta\phi p} \right] \quad [12]
 \end{aligned}$$

4. DIRECTIVE OR ABSOLUTE GAIN OF THE AERIAL

Directivity D is given by

$$D = \frac{4 \pi [E_{\theta}^2 + E_{\phi}^2]_{\max}}{\int_{\theta=0}^{\pi} \int_{\phi=0}^{2\pi} [E_{\theta}^2 + E_{\phi}^2] \sin \theta d\theta d\phi} \quad [13]$$

where E_{θ} and E_{ϕ} are given by equations [7], [8], [9], [10] and [11].

5. NUMERICAL CALCULATIONS

The radiation patterns in the $\phi = 0^\circ$ plane of several dielectric spherical aerials of radius varying from 1.28 cms. to 7.0 cms. have been calculated using eq. (12). n has been taken equal to 50. Values of n upto 300 have been tried, but have not given any better results than 50. The value of K has been varied from 0.8 to 1.8. For each aerial, the best value of K is taken as the value which makes the theoretical radiation pattern agree best with experimental radiation pattern. The best value of K varies with the radius, and is found to decrease from 1.6 for the smallest radius of 1.28 cms. to 1.2 for a radius of 4.45 cms. (See Fig. 3)

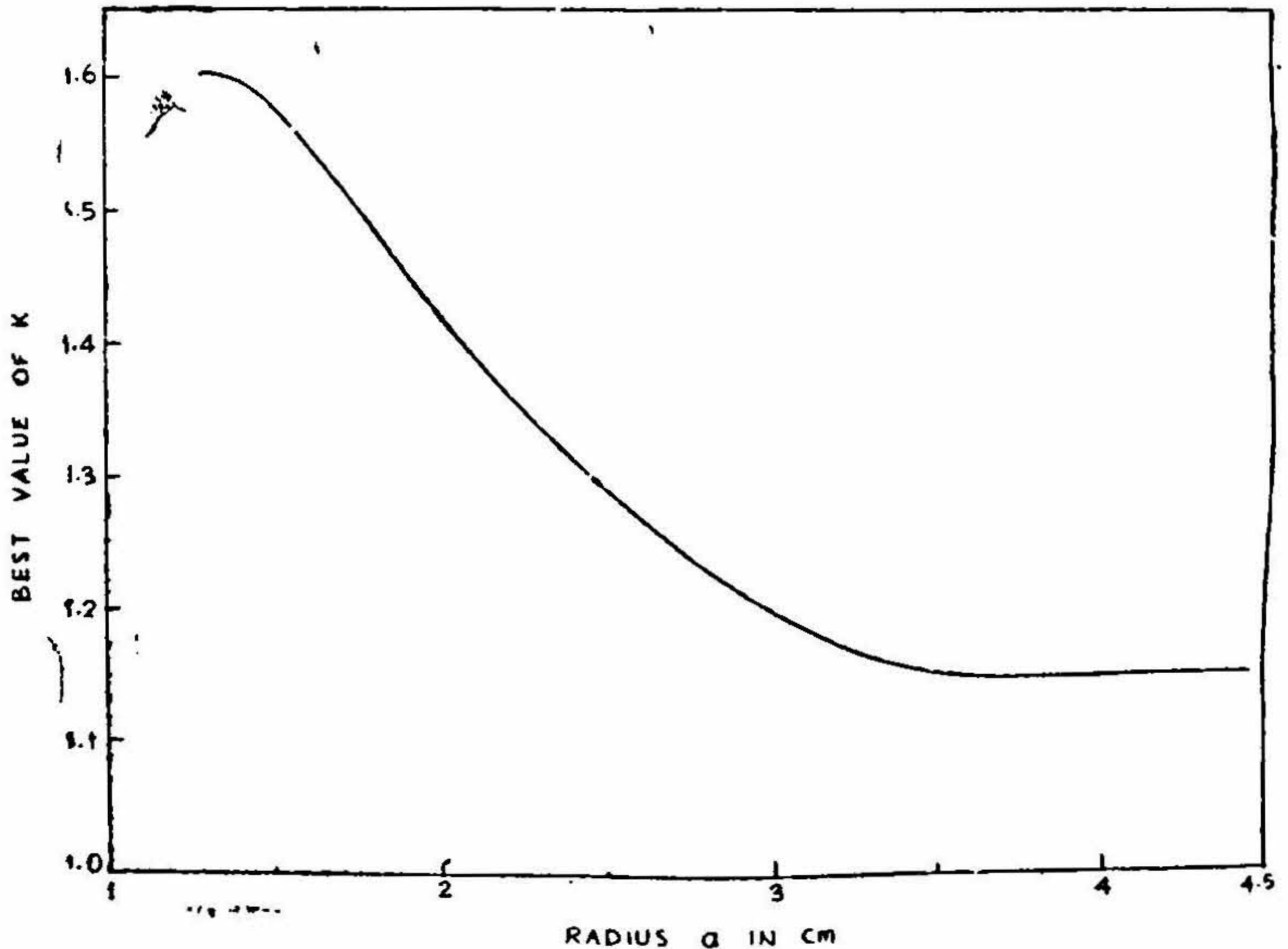


FIG. 3

Best value of K vs. Radius, a of the Sphere.

Fig. 4 and 5 show the radiation patterns in the $\phi = 0^\circ$ plane of aerials of radii 4.45 cms. and 3.5 cms. and θ' equal to 8.2° and 21.4° respectively. The best value of K for these two aerials is 1.2.

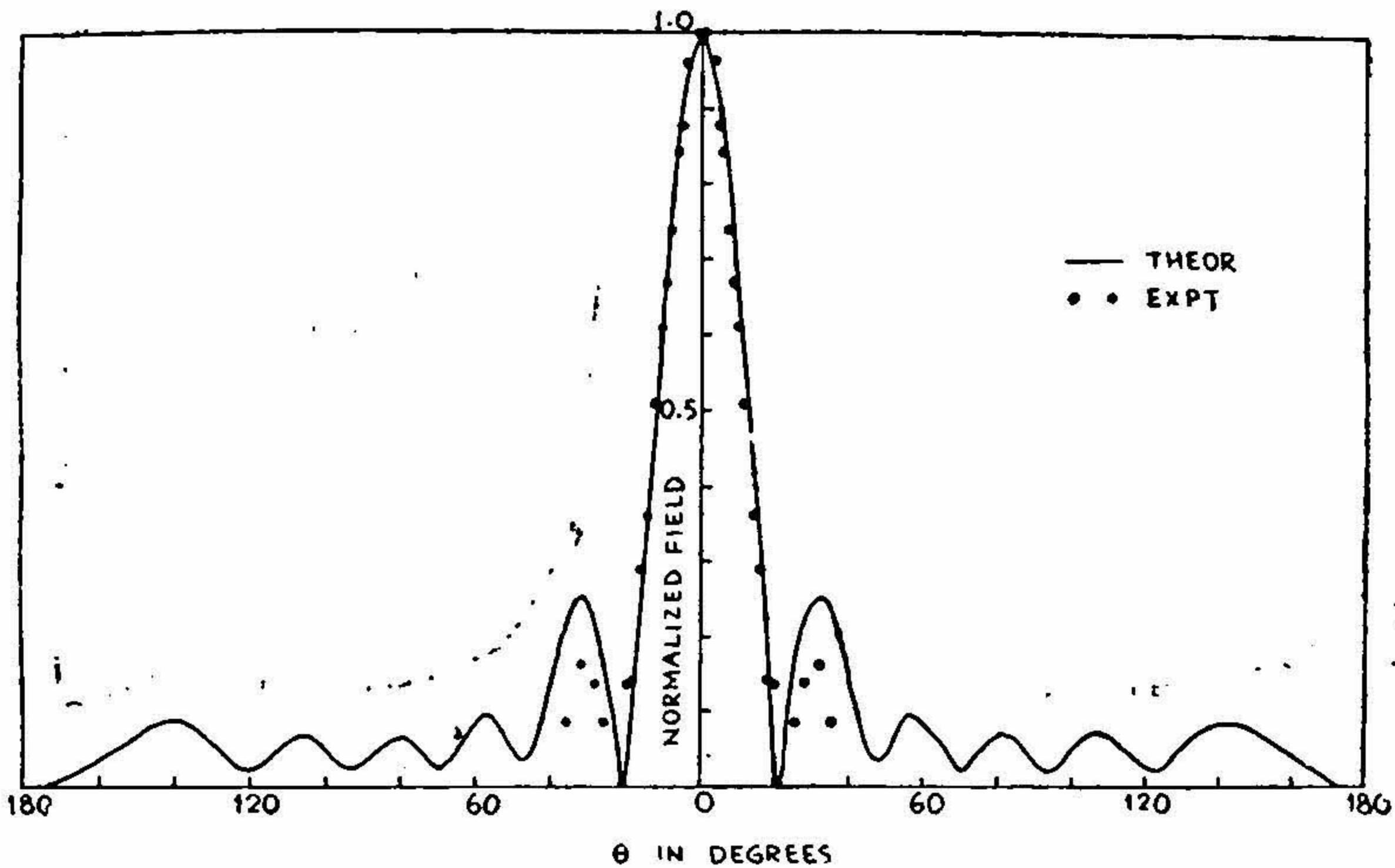


FIG. 4

Calculated and measured Radiation (Field) Patterns in the $\phi = 0^\circ$ Plane of Dielectric Spherical Aerial of Radius 4.45 cms. and $\theta' = 8.2^\circ$.

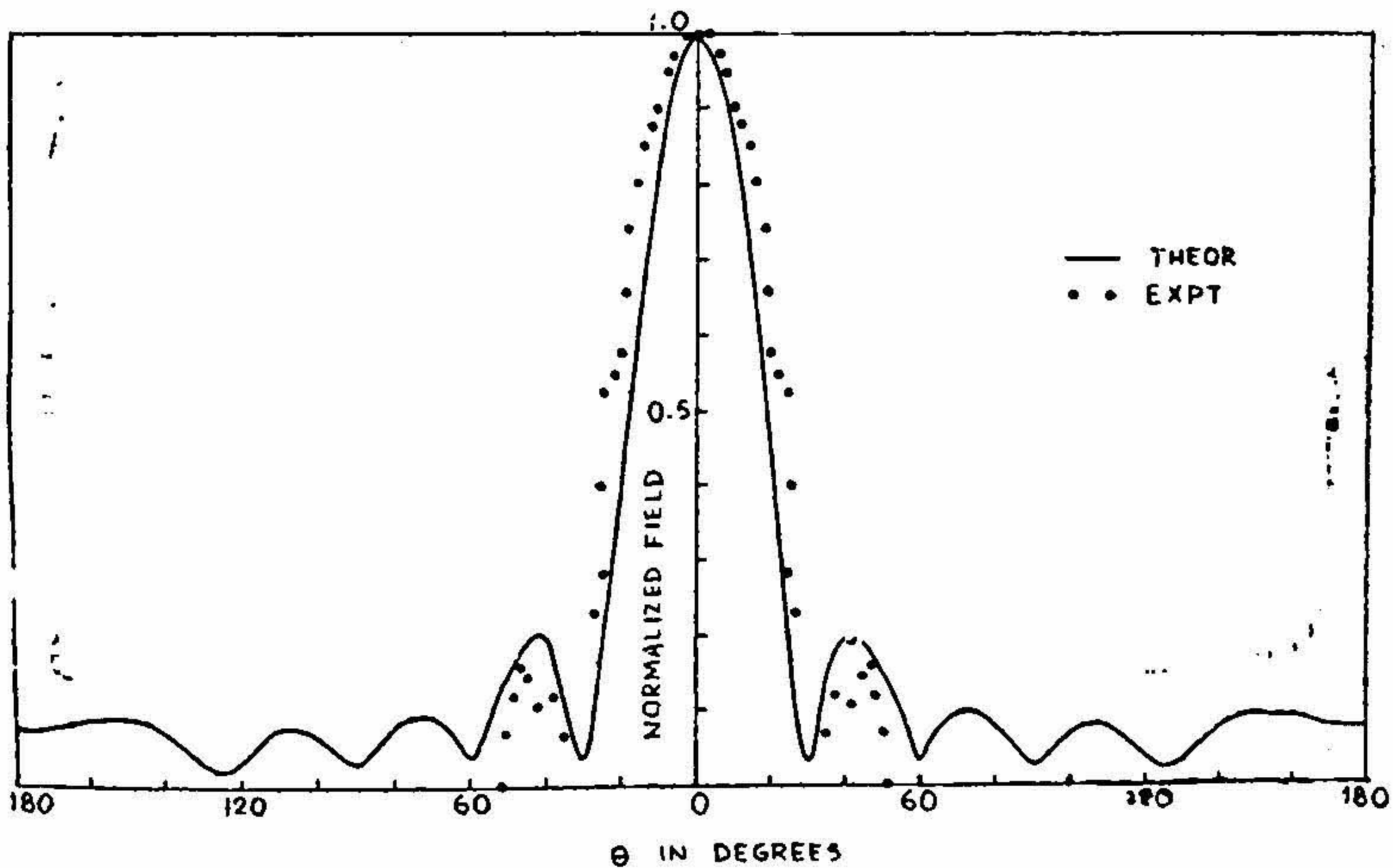


FIG. 5

Calculated and measured Radiation (Field) Patterns in the $\phi = 0^\circ$ Plane of Dielectric Spherical Aerial of Radius 3.5 cms. and $\theta' = 21.4^\circ$.

The gain of these aerials have been calculated using equation [13], using the best value of K for each aerial. Fig. 6 shows the gain of the aerials Vs. radius.

6. EXPERIMENTAL WORK

The radiation patterns of several spherical perspex aerials of radius varying from 1.28 cms. to 4.45 cms. have been measured in the $\phi = 0^\circ$ plane and compared with the theoretical results (Figs. 4 and 5). All these aerials have a radius R_1 of 1.27 cms. at the feed end where they are fitted into the cylindrical waveguide.

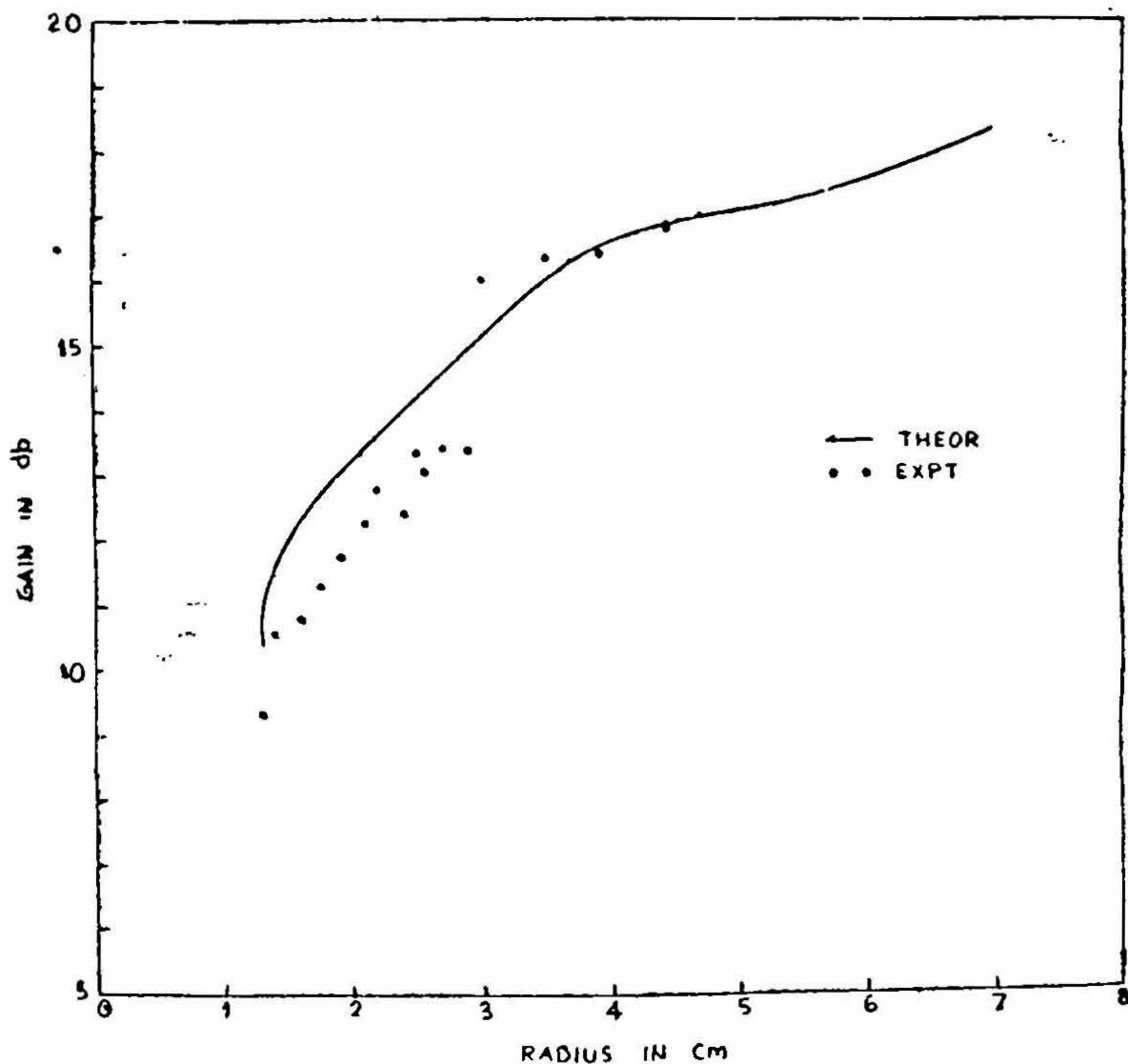


FIG. 6

Calculated and Measured Gain of the Dielectric Spherical Aerial Vs. Radius.

The gain of these aerials have been measured using Purcell's reflection method [9], and compared with the theoretical results. Fig 5 shows the experimental and theoretical curves of gain as a function of the radius of the sphere.

Fig. 7 shows the beam width of the major lobe VS. radius curves (both theoretical and experimental). Fig. 8 shows the theoretical and experimental curves of the positions of the side lobes Vs. radius.

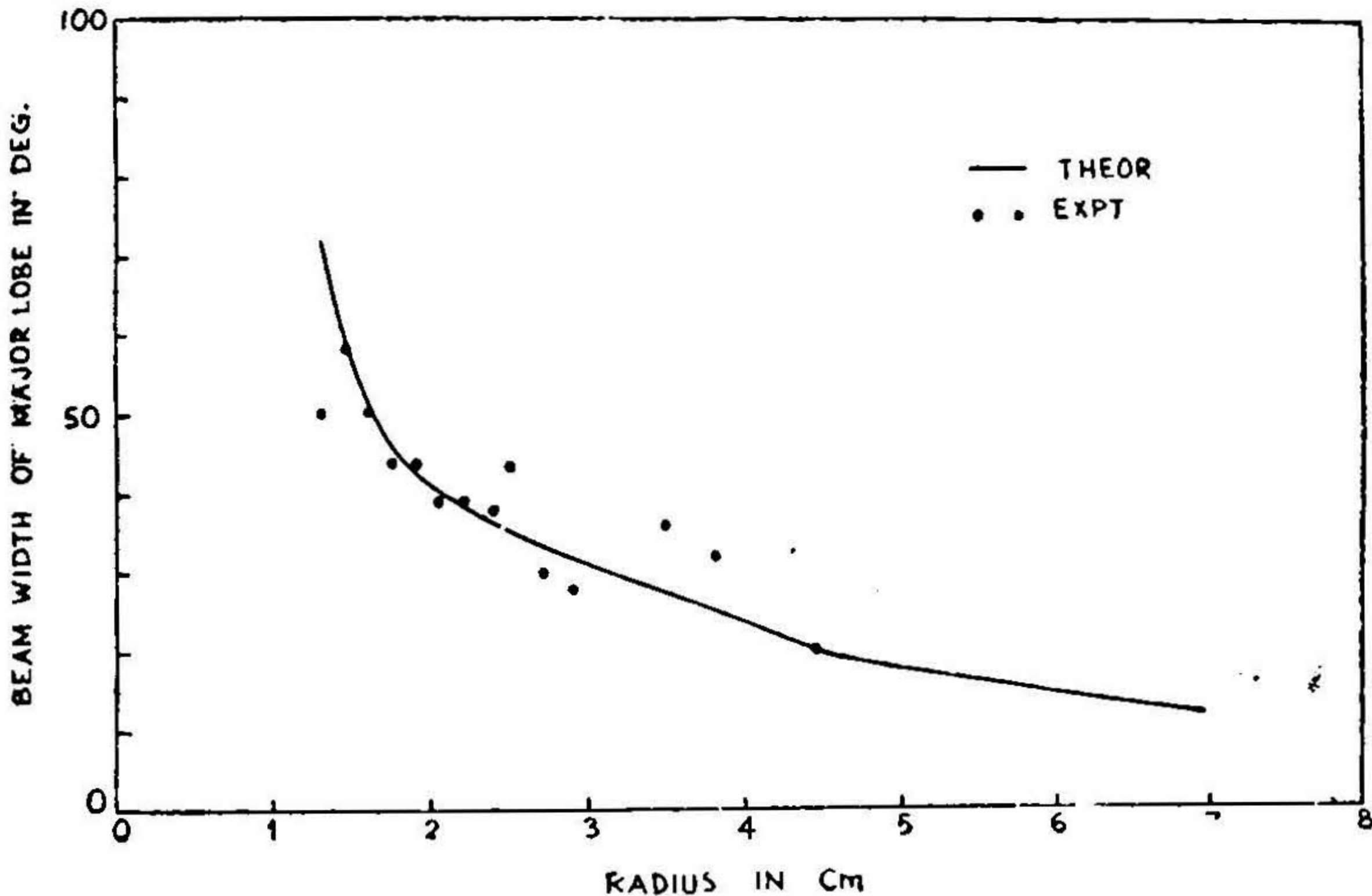


FIG. 7

Beam-Width of the Major Lobe Vs. Radius

The input impedance of the aerials have been measured by the standard method [9]. Fig. 9 shows the variation of the measured input impedance with radius.

7. CONCLUSIONS

The following conclusions can be drawn from the present investigations:

- (1) The dielectric spherical aerial excited by a circular wave guide in the unsymmetric hybrid mode behaves like a directional end-fire aerial with a narrow major lobe along the axis and a number of small minor lobes.
- (2) The beam width of the major lobe decreases and the gain increases as the radius of the dielectric sphere increases.
- (3) The measured input impedance of the aerials varies with the radius in an oscillatory manner.

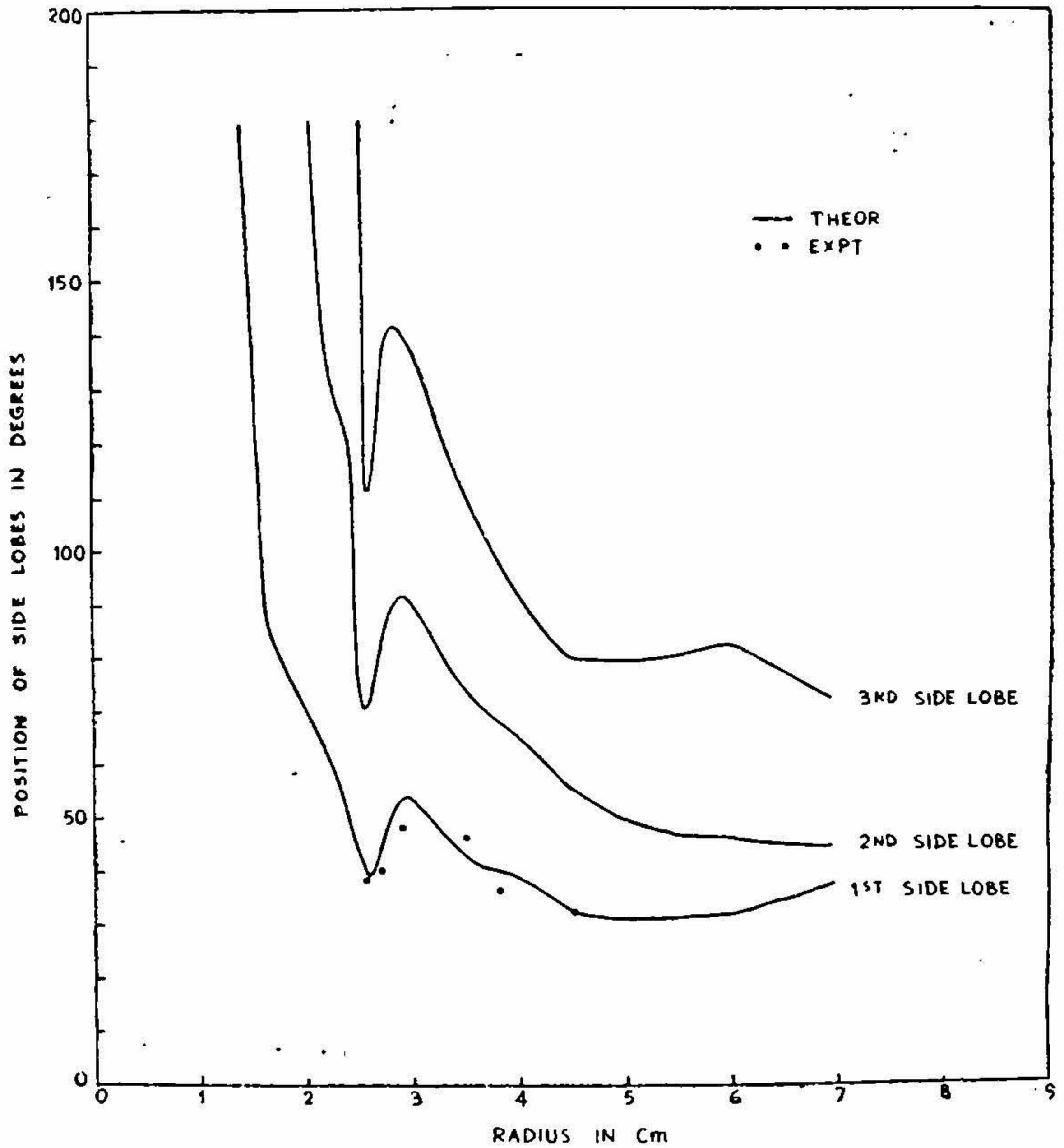


FIG. 8
Position of Side Lobes Vs. Radius

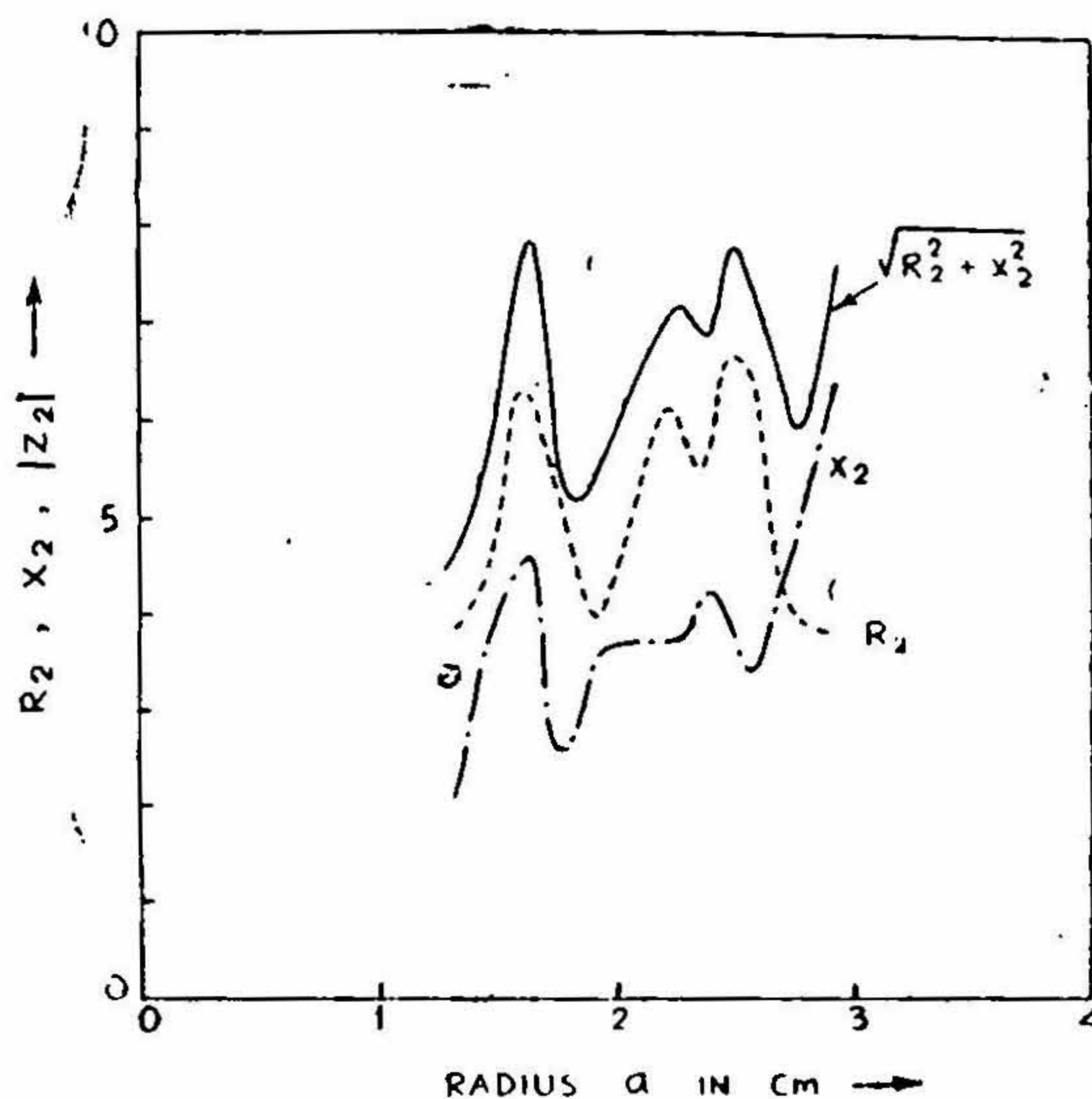


FIG. 9

Relative input Impedance of the Spherical Dielectric Aerial Vs. Radius

8. ACKNOWLEDGEMENT

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