THE METHOD OF EQUIVALENT REPLACEMENTS APPLIED TO THE INVESTIGATION OF FORCE TRANSFER AND POWER EXCHANGE IN A STAGE OF A TURBOMACHINE*

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Abstract

An analysis of the complicated phenomenon of power transfer in a stage of a turbomachine is presented in this paper in the simplest manner by the introduction of an original concept of "An Equivalent Stream" and the connected "Mathematical Profile". This method of investigation is very useful in connection with the development of gas turbine units, having both compressor and turbine stages.

There are two acknowledged theories on arising of force on blade, worked out by the great mathematicians Euler and Joukowski who lived in different ages and had in mind different problems. Both the theories give the same results as regards the integral force on blade, derived by the help of Euler's equation. This paper is a further development of the scientific heritage of Euler and Joukowski. Here is explained as to how both the theories are not to be opposed in favour of one against the other. One of them throws light on some aspects and the other treats the other sides of the same phenomenon. The idea advanced is "any engineering method of design is a method of evaluation by means of replacement scheme of phenomenon". The replacement scheme has to be as simple as possible and comprehensive.

On the basis of Euler's theory, supplemented and developed with ideas on equivalent stream and mathematical profile, it is proved that the method of velocity triangles is the method of hodographs. The very important rule of "the leading and lagging vectors" is formulated and all the possible types of motion of stream are classified and considered from the point of view of theoretical mechanics of equivalent stream.

A general scheme of development of mathematical profile is presented and this gives the scope of enveloping all forms of skeleton of conceivable substantial blade profiles. The scheme is, thus, universal in character. It further characterises the gradual transition through "the mechanically transparent system", from field to field namely turbine regime to compressor regime, the two opposite manners of force interaction. The paper presents the manner of development of turbine profiles, compressor profiles and their attendant characteristics.

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To explain the direct mechanism of power interaction between flow and blades, the concept of "the indicated diagram" is introduced, based on the idea of "mean indicated pressure" acting on the axial projected area of the blade. This idea connects a turbomachine with a reciprocating engine and helps to simplify the method of experimental investigation of a turbomachine stage. Only the moment on the shaft, the inlet and exit total pressures, the total mass flow and the shaft speed are all the ones that are necessary unlike the existing and the commonly used practice of using a stationary system of blades to give data. The idea suggests extraction of simpler experimental data from a dynamic set-up, the real approach to the actual phenomena, to evaluate the characteristics of the profiles used.

One of the basic ideas of the paper is that both the theories of Euler and Joukowski supplement each other covering different aspects of the same question and cannot avoid each other, being only a resulting simplified method of evaluating the integral force arising in the stage of a turbomachine. In the paper it is proved that Euler's theory can give very interesting results by the new concept described. The idea of "indicated diagram" has to be considered as a further development of Joukowski's wing theory applicable to turbomachines.

NOMENCLATURE

The following nomenclature is used in this paper:

A	thermal equivalent $\frac{1}{427}$ Cal./kg.m.
b	breadth of blade.
c	actual absolute velocity (m./sec.).
$\vec{c}$	absolute velocity vector.
$c_{2(m-1)}$	the absolute exit velocity from preceding stage $[c_0{}^2 = \mu_0 \ c_2{}^2{}_{(m-1)}]$.
d	wheel diameter at mid-length of blade.
g	acceleration due to gravity (m./sec.^{2}).
G	total fluid flow rate (kg./sec.).
h_0	stage thermodynamic enthalpy drop along the isentropic path (Cal./kg.).
h_0'	$h_0 + A \cdot \mu_0 \cdot \frac{c_2{}^2{}_{(m-1)}}{2g} \cdot$ (Cal./kg.).
l	length of blade.
$l^t{}_{1,2}$	theoretical work done (or absorbed) per 1 kg. of fluid, from inlet to exit of blades (kg.m./kg.).
M	moment on shaft.
m	mass per second.
$(m-1)$	the stage preceding the considered stage, m.
N	R.P.M.

W_i indicated power on blades.

P_a axial turning force.

P_u circumferential turning force.

P_{ui} impulsive part of P_u.

P_{ur} reactive part of P_u.

p_i mean indicated pressure.

p pressure.

r radius.

u circumferential velocity (m./sec.).

$\vec{u}$ circumferential velocity vector.

w relative velocity (m./sec.).

$\vec{w}$ relative velocity vector.

Δw_{ue} difference between the circumferential components of $\vec{w}$ at exit and inlet of the *exit stream*.

Δw_{ui} difference between the circumferential components of $\vec{w}$ at inlet and exit of the *inlet stream*.

Δw_u difference between the circumferential components of $\vec{w}_1$ and $\vec{w}_2$, *i.e.*, inlet and exit relative velocity vectors of the *equivalent stream*.

W work done (or absorbed) by a fluid flow rate G. kg./sec. (kg.m./sec.).

W_b work done on blading.

Z number of streams; number of blades.

α_1, β_1 angles the velocities c_1 and w_1 respectively make with the circumferential direction.

α_2, β_2 angles the velocities c_2 and w_2 make with the contrary circumferential direction.

β_{2b} exit angle of real blade to the contrary circumferential direction.

η_{rb} efficiency relative to blading.

μ_0 utilisation factor of absolute exit velocity from preceding stage.

ρ degree of thermal reaction.

ρ_{ur} degree of circumferential force reactivity.

ρ_{ui} degree of circumferential force impulsivity.

ψ blade velocity coefficient.

ϕ	nozzle velocity coefficient.
θ	pitch of blades.
Γ	circulation.

SUBSCRIPTS

1	inlet to blade.
2	exit to blade.
0	preceding stage.
a	axial component.
b	blade.
rb	relative to blade.
dem	demarcating.
eff	effective.
i	impulsive part; indicated.
is	isentropic.
n	nozzle.
opt	optimum.
r	reactive part.
t	theoretical; pitch.
u	circumferential component.

INTRODUCTION

This investigation is an attempt to explain the complicated phenomena taking place in a stage of a turbomachine and arrive at expressions, simple but comprehensive, for practical design. An important aspect is the physical explanation as to how the force and the power on blading arise. The basis for the explanation is the Euler's equation connecting the change in momentum of the fluid and the change of the moment of external forces on the blades. The use of this method requires the fixing of the velocity triangles and has been the only method used for a long time in practice by the manufacturers of turbines. Though this method has been universally adopted, during the last few years many have disputed this. Transition from the "fluid theory of Euler" to the "Cascade Theory—the aeromechanical design of flow around the blades" has been recommended. The cascade theory is based on the concept of potential flow with circulation. However, its use is based on experimental results from fixed cascades of blades.

The basic drawback of Euler's theory is that it does not explain the mechanism of force transfer. But Joukowski's wing theory and the cascade theory, based on wing theory, explain the mechanism of force transfer or interaction, which forms the basis of design. Euler's theory, as its opponents point out, supposes the existence of uniform field of velocities and pressures across the flow in any cross-section of the flow annulus. This may not be the actual case and individual streams would differ in magnitude and direction of their velocities and character of change of pressure along the stream. Thus, forces differing in character act around the blades and results in a net pressure difference between the concave and convex surfaces. It, thus, becomes necessary to apply a moment to the turbine shaft equal and opposite to the moment caused by the net pressure difference in the fluid, on the blades. Hence, work has to be performed on the shaft to maintain fluid flow or *vice versa*, *i.e.*, mechanical work can be obtained if the fluid flow is maintained. Thus the concept of potential flow with circulation around wing's profile and the idea of circulation of velocity are introduced in the theory of the turbine. However, the design formulæ take the same form as those derived from Euler's theory. For instance, to determine the force on the blade, the inlet and exit velocity triangles have to be drawn, as is the case in the Euler's theory. To remove the drawback attached to the Euler's theory, *i.e.*, the irregularity of the flow near the blade, a new idea of velocity at infinite distance has to be introduced. It, then, becomes essential to imagine that far in front of the blades and far behind, the flow is uniform, as is the case with the theory of the airplane wing. Referred to a stage of a turbomachine, one cannot really speak of velocities or flow at infinite distance. We can explain the phenomenon of interaction between blading and flow, in an axial turbine, by the following manner:—

The flow in the system of nozzles accelerates due to the transition of the random motion of the molecules, *i.e.*, heat energy, into regular motion of the molecules and has vortex motion, since nozzles are installed inside the cylindrical casing of the turbine. This accelerated vortex flow enters the system of blading, where force interaction, with the resultant power interaction, takes place. The blading system assembled on the periphery of the rotor has the single freedom of movement in the rotary motion. The rotor, thus, rotates as the result of the energy transfer from the flow to the blades, and the flow decelerates due to loss of energy. While the flow is traversing the system of blading,

(1) energy consumption from the turbine shaft, as a result of energy transfer from flow to the blading and thence to rotor and shaft; and

(2) part of energy transition into irregular motion of the molecules, *i.e.*, into heat (due to friction and energy losses in the vortices and waves) occurs.

In a reaction stage, while the fluid traverses the system of blading, the fluid accelerates in its movement relative to the blades due to the continued process of heat energy transition to kinetic energy. At the same time, the absolute movement gets untwisted, *i.e.*, the circumferential component of the momentum (of the absolute movement), decreases. If the absolute exit velocity is axial, maximum energy transfer to the blades takes place and the turbine efficiency will be maximum. The flow through the blades is turning in the direction opposed to the motion of the blades and the range of turn or the magnitude of untwist can be controlled by the value of the blade circumferential velocity, *i.e.*, by the control of the moment on the shaft. If the circumferential velocity is low the effect of untwist is high and is accompanied, consequently, with change of direction of twist of absolute velocity. On the other hand, if the circumferential velocity is increased by reducing the outer moment on shaft, the flow at exit to the blading has insufficient untwist and has high absolute velocity. If the flow had left vortex flow at entry to the blades, it would still have left vortex flow at blade exit but the fair twist would be less.

1. *The interrelation between Fluid Theory of Euler and Theory of Wing*

The problems of research and design of the phenomena taking place in the stage of a turbomachine, *i.e.*, the nozzles (or guide blades) and blades, can be formulated as:

(1) calculation of nozzle annulus cross-sections and the passage between blades;

(2) calculation of the resultant force between flow and blading without examining the precise mechanism of power transfer;

(3) calculation of stage efficiency;

(4) study of the mechanism of force interaction of flow with blades, while the flow is maintained past the blades;

(5) study of the influence of the blade form on flow around blades and the resulting force interaction between flow and blades.

Problems 1, 2 and 3 can easily be evaluated by the use of Euler's theory and expressions derived therefrom. By determining the mean velocity in any cross-section, the dimensions of the cross-section or flow annulus can be calculated for the given design conditions. Mean velocities can be determined from the equations:

$$c = \frac{\int^{(f)} c_i \, df_i}{f} \text{ and } w = \frac{\int^{(f)} w_i \, df_i}{f}$$

This method of designing the cross-sections, by the adoption of the idea of mean velocities, is an accepted procedure, as otherwise even the simplest problems like: "the determination of the required diameter of a round pipe to accommodate a given flow"—will become unnecessarily complicated. Expressions derived from Euler's theory, based on the commonest principles of theoretical mechanics, easily provide data for the calculation of the net force acting on the blading. This force and its components can be calculated to a precise degree of accuracy. It has been shown, over many years of design practice, that these results have tallied with experimental data, wherever it could be had. The mean velocity vectors—absolute inlet and relative exit and the stage efficiency could be determined from experimental data.

Thus three of the five possible problems of research or design could be successfully worked out on the basis of Euler's theory. Whereas, the same theory does not provide means to work out the problems of the mechanism of power transfer between flow and blading and does not explain the influence of blade form. In these cases, it becomes necessary to resort to other methods of investigation—experimental research of flow around blade cascades—the cascade theory based on wing theory. What interrelations must exist between Euler's theory and cascade theory? This question is appropriate since in the application of both the theories, it is always essential to bear in mind the form of velocity triangles of the stage. Evidently, it is the profound link between the two theories—they supplement each other.

2. *The equivalent stream*

Transition from the region of kinematics to that of dynamics can be effected by multiplying all the vectors of the velocity triangle by the factor m, the mass per second of the fluid flowing through the passages between the blades. Thus instead of the velocity triangle, there will be the momentum triangle. The transition can be justified since Euler's fluid theory presupposes that all the particles (any stream) in any cross-section have equal velocities. For this reason, we need draw only one set of velocity triangles, which is common for any stream and for the whole flow, all the streams being identical. To evaluate the force of Joukowski, based on his theorem, we have to bear in mind the possibility of determining the inlet and exit velocity triangles. Here we have however to examine as to which of the streams the velocity triangles refer, especially since the velocity field is very irregular in any cross-section of the flow and the idea of velocities at infinite distance in front of or behind the profile is not applicable to turbomachines.

It is in this connection the authors recommend to use the idea of the "equivalent substantial stream" or more simply "equivalent stream", replacing the total flow in theoretical considerations. Let a mass flow G/g. per second of a fluid (gas or steam) flow through the passage formed between two neighbouring blades. The cross-field of velocities and pressures is irregular along both the circumferential and radial directions in any axial cross-section of the turbine. Each one of the streams is characterised by the local individual velocity, own individual mass per second and individual stream line. Individual momentums are different being proportional to the mass, and the change of momentum of any stream determines its force interaction with the adjacent streams or with the defining surfaces of the blades.

Theoretically it is possible to draw the velocity triangles, the hodographs of absolute and relative velocities and if the mass per second is determinable—the hodograph of momentum. However, it is not an easy matter—practically. "The equivalent stream" (Fig. 1) has to replace the entire lot individual streams and can in some manner bring about the exchange of momentum between the blades and the fluid. (We depart here from the mechanism of energy transfer.)

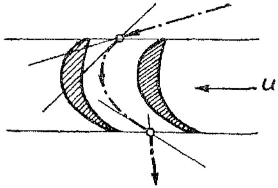

FIG. 1. The Equivalent Stream

The conditions then, for the replacement of the total flow by the equivalent stream are:—

(1) The mass per second of the equivalent stream equals the mass per second of the entire fluid flow (gas or steam).

(2) The force of the equivalent stream on blading equals the force of the entire fluid flow on blading. (The immediate mechanism of force interaction is not considered, and we can assume that the equivalent stream does not touch the sides of the blades.)

(3) The force interaction between the equivalent stream and blading takes place by momentum exchange and equals the total force interaction between the individual streams and the defining surfaces of the blades.

(4) Work transfer by the equivalent stream equals the work transfer by flow.

(5) The circumferential velocity of the turbine wheel due to the action of the equivalent stream flowing in the middle of the flow passage equals the action of the total flow.

(6) The velocity along the equivalent stream in any cross-section of the flow passage between the blades is the mean velocity of the flow in the cross-section.

Thus, the equivalent stream, which replaces the flow in theoretical considerations, is similar to the flow in respect to mass, force and power interactions with blades and to any elementary stream in respect to its cross-section. If it is necessary to calculate the cross-section of the passage, the equivalent stream is replaced, in theoretical considerations, by the flow consisting of a number of streams having the same velocity (or at any rate the same component of velocity along the direction under consideration) and equally distributed mass. The idea of the equivalent stream enables the determination of the single set of velocity triangles and the hodographs; the total force interaction between the flow and the blades; work done by or on the flow and the power output or input on blading. Instead of Z number of streams with mass per second, each G/gZ, we can imagine, the equivalent stream with mass G/g, *i.e.*, the total mass flowing through Z passages and has the same flow line as the equivalent stream through a single passage defined by the two adjacent blades. Figure 1 depicts such an equivalent stream and does not by itself convey the mechanism of action of flow on blading, since the equivalent stream does not contact the blades' surfaces. However, it is apparent that the flow of equivalent stream is directed in a curvilinear manner by the reaction of the blades in having the circumferential motion—velocity u: resulting in transfer of momentum from flow to blades. It is thus possible to confirm that, just as the equivalent stream replaces (for convenience of theoretical considerations) the real substantial flow, consisting of a number of different streams, the transfer of force from flow to blades is realised from the irregular field pressures across the passage between the blades.

There are no contradictions between the theory of Euler and theory of Joukowski. But they differ in the consideration of the method of force transfer. The Euler's theory outlines that the total influence of the blades on flow is that the flow is deflected from the straight line (more exactly spiral or vortical) direction, the flow imparting equal and directly opposite force to blades. Such deviation means that force interaction between flow and blades takes place. But the immediate mechanism of force transfer is not disclosed. The theory of Joukowski tries to answer this problem, *i.e.*, "What is the force and by what manner does the flow apply it to the blades?" However it has not been possible to depart completely from Euler's theory. Joukowski's theory takes into account the irregularity in the field of velocities, pressures, etc., across any axial cross-section of the flow passage. To evaluate the total force Joukowski introduces the idea of circulation of velocity, expressed by the equation: $\Gamma = \int_{(l)} c \cos \widehat{(c, l)}\, dl$. We have to multiply both sides by the mass "m" of the individual streams, to transgress into the realm of dynamics from kinematics. Joukowski's theory makes the replacement thus: the blade is replaced, in theoretical considerations, by a system of vortices and the net resultant flow system, when the system of vortices and steady linear flow are combined, results in circulation of velocities. Such a system of replacements is much more complicated and in case of turbomachines at least this complication is unnecessary. The true picture of the phenomenon is retained, in all its complexity, to explain the arising of force. It is thus clear that by the introduction of the idea of "equivalent stream" the problem of calculating the total force interaction between the flow and the blades is made much simpler than the method based on Joukowski's theory, though the scheme of the phenomenon is very different from the existing one, in theoretical considerations. A true picture of any phenomenon, if it is very complicated, does not help engineering design. A replacement scheme, if it is successfully selected and tallied against sure experimental data without distorting the total result, can simplify calculations to a great extent. Engineering methods should above all be the simplest ones.

The idea of the equivalent stream, replacing the flow, is based on the competence of the commonly used method for calculation of forces and work output or input on blading of a turbomachine stage, by means of velocity triangles. The equivalent stream is only a replacement scheme based on—ideas behind Euler's theory, and it reduces the complicated phenomenon near blades to the problem of deflection of flow by blades and allows us to stray away from the idea of velocity circulation.

3. *Hodographs of absolute and relative velocities along the equivalent stream*

The net force acting on the blades can be evaluated on the basis of Euler's theory, by tracing the motion of the equivalent stream. In Fig. 2 AB and AD represent the absolute and relative stream lines of the equivalent stream in the absolute system of plotting for steady flow. $\vec{w}$ at any point is tangential to the stream line AD and moves with constant transferable velocity $\vec{u}$. $\vec{c}$ is tangential to the fixed absolute stream line AB. At any point on AB, *i.e.*, the absolute stream line, the path traced and to be traced in the relative motion are indicated by dotted and chain lines respectively. The absolute direction $\vec{c}$ deflects substantially from inlet to exit to the blades, due to the curvilinear motion of the relative movement. It is possible to draw the velocity triangle of $\vec{c}$, $\vec{w}$, $\vec{u}$ at any point on the absolute or relative stream line. $\vec{c}$ and $\vec{w}$ are bound vectors since their values depend upon the fixed vector $\vec{u}$ (both in magnitude and direction) and the condition of tangentiality to the respective stream lines at point of application. Moreover both $\vec{c}$ and $\vec{w}$ have variable magnitude. Only one of them can have constant magnitude, for instance, in the case of a theoretical impulse stage, w remains constant.

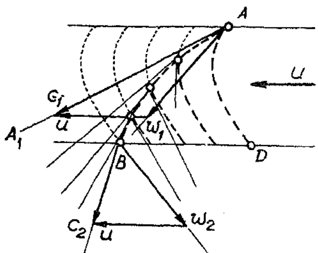

FIG. 2. Motion of Equivalent Stream (Absolute System of Plotting).

The study of the movement of the equivalent stream in the absolute co-ordinate system is difficult. It will be far simpler and convenient to study this movement in the relative system of co-ordinates, moving u, *i.e.*, along with the blades, and the equivalent stream replacing the flow.

In Fig. 3, AD depicts the motion of the equivalent stream in the relative co-ordinate system, *i.e.*, system moving with the blades. At any point on AD, $\vec{w}$ is tangential to AD. As we move along the equivalent stream, $\vec{c}$ changes both in magnitude and direction. *It is clear that the exit velocity triangle is the result of gradual transformation of the inlet triangle.*

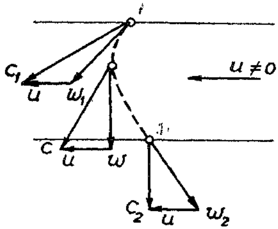

FIG. 3. Motion of Equivalent Stream (Relative System of Plotting).

We can multiply all vectors of the velocity triangle by the mass per second of the total flow, since the equivalent stream concentrates the flow mass, and we get the momentum triangles. As we proceed along the equivalent stream the absolute momentum changes indicating force interaction between flow and blading. As $\vec{u}$ gets lower in magnitude, $\vec{c}$ is drawn closer to $\vec{w}$, till finally, when $\vec{u}$ is zero, $\vec{c}$ is the same as $\vec{w}$ and the velocity triangle, and hence the momentum triangle cannot be formed. Thus, when the blades are stationary ($u = 0$) the transformation of the inlet velocity triangle to the exit one, merely transforms into a simple turn of $\vec{c}$ from $\vec{c}_1$ to $\vec{c}_2$. Change of $\vec{c}$ reflects change in momentum and evidently the existence of force interaction between flow and blades. However no external work is done since the blades are stationary. Since any exit velocity triangle is only the transformed inlet velocity triangle, we can conclude that these two triangles fix the extreme positions of points in hodographs of $\vec{c}$ and $\vec{w}$. Therefore the method of velocity triangle is the method of study of the movement by velocity hodographs. It is possible to represent the character of the stream line of the equivalent stream by the hodographs of the absolute and relative velocity vectors. If we multiply the velocity vectors by the mass per second, we have the momentum hodographs and these give us the means to judge the character of force interaction between the blades and the flow, replaced in theoretical considerations by the equivalent stream.

For practice of design of turbomachines, it is quite enough to find out only the extreme points of velocity hodographs, *i.e.*, only determine the inlet and exit velocity triangles. This gives the possibility to evaluate the net circumferential force applied by the flow to the blades. Besides that, we can assume that in many cases of practice, one of the intermediate steps of transformation of velocity triangle from inlet to exit is the rectangular triangle having one of the cathetii vectors $\vec{w} = \vec{w}_0$ directed along the axis "*a*".

4. *The method of cylindrical cross-sections and its correction*

In the theory of axial turbomachine, as proposed by Joukowski, an elementary stage is considered for simplification of calculations. This elementary stage is defined by two cylindrical co-axial surfaces with radii r and $r + \Delta r$, *i.e.*, Δr the thickness remains constant in the axial direction. This idea is not sufficient for consideration of all problems in the theory and design of turbomachines. Probably an elementary stage defined by conic or curvilinear surfaces co-axial to the machine axis instead of cylindrical co-axial surfaces, is more apt. In this definition the thickness Δr of the elementary stage varies in the axial direction and the change (Fig. 4) will depend upon the constructive scheme of the machine, nature of the substance forming the flow, regime of flow (supersonic, subsonic, etc.) and the form of blades. On this basis, it can easily be observed that the mean relative velocity of flow changes. $\vec{w}$ changes both in magnitude and direction. $\vec{c}$, $\vec{w}$ and $\vec{u}$ form the velocity triangle and if it is possible to draw the inlet triangle, it means that the system of blades is in motion. The velocity hodographs indicate the gradual transition of the inlet to the exit triangle. The transition means not only force interaction but also power interaction between flow and the blades.

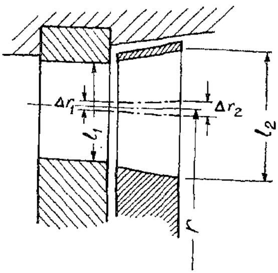

FIG. 4. Elementary Stage defined by Conic Co-axial Surfaces.

The equivalent stream, characterised by the variable $\vec{w}$, can replace the flow and this idea is also consistent for the modified elementary stage with Δr varying. We can now formulate the following:—

(1) If the mean absolute velocity at entrance $\vec{c}_1$ is determined, the entrance velocity triangle is then fixed by u (*i.e.*, by the value of the load in case of the turbine or the forcing moment in case of the compressor, applied to the shaft of the machine).

(2) If the mean relative velocity at blade exit $\vec{w}_2$ is determined, then again, the exit velocity triangle is fixed by u.

Therefore the force and power interaction between flow and blades are determined by: (1) $\vec{c}_1$ both in magnitude and direction (direction determining the circumferential component, *i.e.*, fair twist of $\vec{c}_1$). (2) The circumferential velocity $\vec{u}$ (*i.e.*, the moment-load or forcing- on the shaft).

(3) The change of mean relative $\vec{w}$ both in magnitude and direction (*i.e.*, the form of blade profile and blade length from inlet to exit).

By the study of velocity hodographs, it is possible to interpret the character of power interaction. In Fig. 5 the flow is directed from the top and the blades are moving to the left. Figure 5 *a* represents a turbine stage where: (1) the

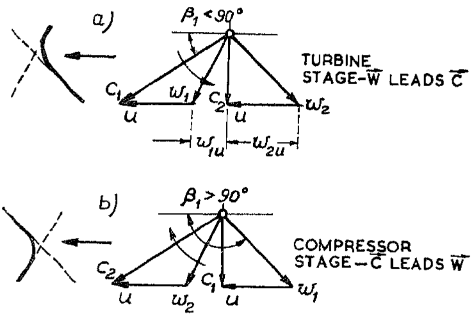

FIG. 5. Stage Characteristics.

vectors $\vec{c}$ and $\vec{w}$ in their hodographs are turning anticlockwise; (2) $\vec{c}$ is lagging (*i.e.*, $\vec{w}$ is leading $\vec{c}$); and (3) the flow gets untwisted and slowed down in its absolute movement (*i.e.*, $\vec{c}$). Under the above conditions power is transmitted from flow to blades and the stage is a "turbine stage".

Figure 5*b* represents a compressor stage, where:

(1) $\vec{c}$ and $\vec{w}$ in their hodographs are turning clockwise;

(2) $\vec{c}$ is leading $\vec{w}$ (*i.e.*, $\vec{w}$ is lagging); and

(3) flow gets twisted and accelerated in its absolute movement. Under these conditions power is transmitted from blades to flow and the stage is a "compressor stage".

We have to examine one other case, the case when there is no transformation of inlet velocity triangle to the exit one (*i.e.*, both are identical). It means that force interaction between flow and blades is reduced to zero. Such a system of blading in which force and power interactions are absent, shall be called a "*mechanically transparent system*". Later on, it will be shown that many thin blades, even bent ones, under definite conditions, become mechanically transparent systems. Apart from this, it is possible to imagine the system of flat thin blades as either a turbine or a compressor stage. In such a case, $\vec{w}$ changes in magnitude only whereas $\vec{c}$ changes both in magnitude and direction. If $\vec{w}$ increases in magnitude ($w_2 > w_1$), without turn we have a turbine stage with flat thin blades. When $\vec{w}$ decreases in magnitude ($w_2 < w_1$) without turn, a compressor stage results. The absence of turning of vector $\vec{w}$ does not violate the rule of leading and lagging vectors.

5. *Theory and design of turbomachines—two basic methods of approach*

Theory and design of turbomachine can be approached by either of the two methods: (*a*) the most profitable form of profile for blade, under given or existing conditions, may be investigated before fabrication of stage; or (*b*) the behaviour of an existing stage with blades having a specific profile and made of metal, may be investigated under conditions of variable regime.

In either of these methods, the phenomena can be analysed by means of the idea of the equivalent stream. In the following parts, a discussion of the efficiency of a turbine stage in general and a rather deeper analysis of the most favourable operating conditions of turbine stage, are given. The discussions are based on the idea of the equivalent stream replacing the flow in theoretical considerations.

6. *Efficiency of blading of turbine stage*

Usually the relative efficiency of blading of a turbine stage is given by the ratio:

$$\eta_{rb} = \frac{A}{h'_0} = \frac{\frac{A}{g}\left[u_1 c_{1u} - u_2 c_{2u}\right]}{h_0 + A\frac{c_0^2}{2g} - \mu_2 A\frac{c_2^2}{2g}}. \tag{2}$$

Using the expressions

$$h_0 = \frac{c_{is}^2}{2g}A \tag{3}$$

$$c_{is}^2 = 8380\, h_0 \tag{4}$$

and

$$c_{it} = \sqrt{8380\,(1-\rho)\,h_0 + c_0^2} \tag{5}$$

expression 2 can be written down for an axial stage as:

$$\eta_{rb} = 2\phi^2 (1 - \rho) \left[\cos \alpha_1 \left(1 + \frac{w_2 \cos \beta_2}{c_1 \cos \alpha_1}\right) - \frac{u}{c_1}\right]$$

$$\times \frac{u}{c_1} \left[\frac{1}{1 - \rho\phi^2 \frac{c_0^2}{c_1^2} - \mu_2\phi^2 (1 - \rho) \frac{c_2^2}{c_1^2}}\right] \quad (6)$$

From expression 6, it is easy to arrive at the following conclusions, known in the theory of turbines:

(*a*) A single impulse stage ($\rho = 0$) with symmetrical deviation of flow has the efficiencies:

(1) $\eta_{rb} = 0$ if $u/c_1 = 0$ and $u/c_1 = \cos \alpha_1$; and

(2) η_{rb} is maximum if $u/c_1 = \cos \alpha_1/2$.

and

(*b*) A congruent reaction stage ($\rho = 0{\cdot}5$, *i.e.*, 50% reaction) has the efficiencies:

(1) $\eta_{rb} = 0$ if $u/c_1 = 0$ and $u/c_1 = 2 \cos \alpha_1$; and

(2) η_{rb} is maximum if $u/c_1 = \cos \alpha_1$.

Usually the analysis of maximum efficiency conditions are left at this stage abruptly, while quite a few points are left without any clarifications. We can naturally ask "In what manner is it possible to get a value $\eta_{rb} = 0$ in impulse stage if the wheel has circumferential velocity $u = c_1 \cos \alpha_1$?" Such a question has never been raised in literature concerning theory of steam and gas turbines, although a formal analysis referring to efficiency on blading was realised by Donath Banky in 1905 (V.D.I.). It is necessary, therefore, to analyse the problem much more deeply and in this connection we must formulate newer ideas.

7. *Generalised system of mathematical profiles of blades*

We shall designate an infinitely thin flat or curved plate, a mathematical profile (or form) of any blade. The scheme of geometrical development from the flat axial blade A, to all possible types of mathematical profiles—flat and bent both for the turbine and compressor—is given in Fig. 6. Profiles will be formed from the axial flat one by proper geometrical alterations. The basis of such a scheme of development is the idea that for each mathematical profile a certain value of circumferential velocity of blades, *i.e.*, u, ensures smooth entry of the equivalent stream into the blades (*i.e.*, zero angle of attack). In other words the form of contour of the profile at entry coincides with the relative inlet velocity $\vec{w}_1$. Similarly the form of the profile at exit is made to coincide with the exit relative velocity $\vec{w}_2$. In this manner any mathematical profile reflects the complete deviation of the equivalent stream on blading.

For each of the flat profiles, given in row 0, it is possible to determine the circumferential demarcating velocity u_{dem}, for the condition of no force interaction between flow and the blades. u_{dem} has to be then maintained by the application of external power to the shaft of the machine. Under this condition the thin flat

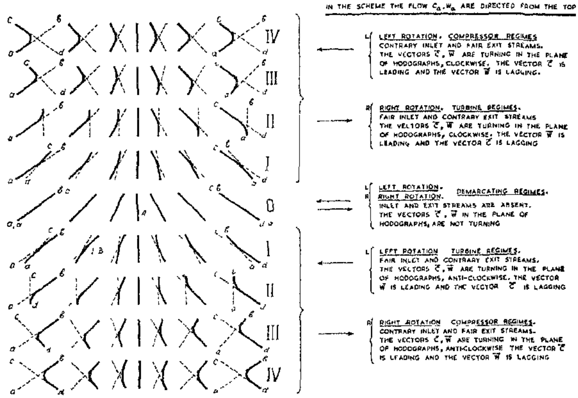

FIG. 6. The General Scheme of Development of Mathematical Profiles of Turbomachines and Character of Currents of Equivalent Stream.

blades become "mechanically transparent" ones. When: (*a*) $u < u_{dem}$ the chosen profile will be a turbine stage profile with transfer of energy from flow to blades; and (*b*) $u > u_{dem}$ the chosen profile will be a compressor profile, with transfer of energy from blades to flow. As indicated above when $u = u_{dem}$ the blades are mechanically transparent, with no energy transfer (see Fig. 7). Absolute velocity $\vec{c}$ does not change both in magnitude and direction during the flow through the blades.

In the scheme of development of mathematical profiles of blades of turbomachines (Fig. 6), the property of leading and lagging velocity vectors, appropriate to the regime of application, and the character of currents of equivalent stream are indicated. The scheme has symmetry of playcard, and with flow from below the turbine and compressor regimes interchange places. In the scheme of Fig. 6 with *flow from above*: (1) with u directed *to the left* the four horizontal rows I to IV *above* row 0 are *COMPRESSOR PROFILES* and the four horizontal rows I to IV *below* row 0 are *TURBINE PROFILES*; and (2) with *u directed to the right* (*i.e.*, by changing the rotation of the shaft) the two regimes get interchanged. The top rows become turbine profiles and the bottom rows compressor profiles.

"A mathematical profile is defined as an infinitely thin profile, characterised by zero angle of attack by the equivalent stream," and may be considered as the

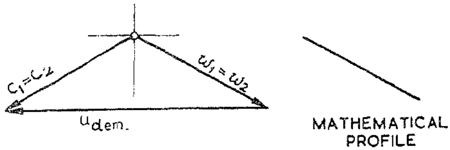

FIG. 7. Mechanically Transparent System. $u = u_{dem}$ (Flat Mathematical Profile).

skeleton of a real profile. A very thin metallic profile can be considered as possessing all the properties of the mathematical one in the presence of a certain circumferential velocity u of the blades, to ensure zero angle of attack. When the external torque acting on the shaft of the machine changes, u changes and under conditions of constant $\vec{c}_1$, the ratio u/c_1 changes, the profile will not, any more, be a mathematical one, since the angle of attack will be other than zero.

Characteristics of mathematical profiles in the general scheme (Fig. 6)

Horizontal Row	Characteristics
0	Flat profiles $\beta_{1b} = \beta_{2b}$ (identical)—cuts *ba* and *cd* coincide.
(top & bottom)	Symmetrical bent profiles. $\beta_{1b} = -\beta_{2b}$ (equal in magnitude but symmetrically oriented to the axial direction)—cut *ba* makes β_{1b} and *cd* makes β_{2b}.
I, II & III (above & below 0)	Unsymmetrical bent profiles. $\beta_{1b} \neq \beta_{2b}$.
0, II & IV	Definite profiles.
I & III	Quantity rows. (Different profiles possible for the same orientation of *ba* & *cd* depending upon the point of intersection of the cuts.)

Any mathematical profile is the result of the combination of the straight cuts *ba* and *cd* forming the inlet and exit elements of the profile, parallel to the directions $\vec{w}_1$, $\vec{w}_2$ in the hodograph of relative velocity. They can cross each other at different angles and may be considered as fixed extreme attitudes of flat profile. Any vertical row can be formed by keeping the direction of one of the two straight cuts constant and varying the inclination of the other. For example—the extreme right vertical row maintains *cd* constant and *ab* is varied;—the mid-profile, in horizontal row being formed by the two cuts coinciding, is flat and mechanically transparent. In this manner it is possible to realise transition in any vertical row from the topmost to the bottommost symmetrical profile, in-between passing through the unsymmetrical, mechanically transparent and again unsymmetrical profile stages.

The scheme of profiles disposed on the right of the middle vertical row of vertical flat profiles, retain the straight cut *cd* constant, whereas the profiles disposed on the left retain *ba* constant.

The profile is drawn using the cuts *ba* and *cd*, and if they cross at an angle a curved profile results. It is easy to realise that any curved profile will result by the junction of flat profiles disposed in the row 0. However it may not be possible to tie down all the variety of mathematical profiles within the frame of the scheme given in Fig. 6, since the profiles are disposed not only in the plane of the scheme but also in space, in parallel planes. Referring to an actual blade profile, any of the surfaces—the concave, the convex or the mean line of the cross-section, can be regarded as a mathematical profile. However only one of these can be considered, at the given moment of time, as a mathematical profile, if by design the necessary condition of zero angle of attack is maintained. The others experiencing incidence, positive or negative, are not mathematical profiles. Even the single surface having the property of a mathematical profile, loses this property when we consider the same profile at any other radius along the length of the blade, since u varies with radius and incidence is no more zero (under the assumption that $\vec{c}_1$ is maintained constant both in magnitude and direction along the length of the thin blade). Further if at any other instant $\vec{c}_1$ varied, the profile is no more mathematical. Thus a face or meanline of a real blade can have the properties of a mathematical profile only at a certain radius and at a certain moment of time.

It can now be seen that, selection of shockless inlet angle, recommended in literature, is not possible. With incidence present, the stream lines get bent while flowing around the blade surface and will promote, more or less, formation of vortices and waves with attendant loss of energy in the form of heat. Streams flowing along the convex and concave surfaces of blade, at exit are no longer identical, and do not remain constant along the length of the blade. The phenomenon is quite complicated and here again, the idea of the equivalent stream flowing at a radius r from the centre of the shaft, is a very successful replacement. It is impossible to refer to shock of the equivalent stream against the blade profile. The mechanism of transfer of forces and power become readily apparent. Inlet and exit elements of the equivalent stream coincide with the elements of the corresponding mathematical profile, in the appropriate work regime of the stage. *Thus it is possible to reflect the variation in the character of the flow around a real aerofoil, by the alteration of the contour of mathematical profile and the corresponding equivalent stream replacing the substantial flow in theoretical considerations, under conditions of variable regime.* Thus, a number of mathematical profiles can represent any selected real blade according to the nature of regimes. Or the other way around, one mathematical profile can represent or reflect the behaviour of a number of real blades.

In the general scheme of mathematical profiles (Fig. 6) any profile can serve both as turbine profile and as a compressor one. (The scheme includes profiles suited for screw propellers as well and forms one of the adjacent field of techniques.) Some of the profiles are more suited for turbine blades, while others could be favoured for blades of compressors. It is quite possible to demarcate regimes, in the general scheme of profiles, enveloping profiles, not adapted from the point of view of theoretical mechanics, for working out the problem of design of turbine or compressor. Actually experimental aerodynamics confirms these considerations as being quite important.

Connecting the real profile with the mathematical one, it is possible to put forth the following idea: " If the real profile of blade is defined, its force and power interaction with flow can be studied by means of the equivalent mathematical profile, contours of the inlet and exit elements of this profile coinciding with the inlet and exit elements of the equivalent stream replacing the flow." If the work regime of a real stage changes, the mathematical profile changes and adapts itself to the altered regime. In this manner study of the variable regime of stage can be limited to the investigation of the character of alteration of mathematical profile under change of regime.

8. *Interaction between flow and blades by Impulse and Reaction types of currents.*

Force interaction between flow and blades is brought about by both impulse and reaction. We shall define a few terms for purposes of easier enunciation of the phenomena:

1. c_u or w_u be termed as *FAIR* if it is in the direction of $\vec{u}$ (*i.e.*, positive direction); and *CONTRARY* if it is opposed to the direction of $\vec{u}$ (*i.e.*, negative direction).

2. The part of the equivalent stream from inlet to blade (1)—to the point on the stream (0) where w_u is zero be termed *INLET STREAM*; we can then have *FAIR INLET STREAM* ($\beta_1 < 90°$ to $\beta_0 = 90°$) and *CONTRARY INLET STREAM* ($\beta_1 > 90°$ to $\beta_0 = 90°$) according to the sign of w_u in this part of the equivalent stream.

Force interaction in the *inlet stream* portion of the equivalent stream is by *IMPULSE.* It can easily be observed that *fair inlet stream corresponds to the turbine regime and contrary inlet stream corresponds to the compressor regime.*

3. The remaining part of the equivalent stream from the point on the stream (0) where w_u is zero, to the exit of blade (2) be termed *EXIT STREAM.* We can then have *FAIR EXIT STREAM* ($\beta_0 = 90°$ to $\beta_2 > 90°$) corresponding to

compressor stage; and *CONTRARY EXIT STREAM* ($\beta_0 = 90°$ to $\beta_2 < 90°$) corresponding to turbine stage.

Force interaction in the *exit stream* is by *REACTION*. Thus, a *completely developed turbine stage has FAIR INLET and CONTRARY EXIT; whereas a completely developed compressor stage has CONTRARY INLET and FAIR EXIT* streams and in both cases *inlet stream interacts by impulse and exit stream by reaction.* We shall define a stage as an *UNDERDEVELOPED* one if the stage does not have one of the two portions of the equivalent stream.

The characteristics of the inlet and exit streams are given in Figs. 8 & 9. In a turbine stage, along the inlet stream (1 to 0) the flow applies an impulse force

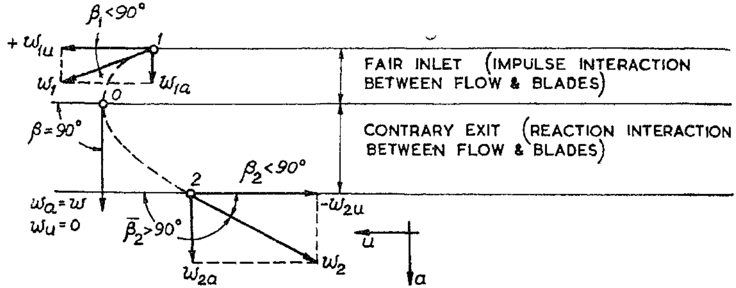

FIG. 8. Turbine Stage (Fully Developed).

in the direction of *u* and reaction opposed to the direction of *a*, to the blades (see Figs. 8 & 10). The flow gives up part of its momentum in the circumferential direction to the blades and receives itself from blades some part of the momentum in the axial direction.

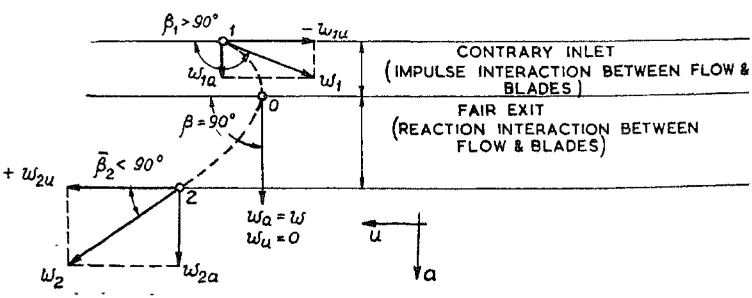

FIG. 9. Compressor Stage (Fully Developed).

In a theoretical axial impulse stage, $\vec{w}$ remains constant in magnitude but changes direction. Along the entire stream (1–0–2) the kinetic energy A $w^2/2g$ remains constant. The momentum vector $\vec{mw}$ retains constant magnitude but changes in direction. Change of components of $\vec{mw}$ (*i.e.*, mw_u & mw_a) specify the method of transfer of energy from flow to blades. Along the inlet stream

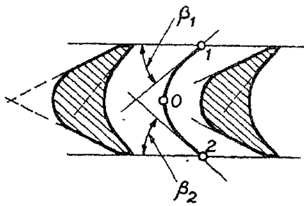

FIG. 10. Turbine Stage—Theoretical Impulse ($\rho = 0$ & $\psi = 1$) Symmetrical Deviation of Flow ($\beta_1 = \beta_2$).

(1–0) mw_u decreases since flow gives up momentum to blades and mw_a increases since flow picks up momentum from blades. In an actual stage, friction in blading transforms a portion of the kinetic energy into thermal energy and hence $\vec{mw}$ decreases in magnitude as the flow proceeds along the equivalent stream. This drop in mw further decreases magnitude of $\vec{mw_u}$ but slows down the otherwise accelerating mw_a. Summing up the flow applies *fair impulse circumferential* and *contrary axial forces* to blades along the *inlet stream* (1–0). Along the *exit stream* (0–2) mw_u increases in magnitude but is contrary in character (while it remained fair in the inlet stream). Axial component of momentum mw_a decreases in magnitude and remains fair as in inlet stream. The flow is accelerated in the direction contrary to the direction of $\vec{u}$ but is slowed down in the axial one. The flow applies fair force to blades by reaction in the circumferential direction, but an impulse force against the blades in the axial direction. If the equivalent stream is accelerated but characterised by the condition that w_a remains constant, the axial impulse force (in the exit stream) does not arise. Thus the flow applies *fair reaction circumferential* and *fair impulse axial* forces to blades along the exit stream (0–2).

Character of force and power interaction between flow and blades can be presented for a typical stage, for example: "Turbine stage—theoretical impulse ($\rho = 0$ & $\psi = 1$), symmetrical deviation of flow ($\beta_1 = \beta_2$)." The equivalent stream 1–0–2 is given in Fig. 10.—1–0 is fair inlet and 0–2 contrary exit streams. Since the stage is theoretical pure impulse, along the equivalent stream mw remains constant (Fig. 11).

Fig. 11.1 gives the variation in the magnitude of mw_u;

Fig. 11.2 gives the variation in the magnitude of mw_a;

Fig. 11.4 shows the nature of the development of turning force, 1–0 impulsive (P_{ui}) and 0–2 reactive (P_{ur}). Both forces add up;

Fig. 11.5 shows the development of axial force on blades, 1–0 reactive (P_{ar}) and 0–2 impulsive (P_{ai}). The two are opposed and for a theoretical impulse turbine stage with symmetrical deviation of flow, $P_{ar} - P_{ai} = 0$.

Data for Fig. 11 is taken from the velocities hodograph of the equivalent stream given in Fig. 12.

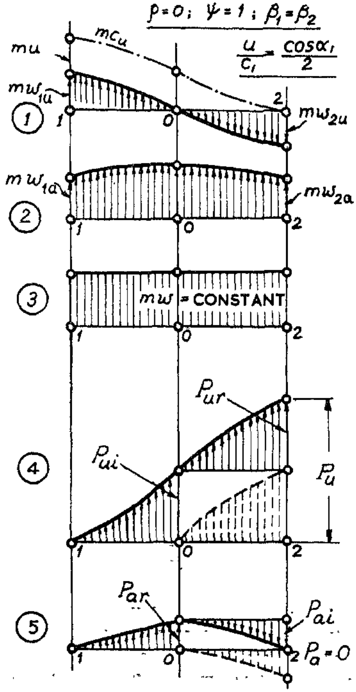

FIG. 11. Character of Force and Power Interaction—Theoretical Impulse Turbine Stage with Symmetrical Deviation of Flow.

For the turbine stage: $\rho = 0$, $\psi = 1{\cdot}0$, $\beta_1 = \beta_2$, hodographs of velocities $\vec{c}$ and $\vec{w}$ (and momentums $\overrightarrow{mc}$ and $\overrightarrow{mw}$) are the circles with centres at A_c and A_w respectively. From these hodographs we can clearly see:

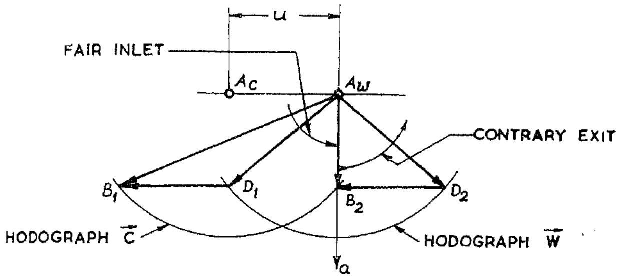

FIG. 12. Turbine Stage ($\rho = 0$; $\psi = 1$; $\beta_1 = \beta_2$ and Optimum Exit).

1. $\vec{c}$ and $\vec{w}$ are turning clockwise in the plane of hodographs, (satisfying the condition for a turbine stage), with blades moving to the left;

2. $\vec{c}$ is lagging behind $\vec{w}$; and

3. inlet velocity triangle (inlet momentum triangle) $A_w\, B_1\, D_1$ is transformed into the exit velocity triangle (exit momentum triangle) $A_w\, B_2\, D_2$ in the case of the most profitable conditions of exit (*i.e.*, axial exit).

Figure 13 shows the hodographs of $\vec{c}$ and $\vec{w}$ for a turbine stage, designed under the condition (all along the equivalent stream) $c_a = w_a =$ Constant. In this case the hodographs are the straight parallels, parallel to u-axis.

From the analysis of the character of force interaction between flow and blades, based on the idea of the equivalent stream, in a stage of a turbomachine, it is possible to evaluate impulse and reactive forces, which can act both in the circumferential and axial directions. From this point of view it is possible and perhaps necessary to refer not only to the degree of thermal reactivity (ρ) but also to degree of mechanical (force) reactivity of a blade, circumferential as well as axial.

9. *Degree of Circumferential Mechanical (Force) Reactivity of Stage*

We can define the *degree of mechanical (force) reactivity* (ρ_{ur}) *of a stage of a turbomachine* as the ratio of: the mechanical reactive force interaction to the total interaction. The reactive turning force is determined by the change of circumferential component of flow momentum along the *resulting exit stream*, whereas

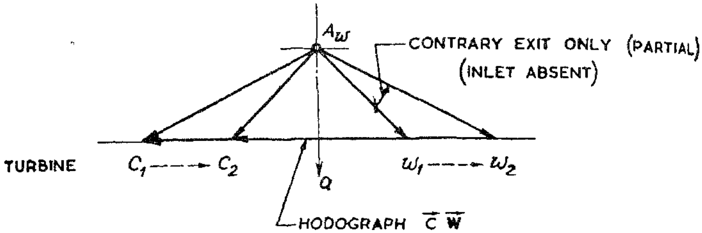

FIG. 13. Turbine Stage ($\rho = 0{\cdot}5$; $c_a = w_a =$ Constant).

the total force is determined by the total change of the same component along the *entire equivalent stream* (Fig. 14).

$$\rho_{ur} = \frac{m.\Delta w_{ue}}{m.\Delta_{wu}} = \frac{\Delta w_{ue}}{\Delta w_u} = \frac{\Delta w_{ue}}{c_{1u} - c_{2u}} = \frac{\Delta w_{ue}}{w_{1u} - w_{2u}} \tag{7}$$

The numerator in expression 7 will be positive or negative according to the character of the exit stream, *i.e.*, contrary or fair. For a turbine stage the numerator and the denominator are always positive and for this reason, a turbine stage can

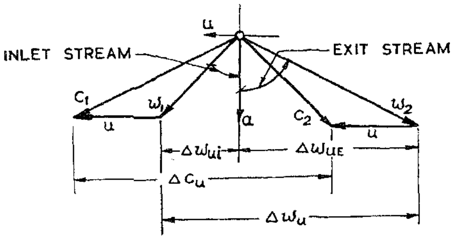

FIG. 14. Turbine Stage.

have, in principle, only positive mechanical (force) reactivity. For any compressor stage both the numerator and the denominator are always negative ($c_{2u} > c_{1u}$) and hence it is physically impossible to get a negative value of circumferential mechanical reactivity. However considering the direction of force applied to blades by flow (fair or contrary), we can conventionally define that: a fair exit stream is characterised by negative force reactivity and a contrary exit stream by positive force reactivity.

Degree of circumferential mechanical (force) impulsivity (ρ_{ui}) can be defined in a similar manner:

$$\rho_{ui} = \frac{m.\Delta w_{ui}}{m.\Delta w_u} = \frac{\Delta w_{ui}}{\Delta c_u} = \frac{\Delta w_{ui}}{c_{1u} - c_{2u}} = \frac{\Delta w_{ui}}{w_{1u} - w_{2u}} \tag{7 a}$$

We have introduced the term *resulting* exit stream in the above definitions. The significance of the term *resulting* becomes clear when we examine the phenomena of force interaction for a complicated contour stream line—shown dotted in Fig. 15, consisting of: 1 H—contrary inlet; H K—fair exit; K B—fair inlet and B 2—contrary exit streams. If the magnitude of $\vec{w}$ is constant, the hodographs of $\vec{c}$ and $\vec{w}$ (and momentums) become circles with centres A_c and A_w as given in Fig. 16. *Along the stream line* 1 *H K* the inlet velocity triangle A_w H H′ is transformed through A_w H H′ to the extreme velocity triangle A_w K K′. During this stage the vectors are turning clockwise and $\vec{c}$ is leading $\vec{w}$, with left rotation of blades. Hence *the regime is that of a compressor transferring momentum from blades to flow.*

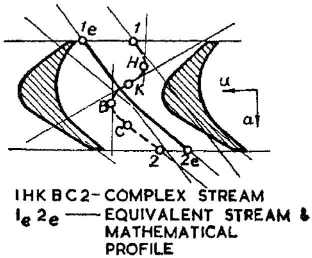

FIG. 15. Force Intaerction between Flow and Blades for a Complex Stream Line.

Along the remaining part of the stream, *i.e.*, K B 2, the velocity triangle A_w K K′ is transformed through A_w B B′ to the exit velocity triangle A_w 2 2′.

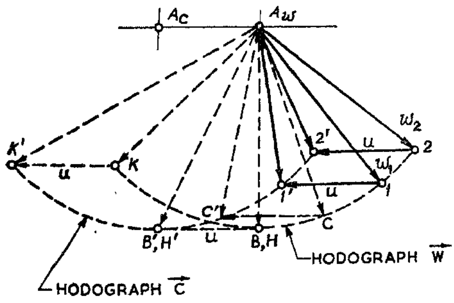

FIG. 16. Velocity Triangles for the Complex Stream of Fig. 15.

Hence the vectors are turning anticlockwise and $\vec{c}$ is lagging behind $\vec{w}$, again with left rotation of blades. The *regime is, hence, a turbine stage, flow transferring momentum to blades.*

The compressor regime transists to the turbine regime and for the phenomena, in the plane of hodographs, A_w I I′—A_w K K′—A_w 1 1′ the energy exchanges cancel each other—shown dotted in Fig. 16. Further transition to A_w 2 2′ determines the net character of the stage. In this case the vectors turn anticlockwise further and turbine stage results. (In case the stream finally left the stage at C, A_w C C′ will be the exit velocity triangle and the later turbine regime will not fully cancel out the compressor regime. Here the net character of the stage would be that of a compressor.) Hence force interaction between a complexly bent stream and blades (or flow and complexly shaped blades as in Fig. 17), as far as the net result is concerned, is similar to that of a much *simpler stream and profile*, which we will call the *resulting equivalent stream and the resulting profile.* The resulting profile coincides with the shape of the resulting equivalent stream. The resulting equivalent streams are shown by the profiles $1_e 2_e$ in Figs. 17 & 18. In Fig. 16, the inlet and exit velocity triangles are shown by thick lines A_w 1 1′ and A_w 2 2′. (For the example shown in Fig. 17, the resulting equivalent mathematical profile is a straight line and the system is mathematically transparent with zero net momentum transfer. External torque on the shaft of the machine maintains the blade system with circumferential velocity *u*.)

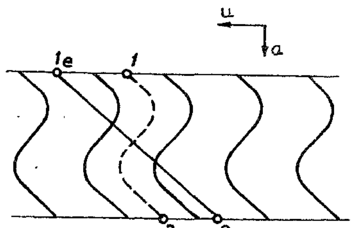

FIG. 17. Complex Blade Profile and Resulting Equivalent Stream.

The resulting equivalent stream and profile (in Fig. 16) belongs to the turbine regime, as indicated earlier, and the entire force transfer is by fair reaction, and a positive moment is applied to the shaft. For this stage:

$$\rho_{ur} = \frac{\Delta w_{ue}}{\Delta w_u} = \frac{w_{1u} - w_{2u}}{w_{1u} - w_{2u}} = 1,$$

i.e., the degree of circumferential mechanical reactivity is unity. And again

$$\rho_{ui} = \frac{\Delta w_{ui}}{\Delta w_u} = \frac{0}{w_{1u} - w_{2u}} = 0,$$

i.e., the grade of circumferential mechanical impulsivity is zero. (The inlet stream of the resultant equivalent stream is absent.)

We can now define the resulting mathematical profile of blade as an infinitely thin plate bent according to the profile of the resulting equivalent stream replacing the flow.

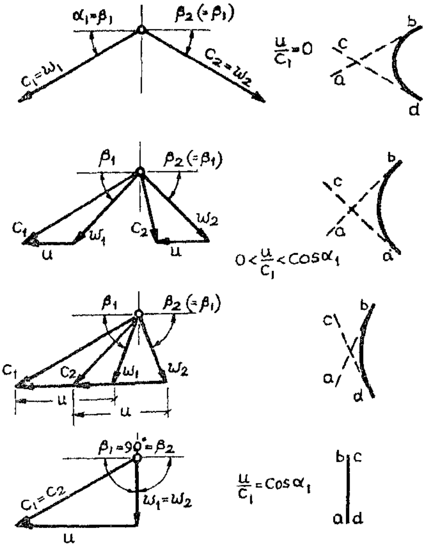

FIG. 18. Impulse Turbine Stage with Symmetrical Deviation of Flow. ($\vec{c}$ Constant and u/c_1 in the Range 0 to $\cos \alpha_1$.)

These ideas very much simplify the investigation of the problem of variabl regime of stage having substantial blades with the given shape. We can consider the problem of variable regime by replacing the actual system of blades by resulting mathematical profiles which have particular forms for each of the regimes.

9. *Analysis of the problem of the most profitable operating conditions of a turbine stage.*

9.1. *An impulse stage with symmetrical deviation of flow.*—The theoretical expression for efficiency of such a stage is usually given in the form:

$$\eta_{rb} = 2\,\phi^2\left(1 + \psi\right)\left(\cos \alpha_1 - \frac{u}{c_1}\right)\frac{u}{c_1} \tag{8}$$

This equation, a particular form of expression 6, envelops all varieties of velocity triangles defined by the range of values of $u/c_1 = 0$ to $u/c_1 = \cos \alpha_1$. (In the scheme of mathematical profiles Fig. 6 the flow is directed from the top and the blades move to the left. The turbine regime is, then, the lower right quarter zone.) The symmetrical profiles ($\beta_1 = \beta_2$) given in the bottommost row IV, are covered by the expression 8 for the range: $\beta_1 = \beta_2 = \alpha_1$ ($u/c_1 = 0$) to $\beta_1 = \beta_2 = 90°$ (*i.e.*, $u/c_1 = \cos \alpha_1$) the profile being flat and oriented along the axis of the stage. This horizontal row is covered, maintaining symmetry and $\vec{c}_1$ constant with change in u. The profile changes with change in u, *i.e.*, u/c_1. Thus expression 8 describes motion of flow through a system of blading which change profile with change in u/c_1. The closer a mathematical profile is to the starting vertical row (α_1 given and $\beta_1 = \alpha_1$) the more will be the deviation of $\vec{c}$ and $\vec{w}$ in their hodographs and more will be the turning force applied to the blades. The turning force becomes zero when $\beta_1 = \beta_2 = 90°$ and in the given case $u = u_{dem} = c_1 \cos \alpha_1$. (The common formula to determine the demarcating circumferential velocity is $u_{dem} = c_1 \cos \alpha_1 - w_1 \cos \beta_1$.) If the wheel is braked until it stops (*i.e.*, $u/c_1 = 0$) the transition of the inlet velocity triangle to the exit one degenerates into the turn of $\vec{c}$ from $\vec{c}_1$ to $\vec{c}_2$ and the turning force reaches a maximum value.

If $u/c_1 = \cos \alpha_1$, the transformation of inlet velocity triangle into the exit one does not happen because along the entire equivalent stream, turning of $\vec{c}$, $\vec{w}$ in the plane of hodographs is absent. The two triangles are identical. For any value of u/c_1 within $0 < u/c_1 < \cos \alpha_1$, the inlet and exit velocity triangles have intermediate contours and any mathematical profile maintaining symmetry is an intermediate one between the bent symmetrical ($\beta_1 = \beta_2 = \alpha_1$) and the flat ($\beta_1 = \beta_2 = 90°$) profiles (see Fig. 18). From Fig. 18 it can easily be observed that the degree of circumferential mechanical reactivity (ρ_{ur}) is 0·5 and the degree of circumferential mechanical impulsivity (ρ_{ui}) is 0·5, for a turbine stage characterised by symmetrical deviation of flow.

9.2. *Connection between the mathematical profile and the profile of substantial blade.*—While designing a turbine stage, based on considerations of optimum operation, we can select any mathematical profile but we must transform it to the substantial blade which ensures the same total effect as the selected mathematical profile. Here arises the problem of reverse equivalent replacement of mathematical profile by the real one. Experimental data is essential to make such a replacement a possibility.

If the field of relative velocities of flow were uniform across any axial cross-section of turbine and the infinitely thin blade were a mathematical profile for the given value of u/c_1, the angle of incidence at inlet to blade would be zero and at exit the flow (all the streams) will have the direction determined by β_2. Actually the blade is solid and the field of velocities is *not uniform*. *The different stream* lines distinguish each other. Effect of friction, waves and vortices are also present. Under conditions of a real stage, it is impossible to make a substantial blade to function with zero angle of incidence by equivalent streams. Hence any mathematical profile is not the profile of the substantial blade. However it characterises the deviation of the equivalent stream replacing the substantial flow. Therefore, theoretically α_1, β_1, β_2, which by cosines enter into the expression 8, are characteristics, not of blade, but of flow or more exactly—of the equivalent stream. It is essential here to stress on the method of determination of the angles α_1 and β_2. This can be done by different conventions. For instance as a first approximation, we can decide, once for all, to make: *(a)* $\alpha_1 = \alpha_{1n}$, *i.e.*, the exit angle of the equivalent stream equals, always, the exit angle from guide blades (α_{1n} is the exit angle of mean line of guide blade); and *(b)* $\beta_2 = \beta_{2b}$, *i.e.*, exit angle of equivalent stream equals exit angle of skeleton of substantial blade. Exit angles are to be defined by the only manner indicated above, though there are other forms as well. The exit angles when defined as effective angles α_{1eff}, β_{2eff} are different from α_{1n} and β_{2b} (see Fig. 19). The effective exit angle must take into consideration the effect of deviation of flow within the region of oblique disposition of nozzles or blades at

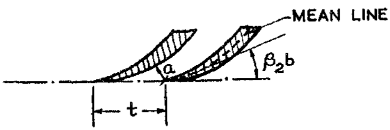

FIG. 19. Sin β_{2eff}. $= a/t$ = Minimum Opening/Pitch.

exit and the deviating effect of the wheel (the influence of clearnace between nozzles and blades). The idea of effective exit angles can reflect the influence of the presence of different twist of individual streams of flow. We can, then, make a clear definition however that $\beta_2 = \beta_{2eff}$ and $\alpha_1 = \alpha_{1eff}$. Based on experimental data, we can have the inlet and exit velocity triangles of stage and evaluate α_1 and β_2. With known α_1 and β_2 we can determine the particular mathematical profile which will be equivalent to the substantial blade with respect to its interaction with flow, under given conditions. Since it is possible to replace a complexly bent equivalent stream by a simple resulting mathematical profile, we can confirm that the mathematical profile, replacing the substantial profile of blade, will have any of the shapes shown in Fig. 6.

9.3. *Impulse compressor stage with symmetrical deviation of flow on blades.*— If a system of blades with symmetrical deviation of flow operates with $u > u_{dem}$ (*i.e.*, $u > c_1 \cos \alpha_1$), the stage is no more a turbine but a compressor. In the scheme of mathematical profiles (Fig. 6) the profile will move to the right upper half (compressor regime) topmost row IV. The fair inlet stream is replaced by the contrary inlet and the contrary exit by the fair exit streams. The profile moves farther away from the middle of the top row IV to the right as the circumferential velocity u increases in magnitude. For such a stage: $\rho_{ur} = 0 \cdot 5$ and $\rho_{ui} = 0 \cdot 5$ remain constant. Energy is transferred from blades to flow, one half by impulse in the contrary inlet and the other half in the fair exit streams.

9.4. *Congruent turbine stage.*—Referring to turbomachines, we shall define a *CONGRUENT STAGE* as a stage in which $\vec{c}_1$ & $\vec{w}_2$ and $\vec{w}_1$ & $\vec{c}_2$ taken in pairs are conjugate complex numbers, if the $\vec{u}$ axis is the imaginary one (see Fig. 20). $\vec{c}_1$ & $\vec{w}_2$ and $\vec{w}_1$ & $\vec{c}_2$ are conjugate in pairs and hence $c_{1a} = w_{1a} = w_{2a} = c_{2a} =$ constant. Velocity hodographs of $\vec{c}$ & $\vec{w}$ are straight lines parallel to the direction of $\vec{u}$. The ends of vectors $\vec{c}$ & $\vec{w}$ slip along this line, during the transformation of the inlet, to the exit triangle.

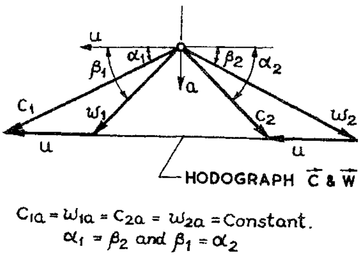

FIG. 20. Congruent Turbine Stage.

We shall consider a congruent turbine stage and analyse the character of change when u/c_1 is altered within the range $0 < u/c_1 < 2 \cos \alpha_1$, change in u/c_1 being realised by changing $\vec{u}$ and maintaining $\vec{c}_1$ constant. The resulting equivalent mathematical profiles are given in Fig. 21. We shall choose a particular value for $\alpha_1 = \alpha_{1n}$ and maintain $\vec{c}_1$ constant.

When $u/c_1 = 0$, the profile is symmetrical and the profile is located in the bottom right half row IV. The force applied to the blades is maximum. But no work is done since u is zero. For the case $u/c_1 = 2 \cos \alpha_1$ (*i.e.*, demarcating value) the blade profile is flat and the profile is located in the right half row 0, and in the same vertical row as the starting profile (*i.e.*, maintains the same α_1). The force interaction is zero and the system mechanically transparent. η_{rb} is zero.

Further if we increase the circumferential velocity above the demarcating value $u/c_1 = 2 \cos \alpha_1$, maintaining $\vec{c}_1$ constant and left rotation of blades, the profile passes higher along the same vertical row but is in the compressor regime, and is no more a turbine stage. Hence, analysis oi the turbine stage, even theoretical, need cover only the range $u/c_1 = 0$ to $u/c_1 = 2\cos\alpha_1$. Considering the turbine regime,

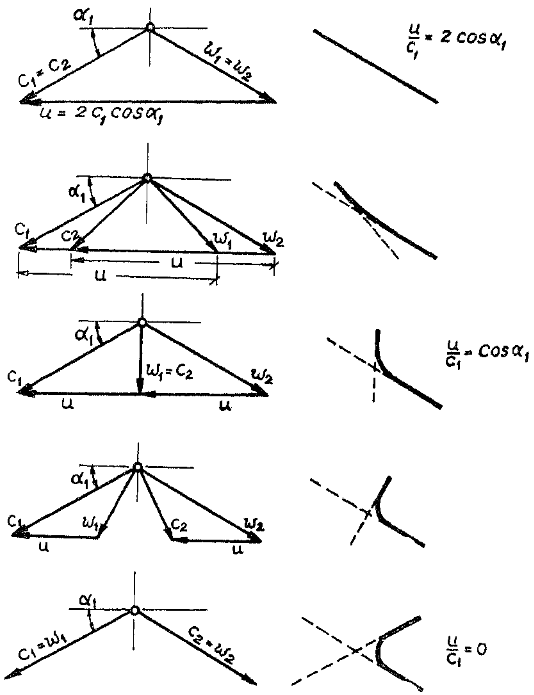

FIG. 21. Congruent Turbine Stage. $0 < u/c_1 < 2\cos\alpha_2$, with $\vec{c_1}$ maintained constant.

the right lower quadrant, all the profiles contained in any vertical row are congruent and maintain a characteristic own angle α_1 and $\beta_2 = \alpha_1$ (in magnitude only). For this reason it is quite enough to consider only any vertical half row, selected by the choice of α_1, to draw very interesting conclusions.

To maintain $\rho = 0{\cdot}5$ at the presence of any numerical value of u/c_1, it is necessary to maintain $c_{1a} = c_{2a}$ by suitable adjustment of the exit areas of nozzles and blades. Just as α_1 and β_2 are given, it is necessary to choose the required exit heights of nozzles (l_1) and blades (l_2). In any one vertical half row (in the turbine regime); since $\vec{c}_1$ is constant, $\vec{c}_1$ and $\vec{w}_2$ are conjugate, the degree of mechanical force reactivity up to the row II is $1 \cdot 0$. In other words along the part of the vertical row of profiles (turbine regime), *i.e.*, for any value of u/c_1 in the range $2 \cos \alpha_1 > u/c_1 > \cos \alpha_1$, $\Delta w_{ue} = \Delta w_u = w_{1u} - w_{2u}$ and hence $\rho_{ur} = 1$ remains constant. In this range the profile transforms itself from the flat to under-developed reactive and thence to the fully-developed reactive one.

Earlier it has been indicated that a symmetrical mathematical profile is typical for impulse stage. However it is shown above that any profile of vertical row has property of congruence, including the symmetrical profile disposed in row IV and belonging to the chosen vertical row. Here the profile has to be a 50% reaction one, since it is congruent and at the same time it has to be an impulse one. This is very true, though superficially there appears to be some contradiction. This contradiction is only apparent and can be proved to be so, in the following manner. For a theoretical congruent stage ($\phi = \psi = 1 \cdot 0$) $c_{1t} = c_1$ and $c_1 = w_2 = w_{2t} = c_{1t}$ and $c_0 = c_2 = w_1$ because of congruency. Again:

$$c_{1t} = \sqrt{8380\,(1 - \rho)\, h_0 + c_0^2} \text{ and } w_{2t} = \sqrt{8380\, \rho\, h_0 + w_1^2}.$$

From these expressions, since $c_{1t} = w_{2t}$ & $c_0 = w_1$, $(1 - \rho) = \rho$, *i.e.*, $\rho = 0 \cdot 5$. *For a congruent stage degree of thermal reactivity is* $0 \cdot 5$.

If u/c_1 is $2 \cos \alpha_1$, the mathematical profile is flat (Fig. 21) and $c_0 = c_1 = w_1 = w_2 = c_2$. ($\vec{c}_1$ and $\vec{w}_1$ coincide with $\vec{c}_2$ and $\vec{w}_2$ respectively.) The available energy h_0 is therefore zero, as otherwise the stage will lose the property of congruence. From this point, the stage is an impulse one, since no energy is transformed in the blades. However it is true that the energy for the stage h_o is itself zero and no energy transformation in the nozzles too. The nozzles merely act as a guide system with no change in the magnitude of the velocity through it, *i.e.*, $c_1 = c_0$. If the theoretical stage maintains the property of congruence for all values of u/c_1, we can state the magnitude of $\vec{c}_1$ for the range $u/c_1 = 0$ to $u/c_1 = 2 \cos \alpha_1$ changes according to the expression:

$$c_1 = c_{1t} = \sqrt{8380 \,.\, 0 \cdot 5\, h_0 + c_0^2} \text{ is constant.}$$

$\vec{c}_0$ and h_0 are thus interconnected. Any increase in c_0 cuts down h_0 and *vice versa*. h_0 is transformed equally in the nozzles and the blades, *i.e.*, ρ is $0 \cdot 5$. The property of congruence maintains $\rho = 0 \cdot 5$ as constant up to the smallest values of h_0. Thus as the mathematical profile moves from the mechanically transparent flat system of profiles to the symmetrical profile, in any vertical row a reaction (in the thermal sense) congruent stage transforms into an impulse one quite regularly.

In the range $\cos \alpha_1 < u/c_1 < 2 \cos \alpha_1$, all mathematical profiles will be underdeveloped reactive turbine profiles and are disposed in the multiform lower horizontal right half row I. Transfer of force from flow to blades is wholly by reaction in the circumferential direction. The equivalent stream line (coinciding with the mathematical profile) is an exit stream one and w_u increases from inlet to exit. Axial force does not arise since $w_{1a} = w_{2a}$. As u/c_1 further decreases, w_1 falls gradually (*i.e.*, c_0 falls since $c_0 = c_2 = w_1$). To maintain $\rho = 0 \cdot 5$, $\vec{c_1}$ = constant and $\vec{w_2}$ = constant, the energy for the stage h_0 increases, the increase equally affecting energies across the nozzle and the blade (*i.e.*, $\Delta h_{01} = \Delta h_{02} = 0 \cdot 5 \, \Delta h_0$).

When $u/c_1 = \cos \alpha_1$, the mathematical profile is disposed in the lower horizontal half row II and is fully developed pure reactive, characterised by the condition $\beta_1 = 90°$. $w_1 = c_0$ will be the smallest and is directed along the stage axis. Hence the stage available energy h_0 and hence $h_{01} = h_{02} = \frac{1}{2} h_0$, will be maximum to provide the required $c_1 = w_2$. It is a well-known fact that under the condition $u/c_1 = \cos \alpha_1$, in a congruent stage, maximum blading efficiency is possible.

If u/c_1 drops further, in the range $0 < u/c_1 < \cos \alpha_1$ all mathematical profiles will have impulse portion of stream lines besides fully developed reactive portion. They are disposed in the multiform lower horizontal half row III. The $c_0 = w_1$ gradually increases and hence the enthalpy drop h_0 decreases. η_{rb} drops from the maximum obtained when $u/c_1 = \cos \alpha_1$. When $u/c_1 = 0$, the mathematical profile will be symmetrical and is disposed in the bottom horizontal half row IV, characterised by $\beta_2 = \alpha_1$. Within this range of u/c_1, w_1 has again increased to the maximum and hence h_0 is zero when $u/c_1 = 0$. η_{rb} gradually reaches zero, turning force and moment on blades gradually reach the maximum and the power has gradually decreased to zero. The reaction stage has regularly transformed into an impulse one with symmetrical profile characterised by $\rho_{ur} = 0 \cdot 5$. Transformation of inlet velocity triangle into the exit one degenerates into the turn of $\vec{c_1}$ to $\vec{c_2}$. $\vec{c_1}$ and $\vec{w_1}$ coincide and so do $\vec{c_2}$ and $\vec{w_2}$. Fig. 22 indicates the turbine stage characteristics with change in u/c_1 discussed above.

9.5. *Congruent stage of axial compressor.*—Theoretically we can provide congruency of compressor stage by different methods; some of which are:

(*a*) maintain $\vec{w_1}$ constant both magnitude and direction and vary $\vec{u}$ (maintain congruency, *i.e.*, $w_{1a} = w_{2a} = c_{1a} = c_{2a}$ and $\vec{c_1}$ & $\vec{c_2}$ conjugate with $\vec{w_2}$ & $\vec{w_1}$ respectively).

(*b*) maintain $\vec{c_1}$ constant both in magnitude and direction and vary $\vec{u}$ (maintain congruency—see *a*).

(c) maintain $\vec{c}_1$ constant with $\alpha_1 = 90°$ and vary $\vec{u}$ (maintain congruency —see a).

(d) maintain $\vec{c}_1$ constant with $\alpha_1 > 90°$ and vary $\vec{u}$ (maintain congruency) and (e) maintain $\vec{u}$ constant and vary $\vec{c}_1$ (maintain congruency).

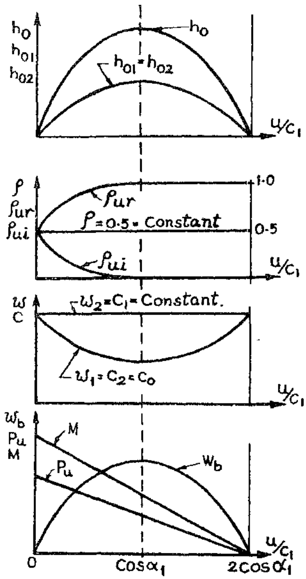

FIG. 22. Congruent Turbine Stage Basic Characteristics. ($0 < u/c_1 < 2 \cos \alpha_1$; $\vec{c}_1$ maintained constant.)

Let us consider any one method, for example e, and analyse the process of transition of mathematical profiles. Choose any profile in the right half horizontal row 0; u is to the left and the flow is directed from the top. Since it is a compressor stage $\vec{w}$ and $\vec{c}$ should rotate clockwise with $\vec{c}$ leading $\vec{w}$. Fig. 23 gives the transition and forms of resulting mathematical profiles. The bottommost profile is flat, mechanically transparent and $u = u_{dem} = 2\ c_1 \cos \alpha_1$. The profile is disposed in row 0. By decreasing the fair-twist of $\vec{c}_1$, maintaining u constant and maintaining conditions of congruency, u/c_1 increases. The mathematical profile begins to form the contrary inlet stream, which is under-developed. The profile is disposed in the multiform upper horizontal half row I. (It is possible to change β_2 and β_1.) When $\alpha_1 = 90° = \beta_2$, the mathematical profile has fully developed contrary inlet stream and is disposed in the half horizontal row II. With $\alpha_1 > 90°$ (*i.e.*, $\beta_2 > 90$), the fair exit stream begins to develop and the profile is disposed in the multiform upper half horizontal row III. It is not possible to reach the symmetrical profile since $u \neq 0$. As $\vec{u} \rightarrow 0$, the mathematical profile can $\rightarrow$ row IV, *i.e.*, a symmetrical profile. At the same time as $u \rightarrow 0$ the mechanically transparent flat profile is disposed nearer to the middle row 0.

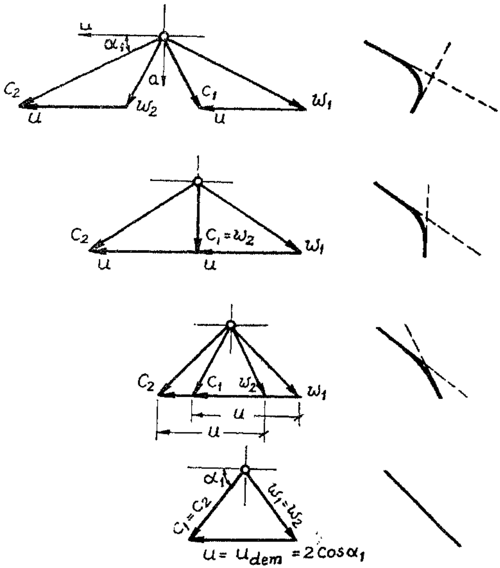

FIG. 23. Congruent Compressor Stage. $u = u_{dem}$ Constant with $\vec{c}_1$ varying ($u/c_1 > 2 \cos \alpha_1$).

The transition of the mathematical profiles for other methods *a*, *b*, *c* and *d* can be studied in a similar manner. From such a study we can draw interesting conclusions:

Mathematical profiles of blades of *congruent compressor stage* can have multiform contours and occupy (for left directed *u* and flow directed from top) the upper right quadrant in the general scheme of mathematical profiles (Fig. 6). From these we can choose any profile, most advantageous from the point of view of theoretical mechanics (diagressing from aerodynamics), which is important practically.

Method *b* gives mathematical profiles (Fig. 24) disposed in the multiform half horizontal row I and are slightly bent. These profiles reflect those commonly employed in practice of compressors. The slight curvature of profile means that the deviation of the equivalent stream, replacing the total flow, is small and hence the force applied to flow by blades is low, as reflected by the expression: $P_u = G/g \times (c_{2u} - c_{1u})$. The power applied to the shaft will be: $W_b = P_u u$. It is hence necessary to have a high u in order to absorb the maximum power. Thereby transferring maximum energy to flow, resulting theoretically in maximum pressure head development. Consideration of ultimate stress allowable in the materials of construction, limits u in design, thereby limiting the ultimate pressure rise in a stage.

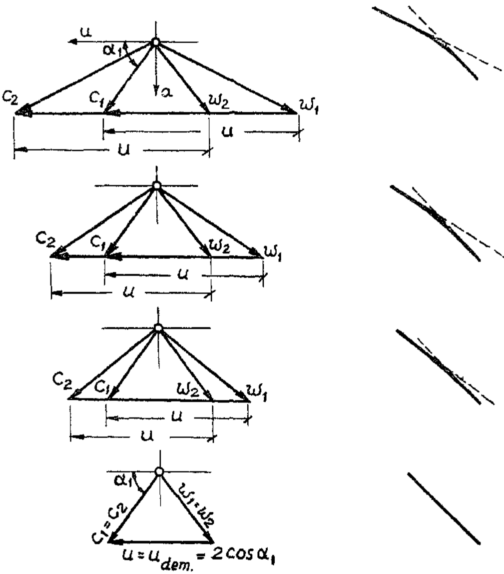

FIG. 24. Congruent Compressor Stage—$\vec{c}_1$ maintained constant ($\alpha_1 < 90$) and u varied ($u > u_{dem}$).

We can compare the profiles resulting from method *e*, given in Fig. 23. The constant magnitude of $\vec{u}$ can be chosen, for instance, from a consideration of the strength of blade material. The mathematical profile gets bent more and more as u/c_1 changes (Fig. 23), and with increasing curvatures of blade, the force applied to flow increases. With a highly curved blade, with a relatively large angle of deviation $\nu = 180 - (\beta_1 + \beta_2)$, the applied force is very large though $\vec{u}$ is moderate. For the same u, pressure rise obtainable with a bent blade is more than a slightly bent one. Though the aerodynamic consideration of flow is quite important, apart from such a consideration, curved blades remove, theoretically, the restrictions in regard to possible rise in pressure imposed by considerations of strength of blade materials. Another limitation arises here. It is necessary to bear in mind that under given moderate velocity u, the absorption of power on blading, and hence the theoretically possible increase in pressure, will be more the more the contrary twist of flow is and more the magnitude of $\vec{c}$. But a high value of c_1 means that in the guide blades ahead of the first stage, pressure decreases considerably and in intermediate guide blades the conversion of kinetic energy into pressure has to be realised only moderately.

A highly curved blade profile is obtained by making the inlet and exit elements of the mathematical profile by straight lines which intersect making as small an inclined angle as possible. In Fig. 25 $\beta_1 + \beta_2$ is made very small. The profile will be fully developed with contrary inlet and fair exit streams, with large or small radius of curvature. The actual profile determines the aerodynamic characteristics

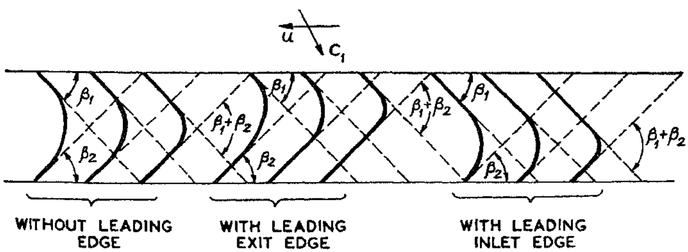

FIG. 25. Multiform Profiles (β_1 and β_2 Remaining Constant).

of the blade. The characteristics can be determined practically only from experimental tests. The shape of the blade can either be determined from experimental data or be based on data by aerodynamical calculation. Experimental data is far from sufficient, at the present time. And it is unfortunate that we have to base the determination of the shape of the actual blade, in insufficient experimental data.

It has been indicated that it would be theoretically more profitable to use highly curved blades in preference to slightly curved ones. However from the experimental aerodynamic point of view, the highly curved blades lack good performance and it may be mentioned here that special adaptation of blade profiles can give better aerodynamic performance.

10. *Reflection of variable regime of stage of turbomachine by general scheme of development of mathematical profiles*

Consider the problems: force transfer from flow to blades of given shape and change of power on blading with change in u/c_1. The ideas developed in earlier sections can be used to study the behaviour of turbine stage under conditions of variable regime. It is also possible to explain the behaviour of turbine during starting, stopping, load throw off, loading, etc.

10 *a*. *The flat blade oriented along the axis of the turbine.*—Consider a theoretical impulse ($\rho = 0$; $\psi = 1$) stage with flat blades oriented along the axis of the turbine. Fig. 26 gives the scheme of arrangement. If we neglect losses, the system is mechanically transparent if $u = u_{dem} = c_1 \cos \alpha_1$. A loading moment M′ applied to the shaft of the machine will be overcome by an equal moment M applied in the reverse direction by force P_u applied by flow to blades. $M = P_u r$, $M = -M'$ and power developed by the moment M is:

$$W_b = M \cdot \omega = P_u r\, u/r = P_u u.$$

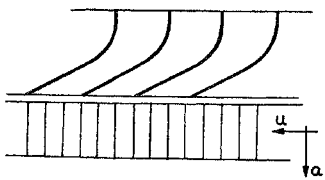

FIG. 26. Scheme of Impulse Turbine Single Stage with Flat Blades Oriented along an Axis.

The circumferential velocity (at mid-blade length) will, now, be $u < c_1 \cos \alpha_1$, and this becomes less and less as M′ on the shaft increases. The inlet and exit velocity triangles will be as shown in Fig. 27. The same figure gives the resulting mathematical profiles of blades. Under these conditions, *i.e.*, $u < c_1 \cos \alpha_1$:

1. The flow gets untwisted (*i.e.*, the fair twist is cut down) while the flow passes through the blade;
2. Force $P_u = G/g\,(c_{1u} - c_{2u})$ is applied to the blades;
3. Power $W_b = P_u u$ is developed by the flow on the blades;
4. The circumferential velocity is u at mid-blade length;
5. Moment on shaft $M = P_u r$; and
6. The angle of incidence between blade and equivalent stream is not zero (*i.e.*, $\beta_{1b} = 90°$ but $\beta_1 \neq 90°$).

In order to maintain zero incidence, we must replace the flat profile by a certain bent profile depending upon the magnitude of u ($u < c_1 \cos \alpha_1$) disposed in the left under horizontal half row II (Fig. 27 *b*). All the profiles are characterised by $\beta_2 = 90°$. With $\vec{c}_1$ and G maintained constant, u decreases more and more as the external loading moment M′ on shaft increases. The velocity triangles change

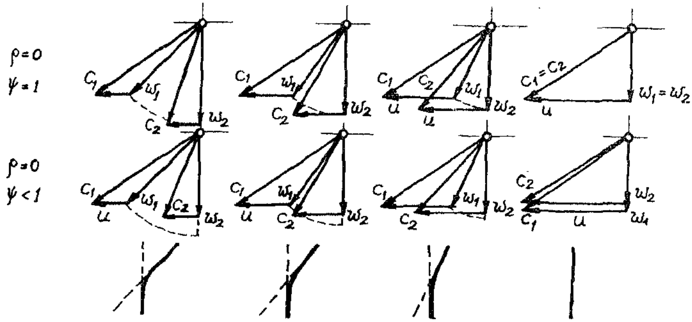

FIG. 27. Velocity Triangles and Equivalent Mathematical Profiles of Scheme in Fig. 26, for $\vec{c}$ Constant and u Varied ($u/c_1 < \cos \alpha_1$).

accordingly and the mathematical profiles, reflecting the deviation of equivalent stream have changed inlet elements though the exit element is maintained constant at $\beta_2 = 90°$. In the scheme (Fig. 6) we can observe that the mathematical profiles are displaced to the left in under half row II.

10 *b*. *Flat blade inclined at an angle to the plane of rotation of wheel.*—Considering force interaction between flow and a flat blade disposed in the left half row 0 under conditions of variable regime, we can deduce that all the regimes are reflected by mathematical profiles disposed in the multiform left half row I. The force interaction between flow and any flat blade disposed in the right half row 0 will be reflected by profiles disposed in the left half rows III, IV and in the right half rows I to IV.

10 *c*. *The bent blade.*—Force interaction between a substantial bent blade and flow can be described by corresponding development of mathematical profile as a function of u/c_1 (scheme in Fig. 6). In this paper we cannot go into the details. However, we can give a brief analysis. In a multistage turbine, some of the stages can transit from the turbine regime to the compressor regime. The transition will be marked by transition of mathematical profiles, reflecting regimes, shifting to the upper rows (*i.e.*, compressor regime) in Fig. 6. As an example we can consider the behaviour of blades in the first or the control stage of a steam turbine governed by nozzle control. The groups of blades drawing steam from a partly

opened valve, at a given moment, will get into a type of force interaction reflected by a complexly bent equivalent stream and a complexly bent mathematical profile of blades. However, we have already seen earlier, that we can replace a complexly bent stream and profile by simple bent ones based on the idea of resulting equivalent stream and resulting mathematical profile. The overall character of force and power interaction can be made clearer by such a replacement. Fig. 28 gives the resulting equivalent stream which consists of the only contrary inlet stream, which is typical for a compressor regime. Therefore in this stage the group of blades, at that particular instant, does not develop power on blading but transfers energy from blades to flow. In this respect it is possible to explain the necessity to have " overlap " of nozzle control valves, though it is compulsory only in the single stage turbine.

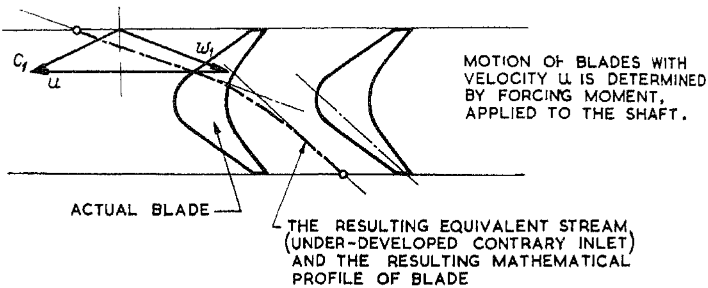

FIG. 28. Turbine Stage Operating under Compressor Regime.

11. *Experimental determination of the connection between the mathematical profile of blade under conditions of given regime and the profile of substantial blade*

The scheme of development of mathematical profiles of blades, Fig. 6, is universal, as has been shown in the general investigation. The scheme reflects all the regimes of the substantial stage with blades, having any given profile, both for left and right rotation. These regimes are: turbine, compressor and demarcating. The profiles shown in the scheme and those belonging to the multiform rows I and III not shown in the figure are the resulting profiles, reflecting the contours of the resulting equivalent streams, replacing flow in theoretical considerations. These profiles represent, in theoretical consideration, a system of blades, and each is characterised by the condition of regime and a certain loading or forcing moment of external forces applied to the shaft of the turbomachine under given conditions at entry to blades. The force interaction between flow and blades is reflected by the equivalent stream. To determine the total force interaction it is sufficient to find out the extreme points of hodographs $\vec{w}, \vec{c}$ of the equivalent stream.

When flow conditions change, total force interaction changes and the new regime will be reflected by a different mathematical profile and equivalent stream.

All possible types of streams can be reflected by the profiles of the scheme. If in the scheme the flow is directed from the top and the direction of profile movement is to the left, all the upper rows above row 0 represent compressor regime characterised by fair inlet and contrary exit streams. Maintaining the flow direction, if the direction of profile movement is changed to the right, the regimes get interchanged, all the upper rows are turbine profiles and the under rows compressor profiles. The row 0 reflects the demarcating regime, the mechanically transparent systems. If the flow connected with such profiles has both axial and rotary movement, the row reflects the regime of screw propellers. When the conditions of the demarcating regime changes the mathematical profile just moves over from row 0 to one of the neighbouring rows, characterising either compressor or turbine regime.

In actual machines the stream of flow can be complexly bent and even then, these can be replaced by complicated equivalent streams and the corresponding mathematical profiles can be got by combinations of simple profiles, taken from different rows. For instance, the symmetrical profile disposed in the upper (or under) row IV can be considered as the combination of two profiles taken from the upper (or under) row II. One of them belongs to the right half row and the other to the left half row. Independent of the combined profile, it is always possible to replace the flow by the resulting mathematical profile, disposed in a certain row, reflecting the total force interaction and power exchange between flow and blades, whether it be a compressor or turbine stage.

Thus the replacement of substantial flow by imaginary equivalent stream and replacement of substantial blades by imaginary mathematical ones, gives the possibility to schematise the phenomena taking place within a stage of a turbomachine and calculate the force interaction and energy exchange by means of the simple and lucid equation of Euler. To make the replacements possible, we must have reliable experimental data which enable us to determine extreme points of hodographs of $\vec{c}$, $\vec{w}$ describing the motion of the equivalent stream. Evidently these initial data could be gathered only by experimental methods.

Once the manner of defining the design exit angles is decided (*i.e.*, conventionally either $\alpha_1 = \alpha_{1n}$ & $\beta_2 = \beta_{2s}$ or $\alpha_1 = \alpha_{1eff}$ & $\beta_2 = \beta_{2eff}$), we can always decide on the necessary experimental set-up to determine the velocity coefficients ϕ, ψ for the equivalent stream. All well-known experimental methods to determine ϕ & ψ (the impulse plate method, reaction method, impact tube method, etc.) are not always correctly directed since the coefficients are separately determined for nozzles and stationary (*i.e.*, stopped) blades. Whereas, in a turbine stage, nozzles and blades are always in pairs and blades are in motion. Hence the coefficients must be determined from experimental set-ups corresponding to the actual conditions, with blades in motion under conditions of any given regime. The regime conditions are defined by:

1. the moment (loading or forcing) of external forces applied to shaft;
2. the pressures in front of nozzles and behind blades;
3. the total flow of steam or gas from nozzles; and
4. the velocity and direction of motion of blades.

The analysis and experimental method can be clarified by following an example:

Stage under analysis: Single impulse ($\rho = 0$) *stage with supersonic velocity* of discharge from nozzles. Figure 29 shows a section through the stage. The nozzle velocity coefficient ϕ is experimentally determined by any of the well-known methods. We can replace the substantial flow and blades by the equivalent stream and the mathematical profile shown in Fig. 29. Let exit angles be defined by $\alpha_1 = \alpha_{1n}$ and $\beta_2 = \beta_{2b}$. α_1, β_1 and radius to mid-length can be measured directly. By loading the shaft carrying the tested wheel, with a brake, given regime of work

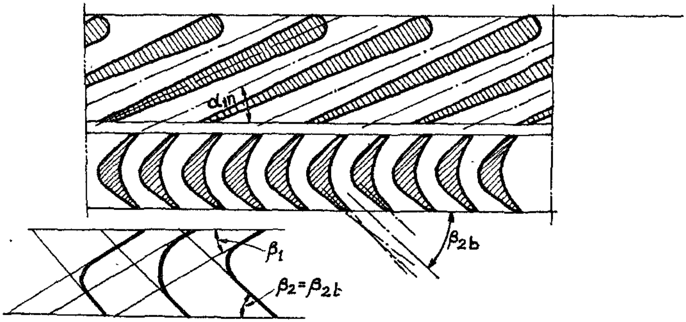

FIG. 29. Impulse Single Turbine Stage ($\rho = 0$) and Multiform Mathematical Profiles (β_1 and β_2 maintained Constant).

can be maintained. The data to be taken from the experimental set-up are: G, N, p_0, p_2 and M. (By an initial run, the friction moment on bearings can be determined and M is adjusted to include this friction moment.) Turning force at (mid-blade length) radius r is given by:

$P_u = M/r \ldots\ldots Kg.$ u m./sec. can be evaluated ($u = 2\pi r N/60$).

Corresponding to radius r, work done on blading is:

$W_b = P_u u = G/g\,(c_{1u} - c_{2u})u = G/g\,(w_{1u} - w_{2u})\,u$; and the value of $(c_{1u} - c_{2u}) = (w_{1u} - w_{2u})$ can be determined. With the above data the blade velocity coefficient ϕ, for the tested blades under conditions of given regime, can be determined by the graphic construction given in Fig. 30.

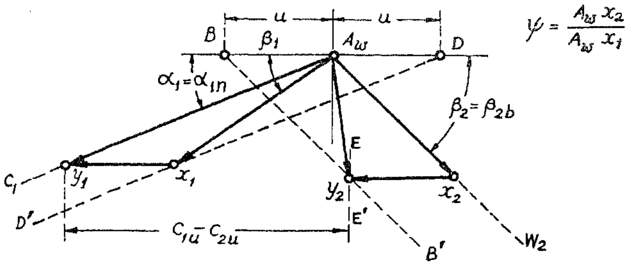

FIG. 30. Graphical Construction for Determination of ψ.

A_w is the pole of hodographs $\vec{w}$, $\vec{c}$. B A_w D is the direction of $\vec{u}$ and set off B A_w = A_wD $= u$. Draw $A_w C_1$ at α_1 ($\alpha_1 = \alpha_{1n}$) and D D′ parallel to $A_w C_1$. Set $A_w W_2$ at β_2 ($\beta_2 = \beta_{2b}$) and draw B B′ parallel to $A_w W_2$. Magnitude of $\vec{c}_1$ can be determined from the isentropic decrease in enthalpy in nozzles and ϕ. $c_1 = \phi\, c_{1t}$. Along $A_w C_1$ set off $A_w Y_1 = c_1$ to the same scale as u and $y_1 x_1$ parallel to B A_w D, cutting D D′ at x_1. The inlet triangle $A_w y_1 x_1$ is determined now, $A_w x_1$ being the relative inlet velocity to blade. y_1 is the initial point of hodograph $\vec{c}$ and x_1 is the initial point of hodograph $\vec{w}$. Draw E E′ perpendicular to B A_w D at a distance ($c_{1u} - c_{2u}$) determined from experiment, in the direction of u, cutting B B′ at y_2. y_2 is the final point of hodograph $\vec{c}$ and $A_w y_2$ is the exit absolute velocity $\vec{c}_2$. Draw $y_2 x_2$ parallel to B A_w D cutting $A_w W_2$ at x_2. x_2 is then the final point of hodograph $\vec{w}$ and $A_w x_2$ is the relative exit velocity $\vec{w}_2$. $A_w y_2 x_2$ is the exit velocity triangle. The magnitude of $\vec{w}_1$, *i.e.*, $A_w x_1$ and $\vec{w}_2$, *i.e.*, $A_w x_2$ determine the value of the blade velocity coefficient ψ:

$$\psi = w_2/w_1 = A_w x_2/A_w x_1 \text{ for the given conditions.}$$

Change of regime can be brought about by altering discharge of steam or gas; speed of wheel; pressures in front of nozzles and behind blades. The inlet and exit velocity triangles will change according to the changed regime. The mathematical profile and the equivalent stream, characterising the regime, will be different in each case, although the shape of the real blade is the same.

The method described above, for the determination of the blade velocity coefficient ψ, applies to an impulse stage with supersonic discharge velocity c_1 from nozzles. In such a case, the flow disturbance from blades cannot travel upstream and conditions of operation of nozzles, thus, conform to the isolated case, under which ϕ was determined. In the absence of experimental verification, it is not possible to decide the merits of the method, when applied to a stage characterised by thermal reactivity $\rho > 0$ and subsonic discharge velocity from nozzles. This is so because flow disturbances downstream in the blades can travel upstream, flow being subsonic, and results in altered conditions of nozzle operation (*i.e.*, ϕ may be disturbed). Some modifications, to the experiment and the graphical method presented, may be necessary to make it more precise, so as to be applicable to any general case.

It is possible to discredit the present method, in that it does not separate the individual losses, such as rotation losses, leakage losses, etc. From the authors' point of view, *it is not a drawback but a merit of this method.* The behaviour of a separate nozzle or blade is not of interest to the engineer, who is more concerned with the problem of utilisation of energy of flow within a real stage under given conditions.

12. "*Indicator Diagram*" *of Turbomachine*

Pressure distribution around the blade profile, plotted from experimental data, is shown in Fig. 31. Let the figure give the circumferential component of pressure in the direction of u, and later when reference is drawn to pressure, the circumferential component acting on an element of blade surface is to be taken into account. The magnitude and pressure direction (positive or negative) are different for the concave and the convex surfaces of the blade. The net pressure difference acts in the direction of motion of blades in a turbine stage (in a compressor it is opposed to the motion of blades) and is the force moving the blades. In the figure curve *i1e* represents the pressure on the concave surface and *i2e* the pressure on the convex surface. Thus the shaded area gives the net force in the direction of u. Joukowski's wing theory explains that the blade motion is caused by the pressure differences acting on the blade.

The shaded area representing the net pressure (*i1e2i*) can be replaced by an equiarea rectangle with side b, blade projected dimension in a plane perpendicular to u and height p_i, *i.e.*, " the mean difference of pressure ". Thus the total pressure acting on the entire blade along its length l, will be $b.p_i.l$.

Power indicated W_i on blades of a turbine stage will, then, be:

$$W_i = p_i blz\pi dN/60 \times 75 \qquad \text{HP}$$

The expression for indicated power of stage is identical with the one used to determine the indicated power of a reciprocating engine. The expression confirms that the work done by a turbine stage equals the acting force multiplied by the velocity of motion. For this reason, we can call the diagram of pressure distribution around the blade profile, as the " Indicated Diagram". Of course, we cannot take such a diagram by the commonly used indicators. A turbine can be considered as an engine with a number of pistons (equal in number to Z blades with flow around) and infinite stroke (rotary motion), with continuous flow of gas. Whereas in a reciprocating engine, the flow is intermittent and there is definite stroke traversed by the piston, which is reciprocating.

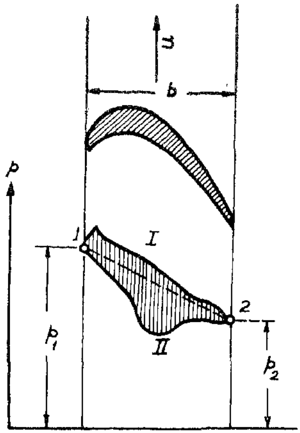

FIG. 31. Pressure Distribution around Blade Profile (Indicated Diagram of Turbomachine Stage).

The expression for indicated power gives a good qualitative interpretation of the mechanism of force transfer from flow to blades of turbine. It shows that the motion of wheel is due to the pressure difference between the two sides of work area of blades. In Fig. 31 the contour of pressures around blade profile varies along the length of the blade and is turning with respect to the stationary nozzles, *i.e.*, pressures along turbine axis and circumferential direction pulsate. The axial gradient of pressure will be controlled by action of centrifugal forces and influence of radial blade clearance. The pulsation of pressure distribution around blade profile at radius r will be determined by the influence of velocity field set by the system of nozzles. Hence, p_i in the expression for indicated power of turbine stage must be the mean values evaluated along the radius, axis and circumference.

$$p_i = \frac{Z \int_\theta \int_l \int_b \phi(\theta, l, b)\, d\theta\, dl\, db}{blZ}$$

The expression for power on blading shows that the power is determined by the pitch of the blades (*i.e.*, the number of blades) and blade length. As the pitch changes the mean indicated pressure p_i changes and will be maximum for certain pitch for blades of given shape. Increase in blade length will indicate greater mass flow and leakage will form a lesser proportion of total flow. Hence increase in blade length gives increased value for p_i. Twist of blades along its length can even out the pressure profiles around blade section at different radii along its length. In other words, a complicated geometrical form of blade will simplify the profile of pressure distribution around the blade, making the net difference of circumferential pressure components more uniform along the blade height. Contrary to this a simple geometrical form of blade (*i.e.*, with no twist), will have the complicated shape of a prism representing the distribution of circumferential component of pressure along the blade. We can partly examine the considerations, given above, by establishing a flow through a system of stationary flat cascades of profiles or aerofoils. Experimentation with rotary wheels needs special equipment. But such experiments are quite possible, although not necesasary from point of view of design practice.

In principle, if abundant experimental data is available, it is possible to offer the following method of design of turbomachines.

Any blade can be characterised by a certain value of p_i under conditions of given regime. The value of p_i depends on many factors—diameter of stage, length of blade, pitch of blade, blade profile, twist of blades along its length, etc. It is very simple to determine the value of p_i for a ready-made turbine by experimental method. Turning force under conditions of given regime must be measured. The regime is determined by the total discharge of steam or gas from nozzles, the wheel speed and the initial and final pressures. Measuring the moment M, from the expression $M = P_u r$, P_u can be evaluated. p_i can now be determined from the expression $p_i = P_u/blZ$. The value of p_i reflects all losses, pulsating character of flow, etc. The experiment can be repeated under varying conditions. For instance, if the total discharge G, pressures in front of nozzles and behind blades are kept constant and the loading moment M changed, we can find the function: $p_i = f(u)$, *i.e.*, $p_i/p_{i\,max} = f(u/c_1)$. Equating the indicated power to the power expressed by Euler's equation, we have: $p_i blZ = (G/g)(c_{1u} - c_{2u})$, *i.e.*,

$$p_i = \frac{G}{g}(c_{1u} - c_{2u})\frac{1}{blZ}.$$

This expression in fact substantiates the idea that Euler's and Joukowski's theories do not contradict but only supplement each other.

Conclusion

Methods of design of turbomachines can be very different, but all are based on experimental data. As a matter of fact every one of the methods of calculation and design is only a method of equivalent replacements. The commonly used method of design, the method based on Euler's theory, is based on equally solid grounds as the method based on the theory of Joukowski. In essence, Joukowski's method is based on the concept " that a system of vortices can replace a solid enveloped by a fluid in motion ". By this replacement substantial blades are removed from flow in theoretical considerations. Joukowski's method of calculation has been very useful in the field of design of wings for aircraft. Applied to turbomachines, this method can be simplified still further by the introduction of the concept of " mean pressure difference " acting on blade, instead of theoretical calculation of difference of pressures distributed unequally on the surface of the same blade. It is the natural replacement.

The concept of " added circulation " is not necessary to explain the cause for pressure difference. The flow mass is distributed unequally, under the influence of centrifugal forces, acting upon the particles, which are forced to move in a curvilinear stream line. It must be taken as an unalterable fact confirmed experimentally.

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References

1. Yourinsky, V. T. .. "The investigation of force and power interaction in a turbomachine stage by the method of equivalent replacements (in Russian)," *Trudi - Novotcherkassk. Polytechn. Inst.*, 1956, **37** (51).

2. ——————— .. "A new form of the equation of power and force on blading of turbomachine stage (in Russian)," *Neutchn Trudi Novotcherhassk. Polytechn. Inst.*, 1957, **50** (64).