LECHER WIRE MEASUREMENTS.

By S. R. Kantebet.

SYNOPSIS.

Experiments made with six-metre 50,000 k.c. stationary waves on parallel wires using various detecting arrangements indicated that the nodal points might be sharply defined and that reasonable accuracy was possible. An account of the apparatus used is given together with graphs of stationary waves observed.

A note is added relating to the effect of end conditions on reflection and the production of stationary waves.

EXPERIMENTAL.

The following is a brief account of some experiments carried out for the purpose of ascertaining the accuracy with which short radio waves could be measured by simple observations on the distribution of stationary waves along wires. It will be seen that wave-lengths of the order of six-metres 50,000 k.c. may be measured to within at least one part in 300 and it is to be concluded that methods may be developed to give very much better results. The main factor limiting accuracy is the degree of definiteness of the *nodal* or *anti-nodal* points in the current or voltage distribution. At first it was expected that this would be very poor and quite unsuitable for accurate measurements; for it was anticipated that the detector deflection when plotted would take the form of a sine squared curve giving indefinite maxima and minima regions. It was observed however, that under the conditions governing these experiments the stationary waves were not represented by the simple sinusoidal waves expected.

It appears that a third harmonic is present in the stationary waves and this results in a pronounced increase in the sharpness of the nodal points and consequently in the reliability of the measurements of the distance between nodes. The gain in definiteness to be expected may be seen from Fig. 1*a* which shows a pure sine wave distribution of current I and also a broken line graph of $(I)^2$ to which the galvanometer deflections should be proportional. The current nodal region is wide as indicated by the figure, and accurate measurements impossible. On the other hand, should the wires carry additional stationary waves which correspond to an odd harmonic of the fundamental frequency, such as the third harmonic, the conditions would then be as shown in Fig. 1*b* where the fundamental and harmonic component waves, shown dotted, give the resultant current wave I shown as a full line and $(I)^2$ shown as a broken line. The narrowing down of the current nodal region is such that reasonably accurate measurements become possible. These were the conditions obtained in the experiments when using a 50-watt valve generator adjusted for six-metres wave-length. The current distribution, measured by means of a crystal detector and Weston galvanometer of the moving coil type, was that given in Table I and Fig. 2 covering nearly two waves-lengths.

TABLE I.

Current distribution measurements. (Crystal detector and galvanometer.)

Distance feet	Galvanometer deflection	Distance feet	Galvanometer deflection	Distance feet	Galvanometer deflection
0·0	5·0	12·0	0·2	24·0	35·0
0·5	3·2	12·5	6·0	24·5	36·1
1·0	2·2	13·0	20·0	25·0	37·0
1·5	1·0	13·5	35·0	25·5	36·5
2·0	0·0	14·0	37·1	26·0	36·0
2·5	3·3	14·5	36·0	26·5	34·4
3·0	5·0	15·0	35·1	27·0	32·0
3·5	30·1	15·5	34·0	27·5	32·9
4·0	34·0	16·0	33·0	28·0	35·0
4·5	35·2	16·5	31·9	28·5	35·6
5·0	35·0	17·0	30·1	29·0	37·0
5·5	33·0	17·5	32·0	29·5	36·5
6·0	29·1	18·0	34·0	30·0	34·0
6·5	29·0	18·5	36·2	30·5	23·0
7·0	30·1	19·0	37·0	31·0	6·0
7·5	32·5	19·5	35·5	31·5	4·0
8·0	34·0	20·0	30·5	32·0	0·6
8·5	34·6	20·5	15·0	32·5	3·2
9·0	36·0	21·0	2·0	33·0	11·0
9·5	35·0	21·5	1·2	33·5	14·2
10·0	28·0	22·0	0·1	34·0	19·0
10·5	17·0	22·5	4·0	34·5	25·1
11·0	3·0	23·0	16·1	35·0	29·2
11·5	1·8	23·5	30·0	35·5	31·0
...	...	...	...	36·0	32·0

In Fig. 2 the galvanometer deflection is plotted to a distance base representing the actual positions of the detecting instrument as it was moved along the Lecher wires, closed at source and open at end. The presence of a third harmonic is most marked and the resulting increase in sharpness observed bears out the above statements. The four nodal points were ten feet apart ($\lambda = 20$ ft. $= 6{\cdot}1$ metres) and it was observed that a movement of the detector one centimetre on either side of the node produced a change in deflection which could be distinguished with certainty.[1] The error therefore was less than 2 cms. in 610 cms. for these waves of 6·10 metres length.

A somewhat similar distribution curve, having even more pronounced central depressions, has been obtained by Messrs. G.

[1] The galvanometer current at a point one centimetre on either side of the node varied between 2 and 4·5 micro-amperes.

Lamm and E. Graham[1] in whose experiments the Lecher wires were joined at one end and the high-frequency E. M. F. induced at a point near the other end where the wires were connected together through a resistance the value of which was adjusted by trial to produce symmetrical stationary waves.

Alternative methods of detecting the nodal points were also tried; in the first case a *neon tube* (Osglim lamp) in series with a 100 to 150 volt biasing battery was connected across the wires. Maximum illumination indicated the potential anti-nodes but it was not found possible to obtain sufficient accuracy in the measurements. In the second case the detector consisted of a vacuum thermo-couple[2] and sensitive galvanometer by means of which the results given in Table II and Fig. 3 were obtained. Whilst in agreement with the former results the latter method was not found as satisfactory as the former crystal detector method.

TABLE II.

Current distribution measurements. (Thermo-couple and galvanometer.)

Distance feet	Galvanometer deflection	Distance feet	Galvanometer deflection	Distance feet	Galvanometer deflection
0·0	5·5	12·0	6·0	24·0	5·2
0·5	4·5	12·5	4·7	24·5	8·0
1·0	3·0	13·0	3·0	25·0	10·5
1·5	1·4	13·5	3·2	25·5	12·9
2·0	0·5	14·0	3·8	26·0	15·0
2·5	0·8	14·5	5·0	26·5	16·5
3·0	3·0	15·0	8·0	27·0	17·0
3·5	6·1	15·5	11·2	27·5	16·6
4·0	9·4	16·0	14·5	28·0	15·0
4·5	12·0	16·5	16·0	28·5	13·5
5·0	14·5	17·0	17·2	29·0	12·5
5·5	15·7	17·5	15·8	29·5	13·0
6·0	16·2	18·0	15·0	30·0	13·0
6·5	16·0	18·5	14·2	30·5	14·0
7·0	15·5	19·0	12·5	31·0	12·1
7·5	15·8	19·5	13·0	31·5	9·4
8·0	16·3	20·0	16·1	32·0	7·0
8·5	16·6	20·5	16·3	32·5	4·1
9·0	17·2	21·0	13·0	33·0	3·0
9·5	18·0	21·5	11·1	33·5	2·8
10·0	18·5	22·0	5·5	34·0	3·6
10·5	16·0	22·5	4·0	34·5	5·0
11·0	11·0	23·0	3·5	35·0	8·1
11·5	9·5	23·5	4·0	35·5	10·2
...	...	...	...	36·0	12·1

[1] *Wireless World and Radio Review*, 1924, 15, 433.
[2] Cambridge Instrument thermo-couple, heater 38 ω; Weston pointer galvanometer, resistance 8 ω, full scale deflection of 30 divisions for 0·6 milliamp.

The Lecher wire system used was adjustable in length and consisted of two No. 18 S.W.G. bare copper wires about 36 feet long stretched tightly between porcelain insulators about 4 feet above the tiled floor of the laboratory. The wires were connected together in the form of a loop at one end, which served to couple the system to the anode circuit of the 50-watt valve generator, the other ends being open-circuited, as shown in Fig. 2.

Some rough experiments were next made to study the effect of proximity of the earth, etc., and the observations of previous experimenters were more or less corroborated. No change in the position of nodal points was observed when tables holding apparatus were brought within a few inches of the wires, although the deflections were reduced. The presence of iron masses was found to have a great effect. The coupling to the generator was changed considerably without alteration of the nodal points within the limits of observation.

It was also ascertained that for the constant length the diameter of the Lecher wires could be changed from 2·03 mm. to 1·21 mm. and the distance apart changed from 2 to 5 cms. without appreciable alteration in the nodal points as was to be expected.[1]

Prior to making the measurements described, the wave-length was estimated by the following rough methods :—

1. The 50-watt valve generator shown in Fig. 2 has a tuned anode circuit comprising the grid-plate inter-electrode capacity estimated from dimensions to be 10 to 20 micro-micro-farads, and a single turn coil having an inductance calculated[2] to be 490 cms. The corresponding wave-length is $\lambda = 59{\cdot}6 \sqrt{L_{\text{cms}} \times C_{\text{mfd}}} = 4{\cdot}2$ to $5{\cdot}9$ metres.

2. A short-wave detector consisting of a single turn of No. 14 S.W.G. bare copper wire connected to the terminals of a 0·1 milli-micro-farad variable air condenser to which were connected in series a crystal and a moving-coil pointer type micro-ammeter as shown in Fig. 4. With this detector loosely coupled to the valve generator and adjusted for maximum deflection, observations were made using four sizes of the coupling coil with the results given in Table III.

[1] The inductance of a pair of parallel wires is proportional, and the capacity inversely proportional to, $\log_e \frac{2D}{d}$, where D is the distance apart and d the diameter of the wires.

[2] *Bulletin of the Bureau of Standards*, No. 4, p. 150.

TABLE III.

Diameter of Loop in centimetres	Calculated Inductance in micro-henries	Estimated capacity in Mfds.	Calculated wave-length in metres
50	4·0	$0{\cdot}03 \times 10^{-6}$	6·5
10	·59	·198 ,,	6·3
5	·25	·339 ,,	5·7
2·5	·103	·683 ,,	5·6

FORMATION OF STATIONARY WAVES.

The formation of stationary waves on wires.—The possibility of forming pronounced stationary waves on Lecher wires depends upon the length of the system and the end conditions, i.e., whether the ends form an open-circuit or are conductively connected. Two types of reflection are encountered, one with reversal of the magnetic field only and the other with reversal of the electric field only, corresponding to the cases of non-conducting and conducting ends respectively.

Reflection at ends.—Consider a system of wires AB, Fig. 5, to have a pulse of potential induced at the end A, the lines of electric strain indicated by arrows gliding along the wires in the plane of the paper towards B. On arriving at B, if the energy associated with the electro-magnetic wave is not entirely absorbed, reflection takes place in one of the following ways :—

Case I. Open ends at B.—The electric charge travelling along the wires to the end B 'piles up' and as the electric lines accumulate because they cannot leave the wires, their momentum and mutual repulsion cause them to spread out as they come to rest. The magnetic field due to the movement of the electric field thus dies out and in so doing gives rise to fresh electric lines which double the electric field density at B. In this way the difference of potential at B is enhanced. A wave is reflected towards the end A owing to the contraction of the electric lines which are in tension. Apart from any radiated field, the mass of strain lines moves back towards A and in so doing causes new magnetic lines of *opposite sign* to come into being, and thus the electro-magnetic wave is re-established after reflection at the open end B of the system. This is usually referred to as reflection *without reversal* of the electric field.

Case II. Ends short-circuited at B.—In this case the lines representing the electric field, on reaching B, which is a perfectly conducting path, collapse and reform *reversed*, thereby generating new magnetic lines which double the magnetic density at B, where the current will be at its maximum value. This magnetic flux in collapsing, forms fresh electric lines which re-establish the electro-magnetic wave which then travels back towards the end A. This is usually referred to as reflection *with reversal* of the electric field, and is indicated in Fig. 6.

For end conditions at B intermediate between case I (open-circuit) and case II (perfect conductivity), i.e., for definite values of resistance at the end B, absorption of energy takes place accompanied by *reduced* reflection which itself may take place according to either of the foregoing modes, preponderance of one over the other being determined by the value of this resistance in comparison with the L and C constants of the line as shown below.

Stationary waves.—Let it be assumed that timed electric impulses are supplied at the end A, for instance an alternating voltage from a continuous wave generator coupled to the system, and that reflection takes place at the remote end B. The 'on-coming' and 'reflected' waves in passing each other will coincide at points $\lambda/2$ apart, where λ is v/n, thus giving rise to stationary waves of potential difference or of current, the latter being displaced by $\lambda/4$ from the former along the same system.

Partial reflection.—The resistance of the path at the reflecting end B may be that of a short-circuiting bridge, an energy absorbing device or the circuits of a measuring instrument and its effect upon the reflection may be stated as follows :—

Let R be the resistance at the end B and Z_0 be the surge-impedance[1] of the line wires, that is, the equivalent impedance by which the whole line may be replaced without altering the current in the zeroth section of the line itself, and which approximates to $\sqrt{\frac{L}{C}}$, in cases where inductance predominates over resistance.

The reflection taking place is expressed numerically by $k = (R - Z_0)/(R + Z_0)$ and the following conditions hold :—

(*a*) *'Open-circuit' at end B, Fig. 5.*—Because $R = \infty$ the reflection factor $k = 1{\cdot}0$ and complete reflection without reversal takes

[1] C. P. Steinmetz, 'Transient electrical oscillations.'

place with the formation of a potential 'loop' or anti-node at the end B.

(*b*) '*Short-circuit*' *at end* B, *Fig.* 6.—Because $R = 0$ the reflection factor $k = -1{\cdot}0$ and complete reflection with potential reversal takes place with the formation of a potential node at the end B.

(*c*) '*Resistance path*' *at end* B.—If $R > Z_0$ or $R < Z_0$ then k is a positive or a negative fraction and partial reflection takes place. On account of the energy loss in the resistance the amplitudes of the stationary waves are generally less than in cases (*a*) and (*b*).

(*d*) '*Perfect absorption*' *at end* B.—If $R = Z_0$ then $k = 0$ and no reflection takes place as the energy arriving at B is completely absorbed by the resistance path with the result that stationary waves are not produced.

It is evident that the exact arrangement of the conductor system including the conditions at the ends and the presence of measuring circuits, must be taken into account when attempting to deduce results from experimental observations made upon stationary waves.

It is also to be noted that in such a conductor system as that shown in Figs. 5 and 6, a second reflection with reversal takes place at the end A on account of the highly conducting path, and in order that this second reflected wave may synchronise with an induced wave from the source, the length of the conductors must be adjusted for correct phasing of the new and the reflected waves.

The propagation of electro-magnetic waves on well-insulated wires of negligible resistance and leakance is determined by the capacity C and inductance L per unit length of the pair of conductors and is represented by the equation

$$\frac{d^2 V}{dx^2} = C.L.\frac{d^2 V}{dt^2} \qquad *$$

where V represents voltage at a distance x and at time t. Integrating, $V = \cos A\left(x + \frac{t}{\sqrt{CL}}\right)$.

Substituting $\left(x + \frac{2\pi}{A}\right)$ for x with t constant, or $\left(t + \frac{2\pi\sqrt{CL}}{A}\right)$ for t with x constant, the value of V remains unchanged, showing that

* J. A. Fleming, 'Propagation of Electric Currents in Telephone and Telegraph Conductors.'

values of V *repeat at space intervals* $\lambda = \frac{2\pi}{A}$ and time intervals $T = \frac{2\pi\sqrt{LC}}{A}$. Therefore the velocity of propagation $u = \frac{\lambda}{T} = \frac{1}{\sqrt{LC}}$.

The following relations between the length of the conductors and wave-length hold :—

'*Open-circuit*' *at* B.—Thus if a continuous E.M.F. of frequency n is applied to the joined ends (at A) of a pair of wires of length (each) $l = \frac{1}{4}\frac{1}{n\sqrt{LC}} = \frac{\lambda}{4}$ a stationary wave of the form indicated in Fig. 7 will be set up and the system is said to be oscillating at its fundamental frequency.

A positive P.D. applied at A will cause a wave to glide along the wire to B where reflection *without reversal* takes place. The unreversed reflected wave returns to the origin A which it reaches in half a period after having left it. At A it is again reflected this time *with reversal*, so that it is now in exact phase with the negative impulse from the source.

Thus the impressed E.M.F. causes the waves reflected to and fro to grow until the energy losses in the system exactly consume the energy input from the source, after which stationary waves are maintained along the conductor having a constant amplitude at a given point. Stationary waves of the above type, i.e., without potential reversal at B, are also formed when the length of each wire is an odd multiple of $\frac{\lambda}{4}$ and the system is then said to be oscillating at a harmonic of its fundamental wave-length.

'*Closed circuit*' *at* B.—In this case reflection at the end B takes place *with reversal* and the fundamental stationary wave for the given frequency can be obtained only by doubling the wire length, so that $l = \frac{\lambda}{2}$, the distance between the node and anti-node still being $\frac{\lambda}{4}$, and the shape of the potential distribution being that shown in Fig. 8.

In this case the potential wave undergoes reflection *with reversal* at both ends A and B, and in order that the reflected wave may be correct in phase after reflection at A, it is necessary for half a cycle to elapse during its passage from B back to A.[1]

[1] A pair of wires of length $\frac{\lambda}{4}$ and short-circuited at both ends, although unable to resonate at fundamental frequency, may carry stationary waves corresponding to the second harmonic of the impressed voltage if the latter is not a simple harmonic wave.

Thus the length of each wire must be an integral multiple of $\frac{\lambda}{2}$, for producing standing waves when both ends are closed.

It is to be noted that even with all the conditions as to length fully satisfied, there can be no stationary waves formed when the ends of the conductors are connected together through a resistance $R = Z_0 = \sqrt{\frac{L}{C}}$, the surge impedance of the wires, because all the energy supplied is absorbed and reflection is completely suppressed.

A further case of interest is that in which the conductors are several wave-lengths long. At each point of maximum current, e.g., at $\frac{\lambda}{2}$ from the perfect conducting bridge at B, the addition of a perfectly conducting bridge does not alter the waves in form or position. It might be thought that an outgoing wave from A on reaching B (Fig. 9), would experience complete reflection with reversal, so that no effect would be observed between B_1 and B_2 or B_2 and B.

Such is not the case, however, because the portion B_1 to B_2 of the circuit would act as a low resistance path to any change of flux, should the field collapse in the bridge B_1. Consequently reflection does not take place at the intermediate bridges B_1, B_2, etc., but only at the final bridge B, as is found by experiment.

The introduction of a resistance path of surge impedance value for the purpose of suppressing reflection gives a method of eliminating the distortion of stationary waves in cases where undesired reflections may arise. Examples of the employment of such resistances are found in measuring circuits and in some receiving aerial circuits, such as the 'Beverage'[1] antenna designed for directional reception. In its simple form such an aerial consists of two horizontal conductors, the lower one of which may be the conducting earth, arranged as shown in Fig. 10, with ends joined at B which provides coupling to the receiving amplifier. At the remote end A the conductors are joined by a non-inductive resistance, having the value $R = \sqrt{\frac{L}{C}}$, which constitutes a practically perfect damper or absorber of energy.

Electric waves arriving at A in the direction shown by the arrow, provided they have a horizontal component, induce a voltage between

[1] *J. Amer. Inst. Elec. Eng.*, 1923, **42**, 261.

the conductors which builds up towards the receiver end B where it is detected.

Waves coming from the opposite direction cause a similar building up of voltage towards the remote end A, but on account of absorption in the surge impedance these waves are *not* reflected back to the receiver end B, with the result that this aerial has good directional properties.

Before concluding, the writer wishes to acknowledge his indebtedness to Prof. J. K. Catterson-Smith, for his help and suggestions.

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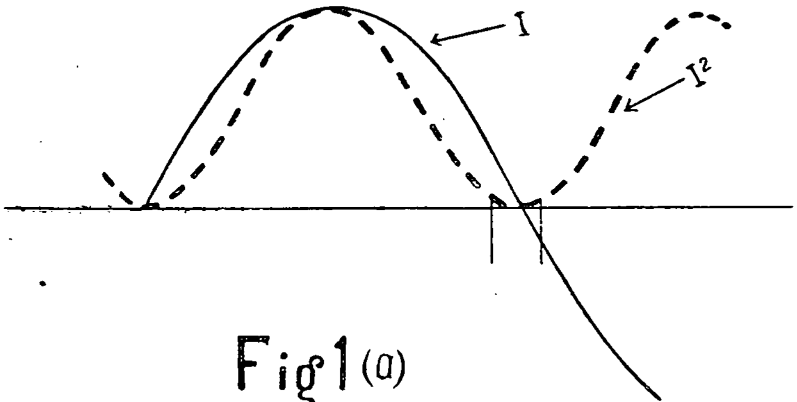


Fig 1 (a)

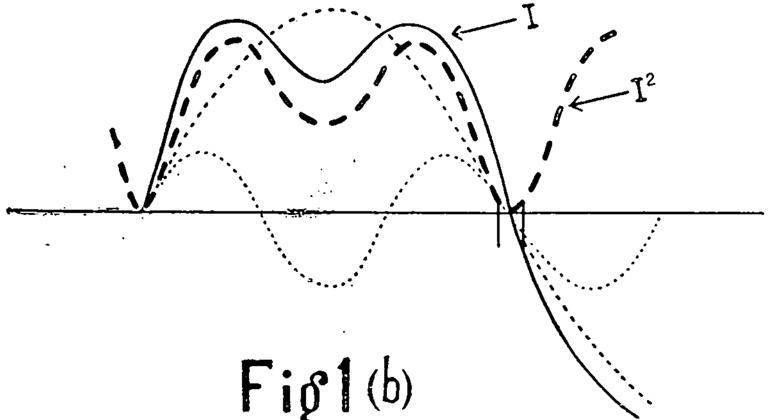


Fig 1 (b)

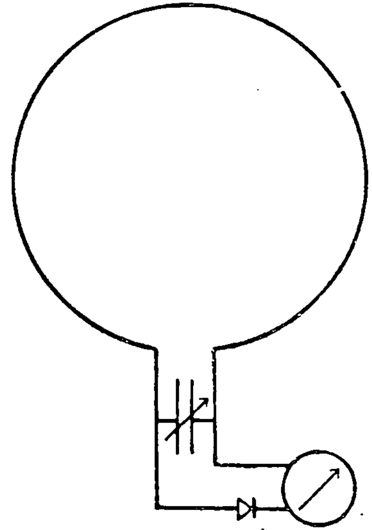


Fig 4

SINGLE TURN WAVE METER.

Current Distribution along Lecher wires.
(Crystal & Calvo)

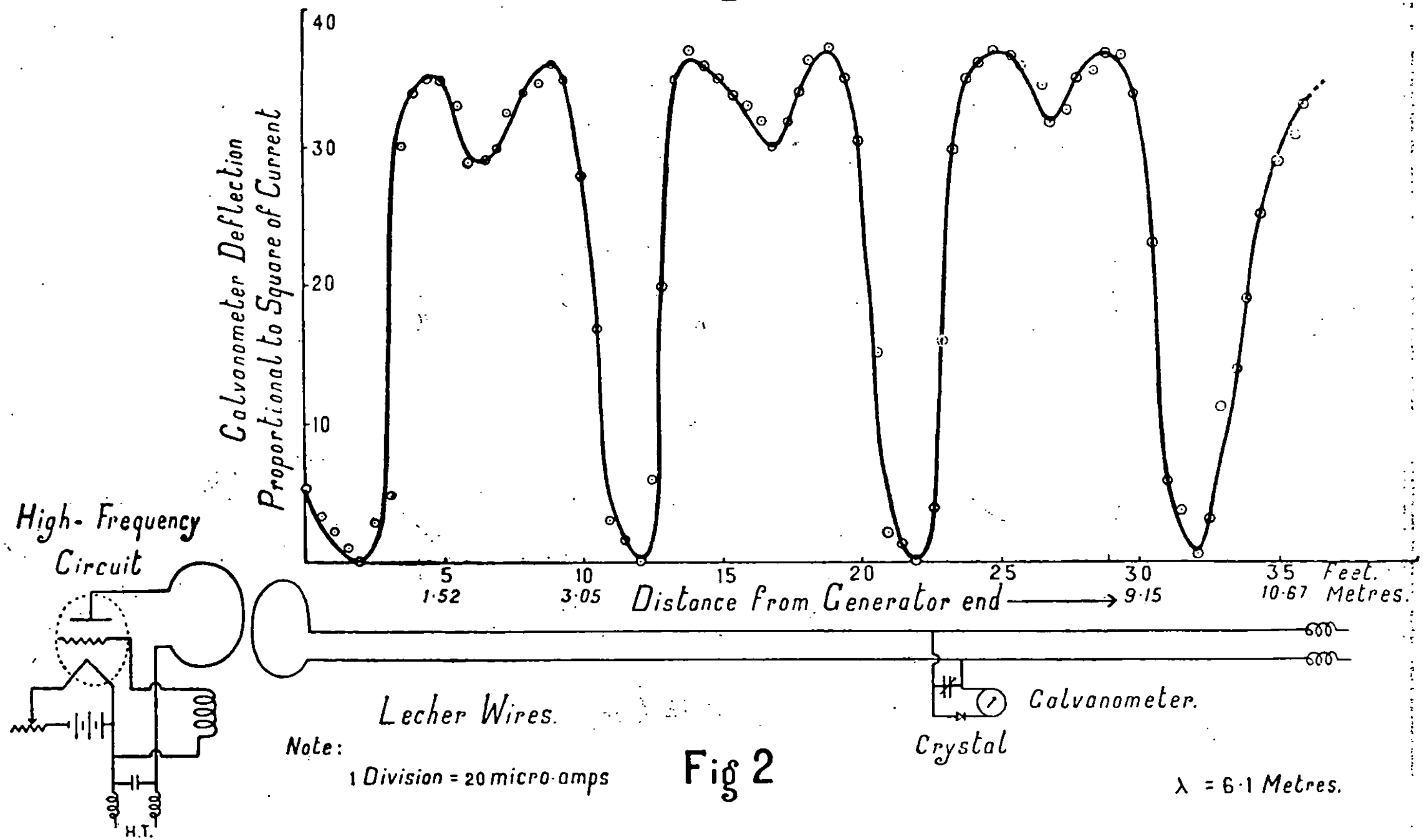
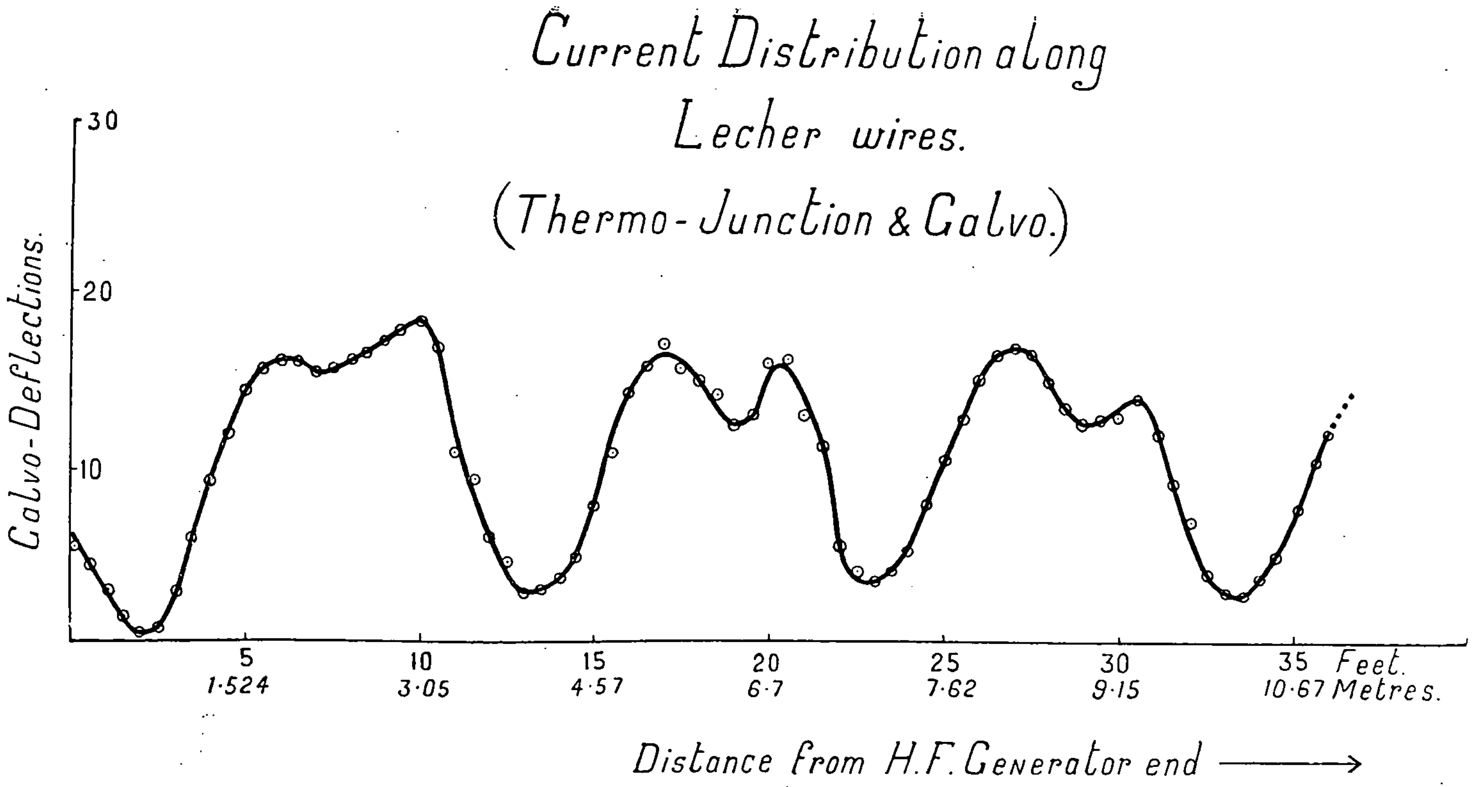


Fig 2



Note: 1 Division = 20 micro amps.

Fig 3

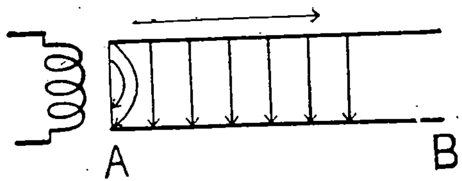


Fig 5

Reflection without reversal of electricfield.

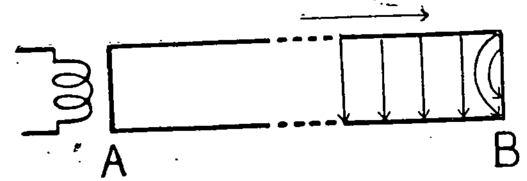


Fig 6

Reflection with reversal of electricfield.

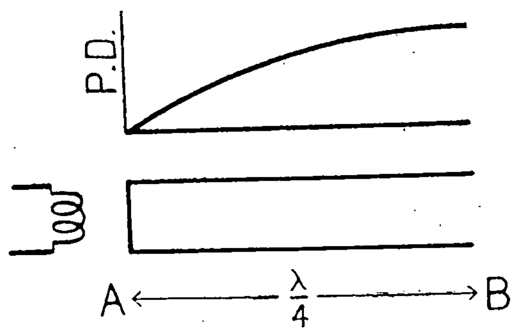


Fig 7

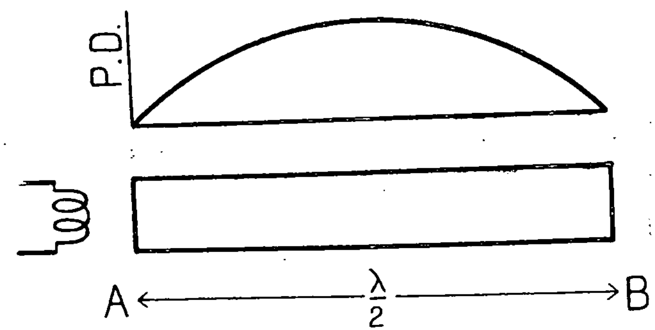


Fig 8

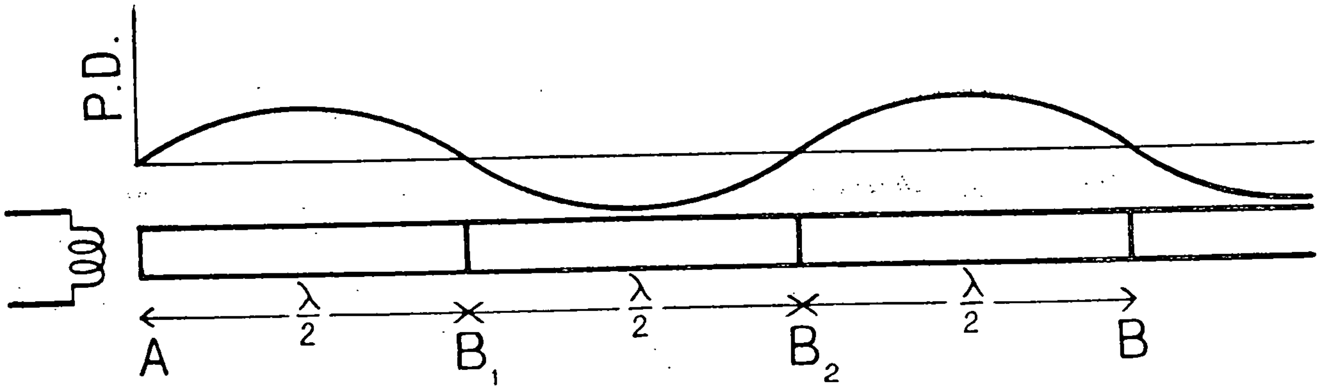

P.D.
$\frac{\lambda}{2}$
$\frac{\lambda}{2}$
$\frac{\lambda}{2}$
A
B_1
B_2
B

Fig 9

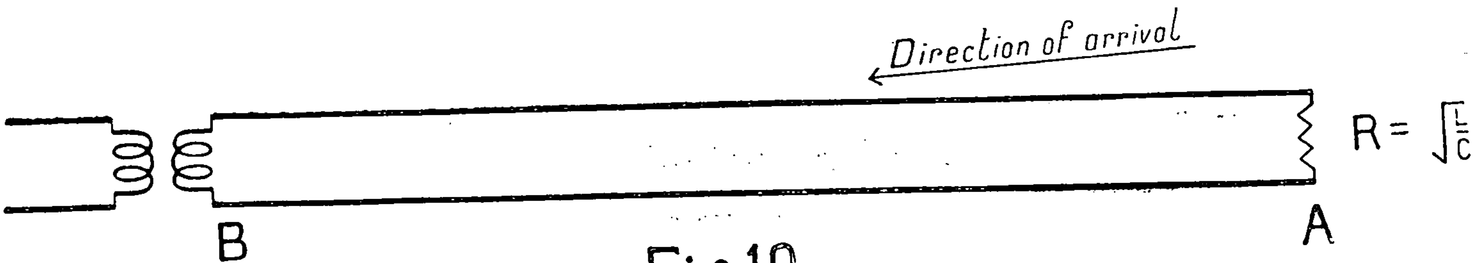

Direction of arrival
$R = \sqrt{\frac{L}{C}}$
B
A

Fig 10