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A STUDY ON CASCADE CONNECTED TRANSFORMERS

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ABSTRACT

The present paper deals with two methods of analysing 2 and 3-unit cascade transformer circuits and gives expressions for output voltage in the case of Dessauer Cascade Connection.

1. INTRODUCTION

No serious attempt seems to have been made in the past to analyse theoretically cascade transformer circuits, especially when two or more transformer units are connected in the Dessauer Cascade Connection similar to Fig. 1. Recently Hagenguth and others,[1] while working with a 5-unit cascade transformer for determining the sixty-cycle spark-over voltage of large gap spacings, have observed that the output voltage of cascade connected transformers is very sensitive to load capacitance. They have also mentioned that the calculations made by Prof. R. R. Benedict of the University of Wisconsin, U.S.A., indicate that the ratio of transformers connected in cascade is calculable on the basis of a series connection of three-winding transformer equivalent circuits. Expressions for open-circuit admittance and short-circuit impedance in the case of n-unit cascade transformer in which 'n' identical transformers are connected in the Dessauer Cascade Connection, have been derived in an earlier paper.[2]

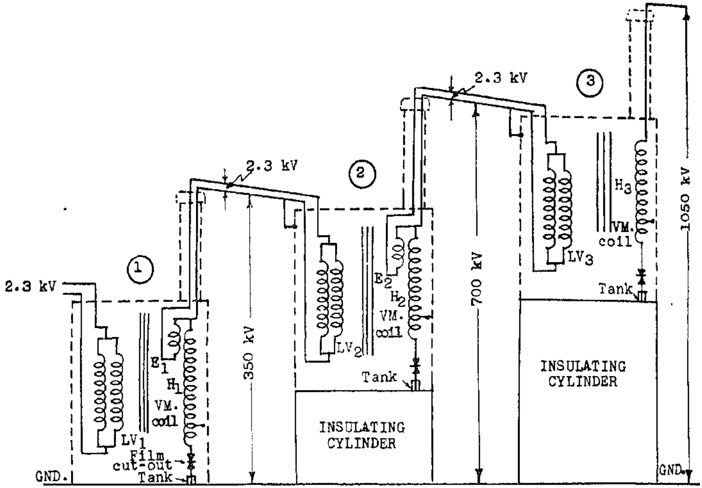

FIG. 1. Connection Diagram of the 3-Unit Cascade Transformer Installed at the Indian Institute of Science[1] for Producing 1050 kv.

2. LIST OF PRINCIPAL SYMBOLS

1. 1st unit = The ground unit of a n-unit cascade transformer.
2. 2nd...kth...$(n-1)$th unit = Intermediate units of a n-unit cascade transformer.
3. nth unit = The last or line unit of a n-unit cascade transformer
4. $LV_1, LV_2 \ldots LV_k \ldots\ldots LV_{(n-1)}$, LV_n = Low-voltage windings of the units 1 to n.
5. $H_1, H_2 \ldots H_k \ldots\ldots H_{(n-1)}$, H_n = High-voltage windings of the units 1 to n.
6. $E_1, E_2 \ldots E_k \ldots\ldots E_{(n-1)}$ = Exciting windings of the units 1 to $(n-1)$.
7. Y_s = open-circuit admittance of the units 1 to n—considered same for all the units as they are identical.
8. Z_{LV} = Leakage impedance associated with the low-voltage winding—considered same for all the units (referred to $l.v.$ side).
9. Z_H = Leakage impedance associated with the high-voltage winding—considered same for all the units (referred to $l.v.$ side).
10. Z_E = Leakage impedance associated with the exciting winding —considered same for all the $(n-1)$ units (referred to $l.v.$ side).

11. $Z_T = Z_{LV} + Z_H + Z_E$

12. $Z_2 = Z_{LV} + Z_E$

13. V_{L1} = Voltage across the input terminals.

14. I_{LV} = Current at the input terminals.

15. $Y = I_{LV}/V_{L1}$ = input admittance.

16. $V_{L2} \ldots V_{Lk} \ldots\ldots V_{L(n-1)}, V_{Ln}$ = Voltages across windings LV_2 to LV_n = voltages across windings E_1 to $E_{(n-1)}$.

17. $I_{12} \ldots I_{(k-1)k} \ldots I_{(n-2)(n-1)}, I_{(n-1)n}$ = Currents in windings LV_2 to LV_n = currents in windings E_1 to $E_{(n-1)}$.

18. $I_{e1}, I_{e2} \ldots I_{ek} \ldots\ldots I_{e(n-1)}, I_{en}$ = Exciting currents of the units 1 to n.

19. $V_{H1}, V_{H2} \ldots V_{Hk} \ldots\ldots V_{H(n-1)}, V_{Hn}$ = Secondary (*h.v.* winding) voltages of the units 1 to n (referred to *l.v.* side).

20. $I_{H1}, I_{H2} \ldots I_{Hk} \ldots\ldots I_{H(n-1)}, I_{Hn}$ = Currents in the windings H_1 to H_n (referred to *l.v.* side).

21. $V_{1G}, V_{2G} \ldots V_{kG} \ldots\ldots V_{(n-1)G}, V_{nG}$ = H.V. terminal voltages with respect to ground of the units 1 to n (referred to *l.v.* side).

22. C_{23} = Capacitance between the tanks of units 2 and 3.

23. $C_{2G} \ldots C_{kG} \ldots\ldots C_{(n-1)G}, C_{nG}$ = Tank-to-ground capacitances of the units 2 to n.

24. $Y_{2G} \ldots Y_{kG} \ldots\ldots Y_{(n-1)G}, Y_{nG}$ = Tank-to-ground admittances of the units 2 to n (refererd to *l.v.* side).

25. $I_{2G} \ldots I_{kG} \ldots\ldots I_{(n-1)G}, I_{nG}$ = Currents through the respective tank-to-ground admittances (referred to *l.v.* side).

26. $Y'_{e2} \ldots Y'_{ek} \ldots\ldots Y'_{e(n-1)}, Y'_{en}$ = Open circuit admittance of 2. k..$(n-1)$, n-unit cascade transformer respectively (referred to *l.v.* side).

27. $Z'_{s2} \ldots Z'_{sk} \ldots\ldots Z'_{s(n-1)}, Z'_{sn}$ = Short circuit impedace of 2..k..$(n-1)$, n-unit cascade transformer respectively (referred to *l.v.* side).

28. $I'_{e2} \ldots I'_{ek} \ldots\ldots I'_{e(n-1)}, I'_{en}$ = No-load currents of 2..k..$(n-1)$, n-unit cascade transformer respectively (referred to *l.v.* side).

29. $V'_{2G} \ldots V'_{kG} \ldots\ldots V'_{(n-1)G}, V'_{nG}$ = output voltages of 2..k..$(n-1)$, n-unit cascade transformer in their respective simple equivalent circuits (referred to *l.v.* side).

30. C_L = Load capacitance.

31. Y_L = Load admittance in the comprehensive equivalent circuit (referred to *l.v.* side).

32. Y_L' = Load admittance in the simple equivalent circuit (referred to *l.v.* side).

33. I_L = Load current in the comprehensive equivalent circuit (referred to *l.v.* side).

34. I_L' = Load current in the simple equivalent circuit (referred to *l.v.* side).

3. Ratio of 2-Unit Cascade Transformer

A single unit can be represented by an equivalent circuit of a two-winding transformer shown in Fig. 2. It can be shown that the output voltage can be expressed in terms of input voltage by the following equation:

$$V_{L1} = V_{1G}\,[1 + Y_x . Z_{LV} + Y_L\,\{Z_H + Z_{LV}\,(1 + Z_H . Y_x)\}] \tag{1}$$

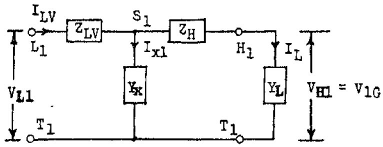

Fig. 2. Equivalent Circuit of a Single Transformer.

(3.1) *Comprehensive equivalent circuit.*—The ratio of 2-unit cascade transformer is calculated from the comprehensive equivalent circuit shown in Fig. 3. The equivalent circuit of the three winding unit-1 and the equivalent circuit of the two-winding unit-2 (the exciting winding of unit-2 being treated as absent as it is not used) are coupled together by an ideal 1 : 1 transformer to preserve the voltage and current relations.[2] As an approximation the exciting current of unit-2 is assumed to flow through the whole leakage impedance of unit-2, *i.e.*, $(Z_{LV} + Z_H)$.

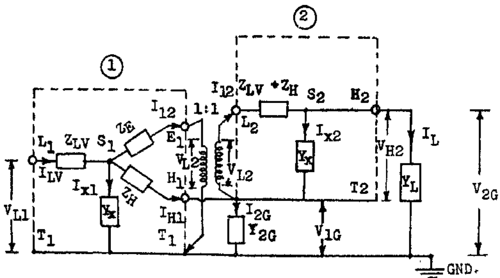

Fig. 3. Comprehensive Equivalent Circuit of 2-Unit Cascade Transformer.

The method of analysing the equivalent circuit shown in Fig. 3 has been described in detail in Appendix A. The following relations are applicable in the case of 2-unit cascade transformer:

$$\frac{V_{2G}}{V_{1G}} = \frac{2 + Y_x Z_T + Y_{2G} . Z_H}{1 + Y_x . Z_T + Y_L\,(Z_T - Z_H)} \tag{2}$$

and

$$V_{L1} = V_{1G}\,[1 + Y_{2G}\,\{Z_H + Z_{LV}\,(1 + Y_x . Z_H)\}]$$
$$+ V_{2G}\,[Y_x . Z_{LV} + Y_L\,\{Z_H + Z_{LV}\,(2 + Y_x . Z_H)\}] \qquad (3)$$

In view of the long calculations, it would be useful to first solve the network for the ratio V_{2G}/V_{1G} and then to extend the analysis to determine the ratio V_{2G}/V_{L1} after evaluating the currents in the different branches.

The equations (2) and (3) are cumbersome from the point of view of practical calculations. They could further be simplified to some extent for practical calculations if the resistances in the circuit can be neglected. In such a case the voltage drops can be added or subtracted arithmetically as they either lead or lag the current by 180°.

(3.2) *The simple equivalent circuit.*—A 2-unit cascade transformer can also be treated as a simple four terminal network, *i.e.*, as a single two-winding transformer with two input and two output terminals. Such an equivalent circuit shown in Fig. 4 would have only two constants, the shunt admittance Y'_{x2} and series impedance Z'_{s2}. The above constants could either be those derived from the general expressions given in the earlier paper[3] for a n-unit cascade transformer or those measured during the open circuit and short-circuit tests on a 2-unit cascade transformer. The ratio V'_{2G}/V_{L1} is given by the simple relationship

$$\frac{V'_{2G}}{V_{L1}} = \left[\frac{1}{1 + (Y'_{x2} + Y'_{L2})\, Z'_{s2}}\right] \qquad (4)$$

The derivation is shown in Appendix B. However, with such a simple equivalent circuit it is not possible to evaluate the individual branch currents and voltages.

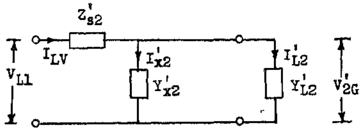

FIG. 4. Simple Equivalent Circuit of 2-Unit Cascade Transformer.

4. Ratio of 3-Unit Cascade Transformer

(4.1) *Comprehensive equivalent circuit.*—The ratio of 3-unit cascade transformer is calculated from the comprehensive equivalent circuit shown in Fig. 5. As before, the exciting current of the line unit (No. 3) is assumed to flow through the whole leakage impedance, *i.e.*, $(Z_{LV} + Z_H)$.

The method of analysis is the same as that used for 2-unit cascade. At first the network is solved for the ratio V_{3G}/V_{1G} and the analysis is extended to determine the ratio V_{3G}/V_{L1} after evaluating the currents in the different branches. The following relations are applicable for calculating the ratio of 3-unit cascade transformer.

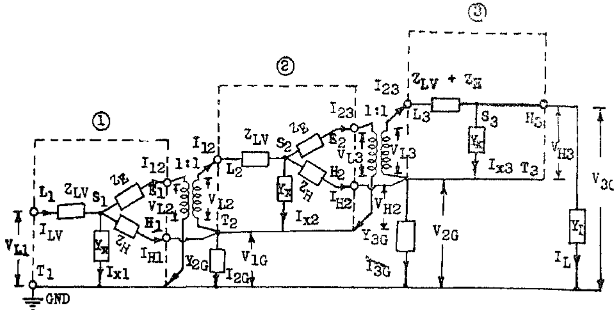

FIG. 5. Comprehensive Equivalent Circuit of 3-Unit Cascade Transformer.

$$
\begin{aligned}
& V_{3G}\,[\{1 + Y_{3G}.Z_2\,(1 + Z_H.Y_x)\}\,(1 + Y_x.Z_T + Y_L.Z_2) \\
& \quad + \{Y_x.Z_2 + Y_L.Z_2\,(2 + Z_H.Y_x)\}\,(2 + Y_{3G}.Z_H + Y_x.Z_T)] \\
& \quad = V_{1G}\,[(2 + Y_{3G}.Z_H + Y_x.Z_T)\,(2 + Y_{3G}.Z_H + Y_x.Z_2) \\
& \qquad \{1 + Y_{3G}.Z_2\,(1 + Z_H.Y_x)\}]
\end{aligned}
\tag{5}
$$

and

$$
\begin{aligned}
V_{H2} = V_{1G} & \left[\frac{1 + Y_{2G}.Z_H + Y_x.Z_2 - Y_{3G}.Z_2\,(1 + Z_H.Y_x)}{1 + Y_{3G}.Z_2\,(1 + Z_H.Y_x)}\right. \\
& - V_{3G}\left[\frac{Y_x.Z_2 + Y_L.Z_2\,(2 + Z_H.Y_x)}{1 + Y_{3G}.Z_2\,(1 + Z_H.Y_x)}\right]
\end{aligned}
\tag{6}
$$

As in the case of cascade transformer, neglecting resistances would make the calculations easier. For further simplification possibility of neglecting Z_H completely can be considered. Such a procedure becomes possible because the general expression which gives the short-circuit impedance of a n-unit cascade transformer[2] reveals that the influence of Z_H on the total impedance decreases with the increase of n.

$$
Z'_{sn} = Z_{LV} + \frac{Z_H}{n} + \frac{\Sigma_1^{(n-1)}\,k^2}{n^2}\,(Z_{LV} + Z_E)
\tag{7}
$$

By these approximations the equivalent circuit is simplified and the calculations made easier.

(4.2) *Simple equivalent circuit.*—It is possible to treat the 3-unit cascade transformer also as a single two-winding transformer and the same represented by an equivalent circuit shown in Fig. 6. The method of analysis is the same as that used for 2-unit cascade and the ratio V'_3/V_1 is given by

$$\frac{V'_{3G}}{V_{L1}} = \left[\frac{1}{1 + (Y'_{x3} + Y'_{L3})\, Z'_{s3}}\right] \tag{8}$$

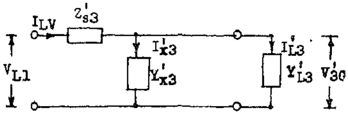

FIG. 6. Simple Equivalent Circuit of 3-Unit Cascade Transformer.

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2. Jayaram B. N. .. "Derivation of constants of a *n*-unit cascade transformer," *J. Ind. Inst. Sci.* 1955, 37, 162.

6. ACKNOWLEDGEMENT

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APPENDIX A

CALCULATION OF THE RATIO OF 2-UNIT CASCADE TRANSFORMER USING THE COMPREHENSIVE EQUIVALENT CIRCUIT

(a) Resistances Included

N.B.—(i) All quantities are vectorial and are referred to low-voltage winding.

(ii) It is assumed that the exciting current of the line unit flows through the whole leakage impedance, *i.e.*, $Z_{LV} + Z_H$.

Referring to Fig. 3

$$V_{H1} = V_{1G} \text{ and } V_{2G} = V_{1G} + V_{H2} \qquad (A.01)$$

Considering the loop $L_2S_2H_2T_2$ in the 2nd unit

$$V_{H2} = V_{L2} - I_{12}(Z_{LV} + Z_H) \qquad (A.02)$$

Again considering the loop $H_1S_1E_1T_1$ in the 1st unit

$$V_{L2} = V_{H1} + I_{H1}.Z_H - I_{12}.Z_E \qquad (A.03)$$

Substituting (A.03) in (A.02)

$$\begin{aligned} V_{H2} &= V_{H1} + I_{H1}.Z_H - I_{12}(Z_{LV} + Z_H + Z_E) \\ &= V_{H1} + I_{H1}.Z_H - I_{12}.Z_T \qquad (A.04) \end{aligned}$$

where

$$Z_T = Z_{LV} + Z_H + Z_E \qquad (A.05)$$

Further at the junction T_2 in the 2nd unit

$$\begin{aligned} I_{H1} &= I_L + I_{2G} \\ &= V_{2G}.Y_L + V_{1G}.Y_{2G} \qquad (A.06) \end{aligned}$$

and at the junction S_2 in the 2nd unit

$$\begin{aligned} I_{12} &= I_L + I_{z2} \\ &= V_{2G}.Y_L + V_{H2}.Y_z \qquad (A.07) \end{aligned}$$

Substituting (A.06) and (A.07) in (A.04)

$$V_{H2} = V_{H1} + (V_{2G}.Y_L + V_{1G}.Y_{2G})Z_H - (V_{2G}.Y_L + V_{H2}.Y_z)Z_T$$

Again from (A.01)

$$\begin{aligned} (V_{2G} - V_{1G}) &= V_{1G} + V_{2G}.Y_L.Z_H + V_{1G}.Y_{2G}.Z_H \\ &\quad - V_{2G}.Y_L.Z_T - (V_{2G} - V_{1G})Y_z.Z_T \end{aligned}$$

Rearranging

$$V_{2G}\{1 + Y_x . Z_T + Y_L (Z_T - Z_H)\}$$
$$= V_{1G} (2 + Y_x . Z_T + Y_{2G} . Z_H)$$

$$\therefore \quad V_{2G} = V_{1G} \left[\frac{2 + Y_x . Z_T + Y_{2G} . Z_H}{1 + Y_x . Z_T + Y_L (Z_T - Z_H)} \right] \qquad \text{(A.08)}$$

Again considering the loop $L_1 S_1 H_1 T_1$ in the 1st unit

$$V_{L1} = V_{1G} + I_{H1} . Z_H + I_{LV} . Z_{LV} \qquad \text{(A.09)}$$

but at the junction S_1 in the 1st unit

$$I_{LV} = I_{12} + I_{H1} + I_{x1} \qquad \text{(A.10)}$$

Substituting from (A.05) and (A.06)

$$I_{LV} + (I_{x2} + I_L) + (I_L + I_{2G}) + I_{x1} \qquad \text{(A.11)}$$

where

$$I_{x2} = V_{H2} . Y_x$$
$$I_L = V_{2G} . Y_L$$
$$I_{2G} = V_{1G} . Y_{2G}$$

and

$$I_{x1} = (V_{1G} + I_{H1} . Z_H) Y_x$$
$$= [V_{1G} + (V_{2G} . Y_L + V_{1G} . Y_{2G}) Z_H] Y_x$$
$$= (V_{1G} . Y_x + V_{2G} . Y_L . Z_H . Y_x + V_{1G} . Y_{2G} . Z_H . Y_x)$$

$$\therefore \quad I_{LV} = V_{H2} . Y_x + 2 . V_{2G} . Y_L + V_{1G} . Y_{2G} + V_{1G} . Y_x$$
$$+ V_{2G} . Y_L . Z_H . Y_x + V_{1G} . Y_{2G} . Z_H . Y_x$$
$$= (V_{2G} - V_{1G}) Y_x + 2 V_{2G} . Y_L + V_{1G} . Y_{2G}$$
$$+ V_{1G} . Y_x + V_{2G} . Y_L . Z_H . Y_x + V_{1G} . Y_{2G} . Z_H . Y_x$$
$$= V_{1G} (Y_{2G} + Y_{2G} . Z_H . Y_x)$$
$$+ V_{2G} (Y_x + 2 Y_L + Y_L . Z_H . Y_x) \qquad \text{(A.12)}$$

Substituting for I_{H1} and I_{LV} in (A.09) from (A.06) and (A.12)

$$V_{L1} = V_{1G} + V_{2G} . Y_L . Z_H + V_{1G} . Y_{2G} . Z_H$$
$$+ V_{1G} (Y_{2G} + Y_{2G} . Z_H . Y_x) Z_{LV}$$
$$+ V_{2G} (Y_x + 2 Y_L + Y_L . Z_H . Y_x) Z_{LV}$$
$$= V_{1G} [1 + Y_{2G} . Z_H + Y_{2G} . Z_{LV} (1 + Y_x . Z_H)]$$
$$+ V_{2G} [Y_x . Z_{LV} + Y_L . Z_H + Y_L . Z_{LV} (2 + Y_x . Z_H)] \qquad \text{(A.13)}$$

(A.08) and (A.13) could be used for determining the ratio V_{2G}/V_{L1}.

APPENDIX B

CALCULATION OF THE RATIO OF 2-UNIT CASCADE TRANSFORMER USING THE SIMPLE EQUIVALENT CIRCUIT

Solving the simple equivalent circuit of the 2-unit cascade shown in Fig. 4

$$\begin{aligned} V_{L1} &= V'_{2G} + I_{LV} \cdot Z'_{s2} \qquad \text{(B.01)} \\ &= V'_{2G} + (I'_{x2} + I'_{L2})\, Z'_{s2} \\ &= V'_{2G} + V'_{2G}\,(Y'_{x2} + Y'_{L2})\, Z'_{s2} \end{aligned}$$

$$\therefore \quad \frac{V'_{2G}}{V_{L1}} = \left[\frac{1}{1 + (Y'_{x2} + Y'_{L2})\, Z'_{s2}}\right] \qquad \text{(B.02)}$$