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INVESTIGATION OF DOUBLE MOTION PRINCIPLE
IN AXIAL MULTISTAGE TURBOMACHINES

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ABSTRACT

This paper presents the investigation of the characteristics of axial double motion turbomachines by the method of equivalent replacements.^{1*} The system of expressions formulated in reference (2) is used for this investigation. Typical schemes are analysed and power coefficients evaluated for comparison with axial single motion turbomachines. It is illustrated that the double motion adoption can intensify the force coefficient, hence the power output and lessen the number of stages for a given total enthalpy drop. However it is illustrated that double motion principle can also weaken the force coefficient, if certain forms of mathematical profiles are adopted for the blading. Investigations so far presented have been particular cases of axial double motion turbomachine. The method of equivalent replacements makes it possible to investigate the general cases, as presented in this paper.

NOMENCLATURE

The following nomenclature is used in this paper:

- c absolute velocity (m./sec.).
- \vec{c} absolute velocity vector.
- c_u circumferential component of c .

* Superscribed numbers refer to respective references.

c_a	axial component of c .
g	acceleration due to gravity (m./sec. ²).
G	total fluid flow rate (kg./sec.).
k_u	circumferential force coefficient.
k_u^{st}	stage k_u .
k_u^t	total k_u .
$M_{(1)}$	moment on shaft by a fluid flow rate 1 kg./sec.
n_1, n_2	characteristic numbers.
P_u	circumferential turning force.
u	circumferential velocity (m./sec.).
u^{st}	stage u .
w	relative velocity (m./sec.).
\vec{w}	relative velocity vector.
$W_{(1)}$	work done (or absorbed) by a fluid flow rate 1 kg./sec. (kg. m./sec.).
W	work done (or absorbed) by a fluid flow rate G kg./sec. (kg. m./sec.).
$W_{(1)}^t$	total $W_{(1)}$.
Z	number of stages.
α_1, β_1	angles c_1 and w_1 respectively make with circumferential direction.
α_2, β_2	angles c_2 and w_2 respectively make with the contrary circumferential direction.
$\overline{\alpha_2}, \overline{\beta_2}$	($180^\circ - \alpha_2$) and ($180^\circ - \beta_2$) respectively.
ρ	degree of thermal reactivity.
ρ_{ur}	degree of circumferential force reactivity.
ρ_{ut}	degree of circumferential force impulsivity.
ω	angular velocity.
ψ	blade velocity coefficient.

SUBSCRIPTS

1	inlet to blade.
2	exit to blade.
s.m.t.	single motion turbine.
d.m.t.	double motion turbine.
con.	congruent.
dm.	double motion.
sm.	single motion.

INTRODUCTION

The first double motion turbine design was evolved in 1912 and it was a radial steam turbine (Ljungstrom type). Considerable interest has, since, been evinced in the application of the double motion principle for various fields. In this paper the investigation of the double motion principle in axial multistage turbomachines by the method of equivalent replacements is presented. The analysis covers the various characteristics quite exhaustively. The double motion principle in axial types of turbomachines has been adopted for aircraft compressors and gas turbines. In such applications the number of stages has been small. This paper illustrates that the number of stages in an axial double motion turbomachine need not necessarily be limited to only a few. The investigation in this paper is limited to the axial type and the investigation of the radial double-motion turbomachine will be presented in a subsequent paper.

Fig. 1 shows one of the possible constructive schemes of an axial double motion turbomachine. The scheme consists, essentially, of a disjointed drum 1 made in two halves carrying the blades at its inner surfaces. This disjointed drum is assembled into the outer casing drum 2.

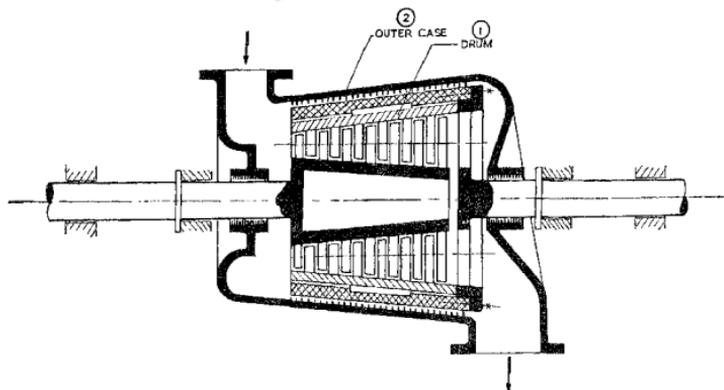


FIG. 1. Possible Constructive Scheme of an Axial D.M. Turbine.

CHARACTERISTICS OF THE DOUBLE MOTION TURBOMACHINE

The theory of the axial double motion turbomachine is analysed in this paper by the method of equivalent replacements used earlier for the single motion turbine.¹ The general expressions for an axial stage, $u_1 = u_2 = u$, when $G = 1$ kg./sec. arc:²

$$M_{(1)} = \frac{u^2}{g\omega} k_u \quad (1)$$

$$P_u = \frac{1}{g} k_u u \quad (2)$$

$$k_u = n_1 - n_2 - 1 \quad (3)$$

$$\rho_{ur} = \frac{-n_2}{n_1 - n_2 - 1} = \frac{-n_2}{k_u} \quad (4)$$

$$\rho_{ur} = \frac{n_1 - 1}{n_1 - n_2 - 1} = \frac{n_1 - 1}{k_u} \quad (5)$$

$$W_{(1)} = \frac{1}{g} k_u u^2 \quad (6)$$

Any stage of a turbomachine can be characterised by values of

$$n_1 = \frac{c_1 \cos \alpha_1}{u}, \quad n_2 = \frac{w_2 \cos \beta_2}{u}$$

and the corresponding k_u , ρ_{ur} , ρ_{ur} , etc. In the plane of hodographs \vec{w} leading \vec{c} characterises a turbine stage, whereas \vec{c} leading \vec{w} characterises a compressor stage. In Fig. 1, the blades of the left rotor move to the left, in the unrolled scheme, and the blades of the right rotor move to the right. Fig. 2 gives the scheme of velocity triangles for an axial multistage double motion turbine characterised by:

- (a) Z number of stages, Z being odd,
- (b) absolute inlet velocity angle for first stage $\alpha_{1I} = 90^\circ$,
- (c) absolute exit velocity angle for last stage $\alpha_{2Z} = 90^\circ$,
- (d) identical power output from the intermediate stages,
- and (e) identical circumferential velocity u in all the stages.

The first stage blades are moving to the left and in the plane of hodographs \vec{c} , \vec{w} turn anticlockwise. The stationary guide blades being absent, c_{1I} is directed axially, i.e., $\alpha_{1I} = 90^\circ$. As the flow passes through the blades, expansion takes place and \vec{w} increases in magnitude. The absolute exit velocity with right twist, from the first stage \vec{c}_{2I} is the inlet absolute velocity to the second stage \vec{c}_{1II} , since the stationary blades are absent. As the second stage blades are moving to the right, w_{1II} is axial ($\beta_{1II} = 90^\circ$) and \vec{c} , \vec{w} are turning clockwise. \vec{c}_{2II} is conjugate to \vec{c}_{1II} (the direction of \vec{u} being the imaginary axis), i.e., $\alpha_{1II} = \alpha_{2II}$, and has left twist. In stage III \vec{c}_{1III} is \vec{c}_{2II} and vectors turn anticlockwise. $\beta_{1III} = 90^\circ$. \vec{c}_{2III} is conjugate to \vec{c}_{1III} and has right twist. In the intermediate stages \vec{c} , \vec{w} turn alternately clockwise (in even stages) and anticlockwise (in odd stages)

and the exit absolute velocity vector c_2 has right twist in odd stages and left twist in even stages. The last stage Z is an odd stage, velocity triangles are as indicated in Fig. 2 with $c_{2Z} = c_{2(Z-1)}$ and $\alpha_{2Z} = 90^\circ$, i.e., the absolute exit flow from the turbine is axial.

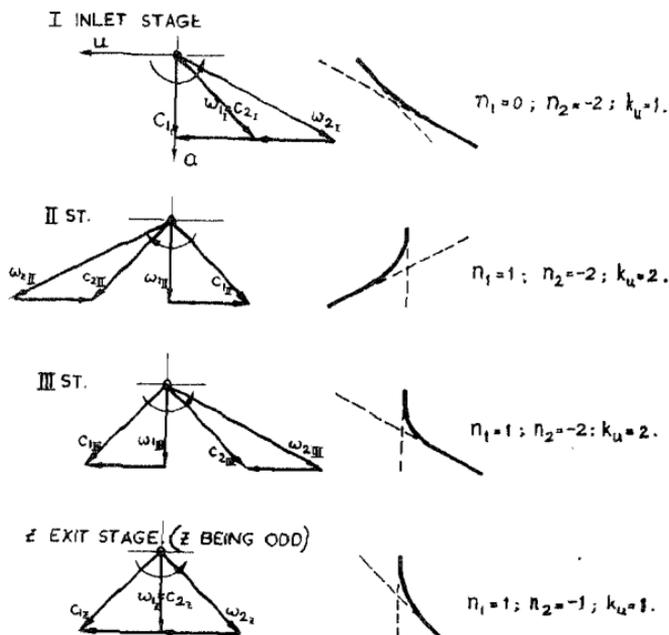


Fig. 2. Scheme of velocity triangles for a double motion turbine with odd number of stages, $\alpha_{11} = 90^\circ$; $k_{u1} = k_{u2} = 1$ and $k_{uII} = k_{uIII} = \dots = k_{u(Z-1)} = 2$; $\alpha_{2Z} = 90^\circ$.

Based on the general expressions 1 to 6, the characteristics of any of the stages can be determined. For the *first stage* $n_1 = 0$; $n_2 = -2$; $k_u = 1$ and $\rho_{ur} = 2$. Therefore the mathematical profile of stage I blade is "under-developed reactive" (Fig. 2) and is disposed in the under right half-row I in the general scheme of development of mathematical profiles (Appendix). The *last stage*, Z, is characterised by $n_1 = 1$; $n_2 = -1$; $k_u = 1$ and $\rho_{ur} = 1$ and hence, the mathematical profile is "fully-developed reactive", disposed in the under right half row II. However as seen from Fig. 2 and Fig. 3 the profile is displaced to the left along the row II, as compared to the profiles of "odd intermediate stages". In Fig. 3, it is shown that the mathematical profile of the last stage can add itself, at the inlet end of the

profile of the first stage. The combination of the two profiles forms a fully developed reactive profile, similar to those of the odd intermediate stages and has the same power effect as any one of the intermediate stages. The *intermediate stages* are

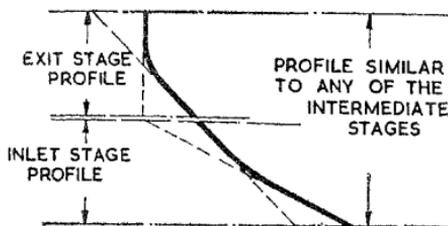


FIG. 3. Combination of exit profile at entry to inlet profile.

characterised by: $n_1 = 1$, $n_2 = -2$, $k_u = 2$ and $\rho_{ur} = 1$, different from those of the inlet and exit stages, and all the intermediate stages have the same power effect as the combined inlet and exit stages. Thus, to organise work process of double motion turbines, according to the scheme in Fig. 2 the number of blade rows, *i.e.*, the number of pressure stages must be odd, if power on blading is to be equally distributed in the two rotors. In regard to force interaction with flow all the stages of a double-motion turbine are "pure reactive" in the circumferential direction ($\rho_{ur} = 1$ and $\rho_{ur} > 1$). The degree of thermal reactivity in all the stages is 1. ($\rho = 1$).

Table I gives the characteristics of the intermediate stages of the double motion turbine. For comparison the same data for an axial impulse stage with symmetrical deviation of flow and an axial congruent stage, is also given in the table, both stages operating under optimum conditions. $k_u = 2$ for any of the intermediate stages of a double motion turbine and a theoretical impulse ($\rho = 0$) stage of usual single motion turbine with symmetrical deviation of flow on blades. Hence, if u is the same, one intermediate stage of the axial double motion turbine replaces one axial impulse stage of a single motion turbine, consisting of two rows of blades (fixed and moving).

The overall k_u of an axial multistage turbine with stages, having the same diameter, is equal to the sum of the k_u^{st} of individual stages:

$$k_u^t = k_{uI} + k_{uII} + \dots = \Sigma k_u^{st} \quad (7)$$

Power on blading of such a multistage turbine, for a flow $G = 1$ kg./sec. and u identical in all stages is:

$$W_{(1)}^t = \frac{k_u^t}{g} u^3 \quad (8)$$

TABLE I

	Symbol	Type of Stage					
		Axial double motion		Axial congruent		Theoretical axial with symmetrical deviation of flow	
1. Degree of thermal reactivity	ρ	1.0		0.5		0	
2. Degree of circumferential force reactivity	ρ_{ur}	1.0		1.0		0.5	
3. Degree of circumferential force impulsivity	ρ_{θ_1}	0		0		0.5	
4. Characteristic number	n_1	1		1		2	
5. do.	n_2	-2		-1		-1	
6. Circumferential force coefficient	k_u	2		1		2	
7. Disposition of mathematical profile in the general scheme (Appendix)	Row	under II right	upper II left	under II right	upper II left	under IV right	upper IV left
8. Number of blade rows		Z		2(Z - 1)		Z - 1	
9. Number of guide blade rows		0		2(Z - 1)		Z - 1	
10. Number of pressure stages		Z		2(Z - 1)		Z - 1	
11. Number of stages with under-developed mathematical profiles, equivalent to a fully-developed stage		2 (inlet and exit stages)		0		0	
12. Total number of blade rows (stationary guide blades and moving blades)		Z		4(Z - 1)		2(Z - 1)	

For a double motion turbine with Z stages, since the first and the last stages together give the same power effect as any intermediate stage, with $k_u = 2$ we can write:

$$k_u^2 = 2(Z - 1) \quad (9)$$

and

$$W_{(1)}' = \frac{2(Z - 1)}{g} u^2 \quad (10)$$

From (9) and (10), it is apparent that as regards power effect, the double motion turbine with Z stages can replace the usual single motion turbine having $(Z - 1)$ impulse stages ($\rho = 0$) characterised by symmetrical deviation of flow on blades, if $\psi = 1$ and unsymmetrical deviation ($\beta_2 < \beta_1$) if $\psi < 1$.

For an axial congruent stage, under optimum conditions $n_2 = -1$, $k_u = 1$. Hence, each intermediate stage of the double motion turbine can replace two congruent stages of the single motion turbine while the first and the last stages (of d.m.t.) can replace only one congruent stage (of s.m.t.). Therefore, as regards power effect, the axial double motion turbine with Z stages is equivalent to an axial single motion turbine having $2(Z - 1)$ congruent stages characterised by $n_2 = -1$. It should be remembered, while making these comparisons, that in the double motion turbine, each stage consists of only one row of moving blades, the stationary row being absent.

To illustrate the influence of double motion turbine principle, in design of turbines, Fig. 4 shows, to the same scale, the schemes for the above compared three types of turbines, with pressure stages having the same u at middle height of blades. Z is taken as five and hence:

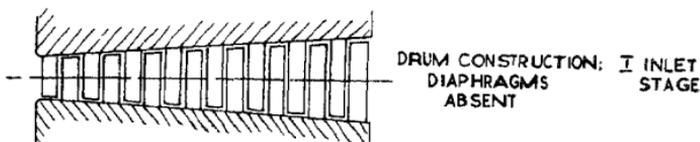
- (a) the double motion turbine has five stages, with five rows of moving blades;
- (b) the axial single motion turbine with impulse stages has $(Z - 1) = 4$ stages, with eight blade rows in all (4 stationary and 4 moving);

and (c) the axial single motion turbine of the usual type having congruent stages will have $2(Z - 1) = 8$ stages, with sixteen blade rows in all (8 stationary and 8 moving). It is quite evident that the double motion scheme is the most economical as regards material investment and the most compact.

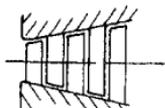
It is interesting to compare the velocity triangles and Fig. 5 shows the velocity triangles under optimum conditions for the three types: (i) axial congruent stage with $n_2 = -1$ (i.e., $\alpha_2 = 90^\circ$); (ii) impulse stage with $\beta_1 = \beta_2$, and (iii) the intermediate stage of the double motion turbine, under the condition that u is the same. The following conclusions can be drawn:

The theoretical ($\psi = 1$) impulse stage is characterised by symmetrical deviation of w , ($\beta_1 = \beta_2$) in the plane of hodographs and unsymmetrical deviation

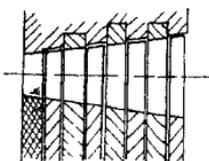
($\alpha_1 \neq \alpha_2$) of \vec{c} . But for an intermediate stage of double motion turbine, the reverse is characteristic, i.e., unsymmetrical deviation of \vec{w} and symmetrical deviation of \vec{c} . The congruent stage is characterised by unsymmetrical deviation of both \vec{w} and \vec{c} .



- C. AXIAL S.M.T. WITH CONGRUENT STAGES:
 NO. OF STAGES = $2(z-1) = 8$;
 NO. OF BLADE ROWS = $4(z-1) = 16$.



- A. AXIAL D.M.T. NO. OF STAGES = $z = 5$;
 NO. OF BLADE ROWS = $z = 5$;
 (DRUM CONSTRUCTION - DIAPHRAGMS ABSENT).

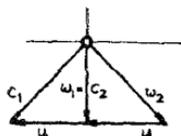


- B. AXIAL S.M.T. WITH IMPULSE STAGES;
 NO. OF STAGES = $(z-1) = 4$;
 NO. OF BLADE ROWS = $2(z-1) = 8$;
 (DISK CONSTRUCTION - DIAPHRAGMS PRESENT).

FIG. 4. Influence of double motion principle as regards material economy and compactness.

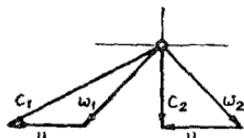
Turning to Fig. 2, we can compare the velocity triangles of the first and the last stages of the double motion turbine. The first stage is characterised by unsymmetrical deviation of both \vec{c} and \vec{w} . c_{1I} has no twist ($\alpha_{1I} = 90^\circ$) and c_{2I} has contrary twist with respect to the blade movement of the first stage, whereas $c_{2I} = c_{1II}$ has fair twist with respect to the motion of blades of the succeeding stage. w_1 for any intermediate stage is thus axial and β_1 for all the stages, excepting the first, becomes 90° . The last stage is, also, characterised by unsymmetrical deviation

tion of \vec{c} and \vec{w} but $\beta_1 = 90^\circ$. The velocity triangle is the same as the usual congruent stage under optimum conditions.



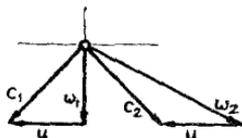
1. AXIAL CONGRUENT STAGE

$$[\alpha_1 = \beta_2; \alpha_2 = \beta_1 = 90^\circ]$$



2. AXIAL IMPULSE STAGE

$$[\gamma = 1; \beta_1 = \beta_2; \alpha_2 = 90^\circ]$$



3. INTERMEDIATE STAGE OF D.M.T

$$[\alpha_1 = \alpha_2; \beta_1 = 90^\circ]$$

FIG. 5. Velocity triangles of compared stages.

Thus, the flow in its absolute motion through the double motion turbine is twisted to the right (according to the scheme in Fig. 2) in the first stage. Through the intermediate stages the flow alternately changes the direction of twist. Twist is contrary for the exit stage, whereas it is fair for the inlet stage. At exit of the last stage of turbine, there is no twist the flow being axial.

We can examine the manner by which the double motion effect can either be increased or decreased, and in the latter case the possibility of the effect being zero, when the complicated arrangement becomes an uneconomical proposition. If the number of blade rows in the double motion turbine is the same as in a single motion turbine, there is no saving in investment of material by the adoption of the double motion principle.

WEAKENING OF DOUBLE MOTION EFFECT

To weaken the double motion effect, it is necessary to decrease the k_u of stages. Fig. 6 shows the scheme by which the weakening can be brought about, along with the mathematical profiles of the various stages. $k_u = 1$ for all the stages. All odd stages, including the first, have similar velocity triangles and likewise, all the even stages have similar velocity triangles. The turbine, as such, consists of a number of pairs of odd and even stages. In any odd stage \vec{c}_1 is axial (*i.e.*, $\alpha_1 = 90^\circ$) and \vec{c}_2 has contrary twist, whereas in the even stages \vec{c}_1 has fair twist and \vec{c}_2 is axial (*i.e.*, $\alpha_2 = 90^\circ$). By using the expressions (1) to (6), we can evaluate the stage characteristics:

(i) *Odd stages.* $n_1 = 0$; $n_2 = -2$; $k_u = 1$ and $\rho_{ur} = 2$. Since the value of ρ_{ur} is greater than 1, the mathematical profile is "under-developed reactive" (Fig. 6) and is disposed in the under right half row I in the general scheme.

(ii) *Even stages.* $n_1 = 1$; $n_2 = -1$; $k_u = 1$ and $\rho_{ur} = 1$. The fully developed mathematical profile is disposed in the upper left half row II.

$$\text{For the scheme (Fig. 6), the total } k_u^t = \Sigma k_u^{st} = Z' \quad (11)$$

where Z' is the number of stages, *i.e.*, number of blade rows, $k_u = 1$ for each of them.

We can compare the scheme of this double motion turbine with the scheme considered earlier (Fig. 2) characterised by $k_u = 2$ in all intermediate stages and $k_u = 1$ in the first and the last stages. In these two cases if the power output and u are the same, the number of stages has to be different and can be determined by equating the right-hand sides of the expressions 9 and 11, *i.e.*,

$$2(Z - 1) = Z' \quad (12)$$

For $Z = 5$, Z' becomes 8, *i.e.*, the number of stages, in the scheme of Fig. 6, will be the same as in the usual single motion turbine with congruent stages characterised by $n_2 = -1$ (*i.e.*, $\alpha_2 = 90^\circ$). However, the overall size of the turbine will be smaller, since each stage of the double motion turbine has only the moving blade row, while the stage of the single motion turbine has two rows (the stationary and the moving).

As illustrated by expression (12), for the same velocity u the double motion principle can be extended to give a design with either lesser or greater number of stages as compared to the usual axial turbine with identical congruent stages under optimum conditions $n_2 = -1$, when $k_{u \text{ con}} = 1$. Let the number of these stages be Z_{con} and Z_{dm} be the number of stages in a double motion turbine. We can now determine the value of $k_{u \text{ dm}}$ for the condition that the number of blade rows in either turbine is the same (*i.e.*, $Z_{\text{dm}} = 2 Z_{\text{con}}$ since the stationary row is absent in the double motion turbine).

$$k_{u \text{ con}} Z_{\text{con}} = k_{u \text{ dm}} Z_{\text{dm}} \quad (13)$$

since

$$k_{u \text{ con}} = 1, k_{u \text{ dm}} = \frac{Z_{\text{con}}}{Z_{\text{dm}}} = \frac{Z_{\text{con}}}{2Z_{\text{con}}} = \frac{1}{2} \quad (14)$$

Hence the minimum value of $k_{u \text{ dm}}$ is 0.5, when the advantage of the double motion principle is completely lost. Fig. 7 shows the scheme of stage mathematical profiles and velocity triangles, characterised by identical value of $k_u = 0.33$ for

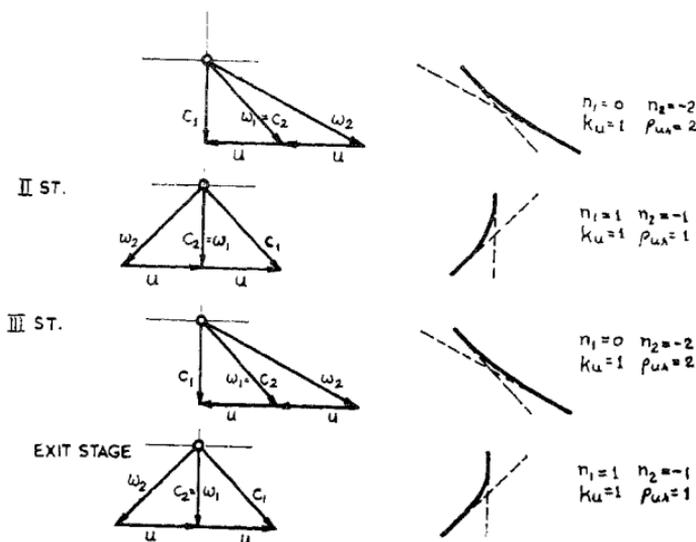


FIG. 6. Scheme of velocity triangles and mathematical profiles for d.m.t. with weakened d.m. effect [$k_u = 1$ for all stages].

all the stages. Odd stages are characterised by: $n_1 = 0$; $n_2 = -1.33$, $k_{u \text{ dm}} = 0.33$ (i.e., less than minimum $k_{u \text{ dm}} = 0.5$); and $\rho_{ur} = 4$; while the even rows have $n_1 = 0.33$, $n_2 = -1$, $k_{u \text{ dm}} = 0.33$ and $\rho_{ur} = 3$. Hence (for this scheme) $Z_{\text{dm}} = Z_{\text{con}}/k_{u \text{ dm}} = Z/0.33$, i.e., the stages in the double motion turbine are thrice as much as in the congruent single motion turbine. The blade rows in the double motion turbine are, however, only half in excess. The double motion principle, in the case considered, gives merely a more complex construction, and increases the number of stages. Thus the double motion principle can guarantee useful results only under certain conditions and cannot always be associated with decrease in number of stages.

INTENSIFICATION OF DOUBLE MOTION EFFECT

As seen above, the effect of double motion can be decreased by incorrect method of design of blades. It is clear, now, by what manner we can theoretically increase

the double motion effect to decrease the number of stages. So long as the stationary blades are absent, the degree of thermal reactivity is always 1 ($\rho = 1$). The decrease of double motion effect is the result of decrease of k_u for profile of blade, i.e., with the decrease in the development of "exit stream" characterised by values of $\rho_{ur} > 1$. To intensify the double motion effect, in addition to the

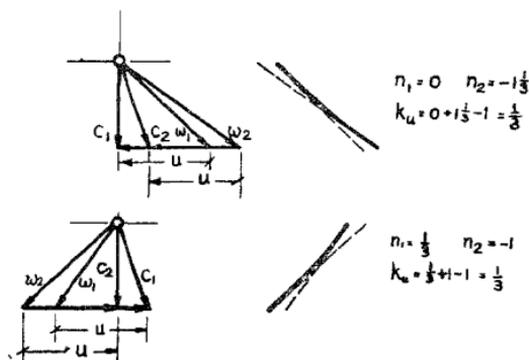


FIG. 7. D.m.t. $k_u = \frac{1}{3}$ for all stages.

"exit stream" being fully developed, i.e., $\beta_1 = 90^\circ$, the "inlet stream" should also be developed (i.e., $\beta_1 < 90^\circ$). The profile with "inlet and exit stream" can absorb higher energies and hence the number of stages (to handle a given total enthalpy drop) decreases.

Fig. 8 shows the velocity triangles and the mathematical profiles of blades for an axial double motion turbine, reflecting increased double motion effect with development of impulse turning force. The *first stage* is characterised by: $n_1 = 0$, $n_2 = -2.33$, $k_u = 1.33$ and $\rho_{ur} = 1.75$ and the mathematical profile is disposed in the right under half row I (being underdeveloped reactive profile $\beta_1 > 90^\circ$). The even stages II, IV, VI, etc., are characterised by mathematical profiles disposed in the left upper half row III and the odd stages III, V, VII, etc., are characterised by profiles of the right under half row III. All these stages have $n_1 = 1.33$, $n_2 = -2.33$, $k_u = 2.66$ and $\rho_{ur} = 0.875$. The *last stage* is an odd one, and the profile though disposed in the right under half row III, is different from the profiles of all the other stages, characterised by $\beta_2 = 90^\circ$, $n_1 = 1.33$, $n_2 = -1$, $k_u = 1.33$ and $\rho_{ur} = 0.75$. The first and the last stages together give the same power effect as any of the intermediate stages. The profile of the last stage can add itself at the inlet of the first stage profile, when it has a similar profile as any of the odd stages.

Comparing the schemes considered above, we can see that in all the cases, the flow at inlet to turbine as well as at exit has axial direction. However, for

each of the variants k_u^{st} is different and accordingly the number of stages are varied, in spite of u remaining the same and double motion principle being adopted.

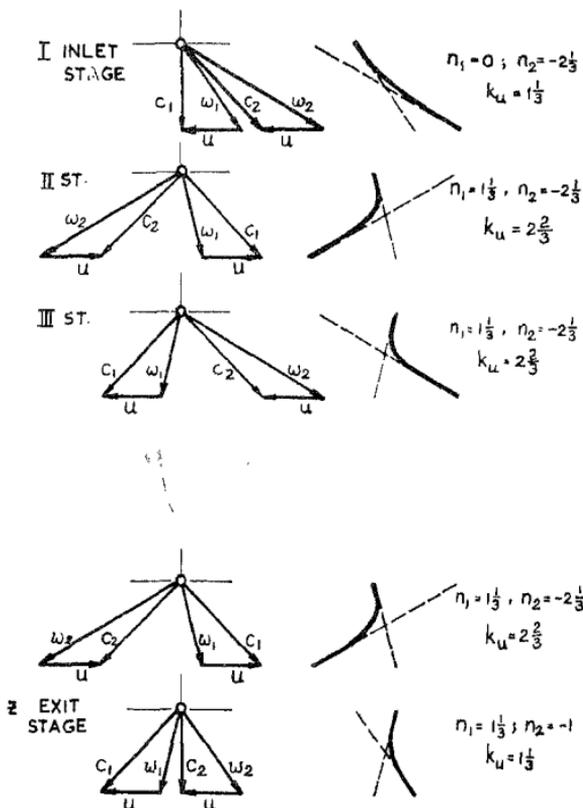


FIG. 8. Scheme of velocity triangles and mathematical profiles for axial d.m.t. with increased d.m. effect.

The double motion principle can, thus, be adopted to give designs with lower or higher total number of stages compared to the conventional single motion turbine.

AXIAL DOUBLE MOTION COMPRESSOR

Detailed investigation of the properties of the axial double motion turbine enables us to consider some aspects of the theory of axial compressor using the double motion principle, without going into the details. It is evident that we can either intensify or decrease the effect of double motion by adequately formed blade

profiles. As in the turbine, the number of stages of a double motion compressor will depend on the type of blade profiles. The number of stages can be brought down compared to the single motion compressor and this aspect gains stature as regards the present tendency of designers, to refrain from the use of supersonic velocities of flow on blading reflecting increased number of stages for a given range of duty.

To illustrate the principle of design of blade profiles of double motion compressor, one of the possible schemes is given in Fig. 9. The scheme provides for

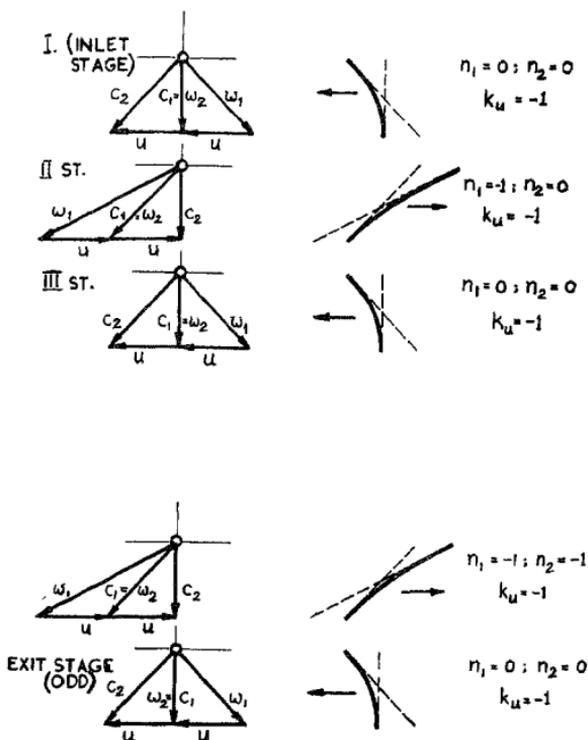


FIG. 9. Scheme of velocity triangles and mathematical profiles for an axial double motion compressor $k_u = -1$ for all stages.

axial inlet to the compressor and equal work consumption, thereby providing theoretically identical increase in enthalpy, in all the stages. In a compressor stage \vec{c} leads \vec{w} in the plane of hodographs and this rule has been applied while drawing

the velocity triangles and mathematical profiles of blades. From Fig. 9 we can see that in all odd stages, the flow gets slowed down with respect to the relative motion ($w_1 > w_2$) and gets accelerated with respect to the absolute motion ($c_1 < c_2$). In the even stages, the flow gets slowed down with respect to both relative and absolute motion ($w_2 < w_1$; $c_2 < c_1$). *Axial exit of absolute flow from compressor can be brought about by making the number of stages an even one.* All the odd stages are characterised by congruence with velocity vectors \vec{c}_1, \vec{w}_2 and \vec{c}_2, \vec{w}_1 taken in pairs being conjugate. The character of congruence does not apply to all the even stages. The characteristics of the odd and even stages are given in Table II. The profiles of *odd stages* are disposed in the upper right half row II, being "fully-developed impulse profiles" with the "inlet stream" fully developed ($\beta_2 = 90^\circ$). Hence for all the odd stages $\rho_{ur} = 0$ and $\rho_{ut} = 1$. *Even stages* are characterised by "under-developed impulse profiles" ($\beta_2 < 90^\circ$) and the mathematical profiles are disposed in the upper left half row I (with $\rho_{ut} = 2$ and $\rho_{ur} = -1$).

TABLE II

	Symbol	Numerical value	
		Odd stage	Even stage
1. Degree of thermal reactivity	.. ρ	1	1
2. Characteristic number	.. n_1	0	-1
3. Do.	.. n_2	0	-1
4. Degree of circumferential force reactivity	ρ_{ur}	0	-1
5. Degree of circumferential force impulsivity	ρ_{ut}	1	+2
6. Circumferential force coefficient	.. k_u	-1	-1
7. Disposition of mathematical profile in the general scheme (Appendix)	row	upper II right	under I left
8. Direction of motion of blades (in the unrolled scheme)		to the left	to the right

CONCLUSION

A few important aspects of the theory of double motion turbomachine were considered in this paper. General theoretical characteristics of such machines were analysed and compared with the characteristics of conventional single motion

turbomachines. The method of equivalent replacements could be utilised to give a general method of investigation of the theory of double motion turbines and compressors. The few correlations between influencing factors enumerated so far have been only particular cases but not general ones. In this paper it has been clearly indicated that: (a) it is possible to increase or decrease the double motion effect by adequate choice of blade profile, (b) the degree of thermal reactivity in all the stages is 1 ($\rho = 1$), (c) the degree of circumferential force reactivity (ρ_{ur}) or impulsivity ($\rho_{u\omega}$) can vary within a wide range.

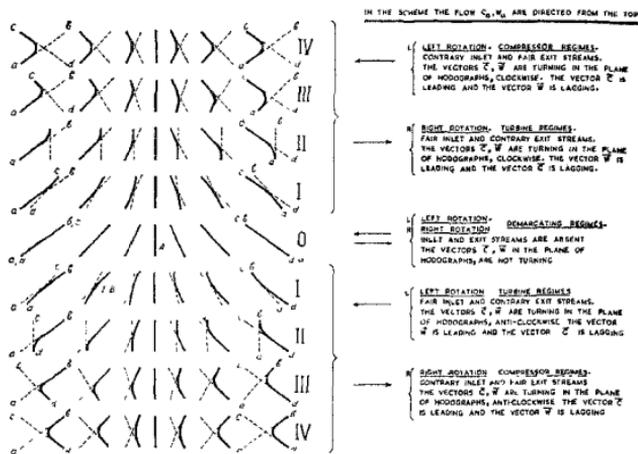
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REFERENCES

1. V. T. Yourinsky and R. G. Narayanamurthi "The Method of Equivalent Replacements applied to the investigation of force transfer and Power Exchange in a stage of a Turbomachine," *Jour. Ind. Inst. Sci.*, 1958, **40** (1).
2. ————— .. "A New form of Expression for Power on Blading in a Stage of Turbomachine, based on the Method of Equivalent Replacements," *ibid.*, 1958, **40** (1).

APPENDIX



The General Scheme of Development of Mathematical Profiles of Turbomachines and Character of Currents of Equivalent Stream.